ETH zürich

Departement Informatik

Programming and Problem-Solving

Object Orientation

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Spring 2021 – May 20, 2021

Classes and Objects

Pyhon Classes

Classes – Technical

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Name

- attribute1
- attribute2
- ...

- method1
- method2
- ...

Classes – Conceptual

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Classes provide **functionality** that allows to perform **queries** based on the data or **operations** on the data

Example

- Coherent measurements
- Functions to read out data
- Functions to modify data

Example – Earthquake Catalog

Schweizerischer Erdbebendienst
Service Sismologique Suisse
Servizio Sismico Svizzero
Swiss Seismological Service

SED > *Earthquake catalog* > Query the catalogue

Earthquake catalog

link	date	time	appraisal	event type	lat [°N]	lon [°E]	source agency	depth	Mw	Ml	Io	Ix	epicentral area
»	2001/01/01	00:03:47.8	certain	earthquake	45.53	6.75	RENASS/BCSF (2009)	5.	1.52	0.9			SSE BEAUFORT(73)
»	2001/01/01	00:20:01.5	uncertain	earthquake	47.51	9.48	LED (2009)	10.	2.17	1.99			
»	2001/01/03	11:11:20.4	certain	earthquake	46.446	9.982	SED (ECOS-09)	4.	2.36	2.3			
»	2001/01/07	18:55:18.3	certain	earthquake	48.05	9.03	LED (2009)	15.	1.82	1.41			
»	2001/01/07	20:55:36.5	certain	earthquake	46.564	10.29	SED (ECOS-09)	5.	1.94	1.6			

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Class for Measurement – First Try

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Python Class `Measurement`

```
class Measurement:
    date = ""
    time = ""

    latitude = 0
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```

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Name of the class / data type

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class Measurement:
    date = ""
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```

Attributes according to CSV header

Class for Measurement – First Try

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Python Class `Measurement`

```python
class Measurement:
    date = ""
    time = ""

    latitude = 0
    longitude = 0

    magnitude = 0
```

Measurement

- `date` (Empty string λ)
- `time` (Empty string λ)
- `latitude` (Number 0)
- `longitude` (Number 0)
- `magnitude` (Number 0)

Classes and Objects

Python Objects

Objects – Instances of Classes

Classes describe the structure of objects, like a **blueprint**
⇨ Comparable with the **header** of the CSV file

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Example

- Variables to store parameters of measurement
- Function to display measurements lucidly
- Function to compare measurements

Object Instantiation

Objects are instances of classes

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```
w = Measurement()
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Measurement w

date	λ
time	λ
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w = Measurement()

w.date = "2001/01/03"
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Measurement w

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Object Instantiation

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w = Measurement()

w.date = "2001/01/03"
```

Dot notation; `instance.attribute`

Measurement `w`

date	2001/01/03
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latitude	0
longitude	0
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w = Measurement()

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Class for Measurement – Second Try

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Class for Measurement – Second Try

Measurement

- `date`
- `time`
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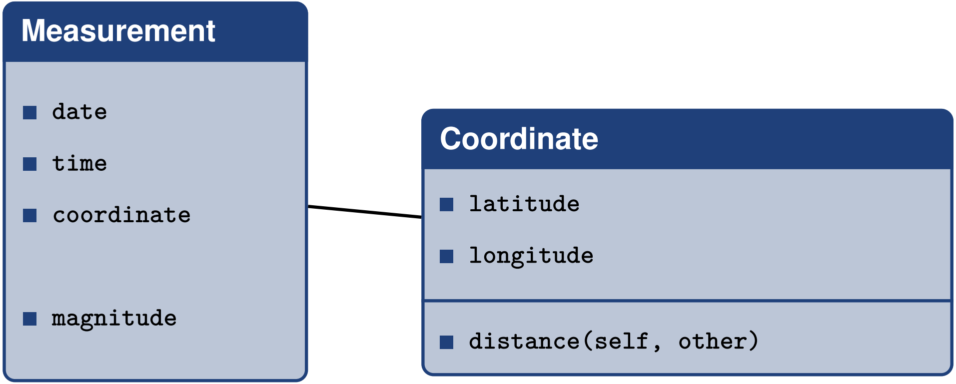
- `longitude`
- `magnitude`

