ETH zürich

Departement Informatik

Programming and Problem-Solving

Sorting 2

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Stacks and Queues

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So far access to arbitrary elements in lists by brackets

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Stack

- Last-In First-Out
- Elements can be inserted at the end
- Elements can be extracted from the same end

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- Last-In First-Out
- Elements can be inserted at the end
- Elements can be extracted from the same end

Queue

- First-In First-Out
- Elements can be inserted at the end
- Elements can be extracted from the front

Queues

Queue – Two Operations

- `append(x)` inserts element `x` at last position
- `pop(0)` removes first element and returns it
- In Python, lists can be used like queues

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data.pop(0)
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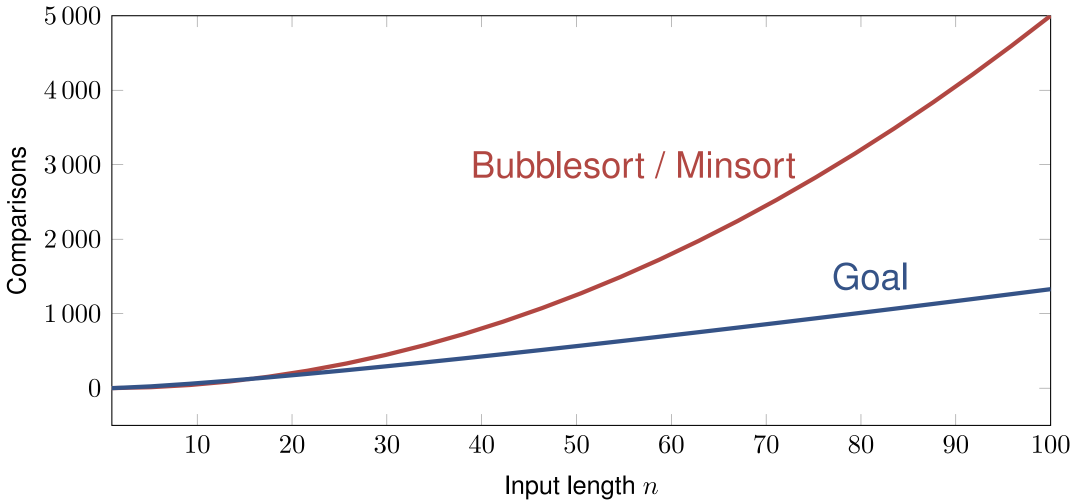
data.append(8)   ←  data = [1, 4, 5, 8]
data.pop()
data.pop()       ←  data = [1, 4]
```

Sorting 2

Mergesort

Time Complexity of Bubblesort



How Fast Can We Sort?

Idea

Merging two sorted list is simple

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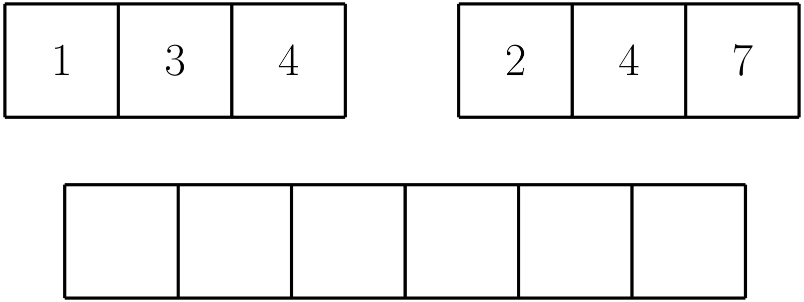
Idea

Merging two sorted list is simple

- First sort small lists
- Merge them
- Repeat

⇨ **Divide and Conquer**

Merging of Sorted Lists



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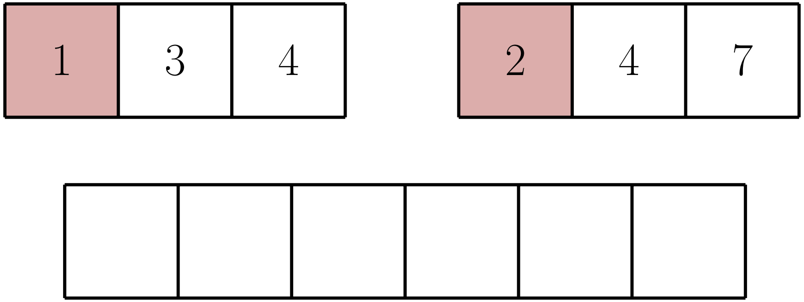

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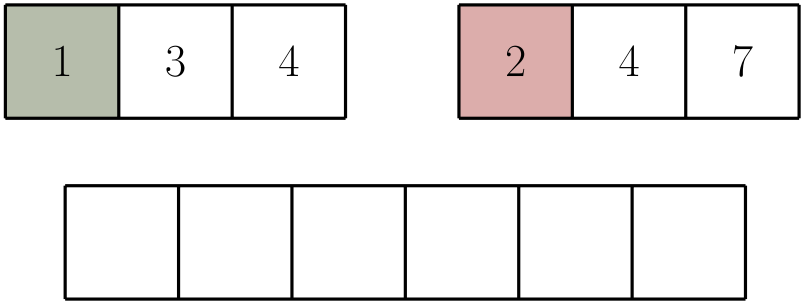

