10. Functions II

Pre- and Postconditions Stepwise Refinement, Scope, Libraries and Standard Functions

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

0^e is undefined for $e < 0$

```
// PRE: e >= 0 || b != 0.0
```

Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Here only value, no effect.

```
// POST: return value is b^e
```

Pre- and Postconditions

- should be correct:

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Funktion **`pow`**: works for all numbers $b \neq 0$

White Lies...

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect

White Lies...

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect:

- Overflow if e or b are too large
- b^e potentially not representable as a double (holes in the value range!)

White Lies are Allowed

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

Mathematical conditions as a compromise between formal correctness and lax practice

Checking Preconditions...

- Preconditions are only comments.

Checking Preconditions...

- Preconditions are only comments.
- How can we ensure that they hold when the function is called?

...with assertions

```
#include <cassert>
...
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e) {
    assert (e >= 0 || b != 0);
    double result = 1.0;
    ...
}
```

Postconditions with Asserts

- The result of "complex" computations is often easy to check.

Postconditions with Asserts

- The result of “complex” computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

Postconditions with Asserts

- The result of “complex” computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

```
// PRE: the discriminant p*p/4 - q is nonnegative
// POST: returns larger root of the polynomial x^2 + p x + q
double root(double p, double q)
{
    assert(p*p/4 >= q); // precondition
    double x1 = - p/2 + sqrt(p*p/4 - q);
    assert(equals(x1*x1+p*x1+q,0)); // postcondition
    return x1;
}
```

Stepwise Refinement

A simple *technique* to solve complex problems

Education P. Wegner Editor

Program Development by Stepwise Refinement

Niklaus Wirth
Eidgenössische Technische Hochschule
Zürich, Switzerland

The creative activity of programming—to be distinguished from coding—is usually taught by examples serving to exhibit certain techniques. It is here considered as a sequence of design decisions concerning the decomposition of tasks into subtasks and of data into data structures. The process of successive refinement of specifications is illustrated by a short but nontrivial example, from which a number of conclusions are drawn regarding the art and the instruction of programming.

Key Words and Phrases: education in programming, programming techniques, stepwise program construction
CR Categories: 1.50, 4.0

1. Introduction

Programming is usually taught by examples. Experience shows that the success of a programming course critically depends on the choice of these examples. Unfortunately, they are too often selected with the prime intent to demonstrate what a computer can do. Instead, a main criterion for selection should be their suitability to exhibit certain widely applicable *techniques*. Furthermore, examples of programs are commonly presented as finished "products" followed by explanations of their purpose and their linguistic details. But active programming consists of the design of *new* programs, rather than contemplation of old programs. As a consequence of these teaching methods, the student obtains the impression that programming consists mainly of mastering a language (with all the peculiarities and intricacies so abundant in modern PL's) and relying on one's intuition to somehow transform ideas into finished programs. Clearly, programming courses should teach methods of design and construction, and the selected examples should be such that a gradual *development* can be nicely demonstrated.

This paper deals with a single example chosen with these two purposes in mind. Some well-known techniques are briefly demonstrated and motivated (strategy of preselection, stepwise construction of trial solutions, introduction of auxiliary data, recursion), and the program is gradually developed in a sequence of *refinement steps*.

In each step, one or several instructions of the given program are decomposed into more detailed instructions. This successive decomposition or refinement of specifications terminates when all instructions are expressed in terms of an underlying computer or programming language, and must therefore be guided by the facilities available on that computer or language. The result of the execution of a program is expressed in terms of data, and it may be necessary to introduce further data for communication between the obtained subtasks or instructions. As tasks are refined, so the data may have to be refined, decomposed, or structured, and it is natural to *refine program and data specifications in parallel.*

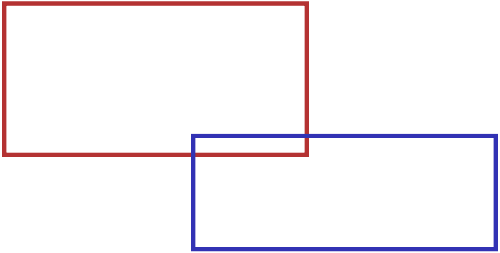
Every refinement step implies some design decisions. It is important that these decision be made explicit, and that the programmer be aware of the underlying criteria and of the existence of alternative solutions. The possible solutions to a given problem emerge as the leaves of a tree, each node representing a point of deliberation and decision. Subtrees may be considered as *families of solutions* with certain common characteristics and structures. The notion of such a tree may be particularly helpful in the situation of changing purpose and environment to which a program may sometime have to be adapted.