Better structuring

- Latitude and longitude belong in their own data type `Coordinate`
- Object of type `Measurement` has an attribute of type `Coordinate`
- "Composition"

Class for Measurement – Second Try



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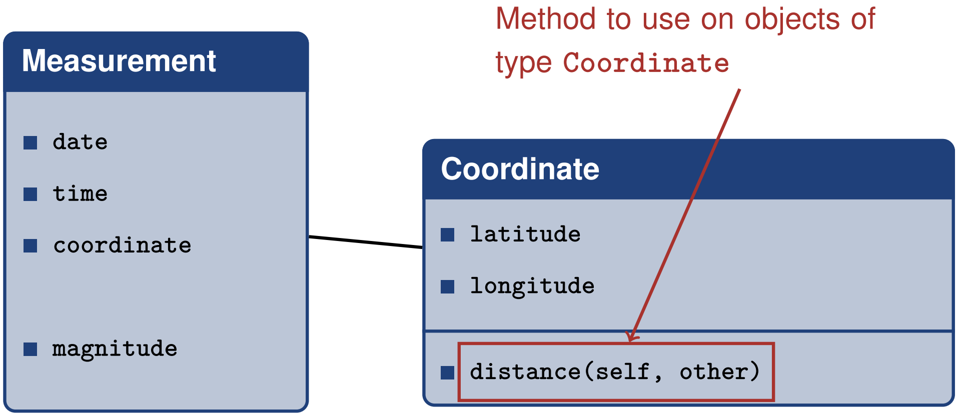

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- Methods are function that are defined within a class
- The first parameter is always `self`, which allows to refer to the current instance
- Again **dot notation**; Call analogously to `append()` for lists
- Pre-defined functions with special functionality
- Function `__str__` defines what happens when instance is given to `print()`

```
class Coordinate:
    def __str__(self):
        return "Dies ist eine Koordinate"
```

Methods in Classes

```
from math import *

class Coordinate:

    latitude = 0
    longitude = 0

    def __str__(self):
        return "Dies ist eine Koordinate"

    # Computes the distance to the provided coordinate 'other' in kilometers
    def distance(self, other):
        dlat = self.latitude - other.latitude
        dlon = self.longitude - other.longitude
        Hav = sin(dlat / 2)**2 + cos(self.latitude) * cos(other.latitude) * sin(dlon / 2)**2
        return 6373 * 2 * atan2(sqrt(Hav), sqrt(1 - Hav))
```

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Enables to access the current object from within a method of that class

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Haversine formula

Classes and Objects

Constructors

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```
k = Coordinate()
k.latitude = 45.97
k.longitude = 7.65
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```
k = Coordinate()
k.latitude = 45.97
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```

Constructors facilitate to easily set the initial values of a newly created object

```
k = Coordinate(45.97, 7.65)
```

Constructors

```
from math import *

class Coordinate:

    def __init__(self, deg_latitude, deg_longitude):
        self.latitude = radians(deg_latitude)              # Conversion from degree measure
        self.longitude = radians(deg_longitude)            # to radians measure

    def distance(self, other):
        dlat = self.latitude - other.latitude
        dlon = self.longitude - other.longitude
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```

```
zurich = Coordinate(47.36667, 8.55)
brisbane = Coordinate(-27.46794, 153.02809)
print(int(zurich.distance(brisbane)))
```

Constructors

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```

Is executed when object is initialized; parameter values are passed to this function

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brisbane = Coordinate(-27.46794, 153.02809)
print(int(zurich.distance(brisbane)))
```

Managing an Earthquake Database

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1. Implement data structure to represent earthquakes
2. Read in CSV file, create objects from the lines, insert them into a dictionary
3. Implement user interface to query data

