ETH zürich

Departement Informatik

Programming and Problem-Solving

Graphs and Graph Algorithms

Manuela Fischer and Dennis Komm

Spring 2021 – May 27, 2021

Graphs

Searching in Networks

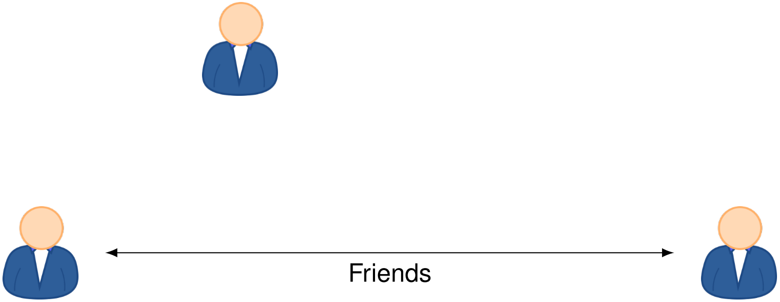
Social Network

Social Network

Social Network

Friends

Social Network



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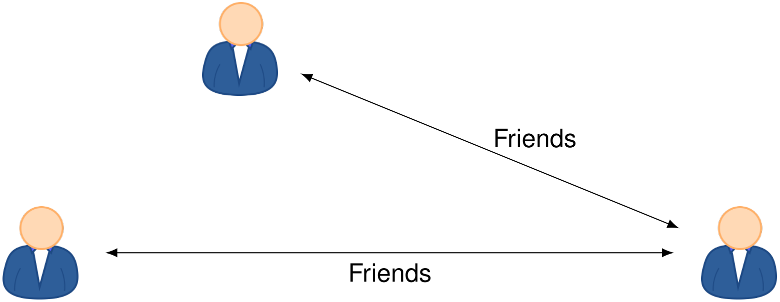

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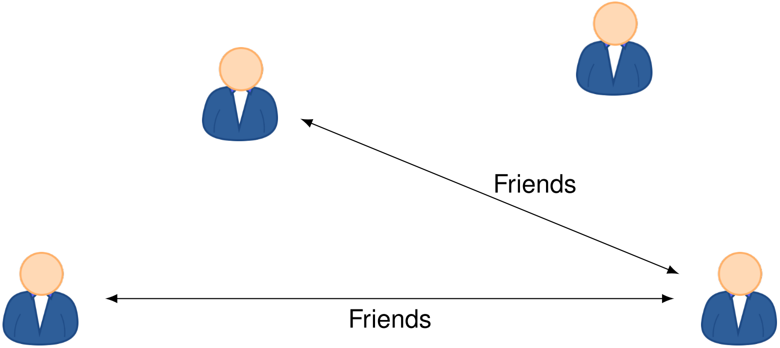

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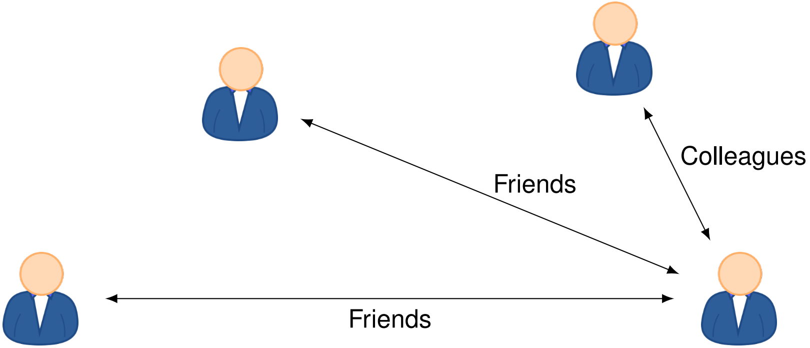

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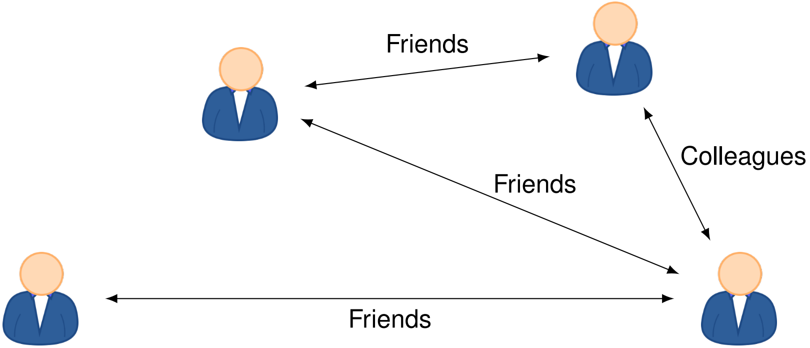

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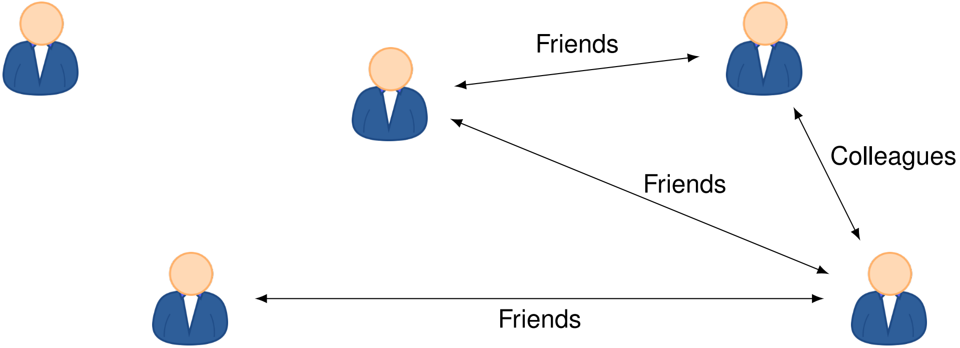

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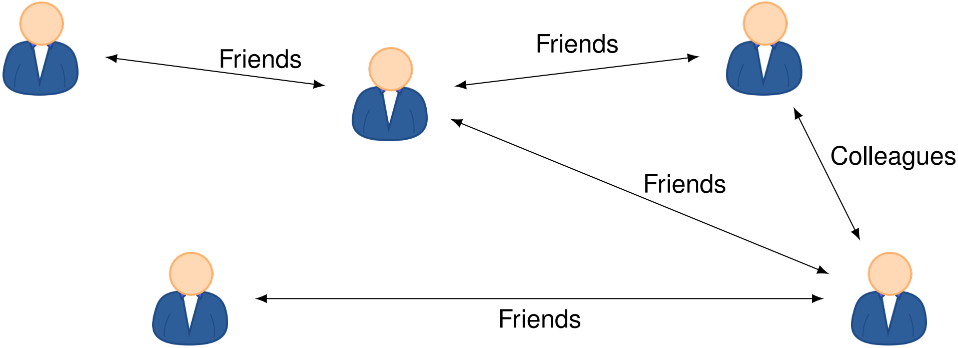

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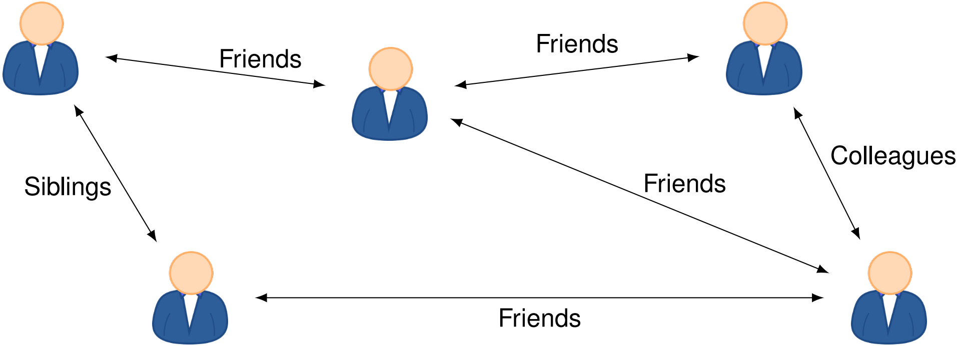

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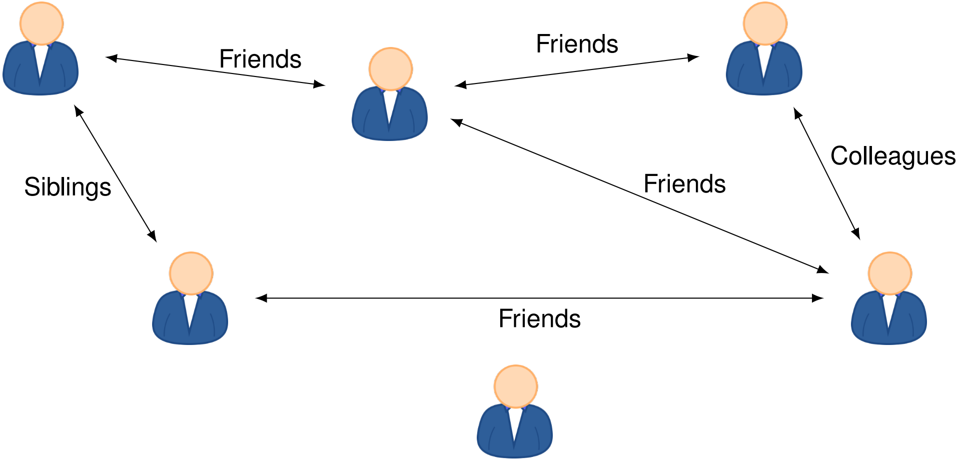

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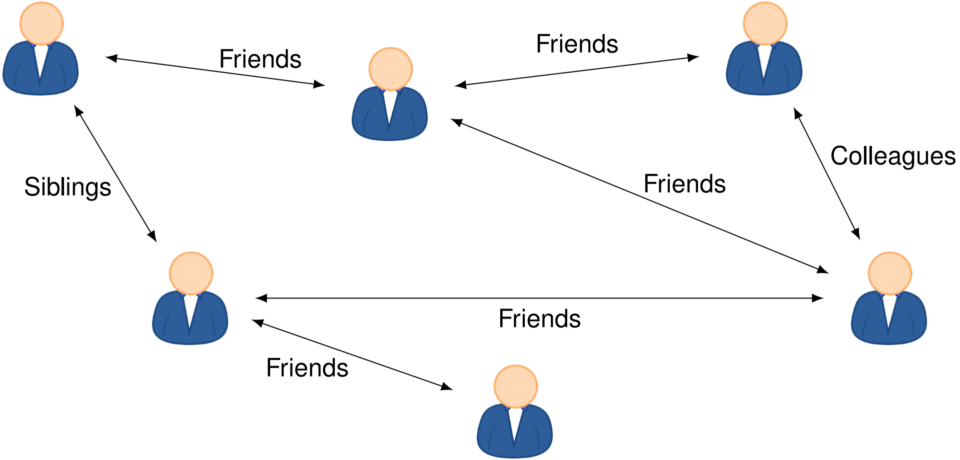

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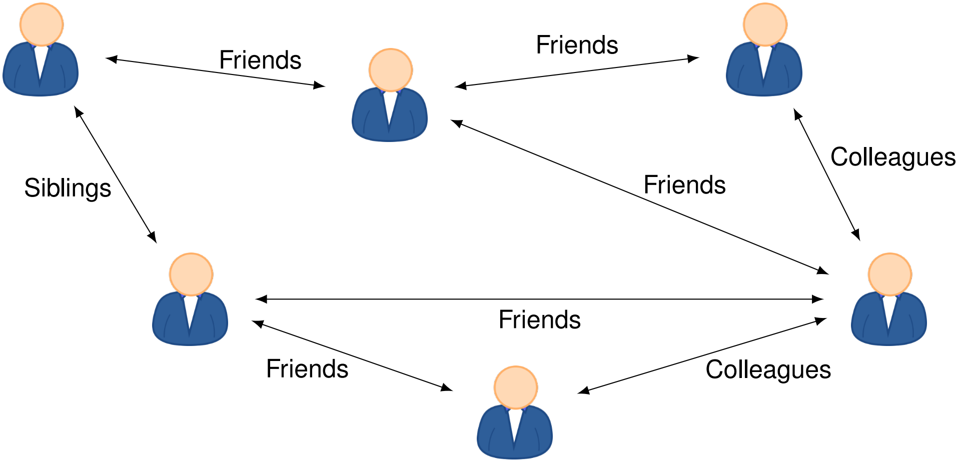

