# Recursion Trees

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- Visualize call structure

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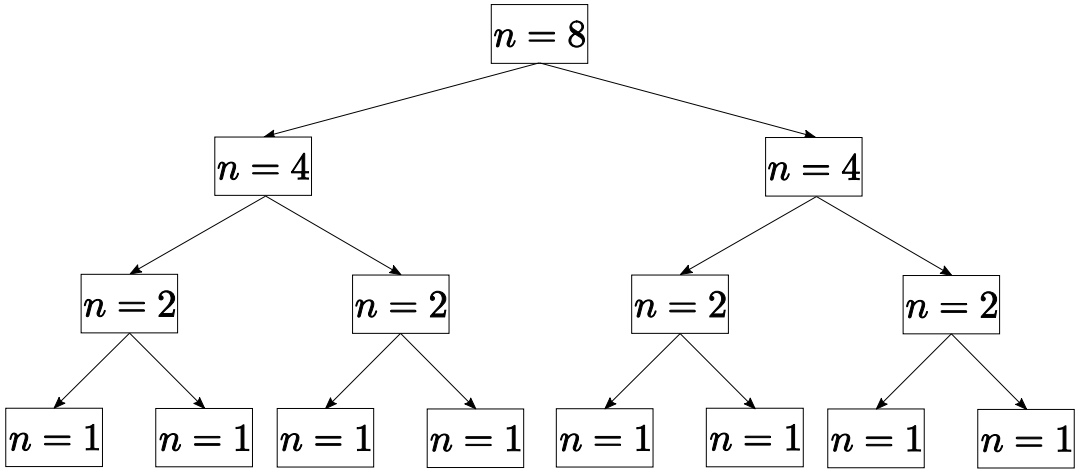
- Example: `fnc(8)`

```
unsigned int fnc (unsigned int n) {
    ...
    return fnc(n/2) + fnc(n/2);
}
```



# Fibonacci Tree Problem

# Fibonacci - Recursion Tree

```
// POST: return value is the n-th
//       Fibonacci number F(n)