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```
30274940.00000; 2001/01/20 15:49:10; certain; earthquake; 45.856; 8.142; "SED (ECOS-09)"; 13.; 2.56; 2.6;
```

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```

Of interest are

- Index 0: Keys for dictionary; is converted to natural number
- Index 1: Date and time; is split at space
- Index 4: Longitude; is converted to floating-point number
- Index 5: Latitude; is converted to floating-point number
- Index 9: Magnitude on Richter scale; is converted to floating-point number

Managing an Earthquake Database

1. Implement data structure to represent earthquakes

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```
class Coordinate:
    def __init__(self, deg_latitude, deg_longitude):
        self.latitude = radians(deg_latitude)
        self.longitude = radians(deg_longitude)
    def __str__(self):
        return str(self.latitude) + ", " + str(self.longitude)

class Measurement:
    def __init__(self, date, time, magnitude, coordinate):
        self.date = date
        self.time = time
        self.magnitude = magnitude
        self.coordinate = coordinate
    def __str__(self):
        return "Erdbeben der Stärke " + str(self.magnitude) + ", gemessen am " \
          + str(self.date) + " um " + str(self.time) + " an Position " + str(self.coordinate)
```

Managing an Earthquake Database

2. Read in CSV, create objects from the lines, insert them into a dictionary

Managing an Earthquake Database

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```
def read_measurements(filename):

    # Datei Zeile für Zeile einlesen
    with open(filename) as file:
        lines = file.read().splitlines()
    measurements = {}

    # Alle Zeilen nacheinander verarbeiten
    for i in range(1, len(lines)):
        tmp = lines[i].split(";")
        tmp_coord = Coordinate(float(tmp[4]), float(tmp[5]))
        tmp_date_time = tmp[1].split(" ")
        tmp_magnitude = float(tmp[9])
        tmp_meas = Measurement(tmp_date_time[1], tmp_date_time[2], tmp_magnitude, tmp_coord)
        measurements[int(float(tmp[0]))] = tmp_meas

    return measurements
```

Managing an Earthquake Database

3. Implement user interface to query data

Managing an Earthquake Database

3. Implement user interface to query data

```
earthquakes = read_measurements("earthquakes.csv")

while True:
    user_input = input("Geben Sie eine Erdbeben-ID ein (Abbrechen mit exit): ")
    if user_input == "exit":
        print("Programm beendet.")
        break
    else:
        quake_id = int(user_input)
        if quake_id not in earthquakes:
            print("Erdbeben-ID nicht gefunden.")
        else:
            print(earthquakes[quake_id])
```

Managing a Student Database

Exercise – Managing a Student Database

Write a class the represents students with attributes

- `student_id`
- `name`
- `grade`

- Enable the user to create student objects using `input()`
- Save them into a dictionary
- Output every student using a `for ... in` loop

Exercise – Managing a Student Database

```python
class Student:

  def __init__(self, s_id, name, grade):
    self.s_id = s_id
    self.name = name
    self.grade = grade

  def __str__(self):
    return "Die / Der Studierende "
      + str(self.name)
      + " (" + str(self.s_id)
      + ") hat die Note "
      + str(self.grade)
      + " erhalten."
```

```python
students = {}

while True:
  user_input = input("Weitere Daten eingeben? [J/N]")
  if user_input == "J":
    tmp_id = int(input(" ID: "))
    tmp_name = input(" Name: ")
    tmp_grade = float(input(" Note: "))
    tmp_student = Student(tmp_id, tmp_name, tmp_grade)
    students[tmp_id] = tmp_student
  elif user_input == "N":
    print("Programm beendet.")
    break
  else:
    print("Ungültige Eingabe.")