A guideline in the process of stepwise refinement should be the principle to decompose decisions as much as possible, to untangle aspects which are only seemingly interdependent, and to defer those decisions which concern details of representation as long as possible. This

Niklaus Wirth. Program development by stepwise refinement. Commun. ACM 14, 4, 1971

Example Problem

Find out if two rectangles intersect!



Top-Down Approach

- Formulate a coarse solution using
  - comments
  - ficticious functions

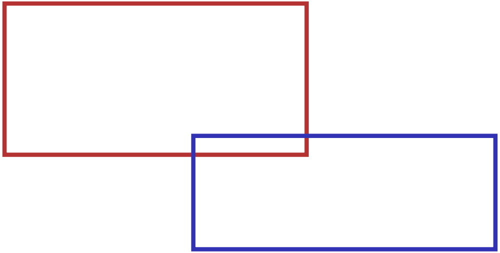
Top-Down Approach

- Formulate a coarse solution using
  - comments
  - ficticious functions
- Repeated refinement:
  - comments $\longrightarrow$ program text
  - ficticious functions $\longrightarrow$ function definitions

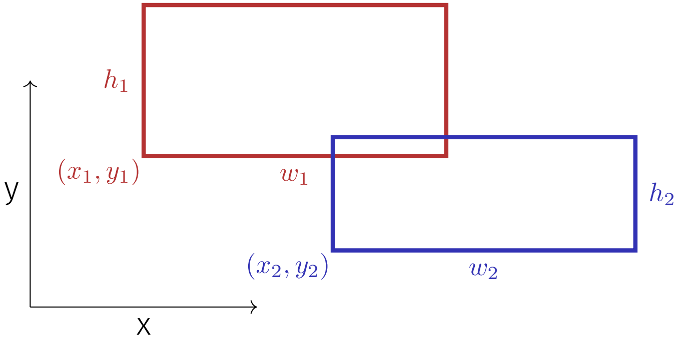
Coarse Solution

```
int main()
{
    // input rectangles

    // intersection?

    // output solution

    return 0;
}
```

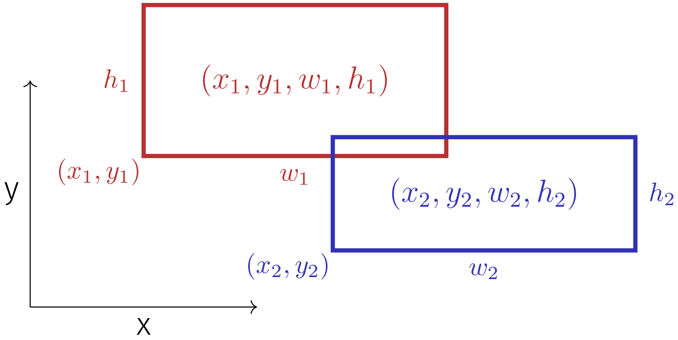
Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

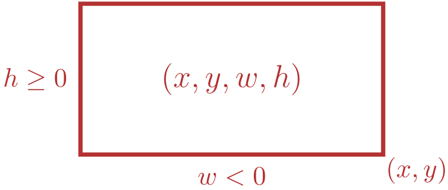


Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

Width w and height h may be negative.



Refinement 1: Input Rectangles

```
int main()
{
    std::cout << "Enter two rectangles [x y w h each] \n";
    int x1, y1, w1, h1;
    std::cin >> x1 >> y1 >> w1 >> h1;
    int x2, y2, w2, h2;
    std::cin >> x2 >> y2 >> w2 >> h2;

    // intersection?

    // output solution

    return 0;
}
```

Refinement 2: Intersection? and Output

```
int main()
{
    input rectangles ✓

    bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);

    if (clash)
        std::cout << "intersection!\n";
    else
        std::cout << "no intersection!\n";

    return 0;
}
```

Refinement 3: Intersection Function...