ifmp::integer fib (const unsigned int n) {
    if (n == 0) return 0;
    if (n == 1) return 1;
    return fib(n-1) + fib(n-2); // n > 1
}
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**fib(4):**

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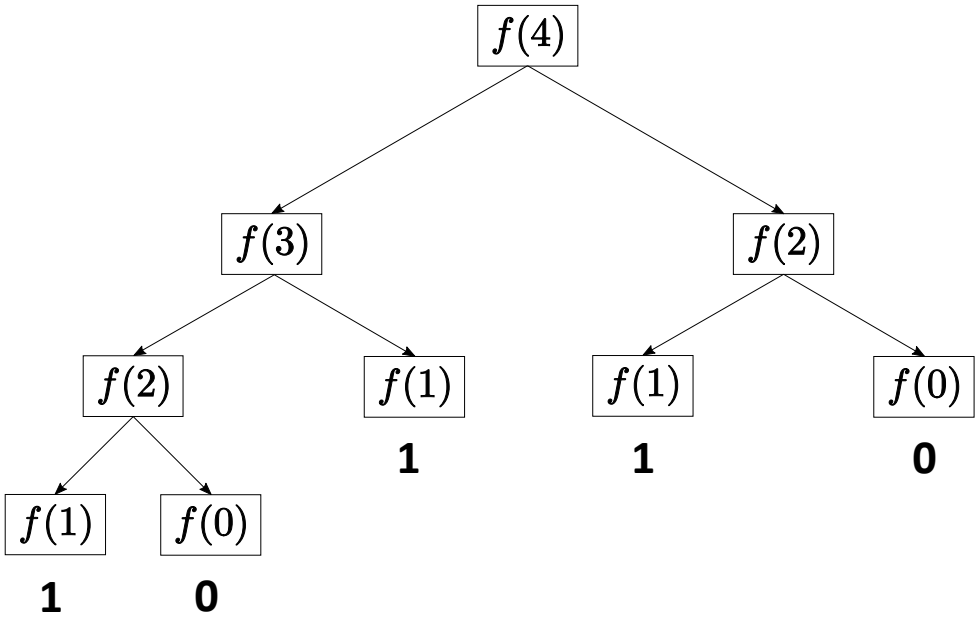
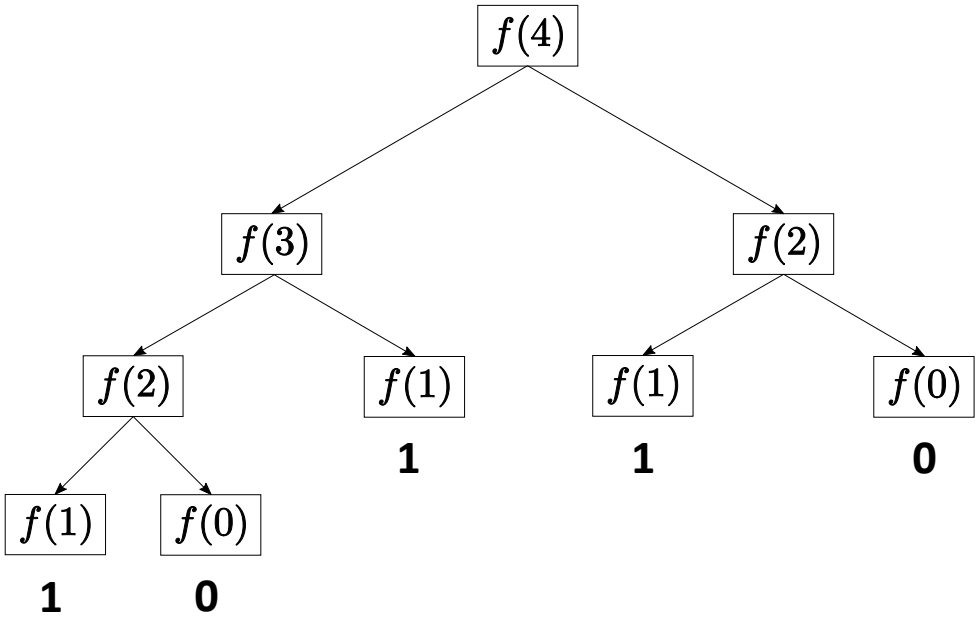


# Fibonacci - Recursion Tree

**Fibonacci-number** VS **function calls**

`n=4:`

**`fib(4):`**

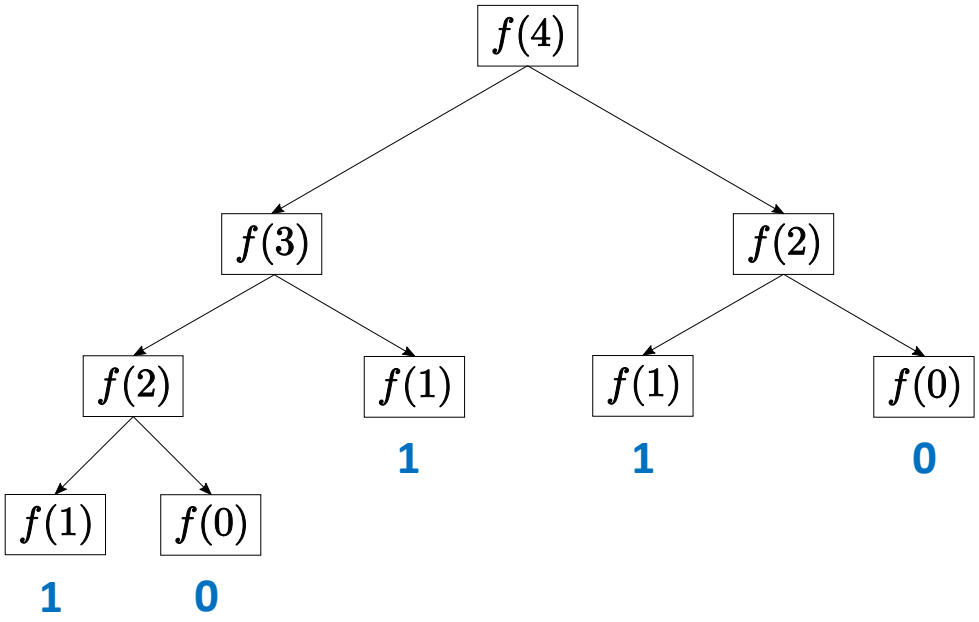