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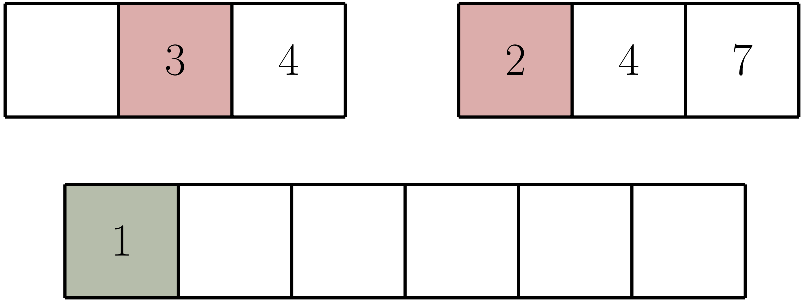

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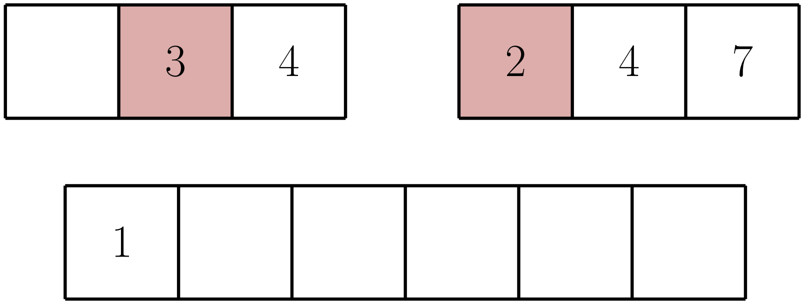

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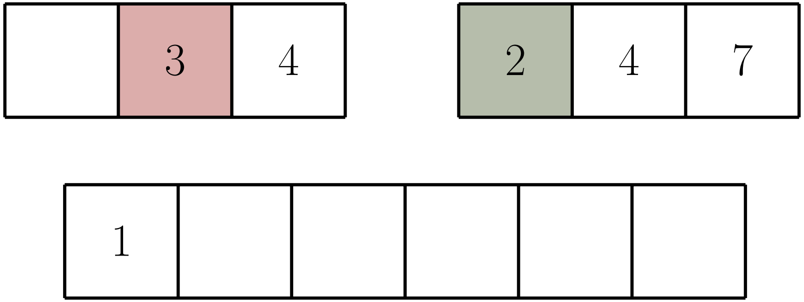

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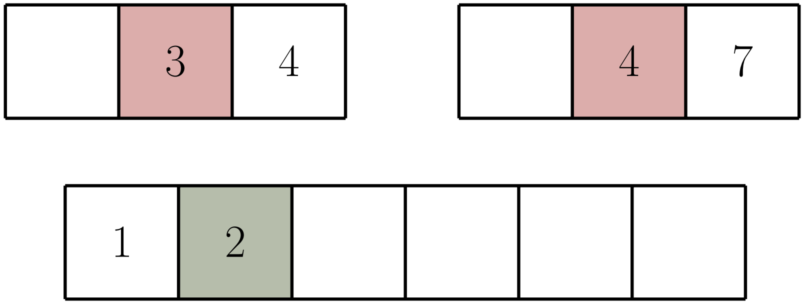

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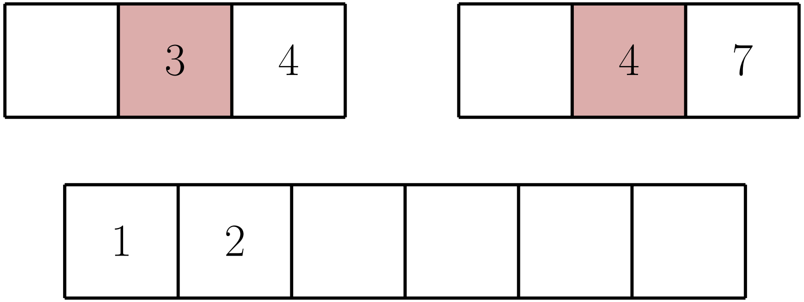

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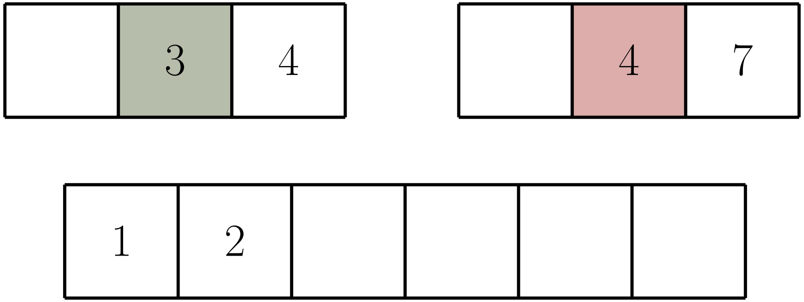

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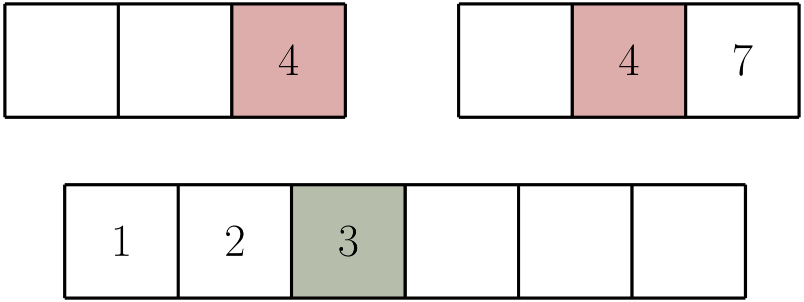
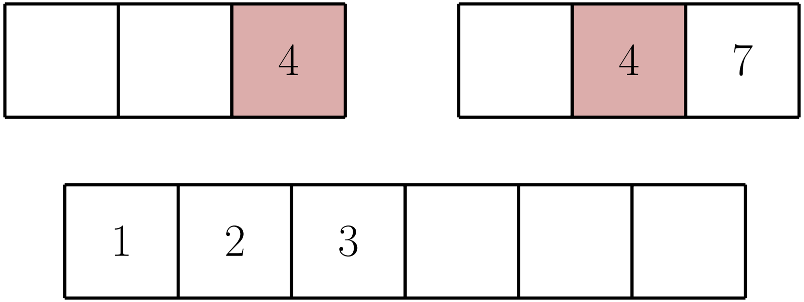
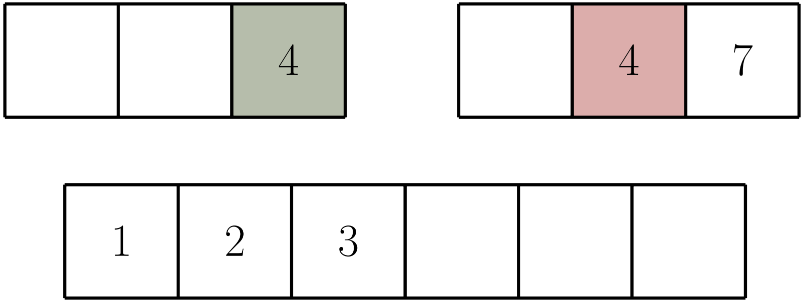



4
4
7
1
2
3

Merging of Sorted Lists



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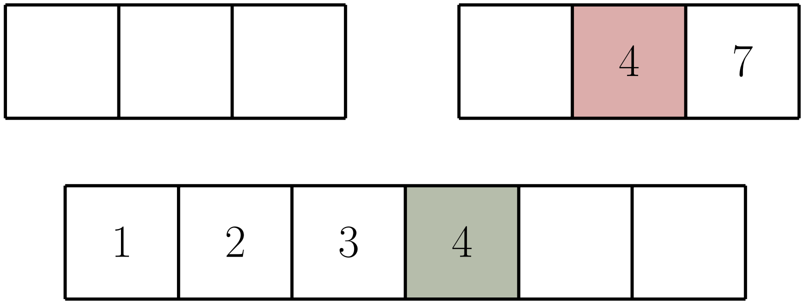

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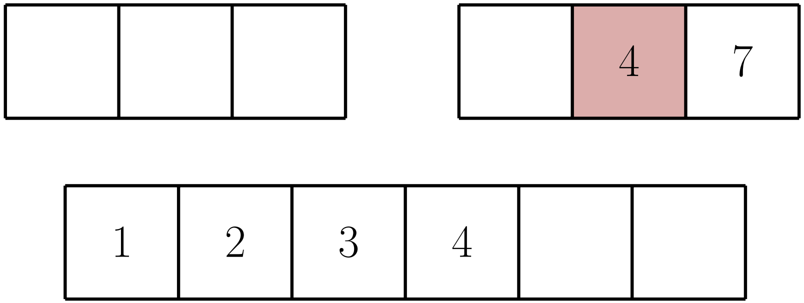

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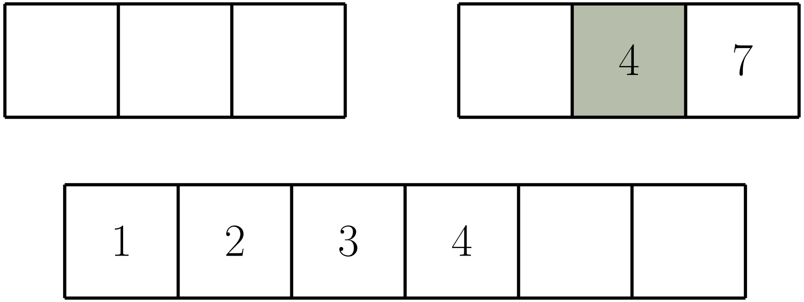

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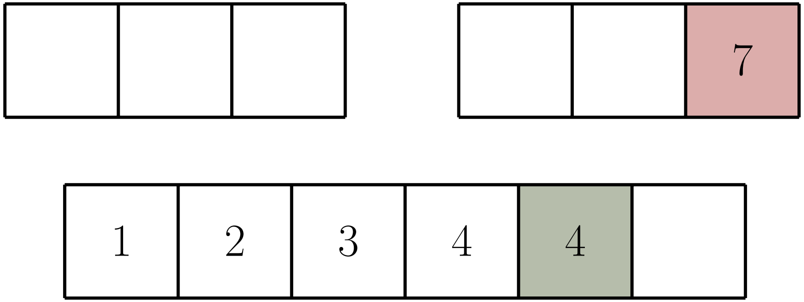

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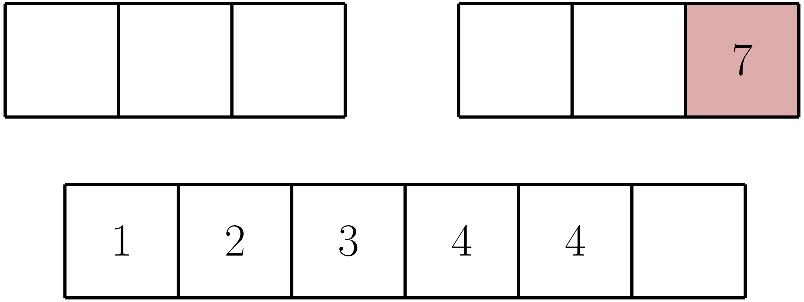

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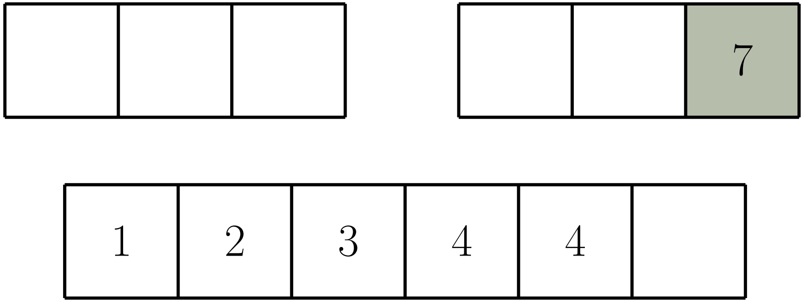

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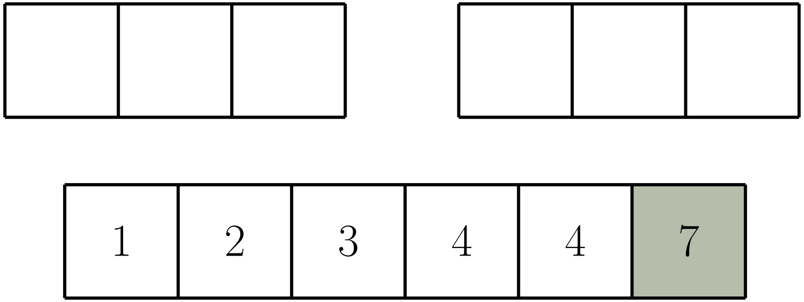

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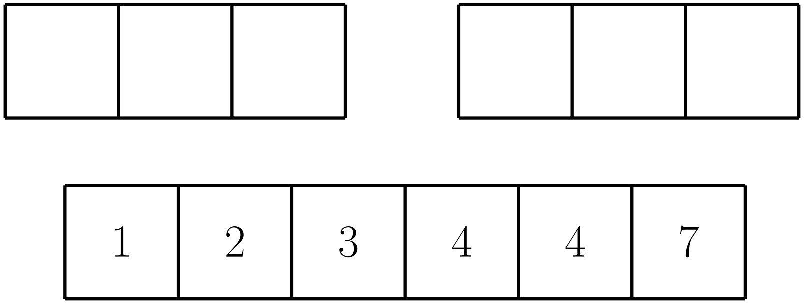

Exercise – Merging of Sorted Lists

Design a function that

- gets two sorted lists
- returns sorted list

Use the functions `pop(0)` and `append()`

Merging of Sorted Lists

