

## 24. Subtyping, Inheritance and Polymorphism

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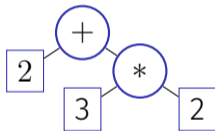
Expression Trees, Separation of Concerns and Modularisation, Type Hierarchies, Virtual Functions, Dynamic Binding, Code Reuse, Concepts of Object-Oriented Programming

# Last Week: Expression Trees

- Goal: Represent arithmetic expressions, e.g.

$$2 + 3 * 2$$

- Arithmetic expressions form a *tree structure*

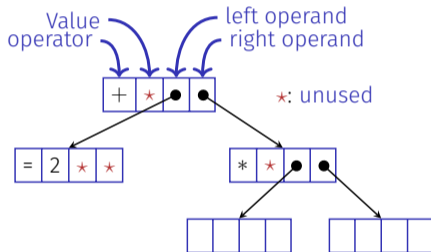


- Expression trees comprise *different* nodes: literals (e.g. 2), binary operators (e.g. +), unary operators (e.g.  $\sqrt{\quad}$ ), function applications (e.g.  $\cos$ ), etc.

# Disadvantages

Implemented via *a single* node type:

```
struct tnode {
    char op; // Operator ('=' for literals)
    double val; // Literal's value
    tnode* left; // Left child (or nullptr)
    tnode* right; // ...
    ...
};
```



**Observation:** `tnode` is the “sum” of all required nodes (constants, addition, ...)  $\Rightarrow$  memory wastage, inelegant

# Disadvantages

**Observation:** `tnode` is the “sum” of all required nodes – and every function must “dissect” this “sum”, e.g.:

```
double eval(const tnode* n) {
    if (n->op == '=') return n->val; // n is a constant
    double l = 0;
    if (n->left) l = eval(n->left); // n is not a unary operator
    double r = eval(n->right);
    switch(n->op) {
        case '+': return l+r; // n is an addition node
        case '*': return l*r; // ...
        ...
    }
}
```

⇒ Complex, and therefore error-prone

# Disadvantages

```
struct tnode {  
    char op;  
    double val;  
    tnode* left;  
    tnode* right;  
    ...  
};
```

```
double eval(const tnode* n) {  
    if (n->op == '=') return n->val;  
    double l = 0;  
    if (n->left) l = eval(n->left);  
    double r = eval(n->right);  
    switch(n->op) {  
        case '+': return l+r;  
        case '*': return l*r;  
        ...  
    }
```

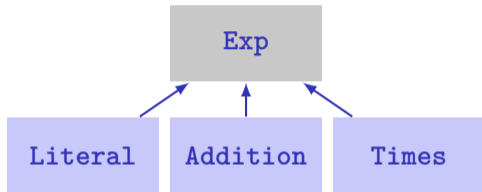
This code isn't *modular* – we'll change that today!

# New Concepts Today

## 1. Subtyping

- Type hierarchy: **Exp** represents general expressions, **Literal** etc. are concrete expression
- Every **Literal** etc. also is an **Exp** (subtype relation)
- That's why a **Literal** etc. can be used everywhere, where an **Exp** is expected:

```
Exp* e = new Literal(132);
```



# New Concepts Today

## 2. Polymorphism and Dynamic Dispatch

- A variable of *static* type **Exp** can “host” expressions of different *dynamic* types:

```
Exp* e = new Literal(2); // e is the literal 2
e = new Addition(e, e); // e is the addition 2 + 2
```

- Executed are the member functions of the *dynamic* type:

```
Exp* e = new Literal(2);
std::cout << e->eval(); // 2

e = new Addition(e, e);
std::cout << e->eval(); // 4
```