for id in students:
  print(students[id])
```

Heaps

Lists and Dictonaries

Complexity on lists and dictionaries with n elements

Lists

Access with `[]`	$\mathcal{O}(1)$
Insertion with `append()`	$\mathcal{O}(1)$
Insertion with `insert()`	$\mathcal{O}(n)$
Removal with `pop(0)`	$\mathcal{O}(1)$
Removal with `pop()`	$\mathcal{O}(1)$
Find minimum	$\mathcal{O}(n)$

Dictionaries

Access with `[]`	$\mathcal{O}(1)$
Insertion with `[]`	$\mathcal{O}(1)$
Find minimum	$\mathcal{O}(n)$

Heaps

Design data structure for special usage

⇨ Minimum can be computed efficiently (Using lists and dictionaries $\mathcal{O}(n)$)

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Data structure with the following operations

Insertion	$\mathcal{O}(\log n)$
Get minimum	$\mathcal{O}(1)$
Pop minimum	$\mathcal{O}(\log n)$

Heaps

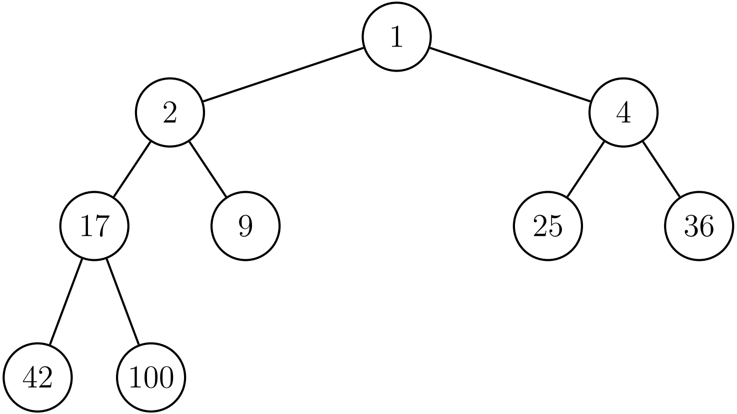
Design data structure for special usage
⇨ Minimum can be computed efficiently (Using lists and dictionaries $\mathcal{O}(n)$)

Data structure with the following operations