```
def merge(left, right):
    result = []
    while len(left) > 0 and len(right) > 0:
        if left[0] > right[0]:
            result.append(right.pop(0))
        else:
            result.append(left.pop(0))
    return result + left + right
```

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```

While not both lists are empty

Merging of Sorted Lists

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```

Append the smaller of both elements

Merging of Sorted Lists

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        else:
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    return result + left + right
```

One of the two given sorted lists may still contain elements

Mergesort

Divide and Conquer

Iteratively merge sorted lists

Mergesort

Divide and Conquer

Iteratively merge sorted lists

- First merge “lists” of length 1 to lists of length 2

Mergesort

Divide and Conquer

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- First merge “lists” of length 1 to lists of length 2
- Merge lists of length 2 to lists of length 4

Mergesort

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- First merge “lists” of length 1 to lists of length 2
- Merge lists of length 2 to lists of length 4
- Merge lists of length 4 to lists of length 8

Mergesort

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Iteratively merge sorted lists

- First merge “lists” of length 1 to lists of length 2
- Merge lists of length 2 to lists of length 4
- Merge lists of length 4 to lists of length 8
- Merge lists of length 8 to lists of length 16

Mergesort

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Iteratively merge sorted lists

- First merge “lists” of length 1 to lists of length 2
- Merge lists of length 2 to lists of length 4
- Merge lists of length 4 to lists of length 8
- Merge lists of length 8 to lists of length 16
- $\ldots$