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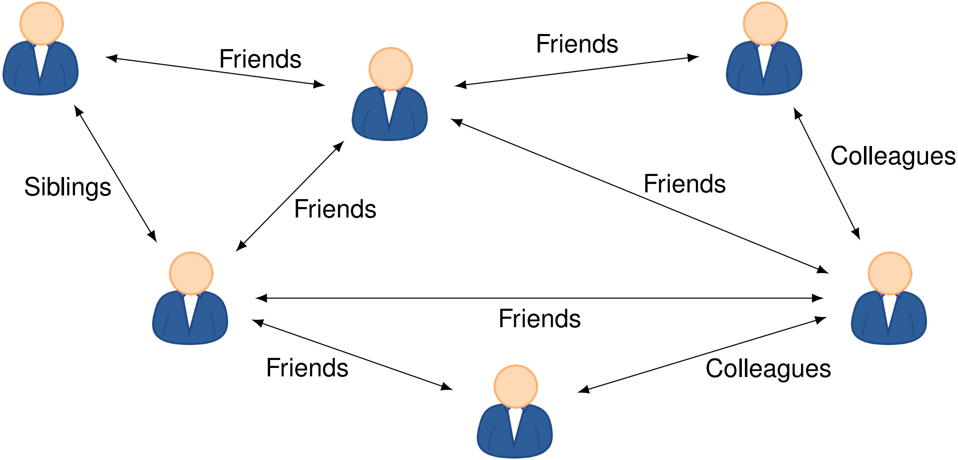

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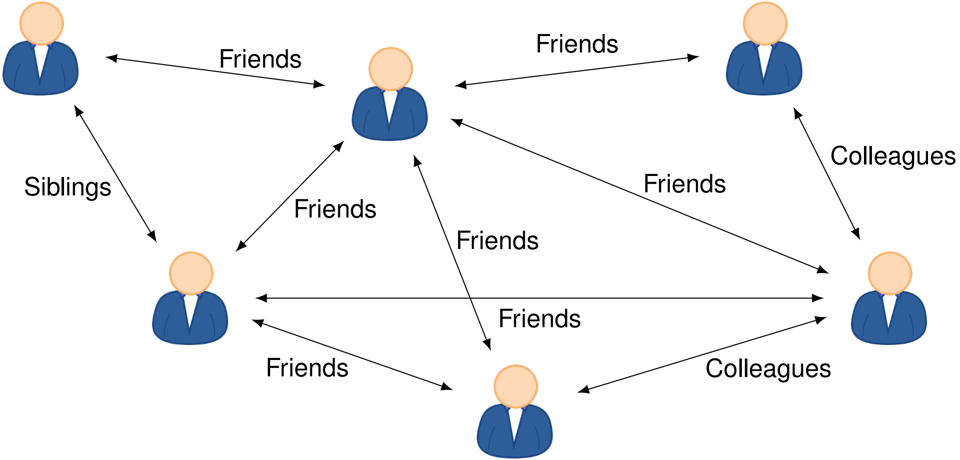
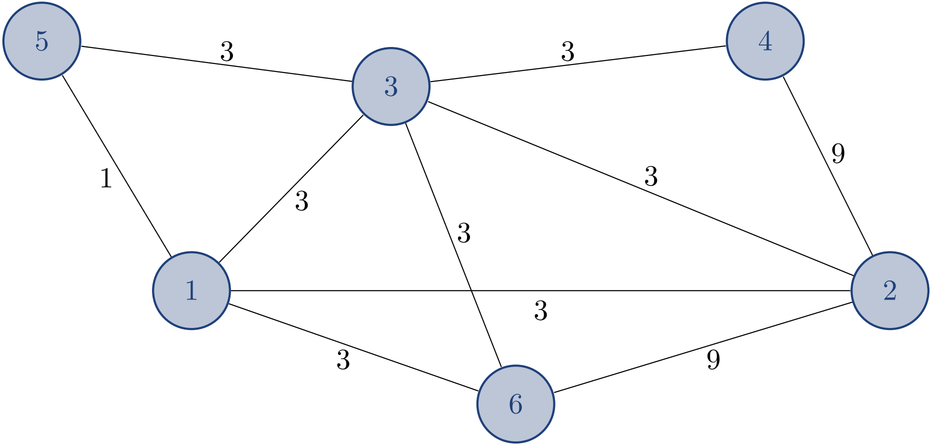


Abstract Modelling



Abstract Modelling

A **graph** $G = (V, E, w)$ consists of

1. a set V of vertices
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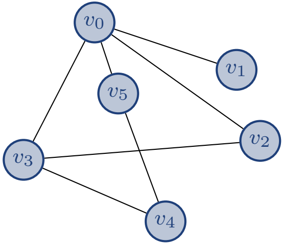

Undirected unweighted graph

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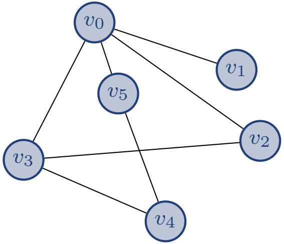

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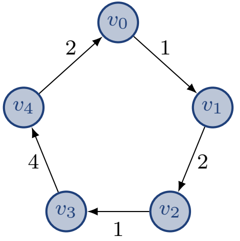

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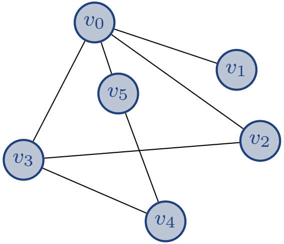

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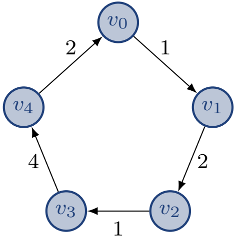

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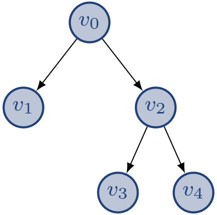

Directed unweighted graph

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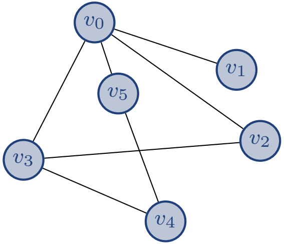

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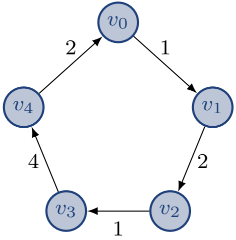

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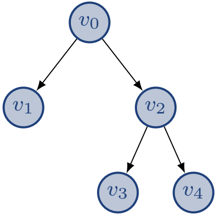

Directed unweighted graph

Which type of graph is used depends on what we want to model

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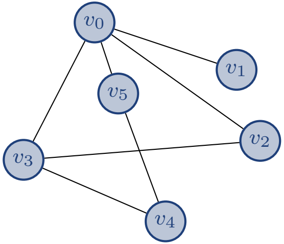

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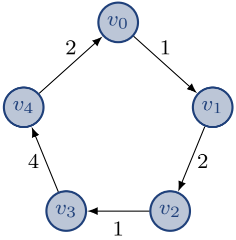

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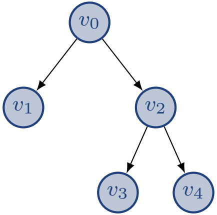


Directed unweighted graph

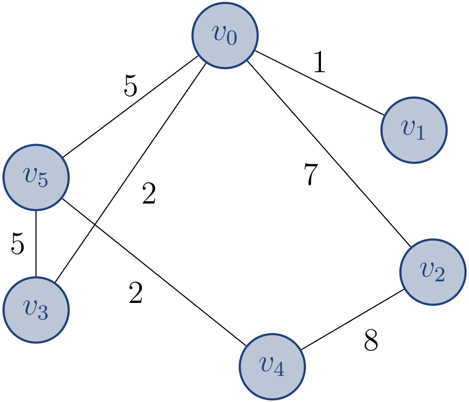
Which type of graph is used depends on what we want to model

We mostly consider undirected, unweighted, connected graphs

Graphs

On the Computer

Adjacency Matrices – Undirected Weighted Graphs



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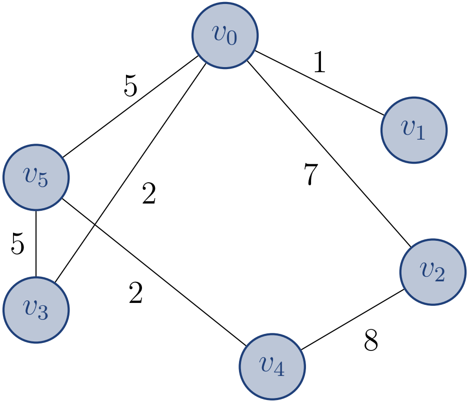

$$\begin{pmatrix} 0 & 1 & 7 & 2 & 0 & 5 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 8 & 0 \\ 2 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 8 & 0 & 0 & 2 \\ 5 & 0 & 0 & 5 & 2 & 0 \end{pmatrix}$$

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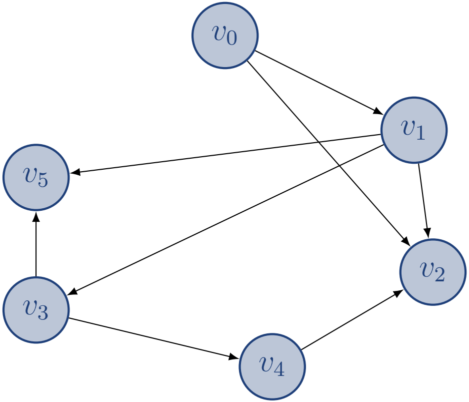

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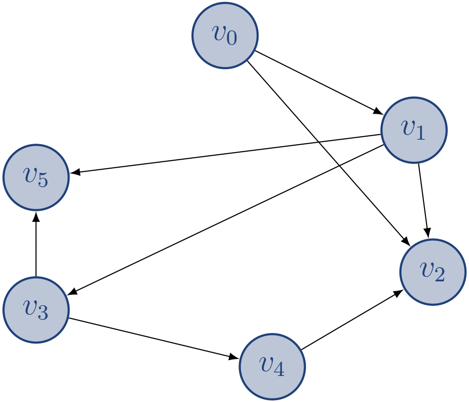
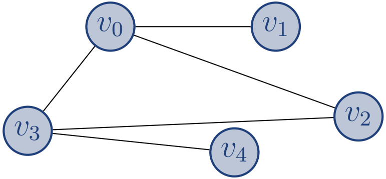


$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency Matrices – Directed / Undirected Graphs

Matrices of undirected graphs
are symmetric



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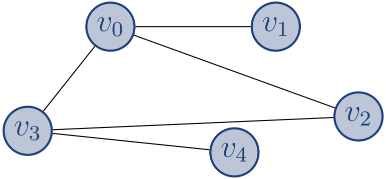

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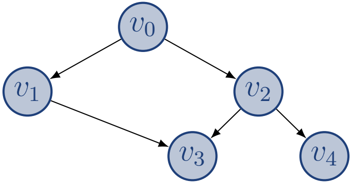
Matrices of directed graphs are not (always) symmetric



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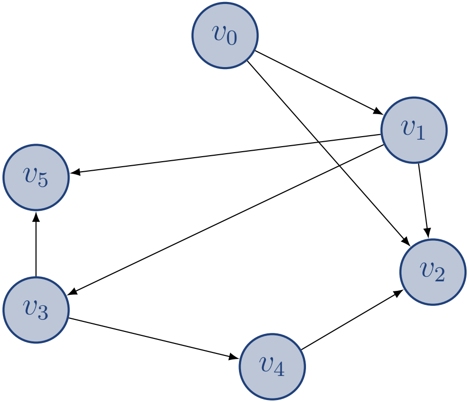
Adjacency Lists – Directed Unweighted Graphs