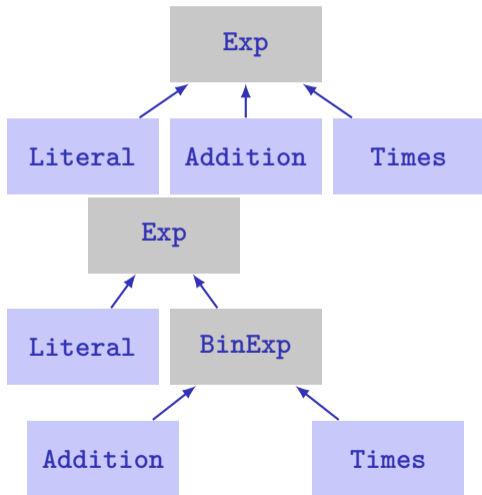
# New Concepts Today

## 3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (**Addition**, **Times**):

$$1 + \text{size}(\text{left operand}) + \text{size}(\text{right operand})$$

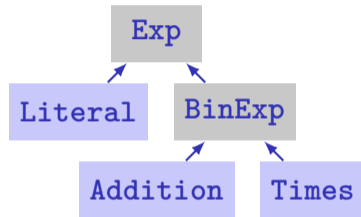
⇒ Implement functionality once, and let subtypes *inherit* it





# Advantages

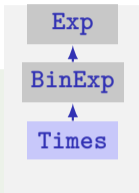
- Subtyping, inheritance and dynamic binding enable *modularisation through specialisation*
- Inheritance enables sharing common code across modules  
⇒ *avoid code duplication*



```
Exp* e = new Literal(2);  
std::cout << e->eval();  
  
e = new Addition(e, e);  
std::cout << e->eval();
```

# Syntax and Terminology

```
struct Exp {  
    ...  
}  
  
struct BinExp : public Exp {  
    ...  
}  
  
struct Times : public BinExp {  
    ...  
}
```



**Note:** Today, we focus on the new concepts (subtyping, ...) and ignore the orthogonal aspect of encapsulation (**class, private** vs. **public** member variables)

# Syntax and Terminology

```
struct Exp {  
    ...  
}  
  
struct BinExp : public Exp {  
    ...  
}  
  
struct Times : public BinExp {  
    ...  
}
```



- **BinExp** is a *subclass*<sup>1</sup> of **Exp**
- **Exp** is the *superclass*<sup>2</sup> of **BinExp**
- **BinExp** *inherits* from **Exp**
- **BinExp** *publicly* inherits from **Exp** (**public**), that's why **BinExp** is a *subtype* of **Exp**
- Analogously: **Times** and **BinExp**
- Subtype relation is transitive: **Times** is also a subtype of **Exp**

<sup>1</sup>derived class, child class

<sup>2</sup>base class, parent class

# Abstract Class Exp and Concrete Class Literal

```
struct Exp {  
    virtual int size() const = 0;  
    virtual double eval() const = 0;  
};
```

← ...that makes `Exp` an *abstract* class

↑ Activates dynamic dispatch

← Enforces implementation by derived classes ...

```
struct Literal : public Exp {  
    double val;  
  
    Literal(double v);  
    int size() const;  
    double eval() const;  
};
```

← `Literal` inherits from `Exp` ...

← ...but is otherwise just a regular class

# Literal: Implementation

```
Literal::Literal(double v): val(v) {}
```

```
int Literal::size() const {  
    return 1;  
}
```

```
double Literal::eval() const {  
    return this->val;  
}
```

# Subtyping: A Literal is an Expression

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);  
Exp* e = lit; // OK: Literal is a subtype of Exp
```

But not vice versa:

```
Exp* e = ...  
Literal* lit = e; // ERROR: Exp is not a subtype of Literal
```

# Polymorphie: a Literal Behaves Like a Literal

```
struct Exp {  
    ...  
    virtual double eval();  
};  
  
double Literal::eval() {  
    return this->val;  
}
```

```
Exp* e = new Literal(3);  
std::cout << e->eval(); // 3
```

- *virtual* member function: the *dynamic* (here: **Literal**) type determines the member function to be executed  
⇒ *dynamic binding*
- Without **Virtual** the *static* type (hier: **Exp**) determines which function is executed
- We won't go into further details

## Further Expressions: Addition and Times

```
struct Addition : public Exp {  
    Exp* left; // left operand  
    Exp* right; // right operand  
    ...  
};
```

```
int Addition::size() const {  
    return 1 + left->size()  
           + right->size();  
}
```

```
struct Times : public Exp {  
    Exp* left; // left operand  
    Exp* right; // right operand  
    ...  
};
```

```
int Times::size() const {  
    return 1 + left->size()  
           + right->size();  
}
```



Separation of concerns



Code duplication



## Extracting Commonalities ...: BinExp