Insertion	$\mathcal{O}(\log n)$
Get minimum	$\mathcal{O}(1)$
Pop minimum	$\mathcal{O}(\log n)$

- Use a "tree"
- Embed this tree into list
- Root (first element of list) contains smallest element
- After removing an element, tree needs to be rearranged
- When inserting element, tree needs to be rearranged as well

Heaps



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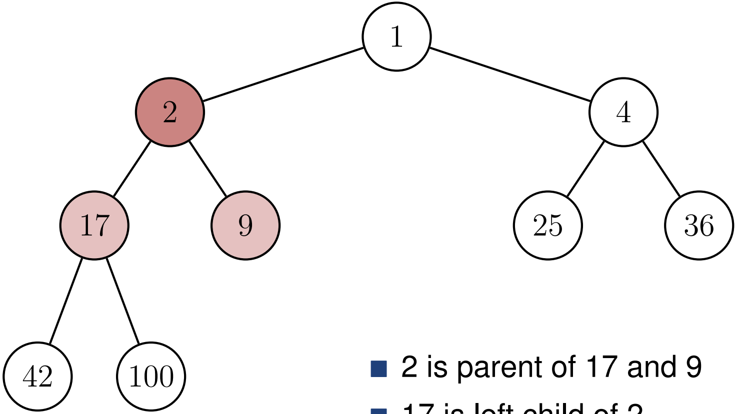
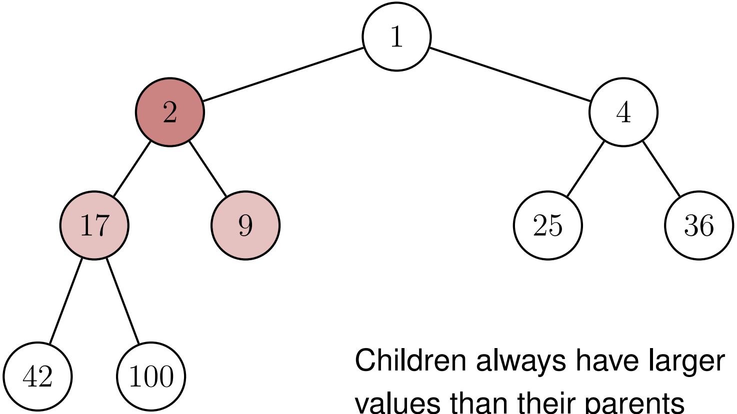


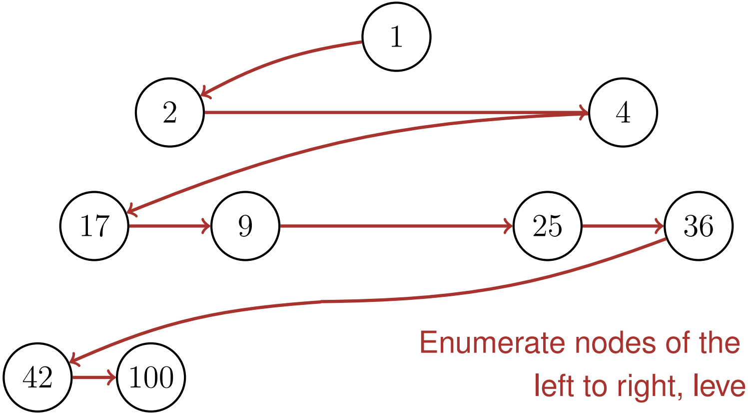
- 2 is parent of 17 and 9
- 17 is left child of 2
- 9 is right child of 2

Heaps



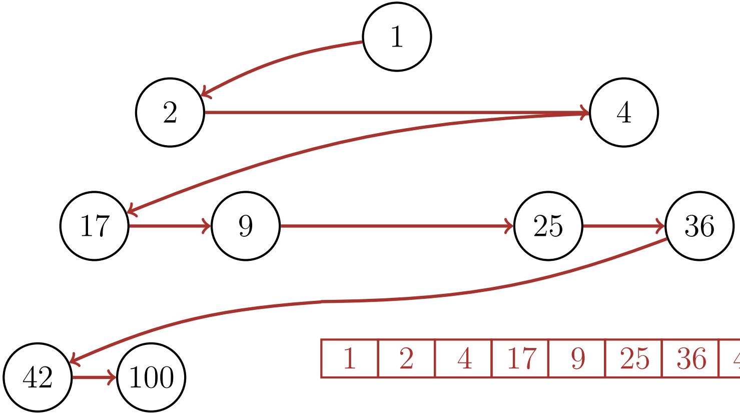
Children always have larger values than their parents

Heaps



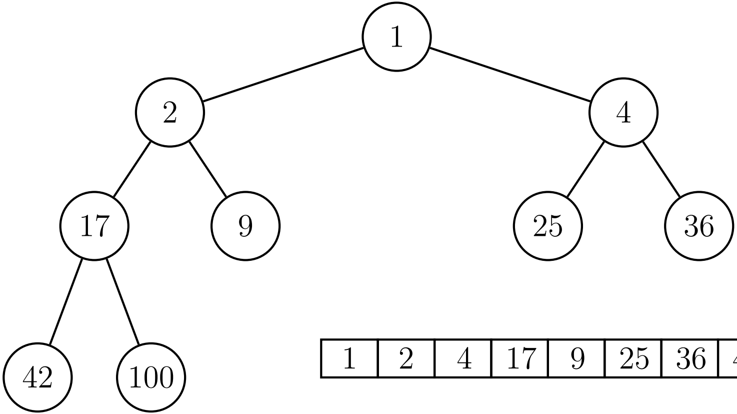
Enumerate nodes of the tree from left to right, level by level, and write values into a list in that order

Heaps



1	2	4	17	9	25	36	42	100

Heaps



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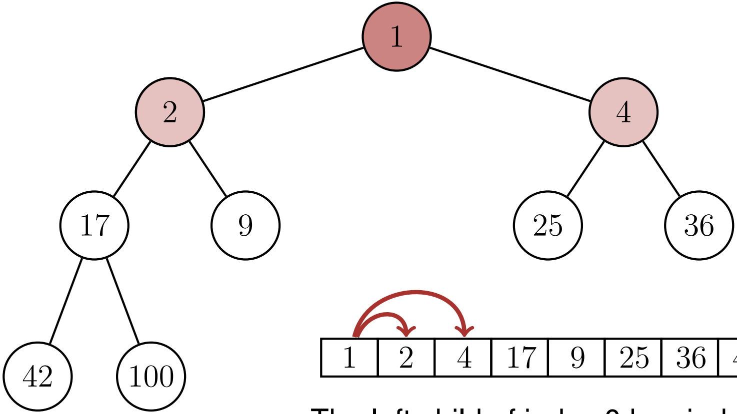
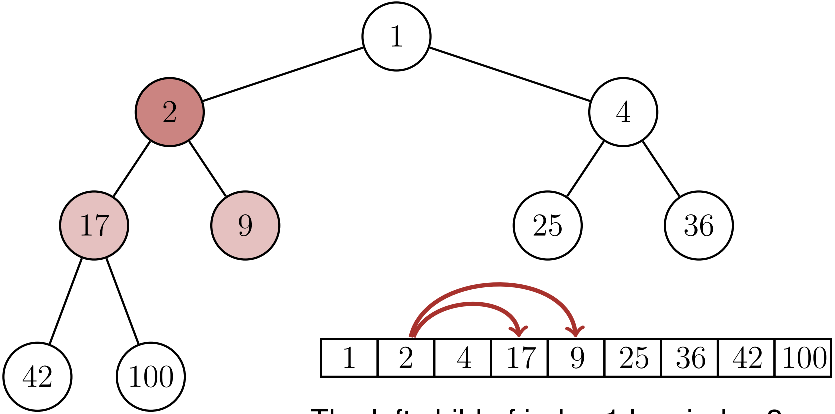