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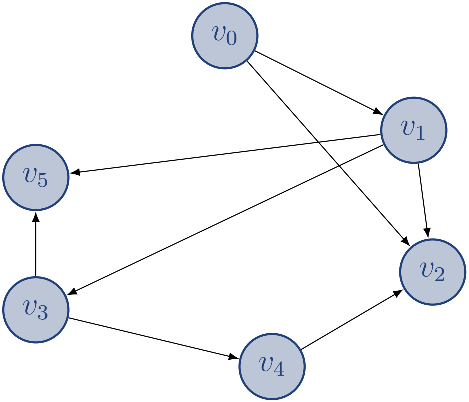

$$\begin{aligned}&((1,2),\\&\ (2,3,5),\\&\ (),\\&\ (4,5),\\&\ (2),\\&\ ())\end{aligned}$$

Adjacency Matrices and Lists in Python

Use 2-dimensional lists

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Matrix: Weighted

```
G = [ [ 0, 1, 7, 2, 0, 5 ],
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```

List: Unweighted

```
G = [ [1,2], [0,2,3,5], [0,1,4], [1,4,5], [2,3], [1,3] ]
```

Graph Algorithms

Breadth-First and Depth-First Search

Breadth-First (BFS) and Depth-First Search (DFS)

Many applications need the systematic exploration of a given graph

- Start and an arbitrary vertex
- Follow edges through graph
- Store vertices in the respective order

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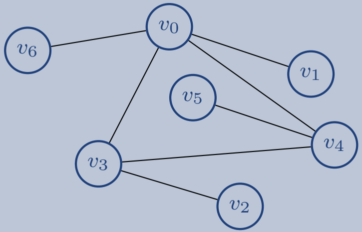
BFS: First go broadly and than deeply, just as with the Heap; break ties in favor of smaller indices



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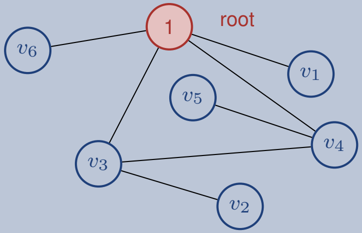

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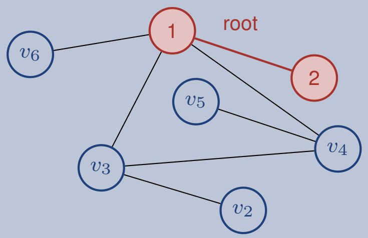

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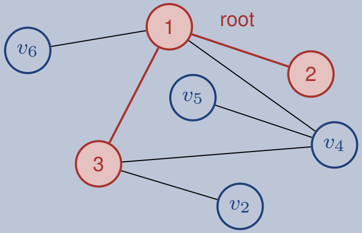

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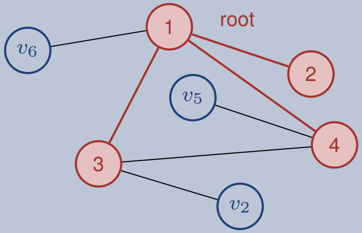

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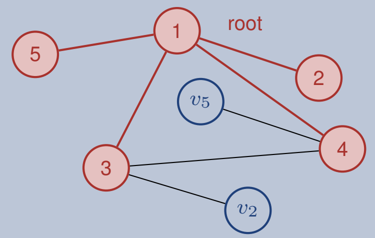

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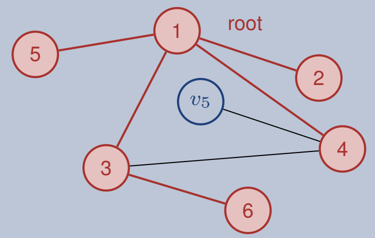

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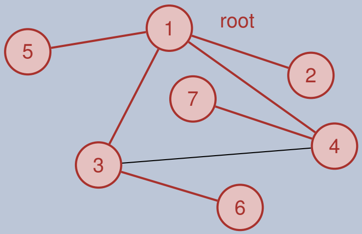

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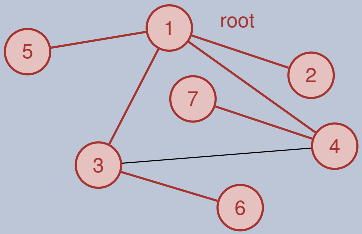
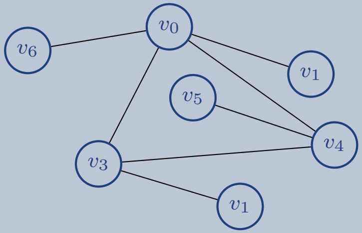


DFS: Go into the graph as deep as possible, then broadly; again break ties in favor of smaller indices



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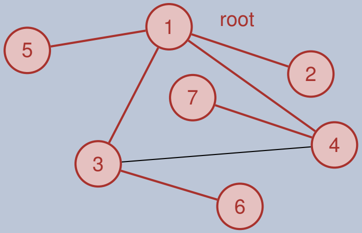

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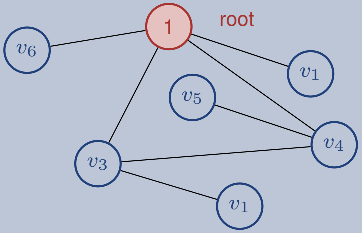

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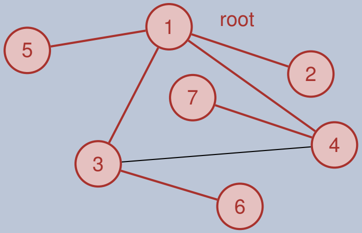

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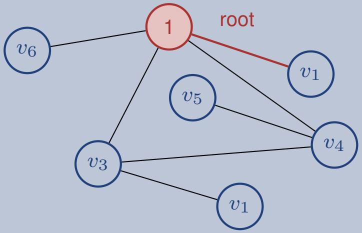

Breadth-First (BFS) and Depth-First Search (DFS)

Many applications need the systematic exploration of a given graph

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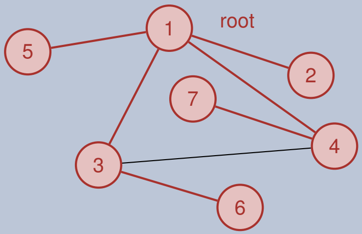

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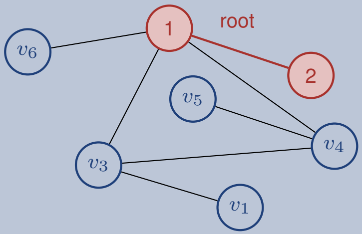

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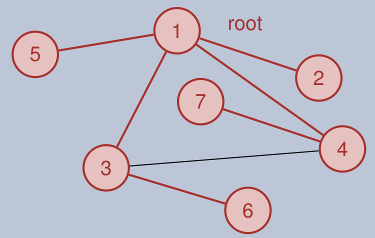

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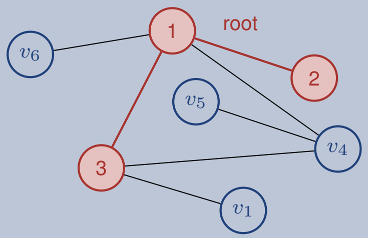

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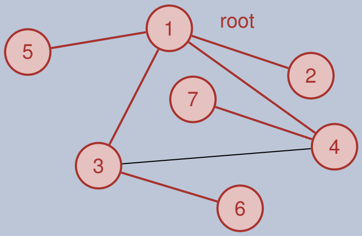

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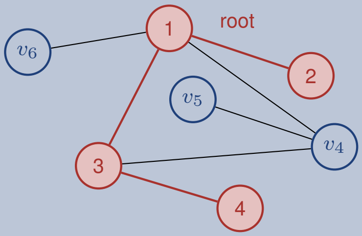

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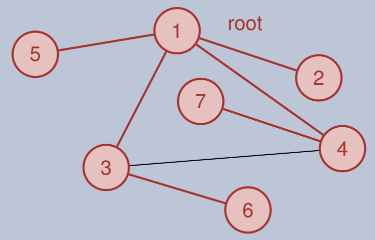

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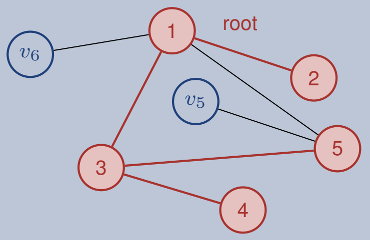

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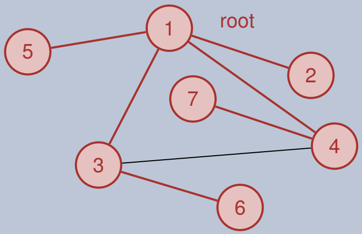
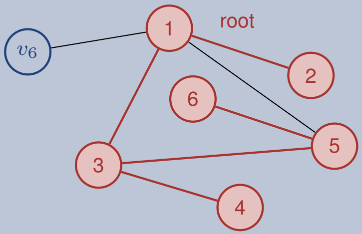


DFS: Go into the graph as deep as possible, then broadly; again break ties in favor of smaller indices



Breadth-First (BFS) and Depth-First Search (DFS)

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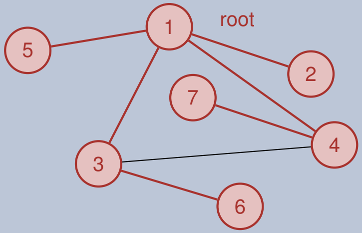

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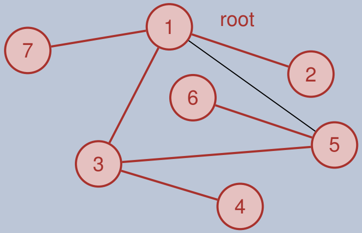
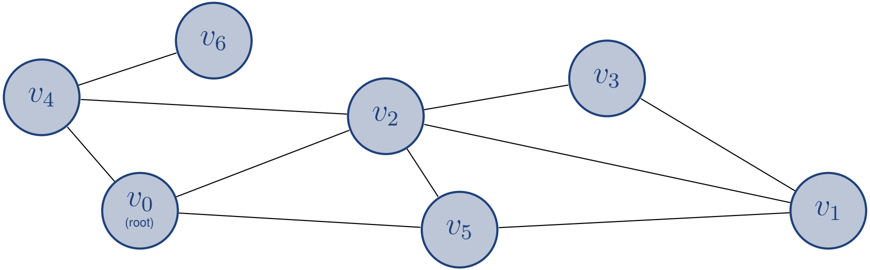


Breadth-First Search

Iteratively with a Queue

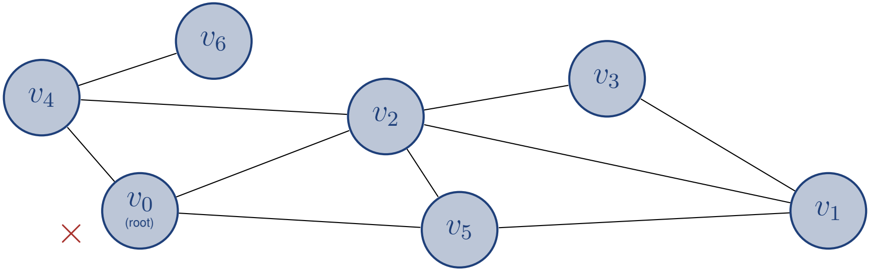
BFS with a Queue



Queue:

Output:

BFS with a Queue

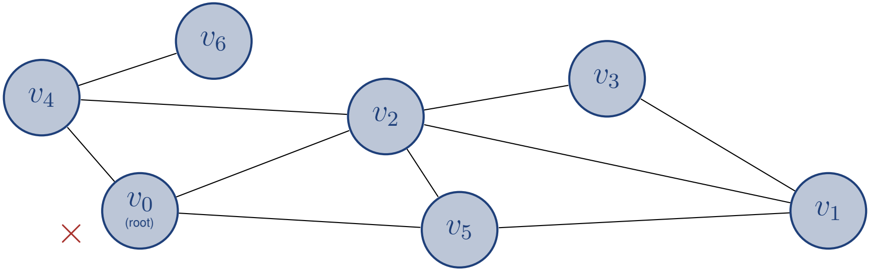


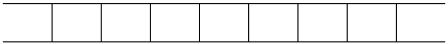
Queue:

v_0								

Output:

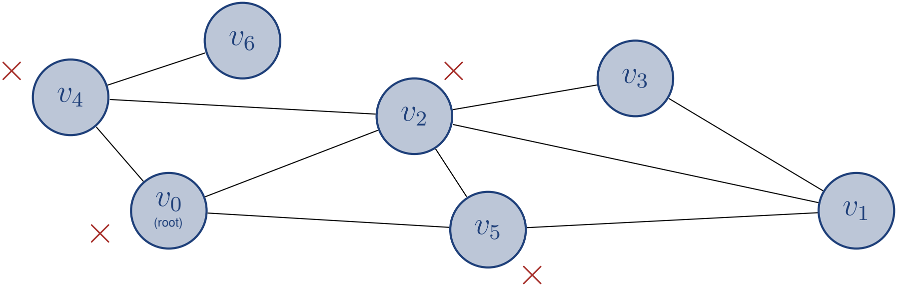
BFS with a Queue




Queue:

Output: v_0

BFS with a Queue

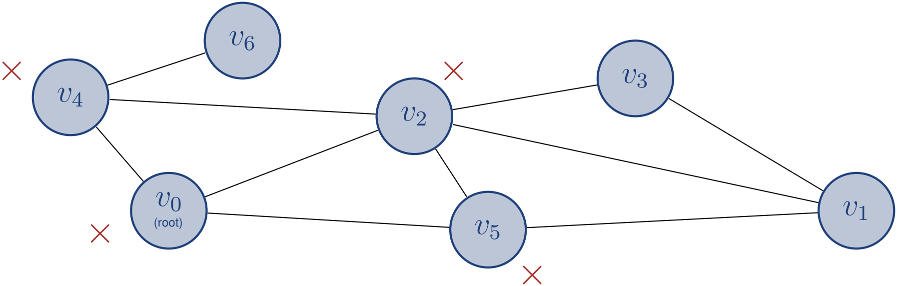


Queue:

v_2	v_4	v_5						

Output: v_0

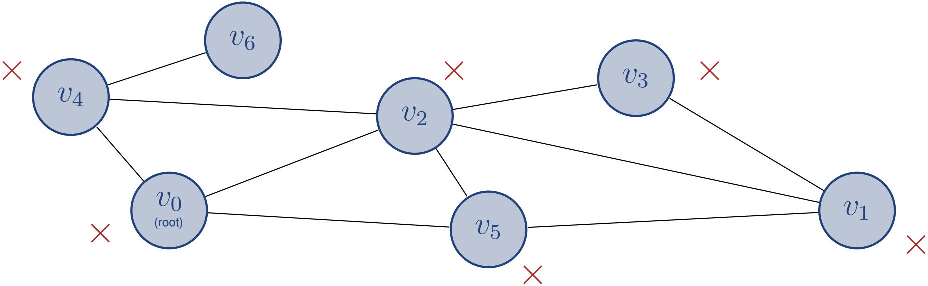
BFS with a Queue



Queue: v_4 v_5

Output: v_0 v_2

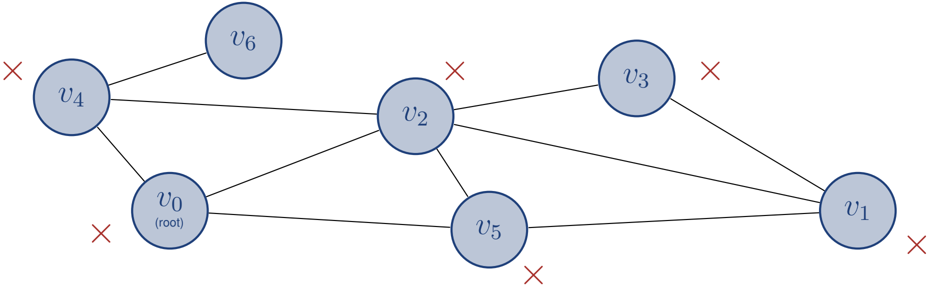
BFS with a Queue



Queue: v_4 v_5 v_1 v_3

Output: v_0 v_2

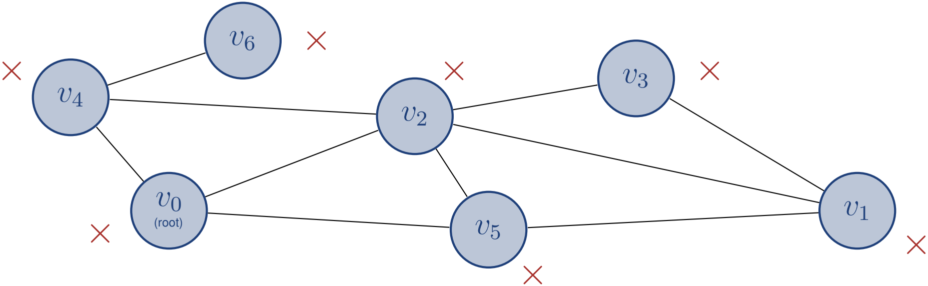
BFS with a Queue



Queue:	v_5	v_1	v_3						

Output: v_0 v_2 v_4

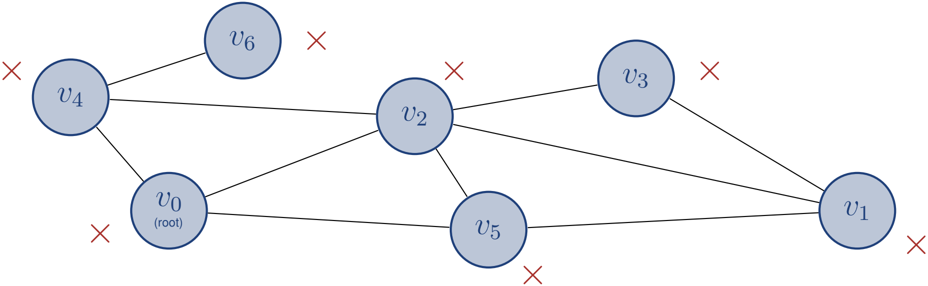
BFS with a Queue



Queue:	v_5	v_1	v_3	v_6					

Output: v_0 v_2 v_4

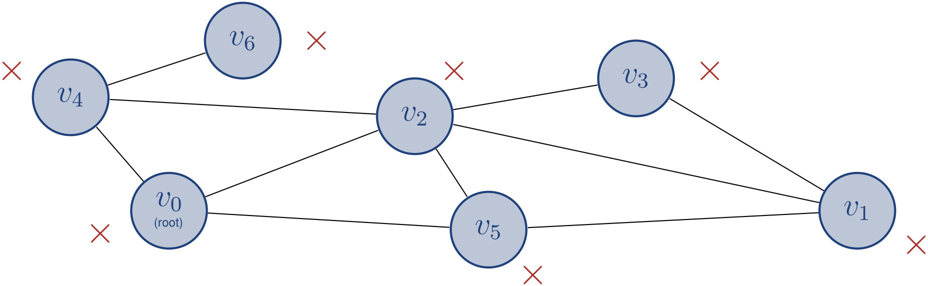
BFS with a Queue



Queue:	v_1	v_3	v_6						

Output: v_0 v_2 v_4 v_5

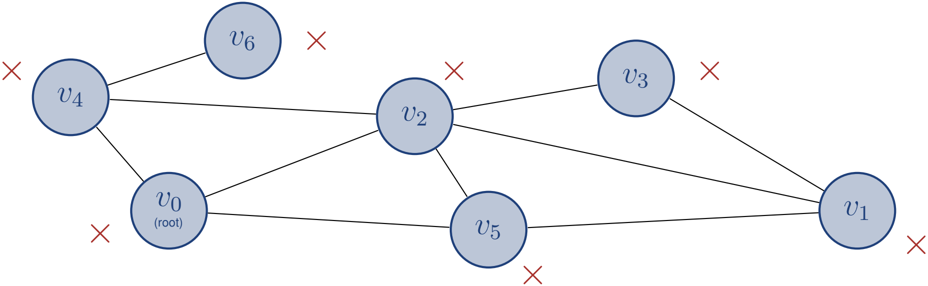
BFS with a Queue



Queue:	v_3	v_6							

Output: v_0 v_2 v_4 v_5 v_1

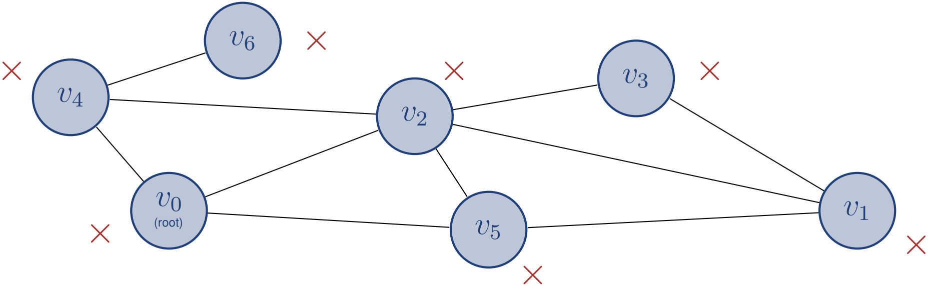
BFS with a Queue



Queue: v_6

Output: v_0 v_2 v_4 v_5 v_1 v_3

BFS with a Queue



Queue:

Output: v_0 v_2 v_4 v_5 v_1 v_3 v_6

BFS with Queue and Adjacency Matrix