```
struct BinExp : public Exp {  
    Exp* left;  
    Exp* right;  
  
    BinExp(Exp* l, Exp* r);  
    int size() const;  
};
```

```
BinExp::BinExp(Exp* l, Exp* r): left(l), right(r) {}
```

```
int BinExp::size() const {  
    return 1 + this->left->size() + this->right->size();  
}
```

Note: `BinExp` does not implement `eval` and is therefore also an abstract class, just like `Exp`

## ...Inheriting Commonalities: Addition

```
struct Addition : public BinExp {  
    Addition(Exp* l, Exp* r);  
    double eval() const;  
};
```

← Addition inherits member variables (`left`, `right`) and functions (`size`) from `BinExp`

```
Addition::Addition(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Addition::eval() const {  
    return  
        this->left->eval() +  
        this->right->eval();  
}
```

↑ Calling the *super constructor* (constructor of `BinExp`) initialises the member variables `left` and `right`

## ...Inheriting Commonalities: Times

```
struct Times : public BinExp {  
    Times(Exp* l, Exp* r);  
    double eval() const;  
};
```

```
Times::Times(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Times::eval() const {  
    return  
        this->left->eval() *  
        this->right->eval();  
}
```

Observation: `Additon::eval()` and `Times::eval()` are very similar and could also be unified. However, this would require the concept of *functional programming*, which is outside the scope of this course.

# Further Expressions and Operations

- Further expressions, as classes derived from **Exp**, are possible, e.g.  $-$ ,  $/$ ,  $\sqrt{\quad}$ ,  $\cos$ ,  $\log$
- A former bonus exercise (included in today's lecture examples on Code Expert) illustrates possibilities: variables, trigonometric functions, parsing, pretty-printing, numeric simplifications, symbolic derivations, ...

# Mission: Monolithic $\rightarrow$ Modular $\checkmark$

```
struct tnode {  
    char op;  
    double val;  
    tnode* left;  
    tnode* right;  
    ...  
}
```

```
double eval(const tnode* n) {  
    if (n->op == '=') return n->val;  
    double l = 0;  
    if (n->left != 0) l = eval(n->left);  
    double r = eval(n->right);  
    switch(n->op) {  
        case '+': return l + r;  
        case '*': return l * r;  
        case '-': return l - r;  
        case '/': return l / r;  
        default:  
            // unknown operator  
            assert (false);  
    }  
}
```

```
int size (const tnode* n) const { ... }
```

```
...
```

```
struct Literal : public Exp {  
    double val;  
    ...  
    double eval() const {  
        return val;  
    }  
};
```

```
struct Addition : public Exp {  
    ...  
    double eval() const {  
        return left->eval() + right->eval();  
    }  
};
```

```
struct Times : public Exp {  
    ...  
    double eval() const {  
        return left->eval() * right->eval();  
    }  
};
```