Mergesort

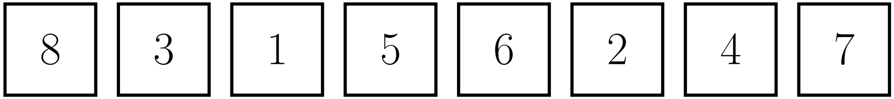
8	3	1	5	6	2	4	7

Mergesort

8	3	1	5	6	2	4	7

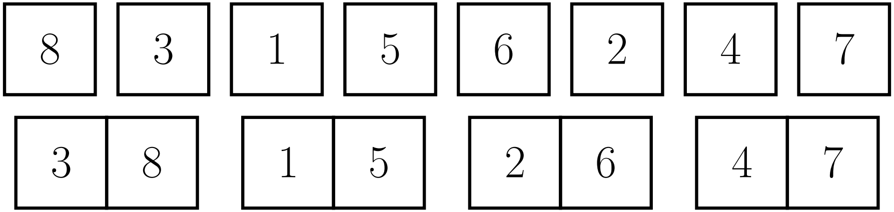
```
[8, 3, 1, 5, 6, 2, 4, 7]
```

Mergesort



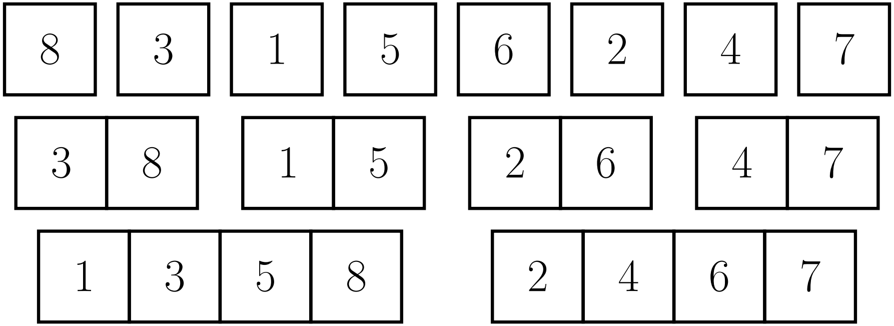
```
[[8], [3], [1], [5], [6], [2], [4], [7]]
```

Mergesort



```
[[3, 8], [1, 5], [2, 6], [4, 7]]
```

Mergesort



```
[[1, 3, 5, 8], [2, 4, 6, 7]]
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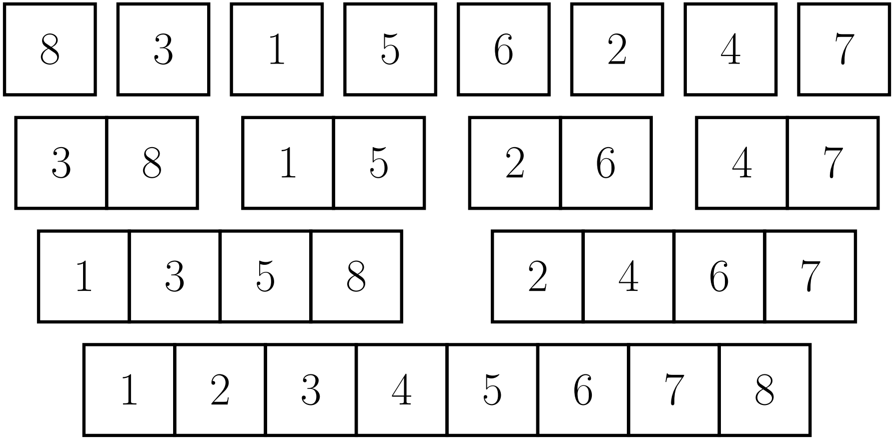
Mergesort



```
[[1, 2, 3, 4, 5, 6, 7, 8]]
```

Merge Step

Single Merge Step

- Get a 2-dimensional list, i.e., list that contains lists
- Each two successive lists are merged using the function `merge()`
- The last list is simply appended if there is an odd number of lists
- The result is again a 2-dimensional list that contains the merged lists

Merge Step

```python
def mergestep(data):
    result = []
    while len(data) > 1:
        left = data.pop(0)
        right = data.pop(0)
        result.append(merge(left, right))
    return result + data
```

Merge Step

```
def mergestep(data):
    result = []
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```

While there are still
at least two lists

Merge Step

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```

Merge the first two lists

Merge Step

```
def mergestep(data):
    result = []
    while len(data) > 1:
        left = data.pop(0)
        right = data.pop(0)
        result.append(merge(left, right))
    return result + data
```

If there is a list left at the end, append it

Mergesort – Complete Algorithm

Complete Algorithm

- Input is given as list `data`
- Convert every element into a list with one element
- This way get 2-dimensional list
- Apply `mergestep()` repeatedly to this list
- At the end, there will only be one element in the list
- This element corresponds to a sorted list

Mergesort – Complete Algorithm

```python
def mergesort(data):
    result = []
    for item in data:
        result.append([item])
    while len(result) > 1:
        result = mergestep(result)
    return result[0]
```

Sorting 2

Time Complexity of Mergesort

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Time complexity of Mergesort is proportional to

Number of merge steps $\times$ Comparisons per merge step

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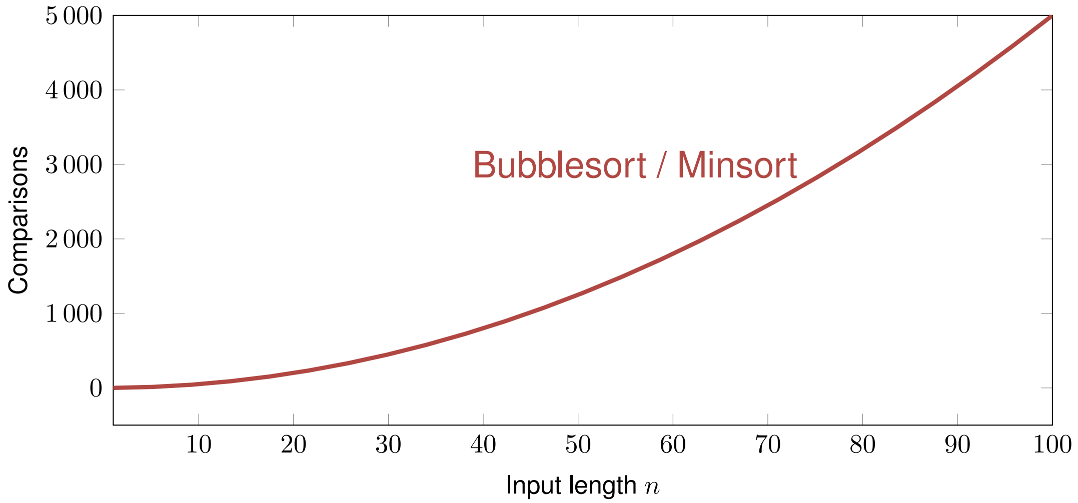
Time complexity of Mergesort is in $\mathcal{O}(n \log_2 n)$