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}

int main() {
    input rectangles ✓

    intersection? ✓

    output solution ✓

    return 0;
}
```

Refinement 3: Intersection Function...

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

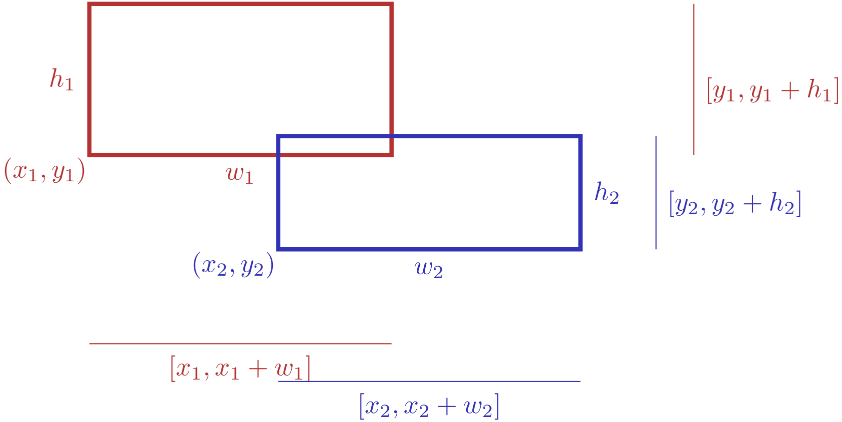
Function main ✓

Refinement 3: ...with PRE and POST

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles,
//      where w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1) and
//      (x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

Refinement 4: Interval Intersection

Two rectangles intersect if and only if their x and y-intervals intersect.



Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2); ✓
}
```

Refinement 4: Interval Intersections

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return false; // todo
}
```

Function rectangles_intersect ✓

Function main ✓

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2);
}
```

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2); ✓
}
```

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

Function `intervals_intersect` ✓

Function `rectangles_intersect` ✓

Function `main` ✓

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

already exists in the standard library

Function intervals_intersect ✓

Function rectangles_intersect ✓

Function main ✓

Back to Intervals

```
// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return std::max(a1, b1) >= std::min(a2, b2)
        && std::min(a1, b1) <= std::max(a2, b2); ✓
}
```

Look what we have achieved step by step!

```
#include <iostream>
#include <algorithm>

// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//     with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
  return std::max(a1, b1) >= std::min(a2, b2)
      && std::min(a1, b1) <= std::max(a2, b2);
}

// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

```
int main ()
{
  std::cout << "Enter two rectangles [x y w h each]\n";
  int x1, y1, w1, h1;
  std::cin >> x1 >> y1 >> w1 >> h1;
  int x2, y2, w2, h2;
  std::cin >> x2 >> y2 >> w2 >> h2;
  bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);
  if (clash)
    std::cout << "intersection!\n";
  else
    std::cout << "no intersection!\n";
  return 0;
}
```

Result

- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

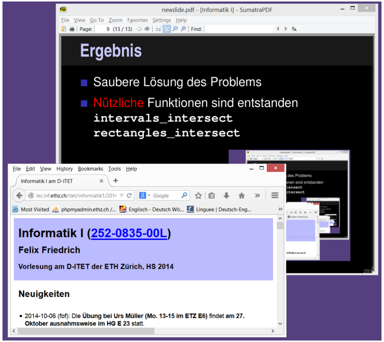
  `rectangles_intersect`

Result

- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

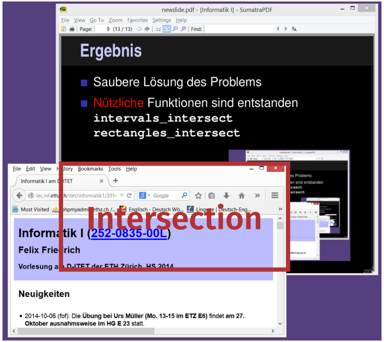
  `rectangles_intersect`



Result

- Clean solution of the problem
- Useful functions have been implemented

```
intervals_intersect
rectangles_intersect
```



Where can a Function be Used?

```
#include <iostream>

int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

Scope of a Function

- is the part of the program where a function can be called

Scope of a Function

- is the part of the program where a function can be called

Extension by **declaration** of a function: like the definition but without `{...}`.