```
struct Cos : public Exp {  
    ...  
    double eval() const {  
        return std::cos(argument->eval());  
    }  
};
```



# And there is so much more ...

Not shown/discussed:

- Private inheritance (`class B : public A`)
- Subtyping and polymorphism without pointers
- Non-virtuell member functions and static dispatch  
(`virtual double eval()`)
- Overriding inherited member functions and invoking overridden implementations
- Multiple inheritance
- ...

# Object-Oriented Programming

In the last 3rd of the course, several concepts of *object-oriented programming* were introduced, that are briefly summarised on the upcoming slides.

*Encapsulation* (weeks 10-13):

- Hide the implementation details of types (private section) from users
- Definition of an interface (public area) for accessing values and functionality in a controlled way
- Enables ensuring invariants, and the modification of implementations without affecting user code

# Object-Oriented Programming

## *Subtyping* (week 14):

- Type hierarchies, with super- and subtypes, can be created to model relationships between more abstract and more specialised entities
- A subtype supports at least the functionality that its supertype supports – typically more, though, i.e. a subtype extends the interface (public section) of its supertype
- That's why supertypes can be used anywhere, where subtypes are required ...
- ...and functions that can operate on more abstract type (supertypes) can also operate on more specialised types (subtypes)
- The streams introduced in week 7 form such a type hierarchy: **ostream** is the abstract supertyp, **ofstream** etc. are specialised subtypes



# Object-Oriented Programming

*Polymorphism* and *dynamic binding* (week 14):

- A pointer of static typ  $T_1$  can, at runtime, point to objects of (dynamic) type  $T_2$ , if  $T_2$  is a subtype of  $T_1$
- When a virtual member function is invoked from such a pointer, the dynamic type determines which function is invoked
- I.e.: despite having the same static type, a different behaviour can be observed when accessing the common interface (member functions) of such pointers
- In combination with subtyping, this enables adding further concrete types (streams, expressions, ...) to an existing system, without having to modify the latter

# Object-Oriented Programming

## *Inheritance* (week 14):

- Derived classes inherit the functionality, i.e. the implementation of member functions, of their parent classes
- This enables sharing common code and thereby avoids code duplication
- An inherited implementation can be overridden, which allows derived classes to behave differently than their parent classes (not shown in this course)

## 25. Conclusion

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# Purpose and Format

Name the most important key words to each chapter. Checklist: “does every notion make some sense for me?”

- Ⓜ motivating example for each chapter
- Ⓒ concepts that do not depend from the implementation (language)
- Ⓕ language (C++): all that depends on the chosen language
- Ⓔ examples from the lectures

# Kapitelüberblick

- 1. Introduction
- 2. Integers
- 3. Booleans
- 4. Defensive Programming
- 5./6. Control Statements
- 7./8. Floating Point Numbers
- 9./10. Functions
- 11. Reference Types
- 12./13. Vectors and Strings
- 14./15. Recursion
- 16. Structs and Overloading
- 17. Classes
- 18./19. Dynamic Datastructures
- 20. Containers, Iterators and Algorithms
- 21. Dynamic Datatypes and Memory Management
- 22. Subtyping, Polymorphism and Inheritance

# 1. Introduction

Ⓜ

- Euclidean algorithm

Ⓒ

- algorithm, Turing machine, programming languages, compilation, syntax and semantics
- values and effects, fundamental types, literals, variables

Ⓕ

- include directive `#include <iostream>`
- main function `int main(){...}`
- comments, layout `// Kommentar`
- types, variables, L-value `a` , R-value `a+b`
- expression statement `b=b*b;` , declaration statement `int a;`, return statement `return 0;`

## 2. Integers

- ① Celsius to Fahrenheit
- ②
  - associativity and precedence, arity
  - expression trees, evaluation order
  - arithmetic operators
  - binary representation, hexadecimal numbers
  - signed numbers, twos complement
- ③
  - arithmetic operators `9 * celsius / 5 + 32`
  - increment / decrement `expr++`
  - arithmetic assignment `expr1 += expr2`
  - conversion `int` ↔ `unsigned int`
- ④ Celsius to Fahrenheit, equivalent resistance

# 3. Booleans

Ⓒ

- Boolean functions, completeness
- DeMorgan rules

Ⓕ

- the type `bool`
- logical operators `a && !b`
- relational operators `x < y`
- precedences `7 + x < y && y != 3 * z`
- short circuit evaluation `x != 0 && z / x > y`
- the `assert`-statement, `#include <cassert>`

Ⓔ

- Div-Mod identity.



## 4. Defensive Programming

- Assertions and Constants
- The `assert`-statement, `#include <cassert>`