Time Complexity of Mergesort



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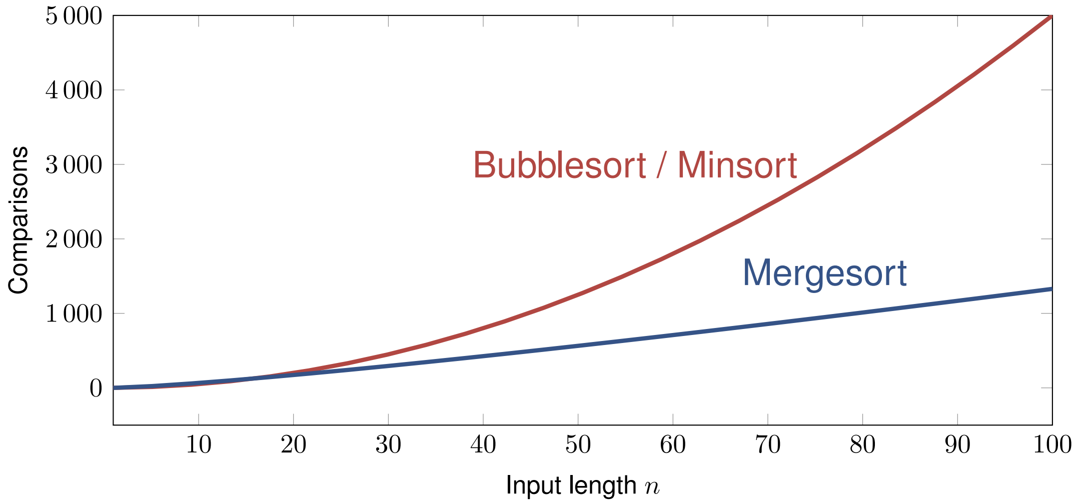

Sorting 2

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For Mergesort (and also Bubble- and Minsort),
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- This is not always the case
- Different best, average, and worst cases
- **Timsort**, for instance, makes use of already sorted sub lists

Sorting 2

Bucketsort

Sorting of Few Elements

Sorting of data sets with respect to **one attribute**

Sorting of Few Elements

Sorting of data sets with respect to **one attribute**

Stable sorting: Elements with same attribute maintain order

Sorting of Few Elements

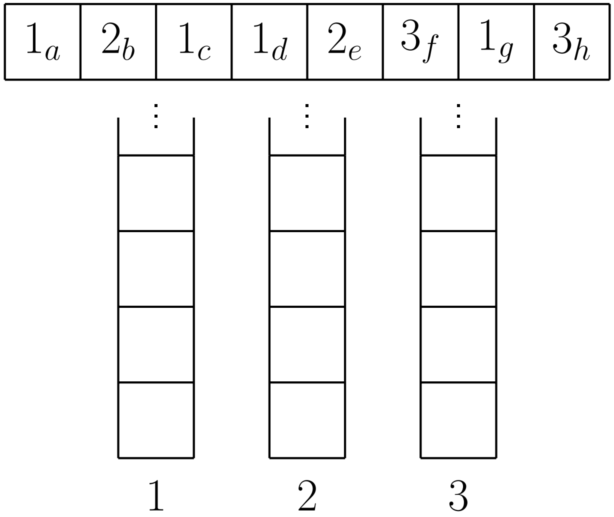
Sorting of data sets with respect to **one attribute**

Stable sorting: Elements with same attribute maintain order

Example

Name	First name	Grade
Adleman	Leonard	6
Caesar	Gaius Julius	3
de Vigenère	Blaise	5
Rivest	Ronald	6
Shamir	Adi	6

Bucketsort



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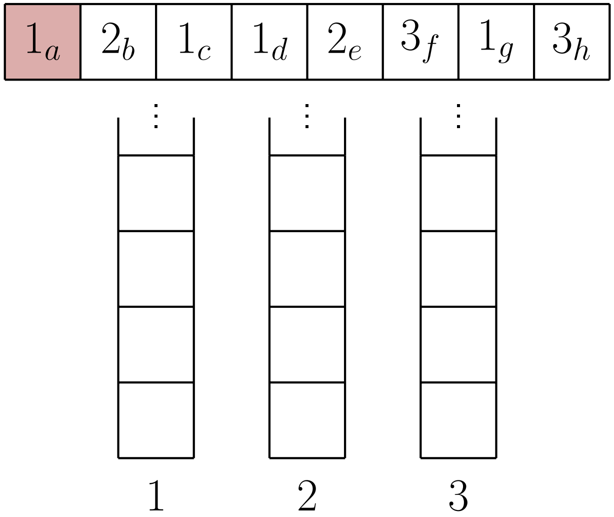

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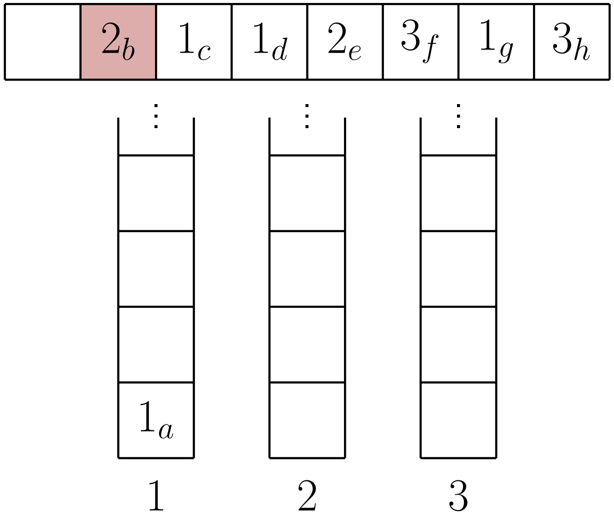

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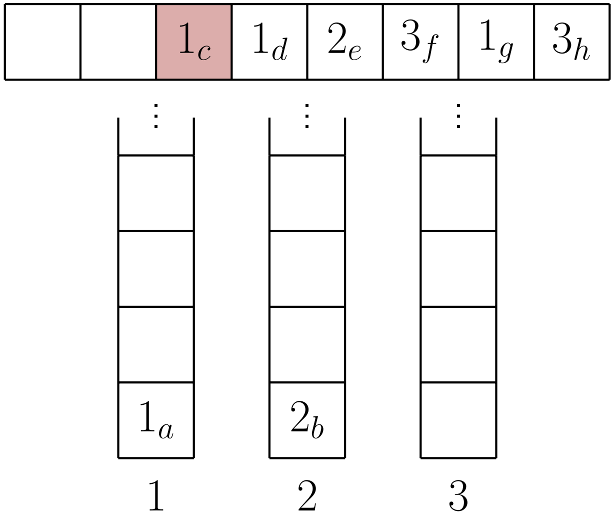