```
double pow(double b, int e);
```

This does not work...

```
#include <iostream>


int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

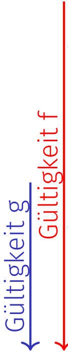
Gültigkeit f

. . . but this works!

```
#include <iostream>
int f(int i); // Gueltigkeitsbereich von f ab hier

int main()
{
    std::cout << f(1);
    return 0;
}

int f(int i)
{
    return i;
}
```

Forward Declarations, why?

Functions that mutually call each other:




```
int f(...) // f valid from here
{
    g(...) // g undeclared
}

int g(...) // g valid from here!
{
    f(...) // ok
}
```

Forward Declarations, why?

Functions that mutually call each other:

```
int g(...); // g valid from here

int f(...) // f valid from here
{
    g(...) // ok
}

int g(...)
{
    f(...) // ok
}
```

Gültigkeit g

Gültigkeit f

Reusability

- Functions such as **`rectangles_intersect`** and **`pow`** are useful in many programs.

Reusability

- Functions such as **`rectangles_intersect`** and **`pow`** are useful in many programs.
- “Solution”: copy-and-paste the source code

Level 1: Outsource the Function

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Level 1: Outsource the Function

`double pow(double b, int e);` in **separate file** `mymath.cpp`

Level 1: Include the Function

```
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"

int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

Level 1: Include the Function

```
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"   <- in working directory

int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

Disadvantage of Including

- **`#include`** copies the file (**`mymath.cpp`**) into the main program (**`callpow2.cpp`**).

Disadvantage of Including

- **`#include`** copies the file (**`mymath.cpp`**) into the main program (**`callpow2.cpp`**).
- The compiler has to (re)compile the function definition for each program

Level 2: Separate Compilation

```
double pow(double b,
           int e)
{
    ...
}
```

mymath.cpp

`g++ -c mymath.cpp` →

001110101100101010
000101110101000111
00010 Funktion **pow**
111100001101010001
111111101000111010
010101101011010001
100101111100101010

mymath.o

Level 2: Separate Compilation

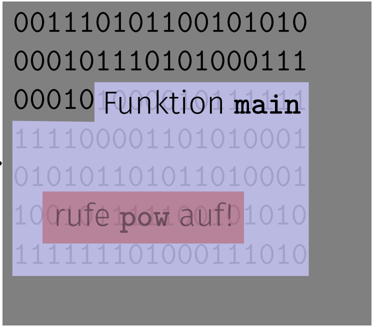
```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e);
```

mymath.h

Level 2: Separate Compilation

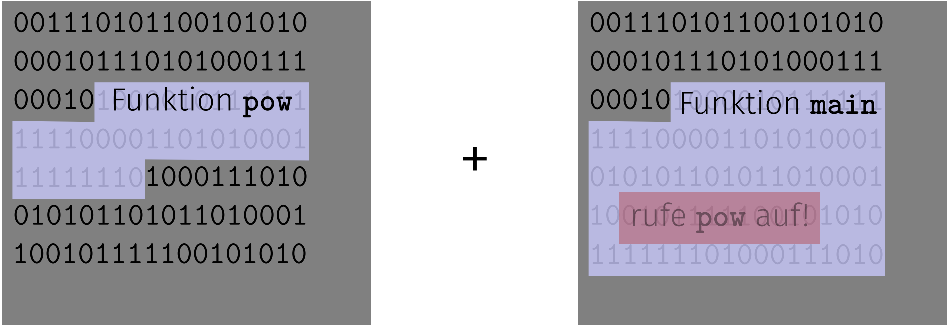
```cpp
#include <iostream>
#include "mymath.h"
int main()
{
  std::cout << pow(2,-2) << "\n";
  return 0;
}
```