# Fibonacci - Recursion Tree

**Fibonacci-number** VS **function calls**

`n=4:` **3**

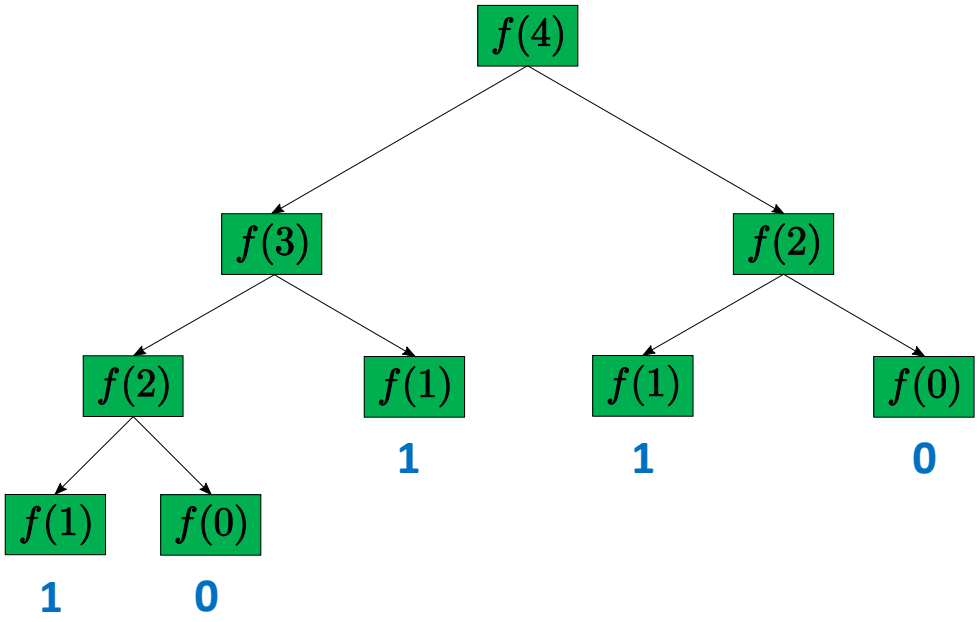
**`fib(4):`**



# Fibonacci - Recursion Tree

| | Fibonacci-number | VS | function calls |
|---|---|---|---|
| n=4: | 3 | | 9 |

**fib(4):**



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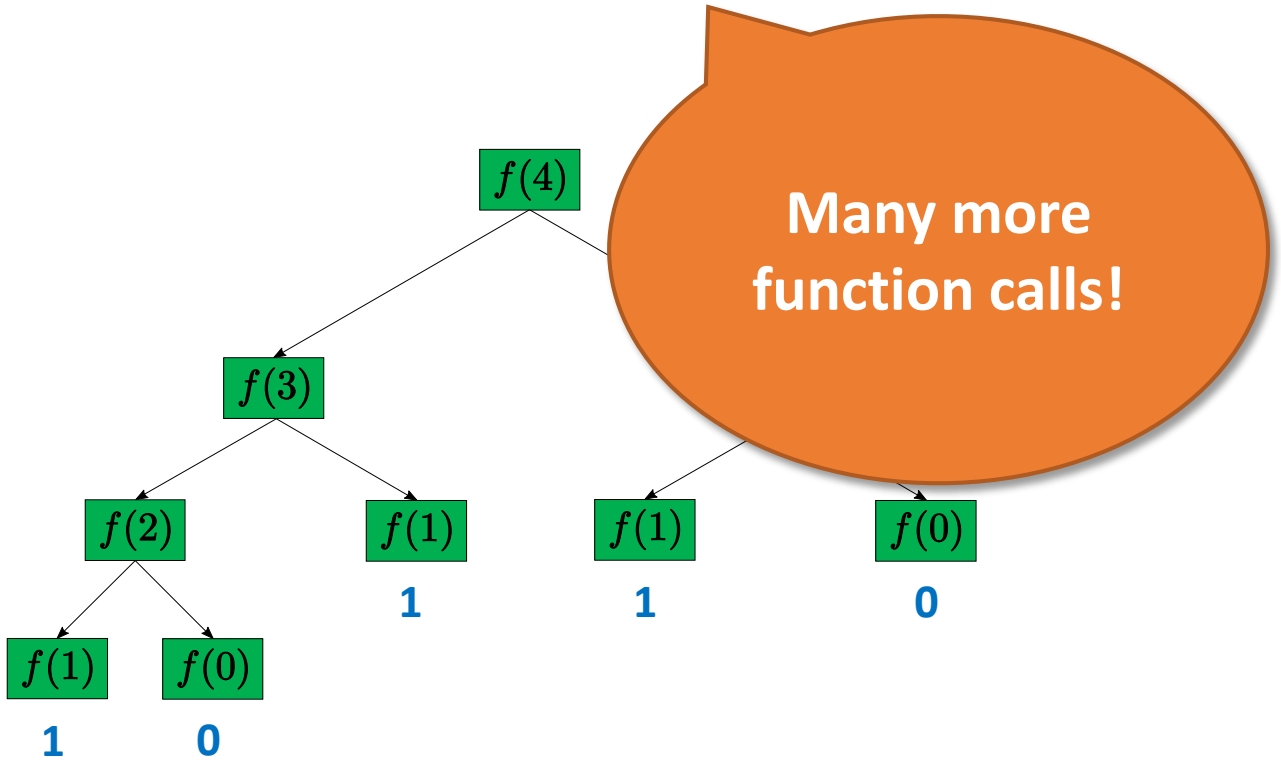


# Fibonacci - Recursion Tree

Fi… number … VS … function calls

