17. Recursion 2

Building a Calculator, Formal Grammars, Extended Backus Naur Form (EBNF), Parsing Expressions

Motivation: Calculator

Input: `3 + 5`
Output: `8`

- binary Operators `+`, `-`, `*`, `/` and numbers

Motivation: Calculator

Input: `3 / 5`
Output: `0.6`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic

Motivation: Calculator

Input: `3 + 5 * 20`
Output: `103`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++

Motivation: Calculator

Input: `(3 + 5) * 20`
Output: `160`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses

Motivation: Calculator

Input: `-(3 + 5) + 20`
Output: `12`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses
- unary operator `-`

Naive Attempt (without Parentheses)

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != ’=’) {
    double rval;
    std::cin >> rval;

    if (op == ’+’)
        lval += rval;
    else if (op == ’*’)
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

Seems to work...

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != ’=’) {
    double rval;
    std::cin >> rval;

    if (op == ’+’)
        lval += rval;
    else if (op == ’*’)
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 1 * 2 * 3 * 4 =
Result 24
```

Oops, Multiplication first...

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != '=') {
    double rval;
    std::cin >> rval;

    if (op == '+')
        lval += rval;
    else if (op == '*')
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 2 + 3 * 3 =
Result 15
```

Analyzing the Problem

Input:

$$13 + \ldots$$

Analyzing the Problem

Input:

$$13 + 4 * \ldots$$

Analyzing the Problem

Input:

$$13 + 4 * (15 - \ldots$$

Analyzing the Problem

Input:

$$13 + 4 * (15 - 7 * \ldots$$

Analyzing the Problem

Input:

$$\underbrace{13 + 4 * (15 - 7 *}_{\text{Needs to be stored such that evaluation can be performed}} 3) =$$

Analyzing the Problem

Result:

$$13 + 4 * (15 - 21)$$

Analyzing the Problem

Result:

$$13+4*(-6)$$

Analyzing the Problem

Result:

$$13 + (-24)$$

Analyzing the Problem

Result:

$$-11$$

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture

Analyzing the Problem

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$$13 + 4 * (15 - 7 * 3)$$

This lecture is

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture is pretty

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture is pretty much

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture is pretty much recursive.

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

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We will store symbols elegantly using recursion.

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

We will store symbols elegantly using recursion.

We need a new formal tool (that is independent of C++).

Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

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A formal grammar defines which strings are valid.

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- Strings: finite sequences of symbols

A formal grammar defines which strings are valid.

To describe the formal grammar, we use:

Extended Backus Naur Form (EBNF)

Short Communications
Programming Languages

What Can We Do about the Unnecessary Diversity of Notation for Syntactic Definitions?

Niklaus Wirth
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Key Words and Phrases: syntactic description language, extended BNF
CR Categories: 4.20

The population of programming languages is steadily growing, and there is no end of this growth in sight. Many language definitions appear in journals, many are found in technical reports, and perhaps an even greater number remains confined to proprietory circles. After frequent exposure to these definitions, one cannot fail to notice the lack of "common denominators." The only widely accepted fact is that the language structure is defined by a syntax. But even notation for syntactic description eludes any commonly agreed standard form, although the underlying ancestor is invariably the Backus-Naur Form of the Algol 60 report. As variations are often only slight, they become annoying for their very lack of an apparent motivation.

Out of sympathy with the troubled reader who is weary of adapting to a new variant of BNF each time another language definition appears, and without any claim for originality, I venture to submit a simple notation that has proven valuable and satisfactory in use. It has the following properties to recommend it:

1. The notation distinguishes clearly between meta-, terminal, and nonterminal symbols.
2. It does not exclude characters used as metasymbols from use as symbols of the language (as e.g. "|" in BNF).
3. It contains an explicit iteration construct, and thereby avoids the heavy use of recursion for expressing simple repetition.
4. It avoids the use of an explicit symbol for the empty string (such as ⟨empty⟩ or ε).
5. It is based on the ASCII character set.

This meta language can therefore conveniently be used to define its own syntax, which may serve here as an example of its use. The word *identifier* is used to denote *nonterminal symbol*, and *literal* stands for *terminal symbol*. For brevity, *identifier* and *character* are not defined in further detail.

```
syntax      =  {production}.
production  =  identifier "=" expression ".".
expression  =  term {"|" term}.
term        =  factor {factor}.
factor      =  identifier | literal | "(" expression ")" |
               "[" expression "]" | "{" expression "}".
literal     =  """" character {character} """".
```

Repetition is denoted by curly brackets, i.e. {a} stands for ε | a | aa | aaa | . . . . Optionality is expressed by square brackets, i.e. [a] stands for a | ε. Parentheses merely serve for grouping, e.g. (a|b)c stands for ac | bc. Terminal symbols, i.e. literals, are enclosed in quote marks (and, if a quote mark appears as a literal itself, it is written twice), which is consistent with common practice in programming languages.