```
G = [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ]
queue = []
visited = [ 0 for i in range(len(G)) ]
```

BFS with Queue and Adjacency Matrix

```
G = [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ]
queue = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in queue and print it

BFS with Queue and Adjacency Matrix

```
G = [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ]
queue = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in queue and print it
- Add unvisited neighbors to queue

BFS with Queue and Adjacency Matrix

```
G = [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ]
queue = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in queue and print it
- Add unvisited neighbors to queue
- `visited` stores which vertices have been visited

BFS with Queue and Adjacency Matrix

```
G = [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ]
queue = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in queue and print it
- Add unvisited neighbors to queue
- `visited` stores which vertices have been visited
- Repeat as long as queue is not empty

Exercise – BFS with Queue and Adjacency Matrix

Implement BFS

- as a Python function
- with a 2-dimensional list as parameter
- using a queue
- and an adjacency matrix

```
G = [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ]
```

BFS with Queue and a Adjacency Matrix

```
def BFS(G):
    queue = []
    visited = [ 0 for i in range(len(G)) ]
    queue.append(0)
    visited[0] = 1
    while len(queue) > 0:
        current = queue.pop(0)
        print(current, end=" ")
        for j in range(len(G)):
            if G[current][j] == 1 and visited[j] == 0:
                visited[j] = 1
                queue.append(j)
BFS([ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ])
```

Exercise – BFS with Queue and Adjacency List

Implement BFS

- as a Python function
- with a 2-dimensional list as parameter
- using a queue
- and an adjacency list

