ETH zürich

Departement Informatik

Programming and Problem-Solving

Binary Search an Recursion

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Searching

Linear Search

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Run once through list from left to right and compare each element to he sought one

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- Most straightforward strategy to search
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- Needs up to n comparisons on list of length n if sought element is at last position (or does not appear)
- Time complexity in $\mathcal{O}(n)$

Linear Search

```python
def linsearch(data, searched):
    index = 0
    while index < len(data):
        if data[index] == searched:
            return index
        index += 1
    return -1
```

Linear Search

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def linsearch(data, searched):
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```python
def linsearch(data, searched):
    if searched in data:
        return True
    else:
        return False
```

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Searching

Binary Search

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Example

Given a list with the first 8 prime numbers, find the position of 17

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2	3	5	7	11	13	17	19
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$\boxed{17}$

Binary Search

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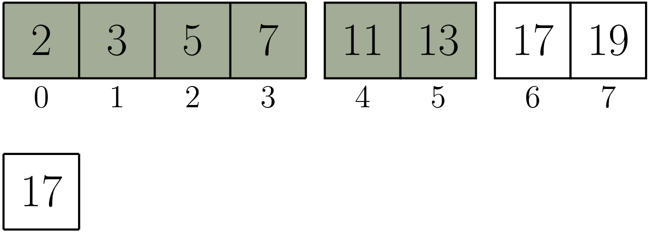

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- These specify **search space**
- Look at value in the middle (index `current`)

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⇨ `left = current + 1`

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- If this value is too small, then also everything left of it is too small

⇨ `left = current + 1`

- If this value is too small, then also everything right of it is too large

⇨ `right = current - 1`

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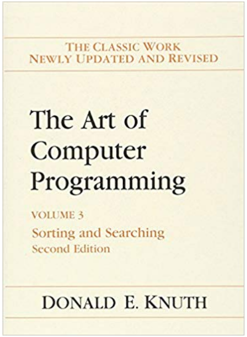
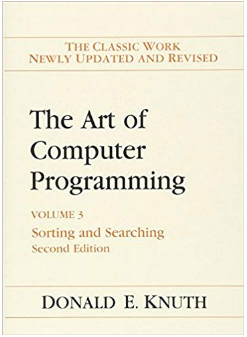


Binary Search

Sorting and searching data are two of the fundamental tasks of computer scientists

The first binary search was published in 1946 (and the principle was known long before), but the first version that works correctly for all n only appeared 14 years later



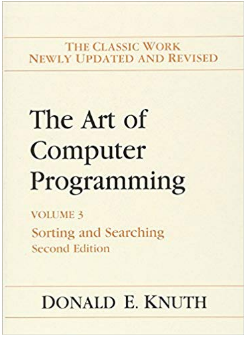
Binary Search

Sorting and searching data are two of the fundamental tasks of computer scientists

The first binary search was published in 1946 (and the principle was known long before), but the first version that works correctly for all n only appeared 14 years later

"Although the basic idea of binary search is comparatively straightforward, the details can be surprisingly tricky..."

–Donald Knuth



Exercise – Binary Search

Implement binary search

- as Python function
- using three “pointers” `left`, `right`, and `current`
- Initially, set `left = 0` and `right = len(data) - 1`
- In every step, shrink search space as described
- If element is found, its position is returned
- Otherwise, -1 is returned

Binary Search

