15. Pointers, Algorithms, Iterators and Containers II

Iterations with Pointers, Arrays: Indices vs. Pointers, Arrays and Functions, Pointers and const, Algorithms, Container and Iteration, Vector-Iteration, Typdef, Sets, the Concept of Iterators

Recall: Pointers running over the Array

Beispiel

```
int a[5] = {3, 4, 6, 1, 2};
for (int∗ p = a; p < a+5; ++p)
   std::cout << ∗p << ' '; // 3 4 6 1 2
```

- An array can be converted into a pointer to its first element.
- Pointers “know” arithmetics and comparisons.
- Pointers can be dereferenced.

⇒ Pointers can be used to operate on arrays.

Array Arguments: *Call by (const) reference*

```
void print_vector (const int (&v)[3]) {
  for (int i = 0; i<3 ; ++i) {
    std::cout << v[i] << " ";
  }
}
void make_null_vector (int (&v)[3]) {
  for (int i = 0; i<3 ; ++i) {
    v[i] = 0;
  }
}
```


Array Arguments: *Call by value (not really ...)*

```
void make_null_vector (int v[3]) {
  for (int i = 0; i<3 ; ++i) {
    v[i] = 0;
  }
}
...
int a[10];
make_null_vector (a); // only sets a[0], a[1], a[2]

int∗ b;
make_null_vector (b); // no array at b, crash!
```

Array Arguments: *Call by value* does not exist

- Formal argument types *T* [n] or *T* [] (array over T) are equivalent to *T* ∗ (pointer to T)
- For passing an array the pointer to its first element is passed
- length information is lost
- Function cannot work on a part of an array (example: search for an element in the second half of an array)

Arrays in Functions

Covention of the standard library: pass an array (or a part of it) using two pointers

- **begin**: pointer to the first element
- **end**: pointer *behind* the last element
- **[begin, end)** designates the elements of the part of the array
- *valid* range means: there are array elements "available" here.
- **[begin, end)** is empty if **begin == end**

Arrays in (mutating) Functions: `fill`

```
// PRE: [begin, end) is a valid range
// POST: every element within [begin, end) will be set to value
void fill (int* begin, int* end, int value) {
  for (int* p = begin; p != end; ++p)
      *p = value;
}
  ...

int a[5];
fill (a, a+5, 1);
for (int i=0; i<5; ++i)
  std::cout << a[i] << " ";
```

expects pointers to the first element of a range

pass the address (of the first element) of a

Pointers are not Integers!

- Addresses can be interpreted as house numbers of the memory, that is, integers
- But integer and pointer arithmetics behave differently.

`ptr + 1` is *not* the next house number but the s-next, where s is the memory requirement of an object of the type behind the pointer `ptr`.

- Integers and pointers are not compatible

```
int* ptr = 5; // error: invalid conversion from int to int*
int a = ptr;  // error: invalid conversion from int* to int
```

Null-Pointer

- special pointer value that signals that no object is pointed to
- represented b the literal **nullptr** (convertible to **T***)

```
int* iptr = nullptr;
```

- cannot be dereferenced (checked during runtime)
- to avoid undefined behavior

```
int* iptr; // iptr points into ''nirvana''
int j = *iptr; // illegal address in *
```


Pointer Subtraction

- If *p1* and *p2* point to elements of the same array **a** with length n
- and $0 \leq k_1, k_2 \leq n$ are the indices corresponding to *p1* and *p2*, then

p1 - *p2* has value k_1 - k_2

↑

Only valid if *p1* and p2 point into the same array.

- The pointer difference describes “how far away the elements are from each other”

Pointer Operators

Description	Op	Arity	Precedence	Associativity	Assignment
Subscript	[]	2	17	left	R-value → L-value
Dereference	*	1	16	right	R-Wert → L-Wert
Address	&	1	16	rechts	L-value → R-value

Precedences and associativities of +, −, ++ (etc.) like in chapter 2

Functions with/without Effects

- Pointers can (like references) be used for functions with effect. Example: **fill**
- But many functions don’t have an effect, they only read the data
- ⇒ Use of **const**

So far, for example:

```
int i = 0;
const int& j = i;

const int zero = 0;
const int& nil = zero;
```