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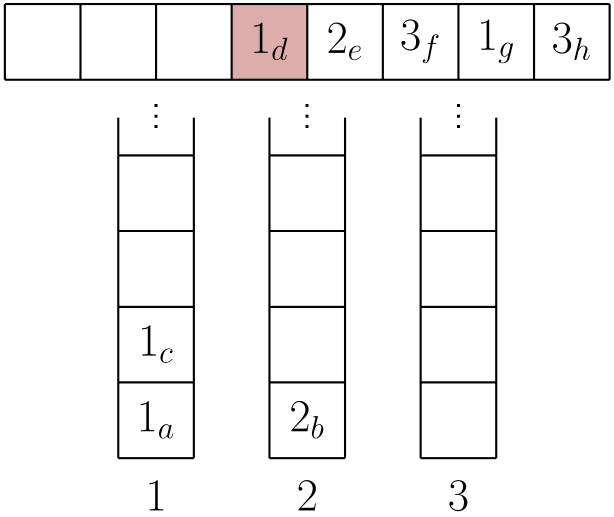

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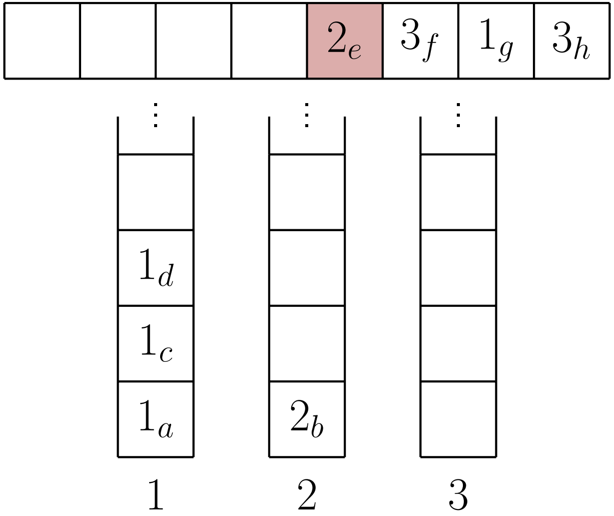

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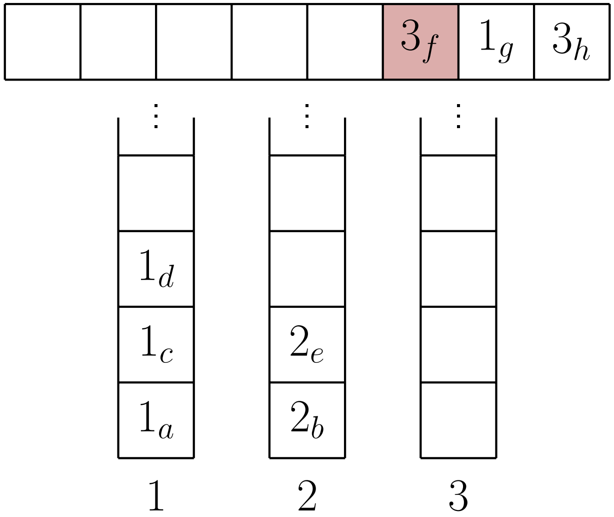

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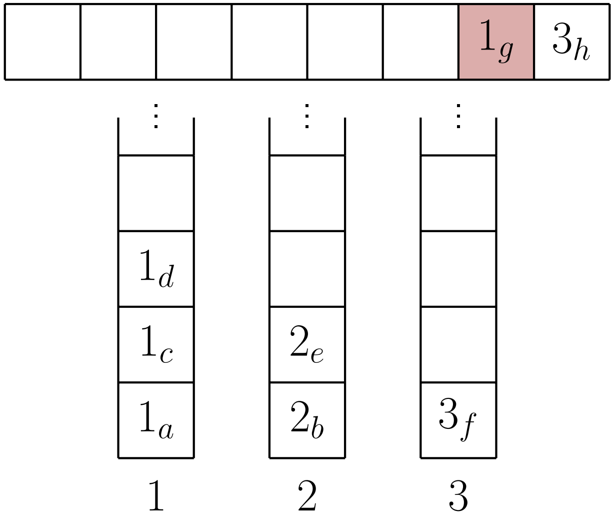

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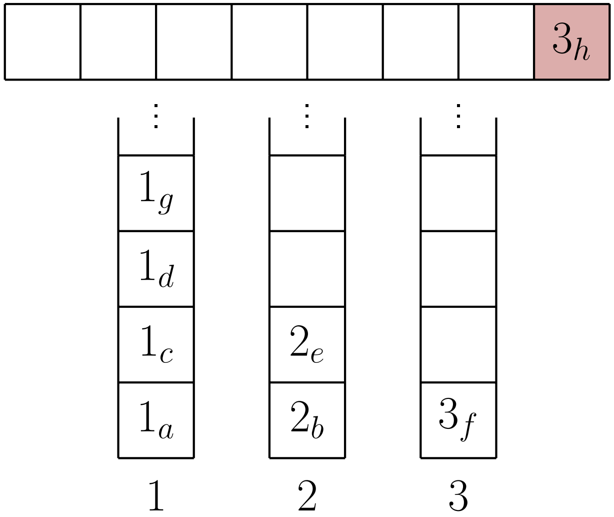

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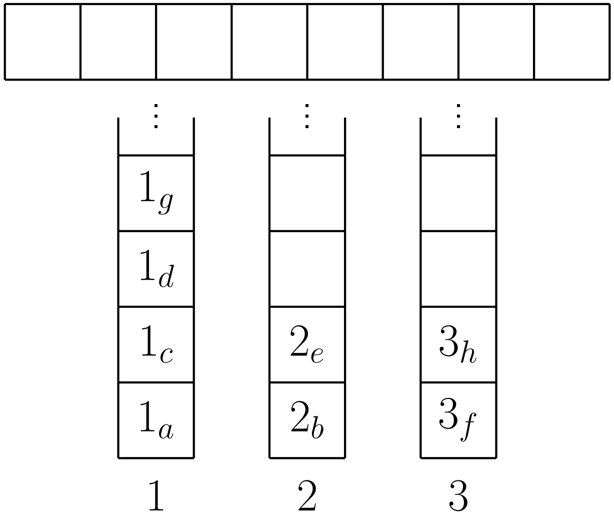

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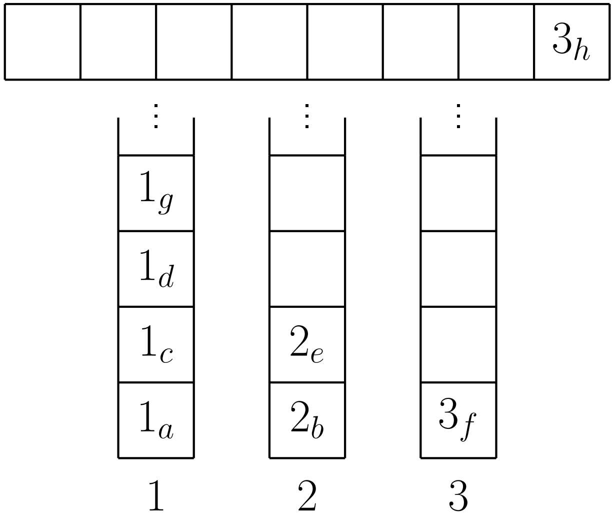