1	2	4	17	9	25	36	42	100

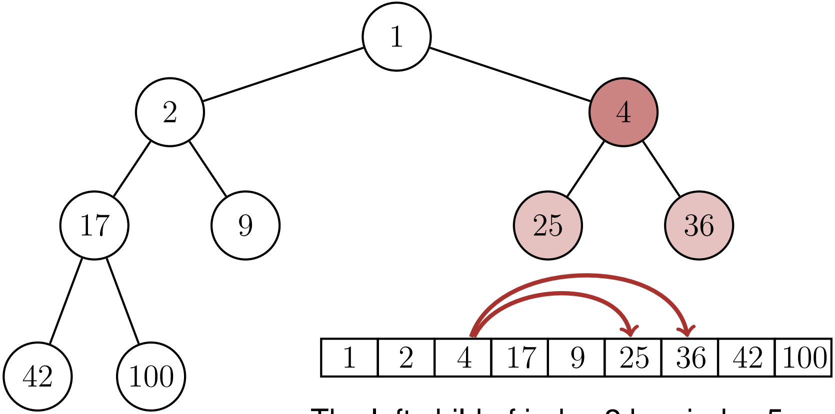
- The left child of index 0 has index 1
- The right child of index 0 has index 2

Heaps



- The left child of index 1 has index 3
- The right child of index 1 has index 4

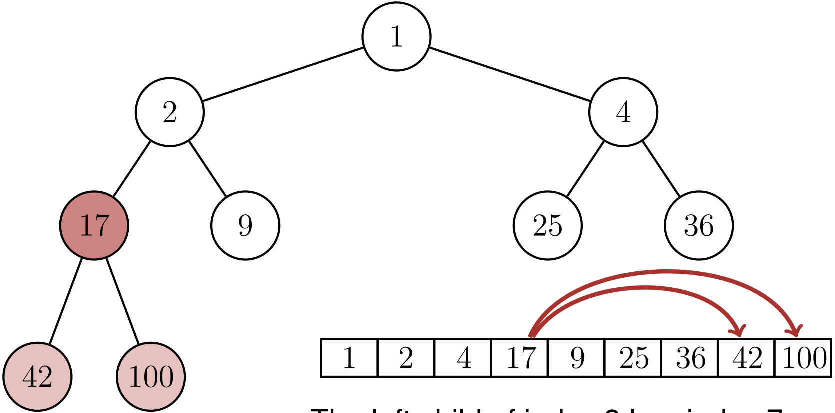
Heaps



1	2	4	17	9	25	36	42	100

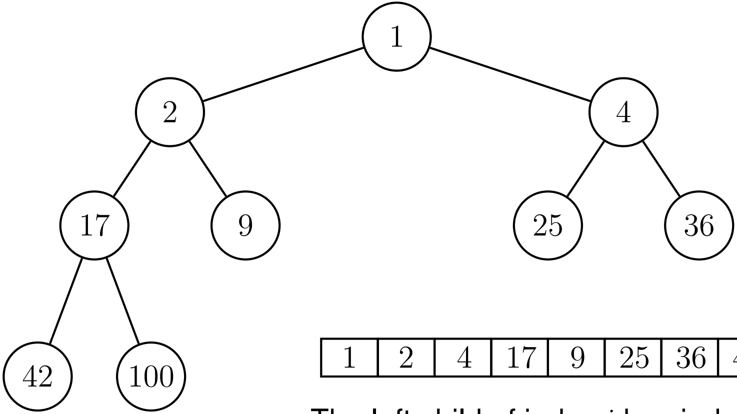
- The left child of index 2 has index 5
- The right child of index 2 has index 6

Heaps



- The left child of index 3 has index 7
- The right child of index 3 has index 8

Heaps



1	2	4	17	9	25	36	42	100

- The left child of index i has index $2i + 1$
- The right child of index i has index $2i + 2$

Heaps

Create class `Heap` with functions

- `add(self, x)` Insert element in $\mathcal{O}(\log n)$
- `getmin(self)` Output element in $\mathcal{O}(1)$
- `popmin(self)` Remove minimum in $\mathcal{O}(\log n)$

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```
class Heap:
  ...
  def add(self, x):
    ...
  def getmin(self):
    ...
  def popmin(self):
    ...
```