```python
def binsearch(data, searched):
    left = 0
    right = len(data) - 1
    while left <= right:
        current = (left + right) // 2
        if data[current] == searched:
            return current
        elif data[current] > searched:
            right = current - 1
        else:
            left = current + 1
    return -1
```

Searching

Complexity of Binary Search

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- $n/2^x = 1 \iff n = 2^x \iff x = \log_2 n$

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- Time complexity in $\mathcal{O}(\log_2 n)$

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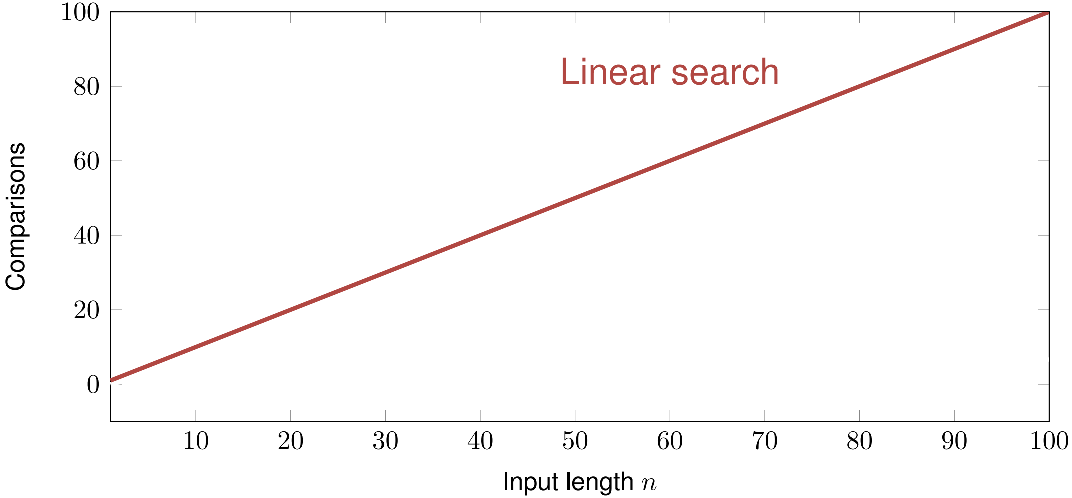

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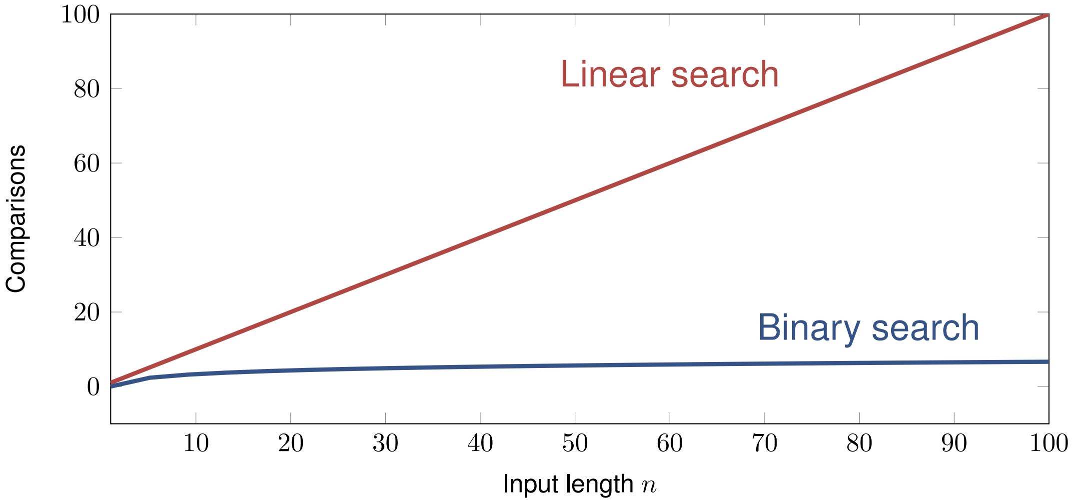

Complexity of Binary Search

- We again use a variable `counter` to count the comparisons
- Algorithm is executed on sorted lists with values 1 to n
- The value of n grows by 1 with every iteration
- Initially, n is 1, at the end $1\,000\,000$
- The first element 1 is always sought
- Results are stored in a list and plotted using `matplotlib`

Complexity of Binary Search

Worst Case

```
values = []
data = [1]


for i in range(1, 1000001):
    data.append(data[-1] + 1)
    values.append(binsearch(data, 1))