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Number

An integer is a sequence of digits. A sequence of digits ist

Number

An integer is a sequence of digits. A sequence of digits ist

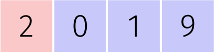
- a digit 2

Number

An integer is a sequence of digits. A sequence of digits ist

- a digit or

- a digit followed by a sequence of digits



Number

An integer is a sequence of digits. A sequence of digits ist

- a digit or 2
- a digit followed by a sequence of digits 2 0 1 9

```
unsigned_integer = digits .
digit    = '0'|'1'|'2'|'3'|'4'|'5'|'6'|'7'|'8'|'9'.
digits   = digit  |  digit  digits.
```

Number

An integer is a sequence of digits. A sequence of digits ist

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2	0	1	9

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unsigned_integer = digits .
digit   = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' .
digits  = digit  |  digit  digits .
```

alternative

terminal symbol

non-terminal symbol

Number (non-recursive)

An integer is a sequence of digits. A sequence of digits ist

- a digit, or 2
- a digit followed by an **arbitrary number of digits** 2 0 1 9

```
unsigned_integer = digits .
digit   = '0'|'1'|'2'|'3'|'4'|'5'|'6'|'7'|'8'|'9'.
digits  = digit { digit }.
```

Number (non-recursive)

An integer is a sequence of digits. A sequence of digits ist

- a digit, or 2
- a digit followed by an **arbitrary number of digits** 2 0 1 9

```
unsigned_integer = digits .
digit    = ’0’|’1’|’2’|’3’|’4’|’5’|’6’|’7’|’8’|’9’.
digits   = digit { digit }.
```

optional repetition

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)

Expressions

$$\underline{-}(3-(4-5))*(3+4*5)/6$$

What do we need in a grammar?

- Number , (?)
  -Number, -(?)

Expressions

$$-(3-(4-5))\underline{*}(3+4\underline{*}5)\underline{/}6$$

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- ? * ?,   ? / ?,   ...

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , `( ? )`
  `-`Number, `-( ? )`
- ? `*` ?, ? `/` ?, …
- ? `-` ?, ? `+` ?, …

Expressions Multiplication/Division

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- ? * ?, ? / ?, ...
- ? - ?, ? + ?, ...

Factor

Multiplication/Division

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor,
  Factor / Factor , ...
- ? - ?, ? + ?, ...

Factor

Addition/Subtraction

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor,
  Factor / Factor , ...
- ? - ?, ? + ?, ...

Factor

Term

Expressions Addition/Subtraction

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- ? - ?, ? + ?, ...

Factor

Term

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term,
  Term - Term, ...

Factor

Term

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term,
  Term - Term, ...

Factor

Term

Expression

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term - Term, ...

Factor

Term

Expression

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (Expression)
  -Number, -(Expression)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term - Term, ...

Factor

Term

Expression

The EBNF for Expressions

A factor is

- a number,

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses

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A factor is

- a number,
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The EBNF for Expressions

A factor is

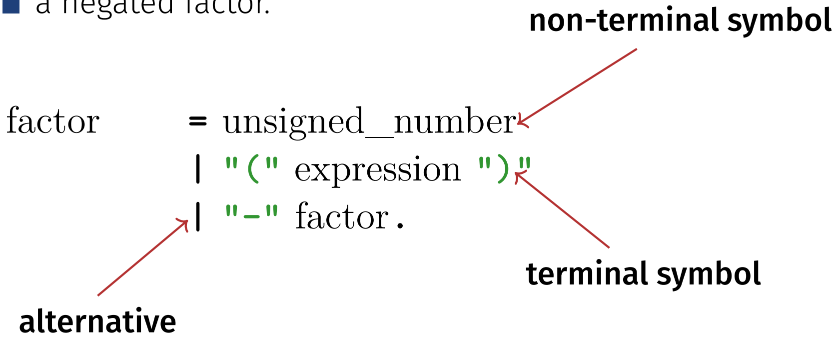
- a number,
- an expression in parentheses or
- a negated factor.

```
factor    = unsigned_number
          | "(" expression ")"
          | "-" factor .
```

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses or
- a negated factor.