```
G = [ [2,4,5], [2,3,5], [0,1,3,4,5], [1,2],
      [0,2,6], [0,1,2], [4] ]
```

BFS with Queue and a Adjacency List

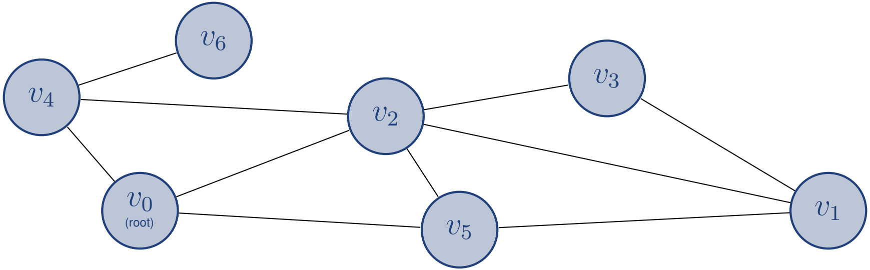
```
def BFS(G):
    queue = []
    visited = [ 0 for i in range(len(G)) ]
    queue.append(0)
    visited[0] = 1
    while len(queue) > 0:
        current = queue.pop(0)
        print(current, end=" ")
        for j in G[current]:
            if visited[j] == 0:
                visited[j] = 1
                queue.append(j)

BFS([ [2,4,5], [2,3,5], [0,1,3,4,5], [1,2], [0,2,6], [0,1,2], [4] ])
```

Depth-First Search

Iteratively with a Stack

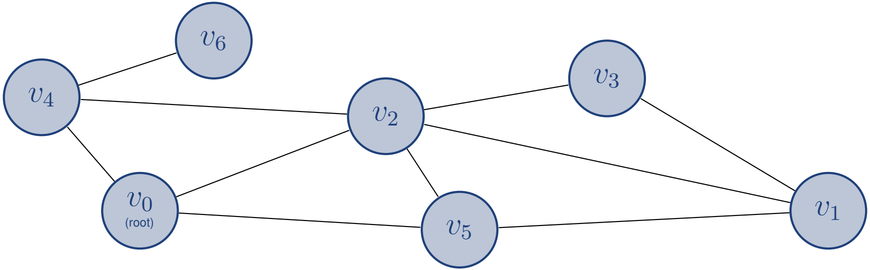
DFS with a Stack



Stack:

Output:

DFS with a Stack

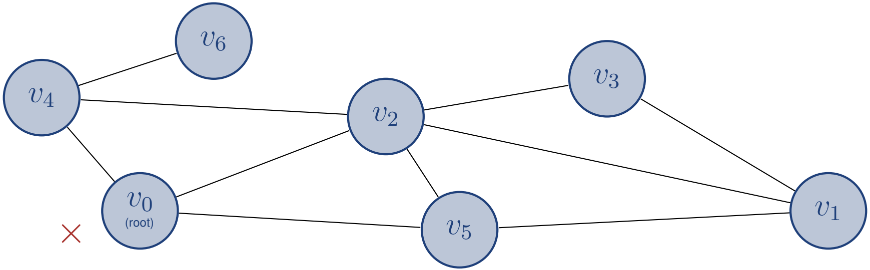


Stack:

v_0								

Output:

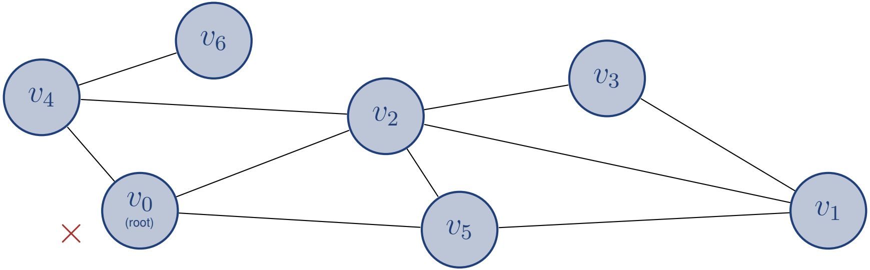
DFS with a Stack



Stack:

Output: v_0

DFS with a Stack

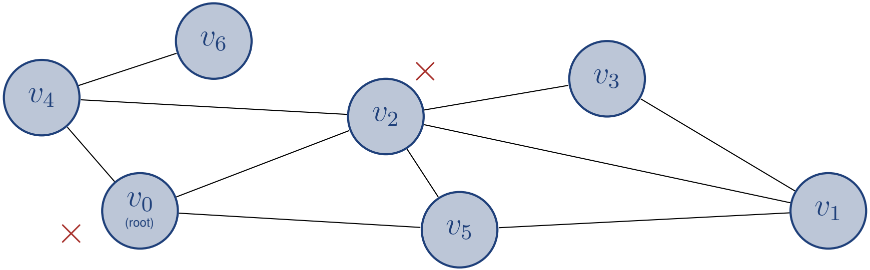


Stack:

v_5	v_4	v_2						

Output: v_0

DFS with a Stack

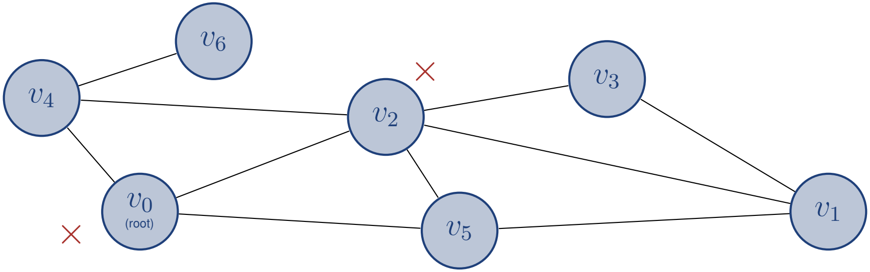


Stack:

v_5	v_4							

Output: v_0 v_2

DFS with a Stack

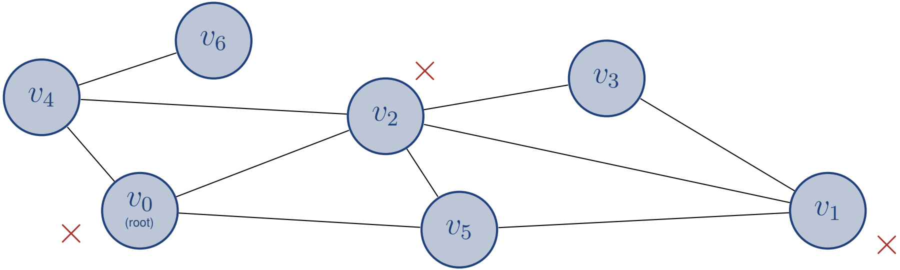


Stack:

v_5	v_4	v_5	v_4	v_3	v_1			

Output: v_0 v_2

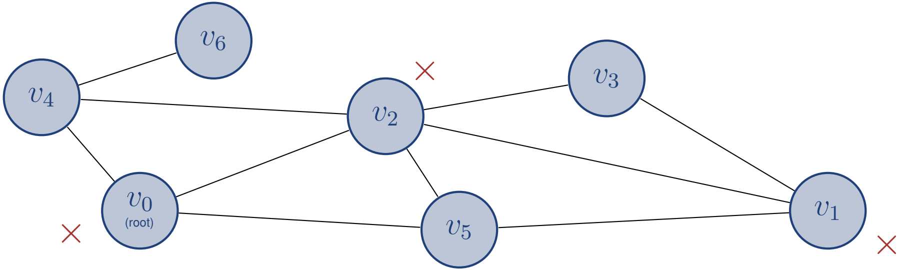
DFS with a Stack



Stack:	v_5	v_4	v_5	v_4	v_3				

Output: v_0 v_2 v_1

DFS with a Stack

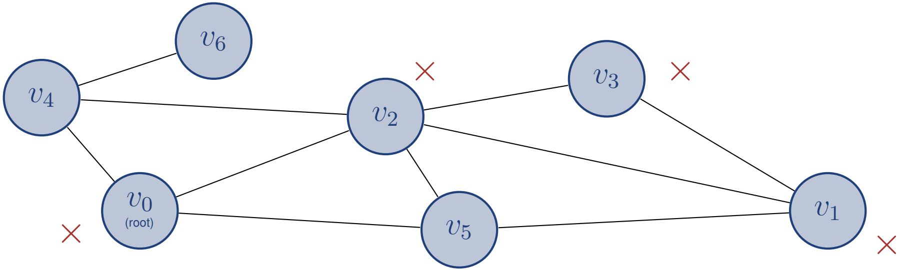


Stack:

v_5	v_4	v_5	v_4	v_3	v_5	v_3		

Output: v_0 v_2 v_1

DFS with a Stack

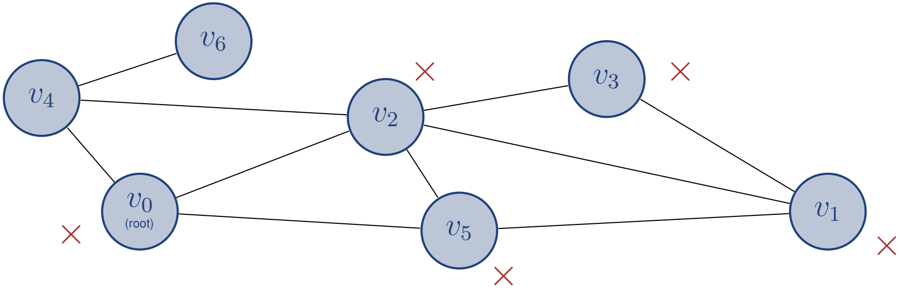


Stack:

v_5	v_4	v_5	v_4	v_3	v_5			

Output: v_0 v_2 v_1 v_3

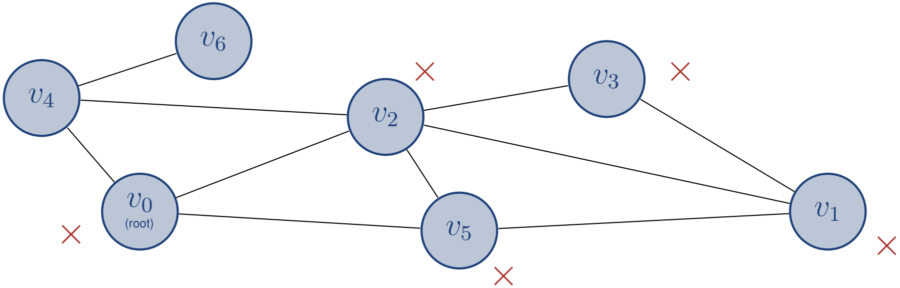
DFS with a Stack



Stack:	v_5	v_4	v_5	v_4	v_3				

Output: v_0 v_2 v_1 v_3 v_5

DFS with a Stack

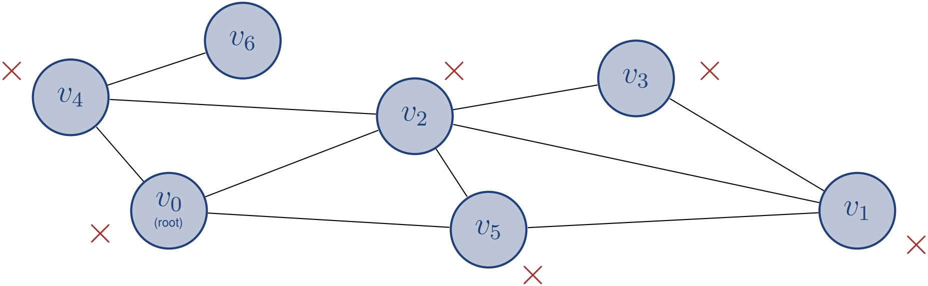


v_5	v_4	v_5	v_4					

Stack: v_5 v_4 v_5 v_4

Output: v_0 v_2 v_1 v_3 v_5

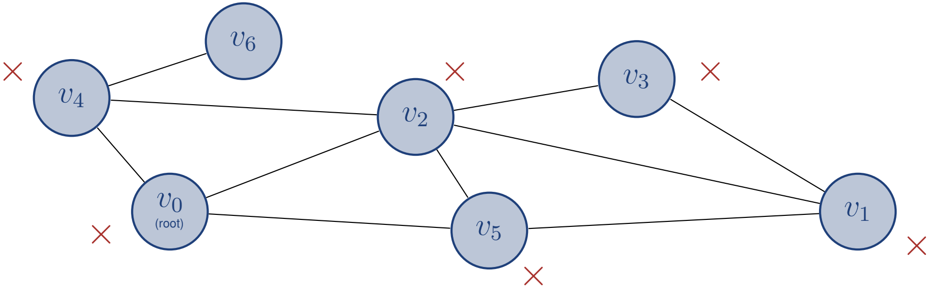
DFS with a Stack



Stack:	v_5	v_4	v_5						

Output: v_0 v_2 v_1 v_3 v_5 v_4

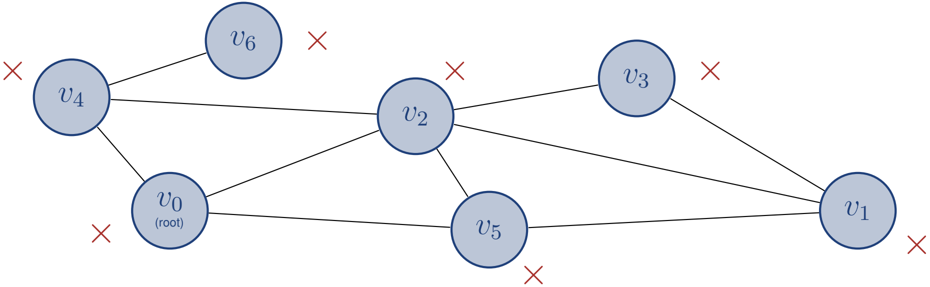
DFS with a Stack



Stack:	v_5	v_4	v_5	v_6					

Output: v_0 v_2 v_1 v_3 v_5 v_4

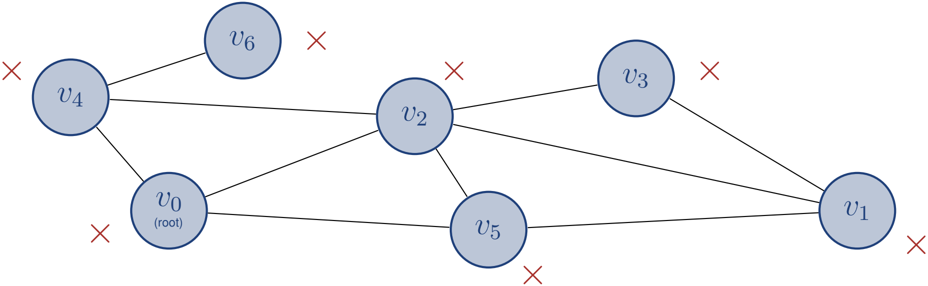
DFS with a Stack



Stack:	v_5	v_4	v_5						

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

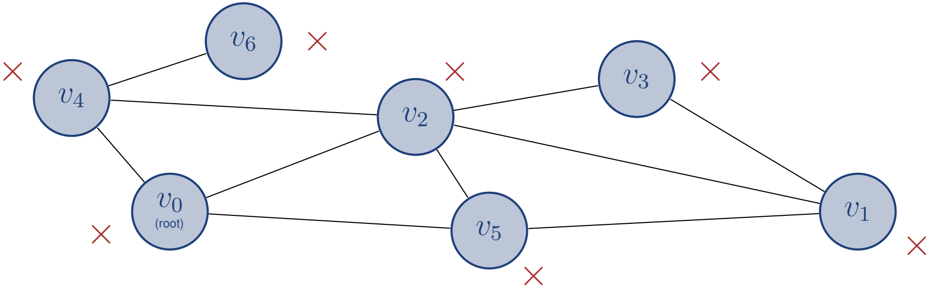
DFS with a Stack



Stack:	v_5	v_4							

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

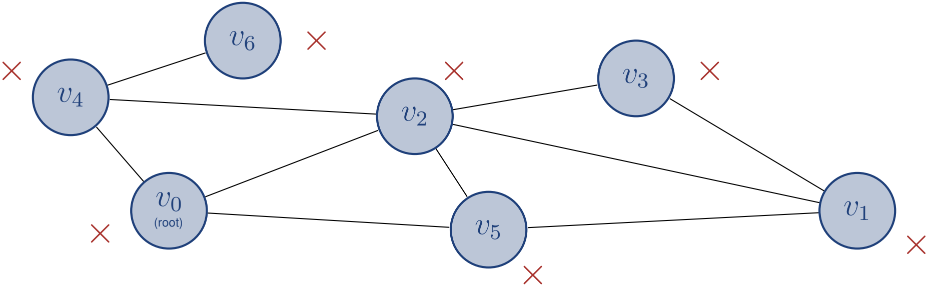
DFS with a Stack



Stack:	v_5								

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

DFS with a Stack



Stack:

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

DFS with Stack and Adjacency Matrix