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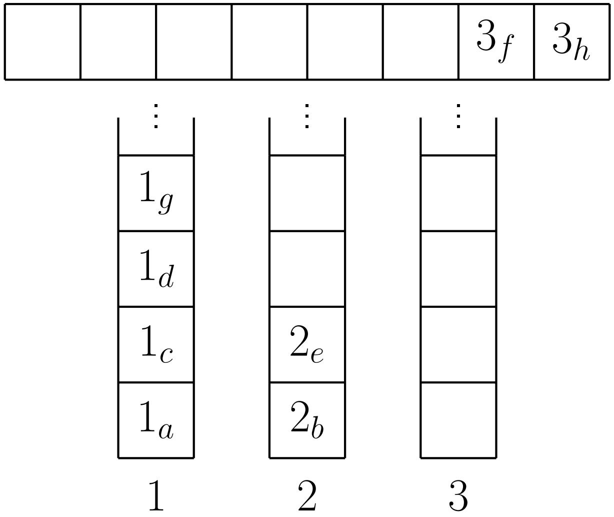

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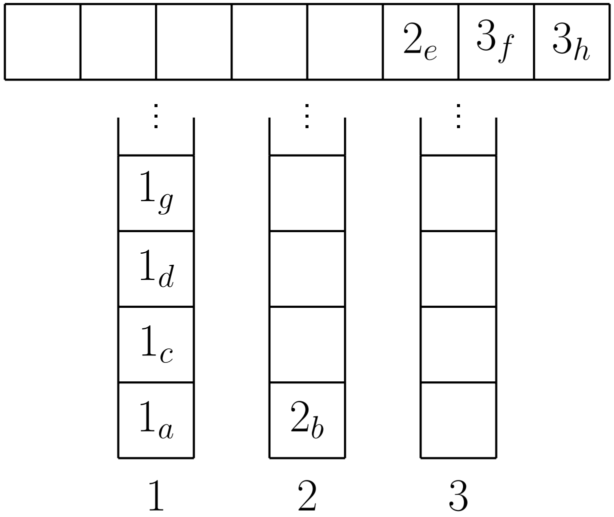
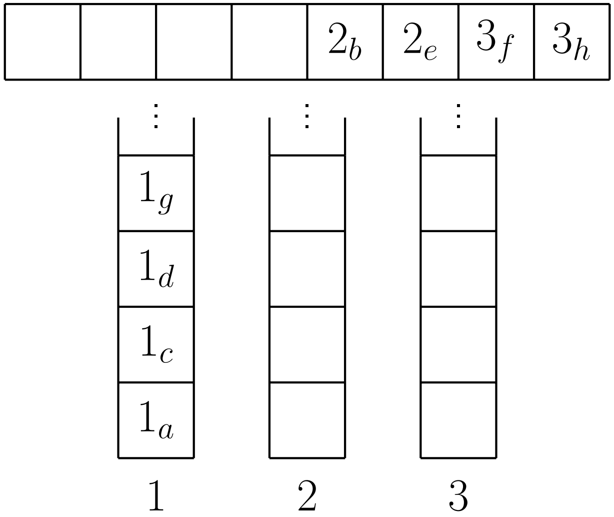
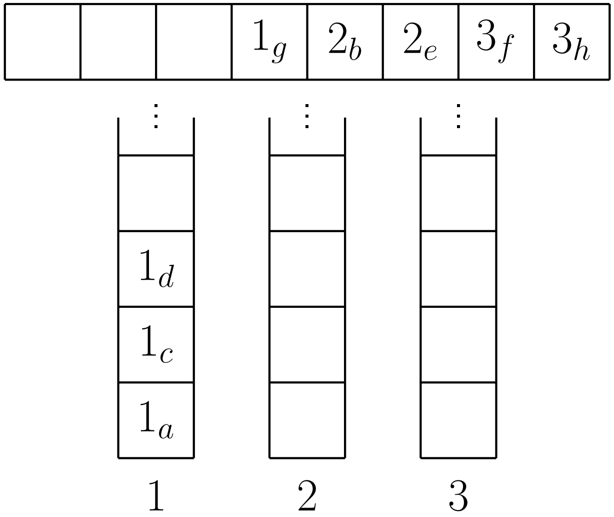