```
factor    = unsigned_number
          | "(" expression ")"
          | "-" factor .
```

non-terminal symbol

terminal symbol

alternative

The EBNF for Expressions

```
factor    = unsigned_number
          | "(" expression ")"
          | "-" factor .
```

Implication: a factor starts with

- a digit, or
- with “(” , or
- with "-"”.

The EBNF for Expressions

A term is

- factor,

The EBNF for Expressions

A term is

- factor,
- factor **∗** factor, factor **/** factor,

The EBNF for Expressions

A term is

- factor,
- factor * factor, factor / factor,
- factor * factor * factor, factor / factor * factor, ...
- ...

The EBNF for Expressions

A term is

- factor,
- factor ∗ factor, factor / factor,
- factor ∗ factor ∗ factor, factor / factor ∗ factor, ...
- ...

term `=` factor `{` `"*"` factor `|` `"/"` factor `}.`

The EBNF for Expressions

A term is

- factor,
- factor ∗ factor, factor / factor,
- factor ∗ factor ∗ factor, factor / factor ∗ factor, ...
- ...

```
term = factor { "*" factor | "/" factor }.
```

optional repetition

The EBNF for Expressions

```
factor     = unsigned_number
             | "(" expression ")"
             | "-" factor .

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term | "-" term }.
```

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.

Parsing

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- **Parser:** A program for parsing.

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.
- **Parser:** A program for parsing.
- **Useful:** From the EBNF we can (nearly) automatically generate a parser:
    - Rules become functions
    - Alternatives and options become **`if`**–statements.
    - Nonterminial symbols on the right hand side become function calls
    - Optional repetitions become **`while`**–statements

Rules

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.


expression = term { "+" term | "-" term }.
```

Functions (Parser)

Expression is read from an input stream.

```
// POST: returns true if and only if in_stream = factor ...
//       and in this case extracts factor from in_stream
bool factor (std::istream& in_stream);

// POST: returns true if and only if in_stream = term ...,
//       and in this case extracts all factors from in_stream
bool term (std::istream& in_stream);

// POST: returns true if and only if in_stream = expression ...,
//       and in this case extracts all terms from in_stream
bool expression (std::istream& in_stream);
```

Functions (Parser with Evaluation)

Expression is read from an input stream.

```
// POST: extracts a factor from in_stream
//       and returns its value
double factor (std::istream& in_stream);

// POST: extracts a term from in_stream
//       and returns its value
double term (std::istream& in_stream);

// POST: extracts an expression from in_stream
//       and returns its value
double expression (std::istream& in_stream);
```

One Character Lookahead…

…to find the right alternative.

```
// POST: the next character at the stream is returned
//       without being consumed. returns 0 if stream ends.
char peek (std::istream& input){
  if (input.eof()) return 0; // end of stream
  return input.peek();  // next character in input
}

// POST: leading whitespace characters are extracted from input
//       and the first non-whitespace character on input returned
char lookahead (std::istream& input) {
  input >> std::ws;        // skip whitespaces
  return peek(input);
}
```

Parse numbers

```
bool isDigit(char ch){
  return ch >= '0' && ch <= '9';
}
// POST: returns an unsigned integer consumed from the stream
// number = digit {digit}.
unsigned int unsigned_number (std::istream& input){
  char ch = lookahead(input);
  assert(isDigit(ch));
  unsigned int num = 0;
  while(isDigit(ch) && input >> ch){ // read remaining digits
    num = num * 10 + ch - '0';
    ch = peek(input);
  }
  return num;
}
```

unsigned_number = digit { digit }.
digit = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9'.

Cherry-Picking

…to extract the desired character.

```
// POST: if expected matches the next lookahead then consume it
//       and return true; return false otherwise
bool consume (std::istream& in_stream, char expected)
{
    if (lookahead(in_stream) == expected){
        in_stream >> expected; // consume one character
        return true;
    }
    return false;
}
```

Evaluating Factors

```
double factor (std::istream& in_stream)
{
  double value;
  if (consume(in_stream, '(')) {
    value = expression (in_stream);
    consume(in_stream, ')');
  } else if (consume(in_stream, '-')) {
    value = -factor (in_stream);
  } else {
    value = unsigned_number(in_stream);
  }
  return value;
}
```

```
factor = "(" expression ")"
       | "-" factor
       | unsigned_number.
```

Evaluating Terms

```
double term (std::istream& in_stream)
{
  double value = factor (in_stream);
  while(true){
    if (consume(in_stream, ’*’))
      value *= factor(in_stream);
    else if (consume(in_stream, ’/’))
      value /= factor(in_stream)
    else
      return value;
  }
}
```

term = factor { "*" factor | "/" factor }.

Evaluating Expressions

```
double expression (std::istream& in_stream)
{
  double value = term(in_stream);
  while(true){
    if (consume(in_stream, '+'))
      value += term (in_stream);
    else if (consume(in_stream, '-'))
      value -= term(in_stream)
    else
      return value;
  }
}
```

expression = term { "+" term | "-" term }.