```
G = [ [0, 0, 1, 0, 1, 1, 0],
      [0, 0, 1, 1, 0, 1, 0],
      [1, 1, 0, 1, 1, 1, 0],
      [0, 1, 1, 0, 0, 0, 0],
      [1, 0, 1, 0, 0, 0, 1],
      [1, 1, 1, 0, 0, 0, 0],
      [0, 0, 0, 0, 1, 0, 0] ]
stack = []
visited = [ 0 for i in range(len(G)) ]
```

DFS with Stack and Adjacency Matrix

```
G = [ [0, 0, 1, 0, 1, 1, 0],
      [0, 0, 1, 1, 0, 1, 0],
      [1, 1, 0, 1, 1, 1, 0],
      [0, 1, 1, 0, 0, 0, 0],
      [1, 0, 1, 0, 0, 0, 1],
      [1, 1, 1, 0, 0, 0, 0],
      [0, 0, 0, 0, 1, 0, 0] ]
stack = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in stack and print it

DFS with Stack and Adjacency Matrix

```
G = [ [0, 0, 1, 0, 1, 1, 0],
      [0, 0, 1, 1, 0, 1, 0],
      [1, 1, 0, 1, 1, 1, 0],
      [0, 1, 1, 0, 0, 0, 0],
      [1, 0, 1, 0, 0, 0, 1],
      [1, 1, 1, 0, 0, 0, 0],
      [0, 0, 0, 0, 1, 0, 0] ]
stack = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in stack and print it
- Add unvisited neighbors to stack

DFS with Stack and Adjacency Matrix

```
G = [ [0, 0, 1, 0, 1, 1, 0],
      [0, 0, 1, 1, 0, 1, 0],
      [1, 1, 0, 1, 1, 1, 0],
      [0, 1, 1, 0, 0, 0, 0],
      [1, 0, 1, 0, 0, 0, 1],
      [1, 1, 1, 0, 0, 0, 0],
      [0, 0, 0, 0, 1, 0, 0] ]
stack = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in stack and print it
- Add unvisited neighbors to stack
- `visited` stores which vertices have been visited

DFS with Stack and Adjacency Matrix

```
G = [ [0, 0, 1, 0, 1, 1, 0],
      [0, 0, 1, 1, 0, 1, 0],
      [1, 1, 0, 1, 1, 1, 0],
      [0, 1, 1, 0, 0, 0, 0],
      [1, 0, 1, 0, 0, 0, 1],
      [1, 1, 1, 0, 0, 0, 0],
      [0, 0, 0, 0, 1, 0, 0] ]
stack = []
visited = [ 0 for i in range(len(G)) ]
```

- Consider first vertex in stack and print it
- Add unvisited neighbors to stack
- `visited` stores which vertices have been visited
- Repeat as long as stack is not empty

Exercise – DFS with Stack and Adjacency Matrix

Implement DFS

- as a Python function
- with a 2-dimensional list as parameter
- using a stack
- and an adjacency matrix

```
G = [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0],
      [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0],
      [0,0,0,0,1,0,0] ]
```

DFS with Stack and Adjacency Matrix

```python
def DFS(G):
    stack = []
    visited = [ 0 for i in range(len(G)) ]
    stack.append(0)
    while len(stack) > 0:
        current = stack.pop()
          if visited[current] == 0:
              visited[current] = 1
              print(current, end=" ")
              for j in reversed(range(len(G))):
                  if G[current][j] == 1 and visited[j] == 0:
                      stack.append(j)

DFS( [ [0,0,1,0,1,1,0], [0,0,1,1,0,1,0], [1,1,0,1,1,1,0], [0,1,1,0,0,0,0],
      [1,0,1,0,0,0,1], [1,1,1,0,0,0,0], [0,0,0,0,1,0,0] ] )
```

Exercise – DFS with Stack and Adjacency List

Implement DFS

- as a Python function
- with a 2-dimensional list as parameter
- using a stack
- and an adjacency matrix

```
G = [ [2,4,5], [2,3,5], [0,1,3,4,5], [1,2], [0,2,6], [0,1,2], [4] ]
```

DFS with Stack and Adjacency List

```
def DFS(G):
    stack = []
    visited = [ 0 for i in range(len(G)) ]
    stack.append(0)
    while len(stack) > 0:
        current = stack.pop()
        if visited[current] == 0:
            visited[current] = 1
            print(current, end=" ")
            for j in reversed(G[current]):
                if visited[j] == 0:
                    stack.append(j)
DFS([ [2,4,5], [2,3,5], [0,1,3,4,5], [1,2], [0,2,6], [0,1,2], [4] ])
```

Depth-First Search

Recursively

Recursive DFS

- Global list `visited`

Recursive DFS

- Global list `visited`
- Function `DFS`, which is called recursively

Recursive DFS

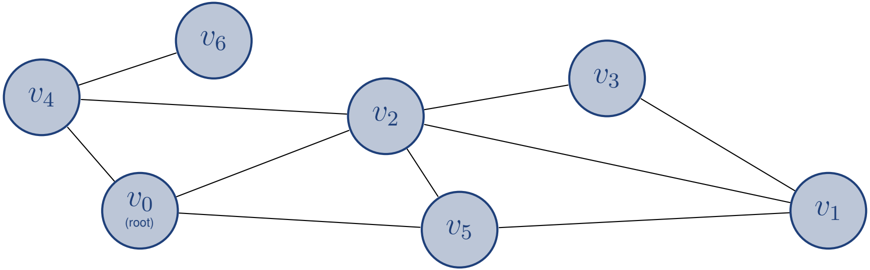
- Global list `visited`
- Function `DFS`, which is called recursively
- Two parameters
  1. graph `G`
  2. Start vertex `current`

Recursive DFS

- Global list `visited`
- Function `DFS`, which is called recursively
- Two parameters
    1. graph `G`
    2. Start vertex `current`

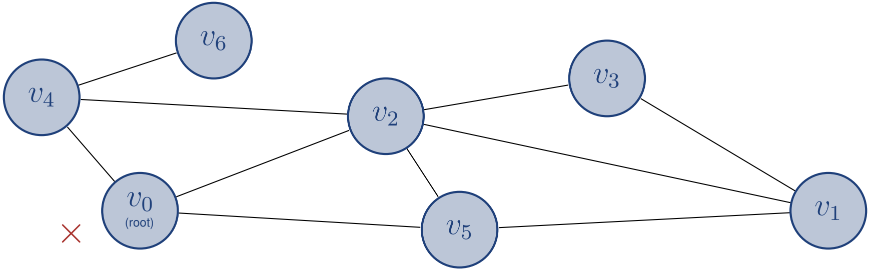
```
visited = [ 0 for i in range(len(G)) ]
def DFS(G, current):
    visited[current] = 1
    print(current, end=" ")
    for i in range(len(G)):
        if G[current][i] == 1 and visited[i] == 0:
            DFS(G, i)
```

Recursive DFS



Output:

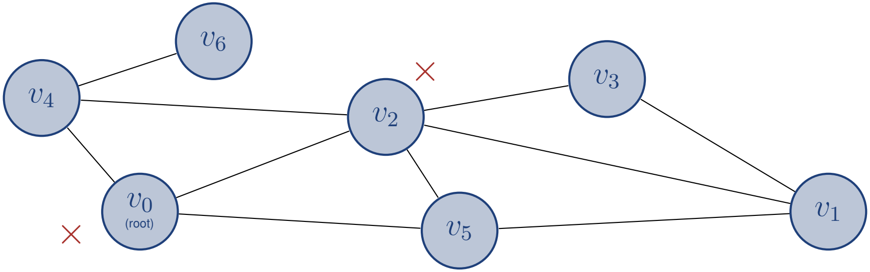
Recursive DFS



`DFS(G,0)`

Output: v_0

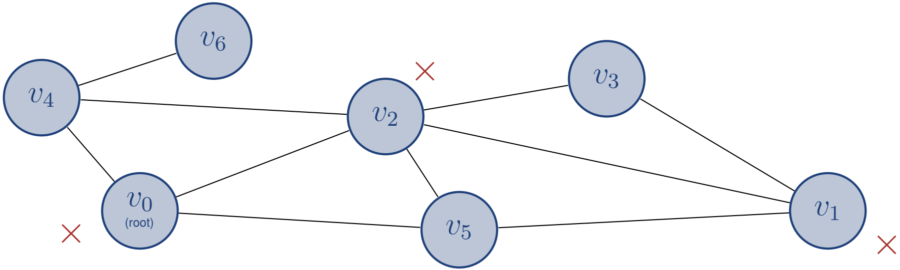
Recursive DFS



`DFS(G,0)` → `DFS(G,2)`

Output: v_0 v_2

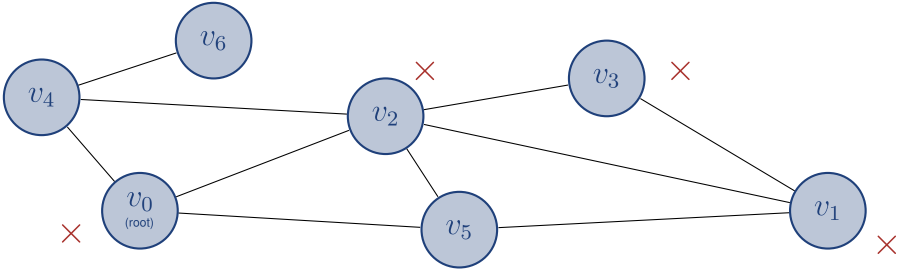
Recursive DFS



`DFS(G,0)` → `DFS(G,2)` → `DFS(G,1)`

Output: v_0 v_2 v_1

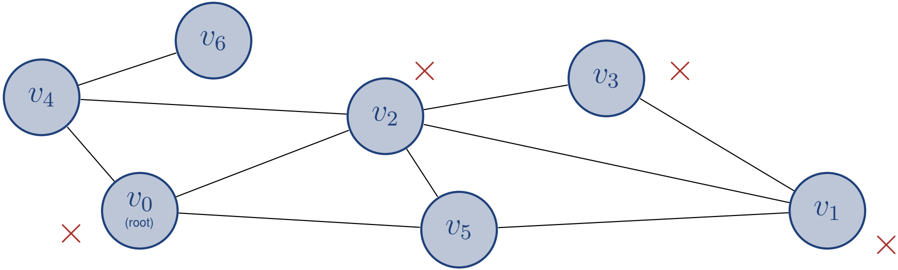
Recursive DFS



`DFS(G,0)` → `DFS(G,2)` → `DFS(G,1)` → `DFS(G,3)`

Output: v_0 v_2 v_1 v_3

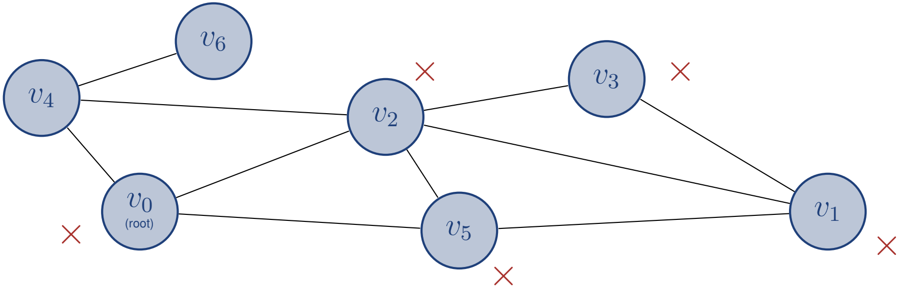
Recursive DFS



`DFS(G,0)` → `DFS(G,2)` → `DFS(G,1)`

Output: v_0 v_2 v_1 v_3

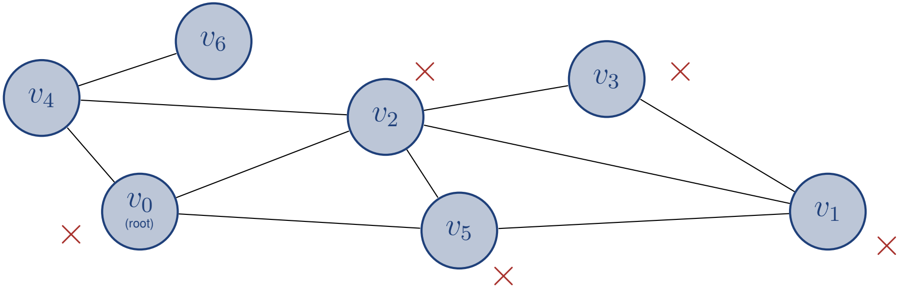
Recursive DFS



`DFS(G,0)` → `DFS(G,2)` → `DFS(G,1)` → `DFS(G,5)`

Output: v_0 v_2 v_1 v_3 v_5

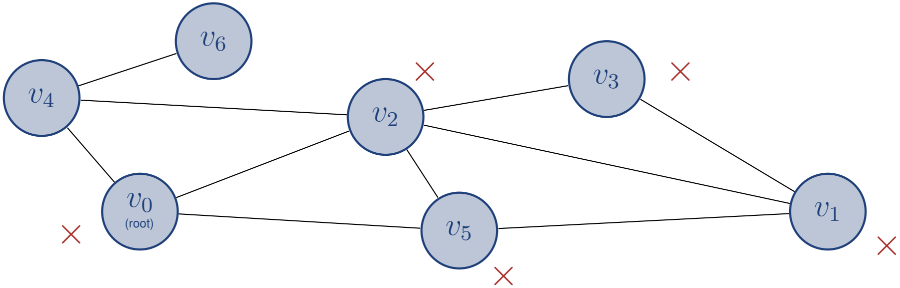
Recursive DFS



`DFS(G,0)` → `DFS(G,2)` → `DFS(G,1)`

Output: v_0 v_2 v_1 v_3 v_5

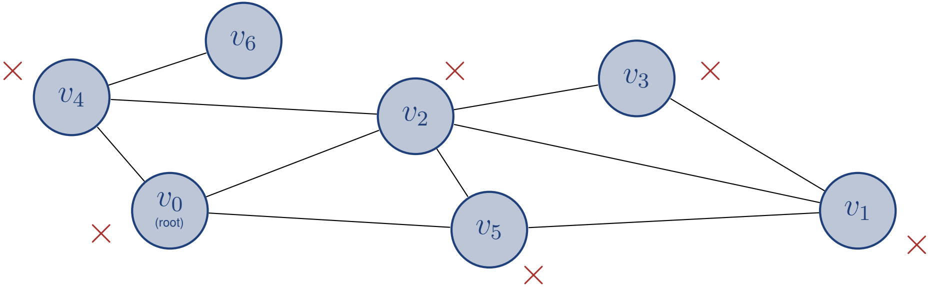
Recursive DFS



DFS(G,0) → DFS(G,2)

Output: v_0 v_2 v_1 v_3 v_5

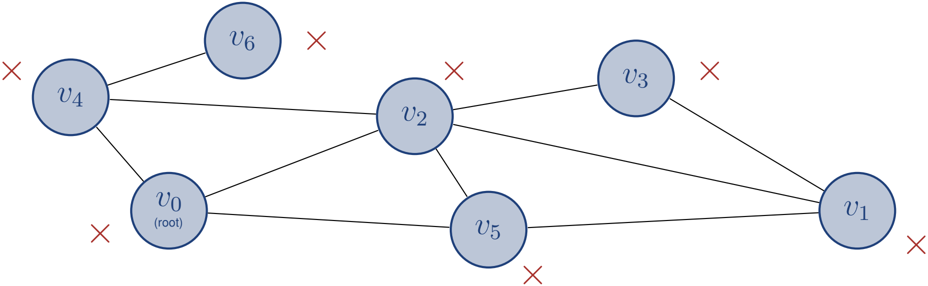
Recursive DFS



`DFS(G,0)` → `DFS(G,2)` → `DFS(G,4)`

Output: v_0 v_2 v_1 v_3 v_5 v_4

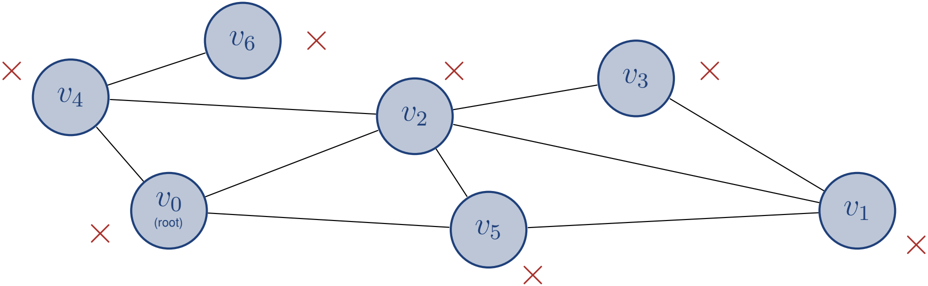
Recursive DFS




DFS(G,0) → DFS(G,2) → DFS(G,4) → DFS(G,6)

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

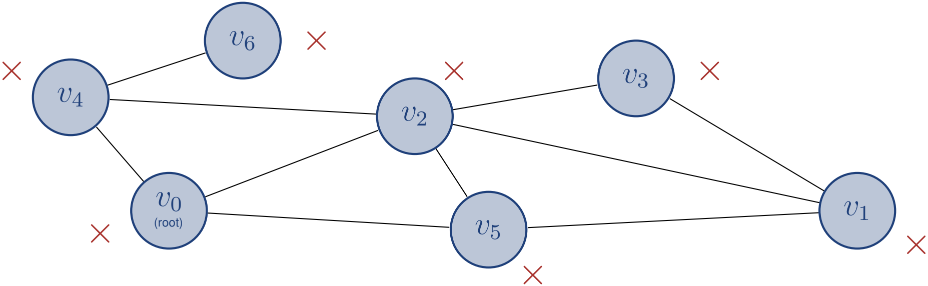
Recursive DFS



DFS(G,0) → DFS(G,2) → DFS(G,4)

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

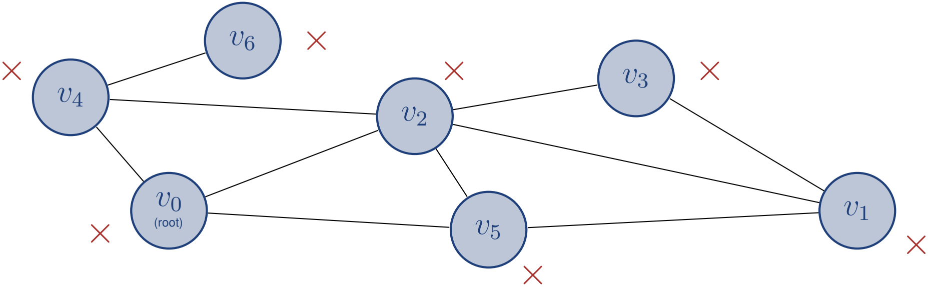
Recursive DFS



`DFS(G,0)` → `DFS(G,2)`

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

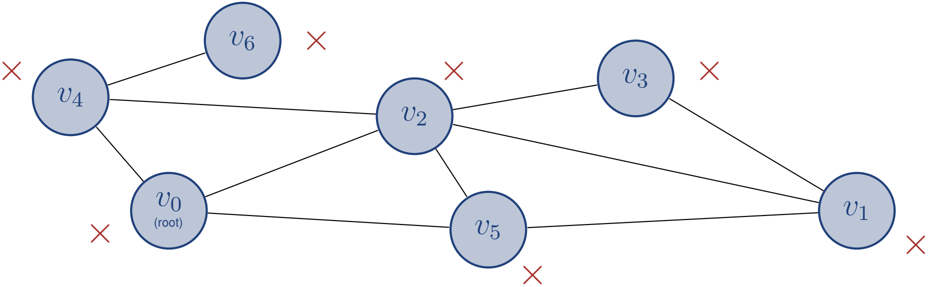
Recursive DFS



`DFS(G,0)`

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

Recursive DFS



Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

Recursive DFS

Applications

Applications

Is graph connected?