Positioning of Const

Where does the `const`-modifier belong to?
const T is equivalent to *T const* and can be written like this

```
const int zero = ...   ⟺   int const zero = ...
const int& nil = ...   ⟺   int const& nil = ...
```


Const and Pointers

Read the declaration from right to left

`int const a;`	a is a constant integer
`int const* a;`	a is a pointer to a constant integer
`int* const a;`	a is a constant pointer to an integer
`int const* const a;`	a is a constant pointer to a constant integer

Non-mutating Functions: `min`

- There are also *non*-mutating functions that access elements of an array only in a read-only fashion

```
// PRE: [begin, end) is a valid and nonempty range
// POST: the smallest value in [begin, end) is returned
int min (const int* begin, const int* end)
{
    assert (begin != end);
    int m = *begin; // current minimum candidate
    for (const int* p = ++begin; p != end; ++p)
        if (*p < m) m = *p;
    return m;
}
```

- mark with `const`: value of objects cannot be modified through such `const`-pointers.

`const` is not absolute

- The value at an address can change even if a `const`-pointer stores this address.

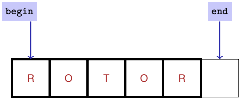
beispiel

```
int a[5];
const int* begin1 = a;
int*       begin2 = a;
*begin1 = 1;    // error *begin1 is constt
*begin2 = 1;    // ok, although *begin will be modified
```

- `const` is a promise from the point of view of the `const`-pointer, not an absolute guarantee

Wow – Palindromes!

```
// PRE: [begin end) is a valid range of characters
// POST: returns true if the range forms a palindrome
bool is_palindrome (const char* begin, const char* end) {
  while (begin < end)
    if (*(begin++) != *(--end)) return false;
  return true;
}
```



Algorithms

For many problems there are prebuilt solutions in the standard library

Example: filling an array

```
#include <algorithm> // needed for std::fill
...

int a[5];
std::fill (a, a+5, 1);

for (int i=0; i<5; ++i)
    std::cout << a[i] << " "; // 1 1 1 1 1
```


Algorithms

Advantages of using the standard library

- simple programs
- less sources of errors
- good, efficient code
- code independent from the data type
- there are also algorithms for more complicated problems such as the efficient sorting of an array

Algorithms

The same prebuilt algorithms work for many different data types.

Example: filling an array

```
#include <algorithm> // needed for std::fill
...

char c[3];
std::fill (c, c+3, '!');

for (int i=0; i<3; ++i)
    std::cout << c[i]; // !!!
```

Excursion: Templates

- Templates permit the provision of a type as argument
- The compiler finds the matching type from the call arguments

Example `fill` with templates

```
template <typename T>
void fill (T* begin, T* end, T value) {
    for (T* p = begin; p != end; ++p)
        *p = value;
}
int a[5];
fill (a, a+5, 1);   // 1 1 1 1 1

char c[3];
fill (c, c+3, '!'); // !!!
```

The triangular brackets we already know from vectors. Vectors are also implemented as templates.

`std::fill` is also implemented as template!

Containers and Traversal

- **Container:** Container (Array, Vector, . . .) for elements
- **Traversal:** Going over all elements of a container
  - Initialization of all elements (`fill`)
  - Find the smallest element (`min`)
  - Check properties (`is_palindrome`)
  - · · ·
- There are a lot of different containers (sets, lists, . . .)

Iteration Tools

- Arrays: indices (random access) or pointers (sequential)
- Array algorithms (`std::`) use pointers

```
int a[5];
std::fill (a, a+5, 1);   // 1 1 1 1 1
```

- How do you traverse vectors and other containers?

```
std::vector<int> v (5, 0); // 0 0 0 0 0
std::fill (?, ?, 1);   // 1 1 1 1 1
```


Vectors: *too sexy for pointers*

- Our `fill` with templates does not work for vectors. . .
- . . . and `std::fill` also does not work in the following way:

```
std::vector<int> v (5, 0);
std::fill (v, v+5, 1);   // Compiler error message !
```

Vectors are snobby. . .