plt.plot(values, color="red")
plt.show()
```

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Add element that is larger by 1 than the currently last

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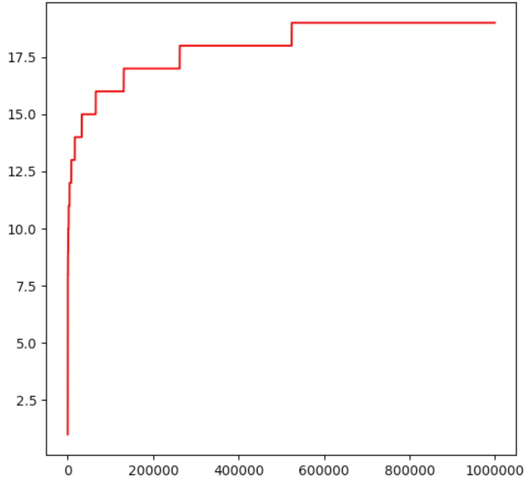

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- Sorting can pay off for multiple searches
- Sorting is in $\mathcal{O}(n \log_2 n)$ and is therefore slower than linear search
- Binary search is in $\mathcal{O}(\log_2 n)$ and is consequently much faster than linear search

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When does sorting pay off?

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When does sorting pay off?

If more than $\log_2 n$ searches are made

Recursive Functions

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`def f():` $\iff$ Python "learns" new word `f`

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`def f():` $\Longleftrightarrow$ Python “learns” new word `f`

From Merriam-Webster dictionary

re·frig·er·a·tor

A room or appliance for keeping food or other items cool

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- This analogy is not entirely correct
- Such functions are called **recursive functions**

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Not from Merriam-Webster dictionary

re·frig·er·a·tor

A refrigerator

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def f():
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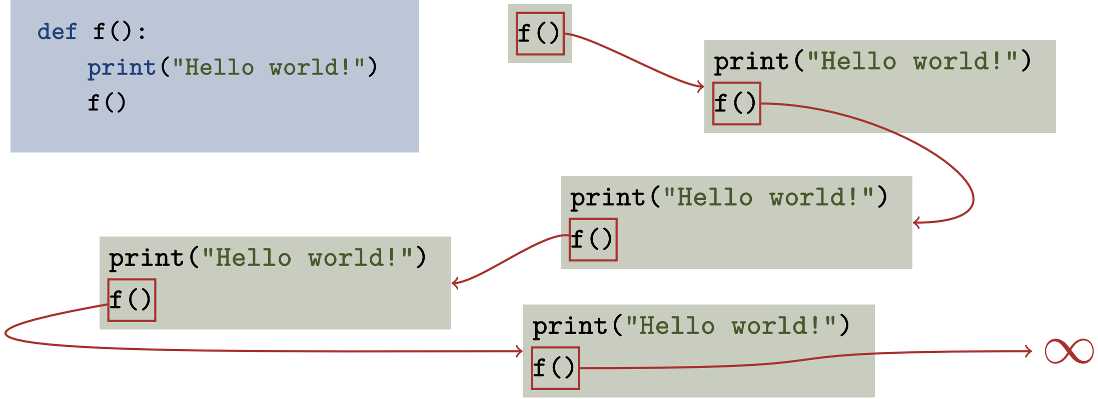

Recursive Functions

We use parameters to end after a finite number of calls

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```
def f(k):
    print(k)
    if k == 1:
        return
    else:
        f(k-1)
```

Recursive Functions

We use parameters to end after a finite number of calls

```
def f(k):
    print(k)
    if k == 1:
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    else:
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```

Parameter (or any local variable) is newly created for every function call

Recursive Functions

We use parameters to end after a finite number of calls

```
def f(k):
    print(k)
    if k == 1:
        return
    else:
        f(k-1)
```

```
f(4)
```

Recursive Functions

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```
def f(k):
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    else:
        f(k-1)
```

```
f(4)
```

```
print(4)
if 4 == 1:
    return
else:
    f(3)
```

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    return
else:
    f(3)
```

```
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if 3 == 1:
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else:
    f(2)
```

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else:
    f(1)
```

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```
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else:
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```

```
print(2)
if 2 == 1:
    return
else:
    f(1)
```

```
print(1)
if 1 == 1:
    return
else:
    f(0)
```

Recursive Functions

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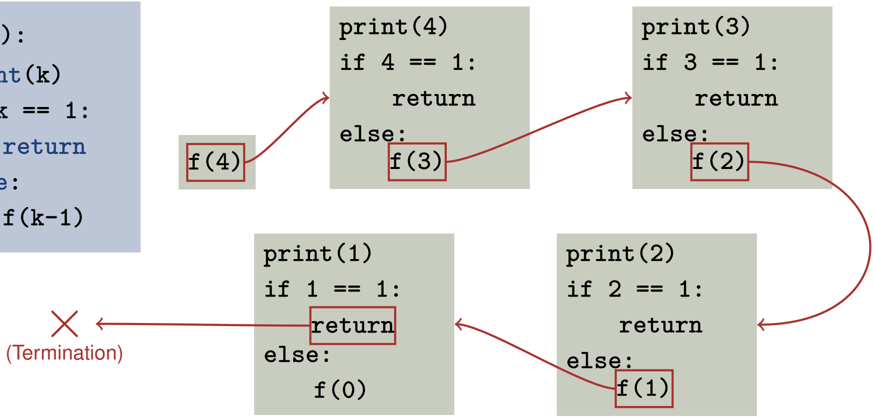