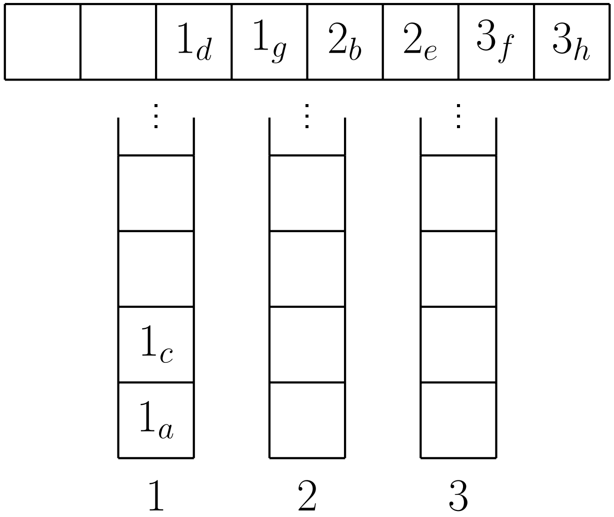
Bucketsort



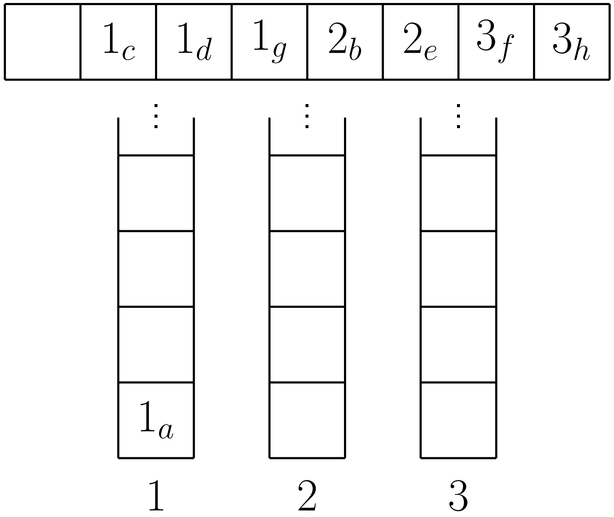
Bucketsort



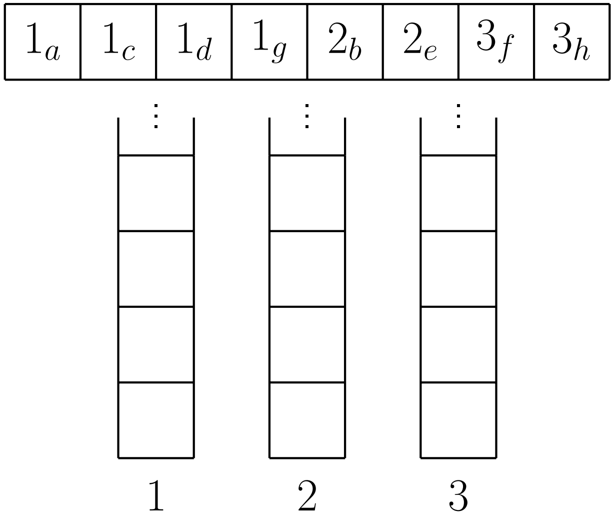
Bucketsort



Bucketsort



Bucketsort



Exercise – Bucketsort

Implement Bucketsort

- as Python function
- using three **stacks** `one`, `two`, and `three` for the possible values 1, 2, and 3
- filling the stacks according to numbers in the list
- concatenating the stacks at the end (this is quite simple in Python using the + operator)

Bucketsort

```python
def bucketsort(data):
    one = []
    two = []
    three = []
    for item in data:
        if item == 1:
            one.append(item)
        else:
            if item == 2:
                two.append(item)
            else:
                if item == 3:
                    three.append(item)
    return one + two + three
```

Bucketsort

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Sorting 2

Time Complexity of Bucketsort

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- Let k denote the number of distinct values

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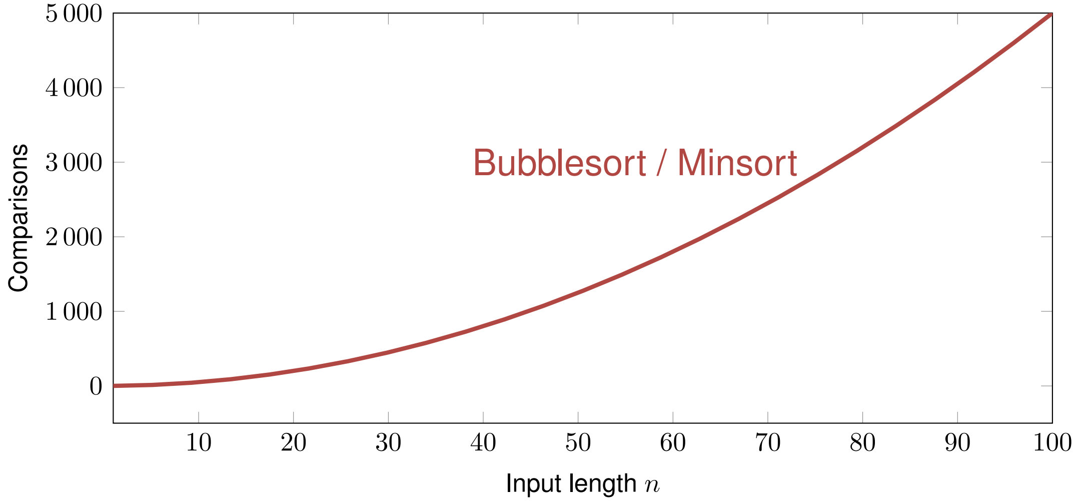
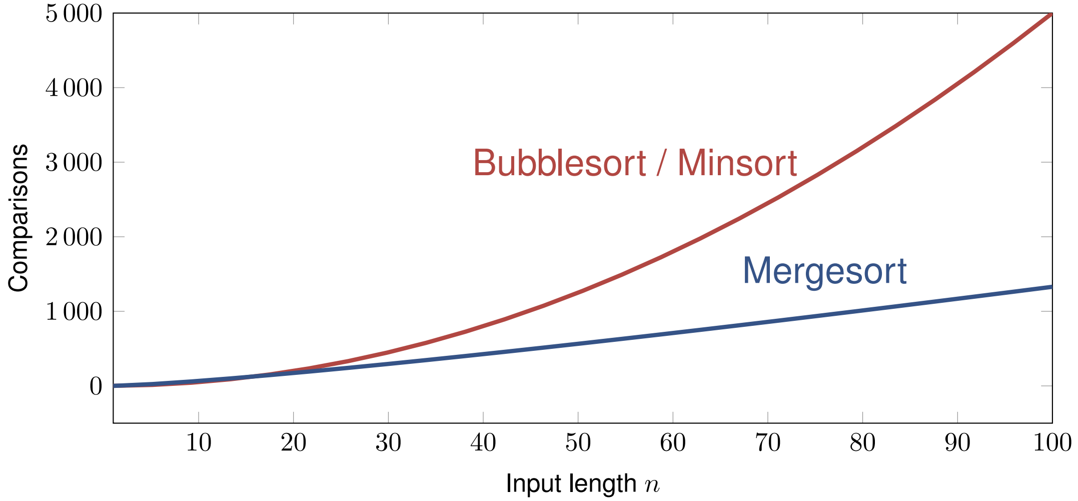
The time complexity of Bucketsort is in $\mathcal{O}(n)$ if there is a constant number of different values

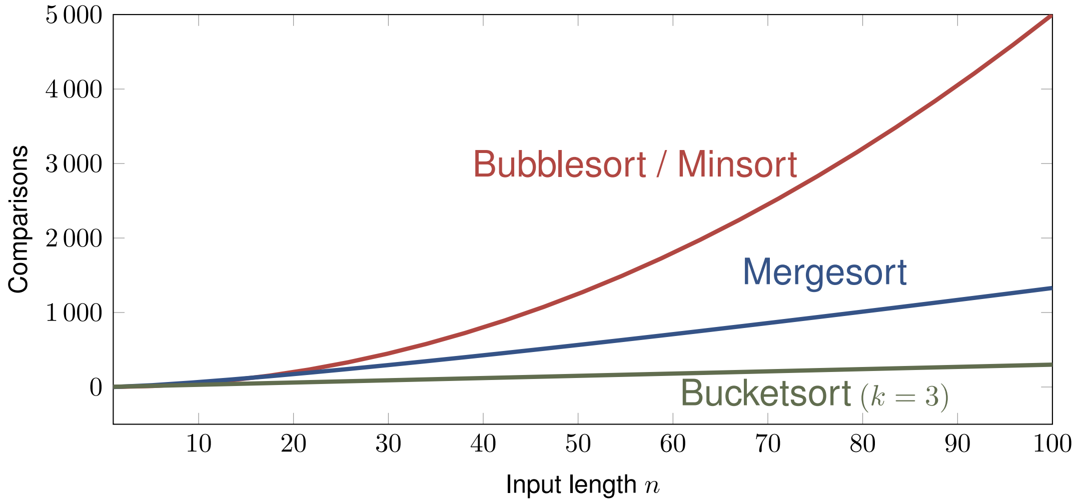
Time Complexity of Bucketsort



Time Complexity of Bucketsort



Time Complexity of Bucketsort



Thanks for your attention