- they refuse to be converted to pointers,. . .
- . . . and cannot be traversed using pointers either.
- They consider this far too primitive.☺

Also in memory: Vector ≠ Array

```
bool a[8] = {true, true, true, true, true, true, true, true};
```

true	*true*	*true*	*true*	*true*	*true*	*true*	*true*

8 Byte (Speicherzelle = 1 Byte = 8 Bit)

```
std::vector<bool> v (8, true);
```

0b11111111

1 Byte

`bool*`-pointer does not fit here because it runs byte-wise and not bit-wise

Vector-Iterators

Iterator: a "pointer" that fits to the container.

Example: fill a vector using `std::fill` – this works

```
#include <vector>
#include <algorithm> // needed for std::fill

...
std::vector<int> v(5, 0);
std::fill (v.begin(), v.end(), 1);
for (int i=0; i<5; ++i)
   std::cout << v[i] << " "; // 1 1 1 1 1
```


Vector Iterators

For each vector there are two *iterator types* defined

- `std::vector<int>::const_iterator`
  - for non-mutating access
  - in analogy with `const int*` for arrays
- `std::vector<int>::iterator`
  - for mutating access
  - in analogy with `int*` for arrays
- A vector-iterator `it` is no pointer, but it behaves like a pointer:
  - it points to a vector element and can be dereferenced (`*it`)
  - it knows arithmetics and comparisons (`++it`, `it+2`, `it < end`,...)

Vector-Iterators: `begin()` and `end()`

- `v.begin()` points to the first element of `v`
- `v.end()` points past the last element of `v`
- We can traverse a vector using the iterator...

```
for (std::vector<int>::const_iterator it = v.begin();
                                       it != v.end(); ++it)
   std::cout << *it << " ";
```

- ...or fill a vector.

```
std::fill (v.begin(), v.end(), 1);
```

Type names in C++ can become looooooong

- `std::vector<int>::const_iterator`
- The declaration of a *type alias* helps with

`using` *Name* = *Typ*;

Name that can now be used to access the type

existing type

Examples

```
using int_vec = std::vector<int>;
using Cvit = int_vec::const_iterator;
```

Syntax prior to C++ 11: `typedef` *Typ Name*;

Vector Iterators work like Pointers

```
using Cvit = std::vector<int>::const_iterator;

std::vector<int> v(5, 0); // 0 0 0 0 0

// output all elements of a, using iteration
for (Cvit it = v.begin(); it != v.end(); ++it)
    std::cout << *it << " ";
```

Vector element pointed to by `it`

Vector Iterators work like Pointers

```
using Vit = std::vector<int>::iterator;

// manually set all elements to 1
for (Vit it = v.begin(); it != v.end(); ++it)
    *it = 1;

// output all elements again, using random access
for (int i=0; i<5; ++i)
    std::cout << v[i] << " ";
```

increment the iterator

short term for `*(v.begin()+i)`

Other Containers: Sets

- A set is an unordered collection of elements, where each element is contained only once.

$$\{1,2,1\} = \{1,2\} = \{2,1\}$$

- C++: `std::set<T>` for a set with elements of type `T`

Sets: Example Application

- Determine if a given text contains a question mark and output all *pairwise different* characters!

Letter Salad (1)

Consider a text as a set of characters.

```
#include<set>
...
using Csit = std::set<char>::const_iterator;
...
std::string text =
"What are the distinct characters in this string?";

std::set<char> s (text.begin(),text.end());
```

Set is initialized with String iterator range `[text.begin(), text.end())`

Letter Salad (2)

Determine if the text contains a question mark and output all characters

Search algorithm, can be called with arbitrary iterator range

```
// check whether text contains a question mark
if (std::find (s.begin(), s.end(), '?') != s.end())
    std::cout << "Good question!\n";

// output all distinct characters
for (Csit it = s.begin(); it != s.end(); ++it)
    std::cout << *it;
```

Ausgabe:

```
Good question!
?Wacdeghinrst
```


Sets and Indices?

- Can you traverse a set using random access? No.

```
for (int i=0; i<s.size(); ++i)
    std::cout << s[i];
```

error message: no subscript operator

- Sets are unordered.
  - There is no "ith element".
  - Iterator comparison `it != s.end()` works, but not `it < s.end()`!