Factorial and Sum

Computing the Factorial Recursively

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$$\mathrm{fact}(n) = n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1$$

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- We observe

$$n! = n \cdot (n-1)! = n \cdot (n-1) \cdot (n-2)! = \ldots$$

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- We observe

$$n! = n \cdot (n-1)! = n \cdot (n-1) \cdot (n-2)! = \dots$$

- Function can be computed recursively by

$$\mathrm{fact}(1) = 1 \quad \text{and} \quad \mathrm{fact}(n) = n \cdot \mathrm{fact}(n-1)$$

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
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```

As before, parameter is newly created for every function call

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Last call returns fixed value 1

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Other calls return
n times “factorial of `n-1`”

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(7)`

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(6)`
`fact(7)`

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

fact(5)

fact(6)

fact(7)

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

```
fact(4)
fact(5)
fact(6)
fact(7)
```

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

```
fact(3)
fact(4)
fact(5)
fact(6)
fact(7)
```

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

fact(2)

fact(3)

fact(4)

fact(5)

fact(6)

fact(7)

Computing the Factorial Recursively – in Python

```python
def fact(n):
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```
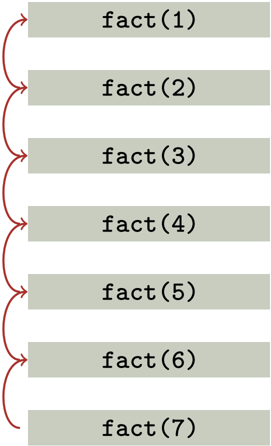

Call Stack



Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

fact(1) return 1

fact(2)

fact(3)

fact(4)

fact(5)

fact(6)

fact(7)

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

fact(2) return 2 * 1

fact(3)

fact(4)

fact(5)

fact(6)

fact(7)

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(2)` `return 2`

`fact(3)`

`fact(4)`

`fact(5)`

`fact(6)`

`fact(7)`

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

```
fact(3)        return 3 * 2
fact(4)
fact(5)
fact(6)
fact(7)
```

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(3)` `return 6`

`fact(4)`

`fact(5)`

`fact(6)`

`fact(7)`

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(4)`   `return 4 * 6`

`fact(5)`

`fact(6)`

`fact(7)`

Computing the Factorial Recursively – in Python

```python
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

```
fact(4)        return 24

fact(5)

fact(6)

fact(7)
```

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

fact(5)	return 5 * 24
fact(6)	
fact(7)	

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(5)`	`return 120`
`fact(6)`	
`fact(7)`	

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

```
fact(6)        return 6 * 120
fact(7)
```

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(6)`	`return 720`
`fact(7)`	

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(7)`	`return 7 * 720`

Computing the Factorial Recursively – in Python

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

Call Stack

`fact(7)` `return 5040`

Exercise – Computing a Sum Recursively

Implement a recursive function that

- takes a parameter `n`
- and returns the sum of the first `n` natural numbers

Exercise – Computing a Sum Recursively

Both recursive functions can be implemented with the same idea

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n * fact(n-1)
```

```
def thesum(n):
    if n == 1:
        return 1
    else:
        return n + thesum(n-1)
```

Recursion vs. Iteration

- There are alternatives using loops

```
def fact(n):
    i = 1
    result = 1
    while i < n:
        i += 1
        result *= i
    return result
```

```
def thesum(n):
    i = 1
    result = 1
    while i < n:
        i += 1
        result += i
    return result
```

Recursion vs. Iteration

- There are alternatives using loops

```
def fact(n):
    i = 1
    result = 1
    while i < n:
        i += 1
        result *= i
    return result
```

```
def thesum(n):
    i = 1
    result = 1
    while i < n:
        i += 1
        result += i
    return result
```

- For the sum, there is also a closed form (from the Bubblesort analysis)