callpow3.cpp



callpow3.o

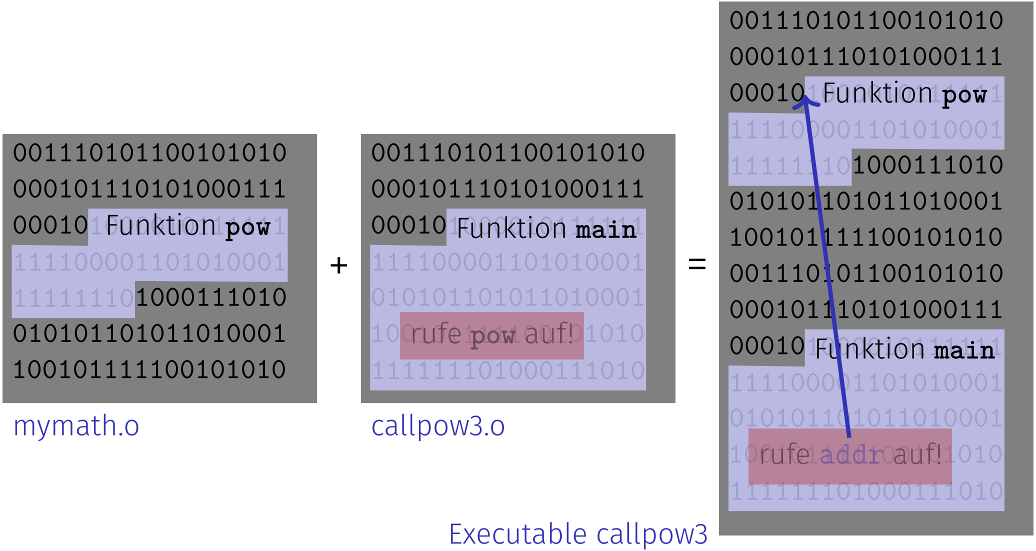
The linker unites...



mymath.o

callpow3.o

… what belongs together



Availability of Source Code?

Observation

`mymath.cpp` (source code) is not required any more when the **`mymath.o`** (object code) is available.

Availability of Source Code?

Observation

`mymath.cpp` (source code) is not required any more when the **`mymath.o`** (object code) is available.

Many vendors of libraries do not provide source code.

Availability of Source Code?

Observation

`mymath.cpp` (source code) is not required any more when the **`mymath.o`** (object code) is available.

Many vendors of libraries do not provide source code.
Header files then provide the *only* readable informations.

Open-Source Software

- Source code is generally available.

Open-Source Software

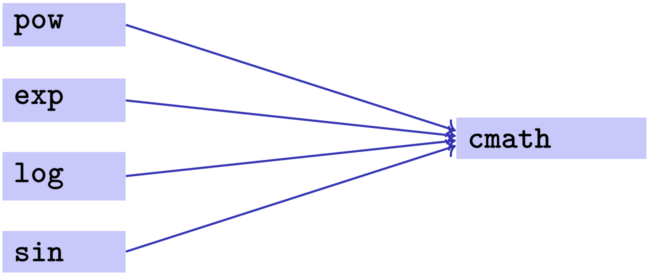
- Source code is generally available.
- Only this allows the continued development of code by users and dedicated “hackers”.

Open-Source Software

- Source code is generally available.

Libraries

- Logical grouping of similar functions



Name Spaces...

```
// cmath
namespace std {

  double pow(double b, int e);

  ....
  double exp(double x);
  ...
}
```

...Avoid Name Conflicts

```cpp
#include <cmath>
#include "mymath.h"

int main()
{
    double x = std::pow(2.0, -2); // <cmath>
    double y = pow(2.0, -2); // mymath.h
}
```

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with **`std::pow`**);
- lead to interesting and efficient programs in a simple way;

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with **`std::pow`**);
- lead to interesting and efficient programs in a simple way;
- guarantee a quality standard that cannot easily be achieved with code written from scratch.

Example: Prime Number Test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, n-1\}$ dividing n .

```
unsigned int d;
for (d=2; n % d != 0; ++d);
```

Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \dots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

- This works because `std::sqrt` rounds to the next representable `double` number (IEEE Standard 754).

```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2);
}
```

Functions Should be More Capable!


```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // fail!  ☹
}
```

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2);
}
```

Functions Should be More Capable! Swap ?

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // ok!
}
```

Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.

Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.
- Not a new concept specific to functions, but rather a new class of types

Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.
- Not a new concept specific to functions, but rather a new class of types