The Concept of Iterators

C++knows different iterator types

- Each container provides an associated iterator type.
- All iterators can dereference (`*it`) and traverse (`++it`)
- Some can do more, e.g. random access (`it[k]`, or, equivalently `*(it + k)`), traverse backwards (`--it`),...

The Concept of Iterators

Every container algorithm is generic, that means:

- The container is passed as an iterator-range
- The algorithm works for all containers that fulfil the requirements of the algorihm
- `std::find` only requires `*` and `++` , for instance
- The implementation details of a container are irrelevant.

Why Pointers and Iterators?

Would you not prefer the code

```
for (int i=0; i<n; ++i)
    a[i] = 0;
```

over the following code?

```
for (int* ptr=a; ptr<a+n; ++ptr)
    *ptr = 0;
```

Maybe, but in order to use the generic `std::fill(a, a+n, 0);`, we *have to* work with pointers.

Why Pointers and Iterators?

In order to use the standard library, we have to know that:

- a static array `a` is a the same time a pointer to the first element of `a`
- `a+i` is a pointer to the element with index i

Using the standard library with different containers: Pointers ⇒ Iterators

Why Pointers and Iterators?

Example: To search the smallest element of a container in the range $[\text{begin}, \text{end})$ use the function call

```
std::min_element(begin, end)
```

- returns an *iterator* to the smallest element
- To read the smallest element, we need to dereference:

```
*std::min_element(begin, end)
```


That is Why: Pointers and Iterators

- Even for non-programmers and "dumb" users of the standard library: expressions of the form
  `*std::min_element(begin, end)`
  cannot be understood without knowing pointers and iterators.
- Behind the scenes of the standard library: working with dynamic memory based on pointers is indispensible. More about this later in this course.

16. Recursion 1

Mathematical Recursion, Termination, Call Stack, Examples, Recursion vs. Iteration

Mathematical Recursion

- Many mathematical functions can be naturally defined recursively.
- This means, the function appears in its own definition

$$n! = \begin{cases} 1, & \text{if } n \leq 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

Recursion in C++: In the same Way!

$$n! = \begin{cases} 1, & \text{if } n \le 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

```
// POST: return value is n!
unsigned int fac (unsigned int n)
{
  if (n <= 1)
    return 1;
  else
    return n * fac (n-1);
}
```


Infinite Recursion

- is as bad as an infinite loop. . .
- . . . but even worse: it burns time and memory

```
void f()
{
  f(); // f() -> f() -> ... stack overflow
}
```


Recursive Functions: Termination

As with loops we need

- progress towards termination

`fac(n)`:
terminates immediately for $n \le 1$, otherwise the function is called recusively with < n .

„n is getting smaller for each call.”

Recursive Functions: Evaluation

Example: `fac(4)`

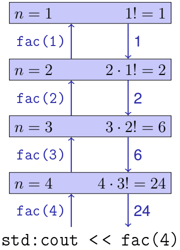
```
// POST: return value is n!
unsigned int fac (unsigned int n)
{
  if (n <= 1) return 1;
  return n * fac(n-1); // n > 1
}
```

Initialization of the formal argument: $n = 4$
recursive call with argument $n - 1 == 3$

The Call Stack

For each function call:

- push value of the call argument onto the stack
- always work with the top value
- at the end of the call the top value is removed from the stack



Euclidean Algorithm

- finds the greatest common divisor $\mathrm{gcd}(a, b)$ of two natural numbers a and b
- is based on the following mathematical recursion (proof in the lecture notes):

$$\mathrm{gcd}(a, b) = \begin{cases} a, & \text{if } b = 0 \\ \mathrm{gcd}(b, a \bmod b), & \text{otherwise} \end{cases}$$

Euclidean Algorithm in C++

$$\mathrm{gcd}(a, b) = \begin{cases} a, & \text{if } b = 0 \\ \mathrm{gcd}(b, a \bmod b), & \text{otherwise} \end{cases}$$

```
unsigned int gcd
 (unsigned int a, unsigned int b)
{
  if (b == 0)
    return a;
  else
    return gcd (b, a % b);
}
```

Termination: $a \bmod b < b$, thus b gets smaller in each recursive call.