```
def thesum(n):
    return n * (n+1) / 2
```

Recursion vs. Iteration

If repeated statements are implemented using loops, we speak of
iterative programming

Recursion vs. Iteration

If repeated statements are implemented using loops, we speak of **iterative programming**

- For all problems, there exist both iterative and recursive solutions

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- The recursive solution can often be viewed as more “elegant”

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If repeated statements are implemented using loops, we speak of
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- For all problems, there exist both iterative and recursive solutions
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- The implementation using recursion is often shorter (more concise) to write
- . . . but almost never faster to execute

Recursion vs. Iteration

If repeated statements are implemented using loops, we speak of **iterative programming**

- For all problems, there exist both iterative and recursive solutions
- The recursive solution can often be viewed as more “elegant”
- The implementation using recursion is often shorter (more concise) to write
- . . . but almost never faster to execute
- What should be used, depends on multiple factors

Euclid’s Algorithm Recursively

Euclid's Algorithm Recursively

Euclid's Algorithm

known from the first lecture

- Input: integers $a > 0$, $b > 0$
- Output: gcd of a and b

```
def euclid(a, b):
    while b != 0:
        if a > b:
            a = a - b
        else:
            b = b - a
    return a
```

Euclid's Algorithm Recursively

Euclid's Algorithm

known from the first lecture

- Input: integers $a > 0$, $b > 0$
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```python
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```

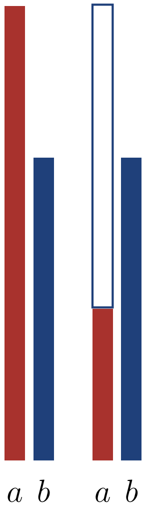

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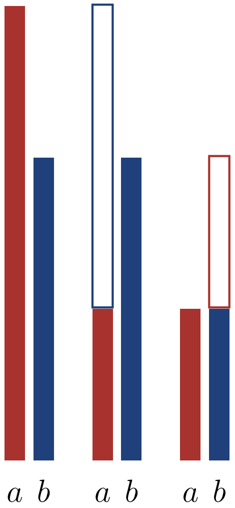

Euclid's Algorithm Recursively

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    return a
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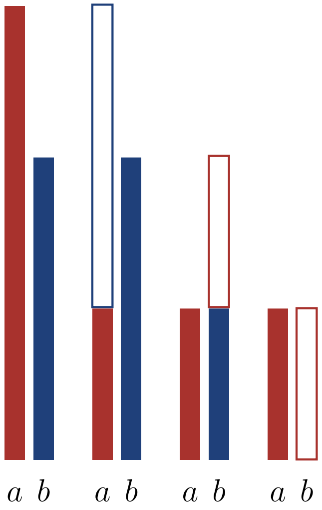

Exercise – Computing the GCD Recursively

Implement Euclid's Algorithm

- as a recursive Python function
- that takes two parameters `a` and `b`

```python
def euclid(a, b):
    while b != 0:
        if a > b:
            a = a - b
        else:
            b = b - a
    return a
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
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```

```python
def euclid(a, b):
    while b != 0:


        if a > b:
            a = a - b
        else:
            b = b - a
    return a
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
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```

Computing the GCD Recursively

```
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

`euclid(119, 68)`

Computing the GCD Recursively

```
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

```
euclid(51, 68)
euclid(119, 68)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

`euclid(51, 17)`

`euclid(51, 68)`

`euclid(119, 68)`

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

```
euclid(34, 17)
euclid(51, 17)
euclid(51, 68)
euclid(119, 68)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

`euclid(17, 17)`

`euclid(34, 17)`

`euclid(51, 17)`

`euclid(51, 68)`

`euclid(119, 68)`

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

```
euclid(17, 0)
euclid(17, 17)
euclid(34, 17)
euclid(51, 17)
euclid(51, 68)
euclid(119, 68)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

```
euclid(17, 0)        return 17
euclid(17, 17)
euclid(34, 17)
euclid(51, 17)
euclid(51, 68)
euclid(119, 68)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

```
euclid(17, 17)      return 17
euclid(34, 17)
euclid(51, 17)
euclid(51, 68)
euclid(119, 68)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

```
euclid(34, 17)      return 17
euclid(51, 17)
euclid(51, 68)
euclid(119, 68)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

`euclid(51, 17)`   `return 17`

`euclid(51, 68)`

`euclid(119, 68)`

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

```
euclid(51, 68)      return 17
euclid(119, 68)
```

Computing the GCD Recursively

```python
def euclid(a, b):
    if b == 0:
        return a
    else:
        if a > b:
            return euclid(a - b, b)
        else:
            return euclid(a, b - a)
```

Call Stack

`return` value is passed through

`euclid(119, 68)` `return 17`

Recursive Sorting and Searching

Binary Search

Iterative Binary Search