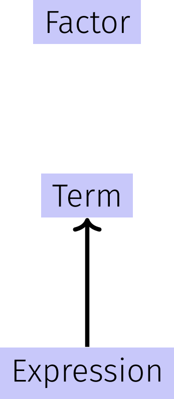
Recursion!

Factor

Term

Expression

Recursion!

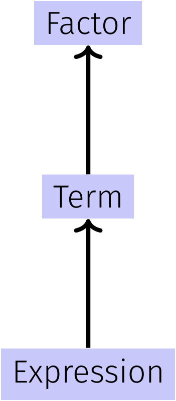


Factor

Term

Expression

Recursion!



Factor

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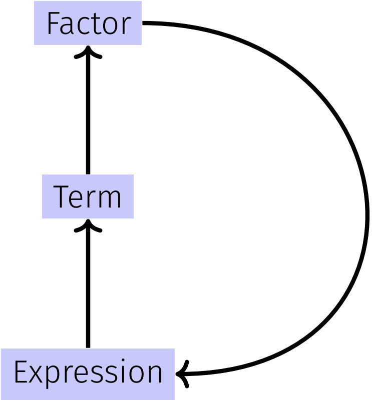
Expression

Recursion!



Factor

Term

Expression

EBNF — and it works!

EBNF (`calculator.cpp`, Evaluation from left to right):

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term | "-" term }.
```

```
std::stringstream input ("1-2-3");
std::cout << expression (input) << "\n"; // -4
```

18. Structs

Rational Numbers, Struct Definition

Calculating with Rational Numbers

- Rational numbers ($\mathbb{Q}$) are of the form $\frac{n}{d}$ with n and d in $\mathbb{Z}$
- $\mathrm{C}{+}{+}$does not provide a built-in type for rational numbers

Calculating with Rational Numbers

- Rational numbers ($\mathbb{Q}$) are of the form $\frac{n}{d}$ with n and d in $\mathbb{Z}$
- C++does not provide a built-in type for rational numbers

Goal

We build a C++-type for rational numbers ourselves! 🙂

Vision

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;
std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

A First Struct

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

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```

member variable (**n**umerator)

member variable (**d**enominator)

A First Struct

member variable

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struct rational {
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member variable

- `struct` defines a new **type**

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- `struct` defines a new **type**
- formal range of values: *cartesian product* of the value ranges of existing types

A First Struct

member variable

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

member variable

- `struct` defines a new **type**
- formal range of values: *cartesian product* of the value ranges of existing types
- real range of values: `rational` $\subsetneq$ `int` $\times$ `int`.

Accessing Member Variables

```
struct rational {
    int n;
    int d; // INV: d != 0
};

rational add (rational a, rational b){
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
```

$$\frac{r_n}{r_d} := \frac{a_n}{a_d} + \frac{b_n}{b_d} = \frac{a_n \cdot b_d + a_d \cdot b_n}{a_d \cdot b_d}$$

Input

```
// Input r
rational r;
std::cout << "Rational number r:\n";
std::cout << " numerator =? ";
std::cin >> r.n;
std::cout << " denominator =? ";
std::cin >> r.d;

// Input s the same way
rational s;
...
```

Vision comes within Reach ...

```
// computation
const rational t = add (r, s);

// output
std::cout << "Sum is " << t.n << "/" << t.d << ".\n";
```

Struct Defintions: Examples

```
struct rational_vector_3 {
  rational x;
  rational y;
  rational z;
};
```

underlying types can be fundamental or *user defined*

Struct Definitions: Examples

```
struct extended_int {
  // represents value if is_positive==true
  // and -value otherwise
  unsigned int value;
  bool is_positive;
};
```

the underlying types can be *different*

Structs: Initialization and Assignment

```
rational s;
```

← member variables are uninitialized

```
rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

← *member-wise* initialization:
`t.n = 1, t.d = 5`

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;
```

← member-wise copy

```
t = u;

rational v = add (u,t);
```

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;
```

`t = u;` ← member-wise copy

```
rational v = add (u,t);
```

Structs: Initialization and Assignment

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rational s;

rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

⟵ member-wise copy

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=`, not so for structs! Why?

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For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

- member-wise comparison does not make sense in general...

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

- member-wise comparison does not make sense in general...
- ...otherwise we had, for example, $\frac{2}{3} \neq \frac{4}{6}$

User Defined Operators

Instead of

```
rational t = add(r, s);
```

we would rather like to write

```
rational t = r + s;
```

User Defined Operators

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rational t = r + s;
```

This can be done with *Operator Overloading* (→ *next week*).