Fibonacci Numbers

$$F_n := \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{if } n > 1 \end{cases}$$

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots$

Fibonacci Numbers in C++

Laufzeit

`fib(50)` takes “forever” because it computes
F_{48} two times, F_{47} 3 times, F_{46} 5 times, F_{45} 8 times, F_{44} 13 times, F_{43} 21 times ... F_1 ca. 10^9 times (!)

```
unsigned int fib (unsigned int n)
{
  if (n == 0) return 0;
  if (n == 1) return 1;
  return fib (n-1) + fib (n-2); // n > 1
}
```

Correctness and termination are clear.

Fast Fibonacci Numbers

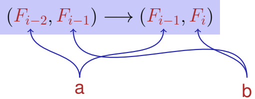
Idea:

- Compute each Fibonacci number only once, in the order $F_0, F_1, F_2, \ldots, F_n$!
- Memorize the most recent two numbers (variables `a` and `b`)!
- Compute the next number as a sum of `a` and `b`!

Fast Fibonacci Numbers in C++

```
unsigned int fib (unsigned int n){
  if (n == 0) return 0;
  if (n <= 2) return 1;
  unsigned int a = 1;  // F_1
  unsigned int b = 1;  // F_2
  for (unsigned int i = 3; i <= n; ++i){
    unsigned int a_old = a;  // F_i-2
    a = b;                   // F_i-1
    b += a_old;              // F_i-1 += F_i-2 -> F_i
  }
  return b;
}
```

very fast, also for `fib(50)`



Recursion and Iteration

Recursion can *always* be simulated by

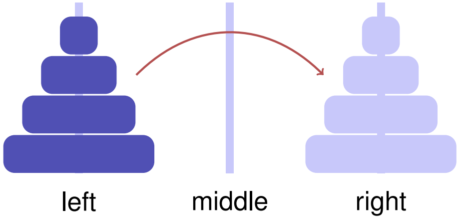
- Iteration (loops)
- explicit “call stack” (e.g. array)

Often recursive formulations are simpler, but sometimes also less efficient.

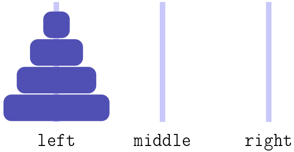
The Power of Recursion

- Some problems appear to be hard to solve without recursion. With recursion they become significantly simpler.
- Examples: *The towers of Hanoi*, The n-Queens-Problem, *Sudoku-Solver*, Expression Parsers, Reversing In- or Output, Searching in Trees, Divide-And-Conquer (e.g. sorting) $\rightarrow$ Engineering Tool III-IV

Experiment: The Towers of Hanoi



The Towers of Hanoi – Code



Move 4 discs from `left` to `right` with auxiliary staple `middle`:

```
move(4,"left","middle","right")
```


The Towers of Hanoi – Code

move$(n,\ src,\ aux,\ dst)\quad\Rightarrow$

1. Move the top $n-1$ discs from src to aux with auxiliary staple dst:
   move$(n-1,\ src,\ dst,\ aux)$;
2. Move 1 disc from src to dst
   move$(1,\ src,\ aux,\ dst)$;
3. Move the top $n-1$ discs from aux to dst with auxiliary staple src:
   move$(n-1,\ aux,\ src,\ dst)$;

The Towers of Hanoi – Code

```
void move(int n, const string &src, const string &aux, const string &dst){
    if (n == 1) {
        // base case ('move' the disc)
        std::cout << src << " --> " << dst << std::endl;
    } else {
        // recursive case
        move(n-1, src, dst, aux);
        move(1, src, aux, dst);
        move(n-1, aux, src, dst);
    }
}
int main() {
    move(4, "left", "middle", "right");
    return 0;
}
```


The Towers of Hanoi – Code Alternative

```
void move(int n, const string &src, const string &aux, const string &dst){
    // base case
    if (n == 0) return;

    // recursive case
    move(n-1, src, dst, aux);
    std::cout << src << " --> " << dst << "\n";
    move(n-1, aux, src, dst);
}


int main() {
    move(4, "left", "middle", "right");
    return 0;
}
```