  - `const int speed_of_light=2999792458`
- Assertions for the GCD

# 5./6. Control Statements

- linear control flow vs. interesting programs
- selection statements, iteration statements
- (avoiding) endless loops, halting problem
- Visibility and scopes, automatic memory
- equivalence of iteration statement
- if statements `if (a % 2 == 0) {...}`
- for statements `for (unsigned int i = 1; i <= n; ++i) ...`
- while and do-statements `while (n > 1) {...}`
- blocks and branches `if (a < 0) continue;`
- Switch statement `switch(grade) {case 6: }`
- sum computation (Gauss), prime number tests, Collatz sequence, Fibonacci numbers, calculator, output grades

# 7./8. Floating Point Numbers

- (M) correct computation: Celsius / Fahrenheit
- (C) fixpoint vs. floating point
  - holes in the value range
  - compute using floating point numbers
  - floating point number systems, normalisation, IEEE standard 754
  - *guidelines for computing with floating point numbers*
- (L) types `float`, `double`
  - floating point literals `1.23e-7f`
- (E) Celsius/Fahrenheit, Euler, Harmonic Numbers

## 9./10. Functions

- ① ■ Computation of Powers
- ② ■ Encapsulation of Functionality
  - functions, formal arguments, arguments
  - scope, forward declarations
  - procedural programming, modularization, separate compilation
  - *Stepwise Refinement*
- ③ ■ declaration and definition of functions
  - `double pow(double b, int e){ ... }`
  - function call `pow (2.0, -2)`
  - the type `void`
- ④ ■ powers, perfect numbers, minimum, calendar

# 11. Reference Types

- ① ■ Swap
- ② ■ value- / reference- semantics, pass by value, pass by reference, return by reference
  - lifetime of objects / temporary objects
  - constants
- ③ ■ reference type `int& a`
  - call by reference, return by reference `int& increment (int& i)`
  - const guideline, const references, reference guideline
- ④ ■ swap, increment

## 12./13. Vectors and Strings

- Iterate over data: sieve of Erathosthenes
- vectors, memory layout, random access
- (missing) bound checks
- vectors
- characters: ASCII, UTF8, texts, strings
- vector types `std::vector<int> a {4,3,5,2,1};`
- characters and texts, the type `char c = 'a';`, Konversion nach `int`
- vectors of vectors
- Streams `std::istream`, `std::ostream`
- sieve of Erathosthenes, Caesar-code, shortest paths

# 14./15. Recursion

- ① recursive math. functions, the n-Queen problem, Lindenmayer systems, a command line calculator
- ②
  - recursion
  - call stack, memory of recursion
  - correctness, termination,
  - recursion vs. iteration
  - Backtracking, EBNF, formal grammars, parsing
- ③
  - factorial, GCD, sudoku-solver, command line calculator

# 16. Structs and Overloading

- ① ■ build your own rational number
- ② ■ heterogeneous data types
  - function and operator overloading
  - encapsulation of data
- ③ ■ struct definition `struct rational {int n; int d;};`
  - member access `result.n = a.n * b.d + a.d * b.n;`
  - initialization and assignment,
  - function overloading `pow(2)` vs. `pow(3,3);`, operator overloading
- ④ ■ rational numbers, complex numbers



# 17. Classes

- rational numbers with encapsulation
- Encapsulation, Construction, Member Functions
- classes `class rational { ... };`
  - access control `public: / private:`
  - member functions `int rational::denominator () const`
  - The implicit argument of the member functions
- finite rings, complex numbers

# 18./19. Dynamic Datastructures

- ① ■ Our own vector
- ② ■ linked list, allocation, deallocation, dynamic data type
- ③ ■ The **new** statement
  - pointer `int* x;`, Null-pointer `nullptr.`
  - address and derference operator `int *ip = &i; int j = *ip;`
  - pointer and const `const int *a;`
- ④ ■ linked list, stack

## 20. Containers, Iterators and Algorithms

- ① ■ vectors are containers
- ② ■ iteration with pointers
  - containers and iterators
  - algorithms
- ③ ■ Iterators `std::vector<int>::iterator`
  - Algorithms of the standard library `std::fill (a, a+5, 1);`
  - implement an iterator
  - iterators and const
- ④ ■ output a vector, a set

## 21. Dynamic Datatypes and Memory Management

- ①
  - Stack
  - Expression Tree
- ②
  - Guideline "dynamic memory"
  - Pointer sharing
  - Dynamic Datatype
  - Tree-Structure
- ③
  - **new** and **delete**
  - Destructor `stack::~~stack()`
  - Copy-Constructor `stack::stack(const stack& s)`
  - Assignment operator `stack& stack::operator=(const stack& s)`
  - Rule of Three
- ④
  - Binary Search Tree

## 22. Subtyping, Polymorphism and Inheritance

- ① ■ extend and generalize expression trees
- ② ■ Subtyping
  - polymorphism and dynamic binding
  - Inheritance
- ③ ■ base class `struct Exp{}`
  - derived class `struct BinExp: public Exp{}`
  - abstract class `struct Exp{virtual int size() const = 0...}`
  - polymorphie `virtual double eval()`
- ④ ■ expression node and extensions

# The End

End of the Course