```
def binsearch(data, searched):
    left = 0
    right = len(data) - 1
    while left <= right:
        current = (left + right) // 2
        if data[current] == searched:
            return current
        elif data[current] > searched:
            right = current - 1
        else:
            left = current + 1
    return -1
```

Recursive Binary Search

Recursive Implementation

- Function again takes parameters `data` and for the given list and the searched element

Recursive Binary Search

Recursive Implementation

- Function again takes parameters `data` and for the given list and the searched element
- Two parameters `left` and `right` define the current **search space**
- In a single call, `left` and `right` are not changed

⇨ **No** loop

Recursive Binary Search

Recursive Implementation

- Function again takes parameters `data` and for the given list and the searched element
- Two parameters `left` and `right` define the current **search space**
- In a single call, `left` and `right` are not changed

⇨ **No** loop

- `current` is again computed as `(left + right) // 2`
- Again consider position `data[current]`

Recursive Binary Search

Recursive Implementation

- Function again takes parameters `data` and for the given list and the searched element
- Two parameters `left` and `right` define the current **search space**
- In a single call, `left` and `right` are not changed

⇨ **No** loop

- `current` is again computed as `(left + right) // 2`
- Again consider position `data[current]`
- If `searched` is not found, call the function recursively and either adjust `left` or `right` accordingly

Exercise – Recursive Binary Search

Implement binary search

- as a recursive Python function
- with four parameters
  `data`, `left`, `right`, and `searched`
- Follow the ideas of the iterative variant

```
def binsearch(data, searched):
    left = 0
    right = len(data) - 1
    while left <= right:
        current = (left + right) // 2
        if data[current] == searched:
            return current
        elif data[current] > searched:
            right = current - 1
        else:
            left = current + 1
    return -1
```

Recursive Binary Search

```python
def binsearch(data, left, right, searched):


    if left <= right:
        current = (left + right) // 2
        if data[current] == searched:
            return current
        elif data[current] > searched:
            return binsearch(data, left, current-1, searched)
        else:
            return binsearch(data, current+1, right, searched)
    else:
        return -1
```

Recursive Binary Search

```
def binsearch(data, left, right, searched):


    if left <= right:
        current = (left + right) // 2
        if data[current] == searched:
            return current
        elif data[current] > searched:
            return binsearch(data, left, current-1, searched)
        else:
            return binsearch(data, current+1, right, searched)
    else:
        return -1
```

```
def binsearch(data, searched):
    left = 0
    right = len(data) - 1
    while left <= right:
        current = (left + right) // 2
        if data[current] == searched:
            return current
        elif data[current] > searched:
            right = current - 1
        else:
            left = current + 1

    return -1
```

Recursive Binary Search

Call Stack

Recursive Binary Search

Call Stack

```
binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)
```

Recursive Binary Search

Call Stack

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 7, 12, 45)`

`current` $= 6$,
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)`

Recursive Binary Search

Call Stack

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 10, 12, 45)`

current $= 9$,
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 7, 12, 45)`

current $= 6$,
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)`

Recursive Binary Search

Call Stack

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 12, 12, 45)`

`current` $= 11$, recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 10, 12, 45)`

`current` $= 9$, recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 7, 12, 45)`

`current` $= 6$, recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)`

Call Stack

```
binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 13, 12, 45)
```

current $= 12$, recursive call

```
binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 12, 12, 45)
```

current $= 11$, recursive call

```
binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 10, 12, 45)
```

current $= 9$, recursive call

```
binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 7, 12, 45)
```

current $= 6$, recursive call

```
binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)
```

Recursive Binary Search

Call Stack

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 13, 12, 45)` `return -1`

current $= 12$,
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 12, 12, 45)`

current $= 11$,
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Recursive Binary Search

Call Stack

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recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 7, 12, 45)`

`current` $= 6$,
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)`

Recursive Binary Search

Call Stack

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 10, 12, 45)` `return -1`

current $= 9$,
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 7, 12, 45)`

current $= 6$,
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)`

Recursive Binary Search

Call Stack

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 7, 12, 45)` **return -1**

$\texttt{current} = 6,$
recursive call

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)`

Recursive Binary Search

Call Stack

`binsearch([2, 3, 5, 8, 10, 19, 21, 25, 28, 32, 36, 37, 42], 0, 12, 45)` `return -1`

Thanks for your attention