Applications

Is graph connected?

⇨ DFS from arbitrary vertex; are all vertices visited when done?

Applications

Is graph connected?

⇨ DFS from arbitrary vertex; are all vertices visited when done?

Is vertex w reachable from vertex v?

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Is vertex w reachable from vertex v?

⇨ DFS from v; is w visited when done?

Is a graph 2-colorable?

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Is vertex w reachable from vertex v?

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Is a graph 2-colorable?

⇨ DFS from arbitrary vertex and color levels differently

Applications

Is graph connected?

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Is vertex w reachable from vertex v?

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Is a graph 2-colorable?

⇨ DFS from arbitrary vertex and color levels differently

Does a graph contain a cycle?

Applications

Is graph connected?

⇨ DFS from arbitrary vertex; are all vertices visited when done?

Is vertex w reachable from vertex v?

⇨ DFS from v; is w visited when done?

Is a graph 2-colorable?

⇨ DFS from arbitrary vertex and color levels differently

Does a graph contain a cycle?

⇨ DFS from arbitrary vertex; is there a back edge?

Recursive DFS

Graph Coloring

Graph Coloring

Consider arbitrary graph

- Can it be colored with two colors?
- Connected vertices (“neighbors”) have different color

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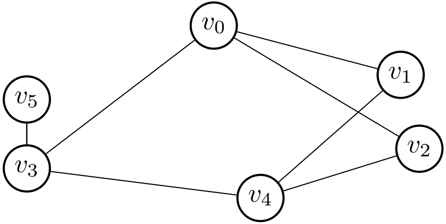

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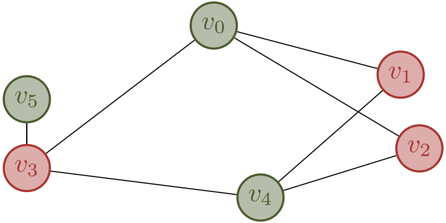

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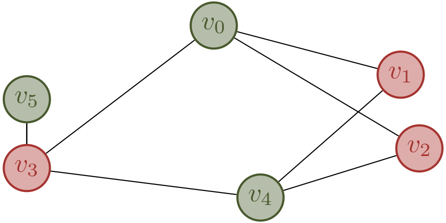

- List `color` instead of `visited`

Graph Coloring

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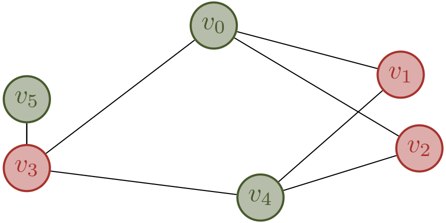
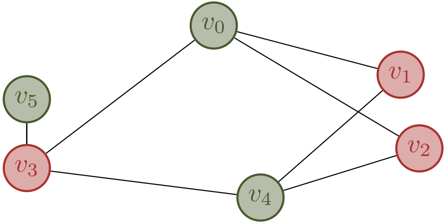


- List `color` instead of `visited`
- 0: not yet visited

Graph Coloring

Consider arbitrary graph

- Can it be colored with two colors?
- Connected vertices (“neighbors”) have different color
- Compute recursively

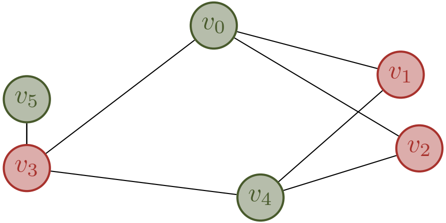


- List `color` instead of `visited`
- 0: not yet visited
- 1: colored green

Graph Coloring

Consider arbitrary graph

- Can it be colored with two colors?
- Connected vertices (“neighbors”) have different color
- Compute recursively



- List `color` instead of `visited`
- 0: not yet visited
- 1: colored green
- 2: colored red

Graph Coloring

We use recursive DFS

- All neighbors of `current` get a color different from that of `current`

Graph Coloring

We use recursive DFS

- All neighbors of `current` get a color different from that of `current`
- If neighbor already has same color as `current`, coloring is invalid

Graph Coloring

We use recursive DFS

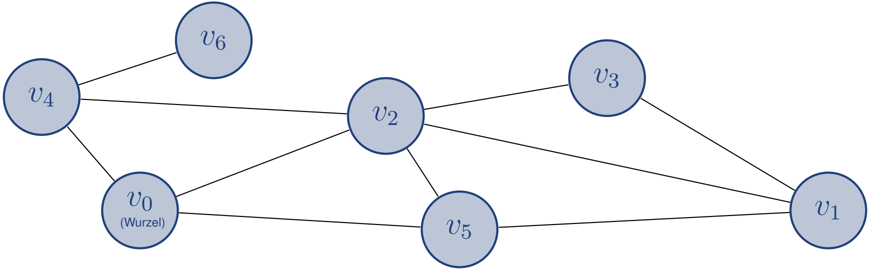
- All neighbors of `current` get a color different from that of `current`
- If neighbor already has same color as `current`, coloring is invalid

```python
def coloring(G, current):
    for i in range(len(G)):
        if G[current][i] == 1 and color[i] == 0:
            color[i] = 3 - color[current]
            coloring(G, i)
        elif G[current][i] == 1 and color[i] == color[current]:
            print("Coloring impossible.")
            return
```

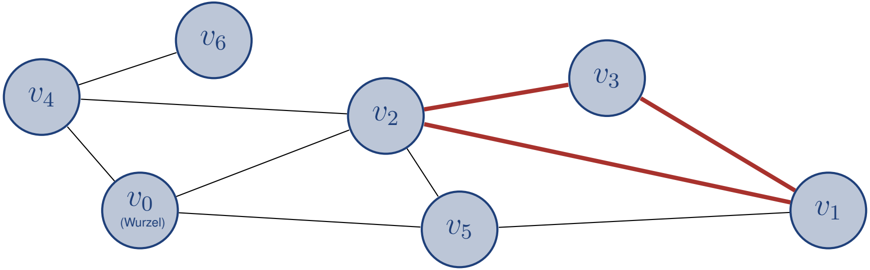
Recursive DFS

Finding Cycles

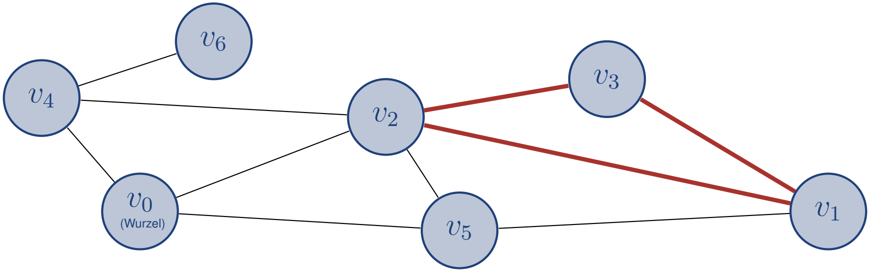
Finding Cycles



Finding Cycles

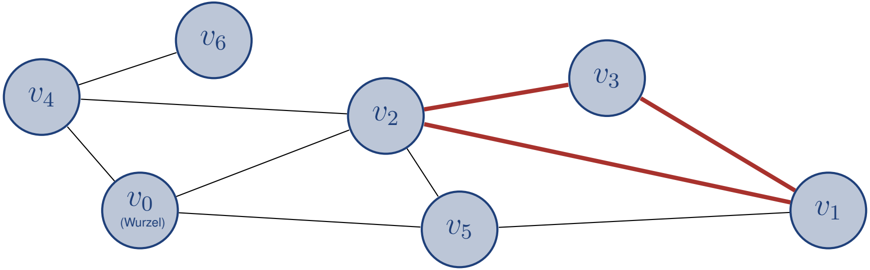


Finding Cycles



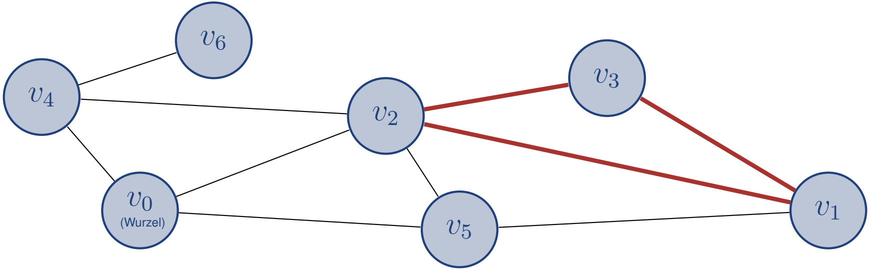
- DFS computes this

Finding Cycles



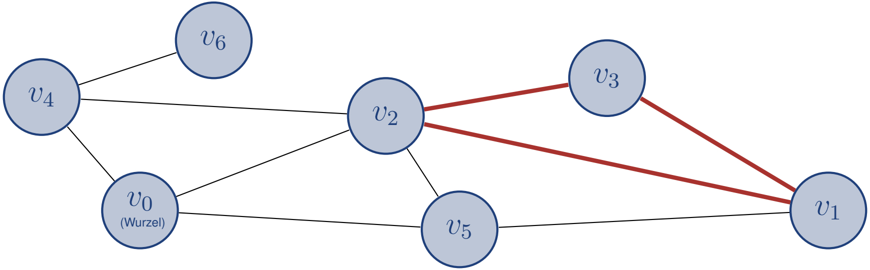
- DFS computes this
- Traverse graph as before

Finding Cycles



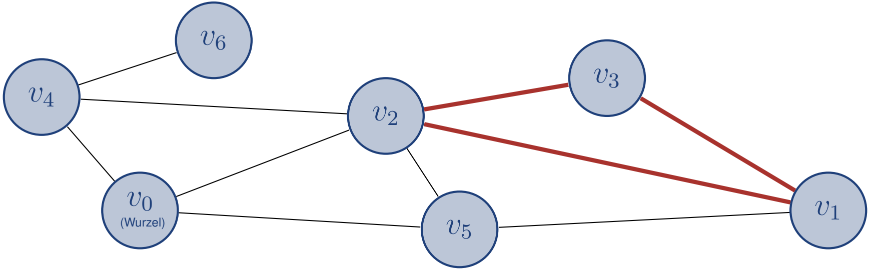
- DFS computes this
- Traverse graph as before
- Is there an edge to a vertex we already visited?

Finding Cycles



- DFS computes this
- Traverse graph as before
- Is there an edge to a vertex we already visited?
- Back-Edge

Finding Cycles



- DFS computes this
- Traverse graph as before
- Is there an edge to a vertex we already visited?
- Back-Edge
- **Attention:** Single edge is not a cycle

Finding Cycles

Compute whether graph contains a cycle

Finding Cycles

Compute whether graph contains a cycle

Extend DFS such that parent is considered

```
def find_cycle(G, current, parent):
    visited[current] = 1
    print(current, end=" ")
    for i in range(len(G)):
        if G[current][i] == 1 and visited[i] == 0:
            find_cycle(G, i, current)
        elif G[current][i] == 1 and visited[i] == 1 and i != parent:
            print("Found cycle.")
            return
```

Thanks for your attention