Heaps – Initialization

- Constructor creates list

```
def __init__(self):
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- Create helper functions; the underscore at the beginning indicates that they are for “internal use” only

```
def _swap(self, i, j):
    self.data[i], self.data[j] = self.data[j], self.data[i]
```

```
def _parent(self, i):
    return (i-1) // 2
```

```
def _left_child(self, i):
    return 2 * i + 1
```

```
def _right_child(self, i):
    return 2 * i + 2
```

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`getmin(self)` – Return smallest element

- Return the first element of list `data`

Heaps – Insert Element

```
def add(self, x):
    self.data.append(x)
    a = len(self.data) - 1
    while a > 0 and self.data[a] < self.data[self._parent(a)]:
        self._swap(a, self._parent(a))
        a = self._parent(a)
```

Heaps – Insert Element

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```
def getmin(self):
    return self.data[0]
```

Heaps – Remove Minimum

`pop_min(self)` – Remove smallest element

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- We cannot simply remove it and leave the remainder

- Overwrite first element with last element and remove the latter using `data.pop()`
- Now there is a wrong element located at the root
- Reorder tree from top to bottom
- To this end, swap root with larger child
- Now look at position of child and repeat

Heaps – Remove Minimum

```python
def popmin(self):
    self.data[0] = self.data[-1]
    self.data.pop()
    a = 0
    while True:
        m = a
        if self._left_child(a) < len(self.data) and \
            self.data[self._left_child(a)] < self.data[m]:
            m = self._left_child(a)
        if self._right_child(a) < len(self.data) and \
            self.data[self._right_child(a)] < self.data[m]:
            m = self._right_child(a)
        if m > a:
            self._swap(a, m)
            a = m
        else:
            return
```

Heapsort

Sorting with Heaps

Heaps – Complexity

- Suppose there are n elements in the heap
- Then the heap has roughly height $\log n$

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- Then, the respective minimum can be extracted n times in $\mathcal{O}(n \log n)$
- **Heapsort:** With this strategy we can sort in $\mathcal{O}(n \log n)$

Heapsort

```
def heapsort(data):
    tmp = Heap()
    sorted_data = []
    for element in data:
        tmp.add(element)
    for i in range(len(data)):
        sorted_data.append(tmp.getmin())
        tmp.popmin()
    return sorted_data
```

Thanks for your attention