n=4:

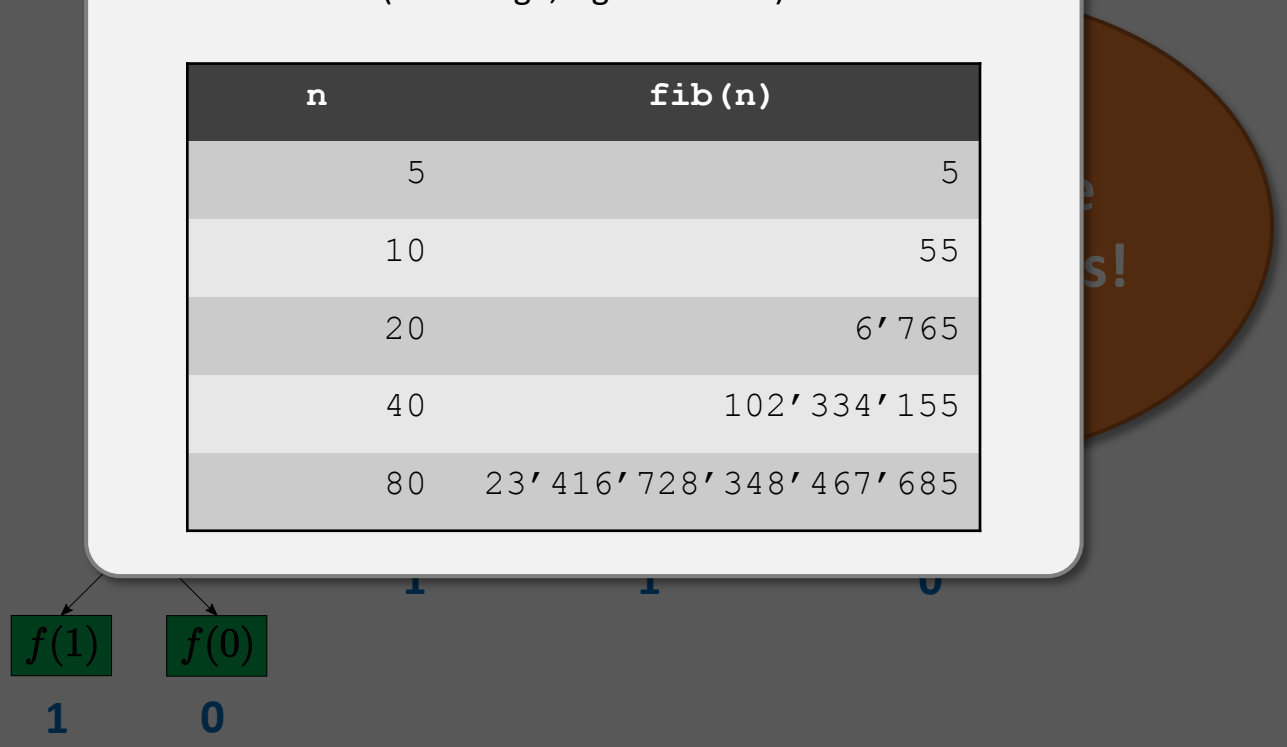
**Fibonacci growth:**

$\texttt{fib(n)} \sim c^n$

(for $n$ large, c golden ratio)

| n | fib(n) |
|---|---|
| 5 | 5 |
| 10 | 55 |
| 20 | 6'765 |
| 40 | 102'334'155 |
| 80 | 23'416'728'348'467'685 |

**fib(4):**



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**fib(4) :**

Requires **unbelievably** many recursive calls!

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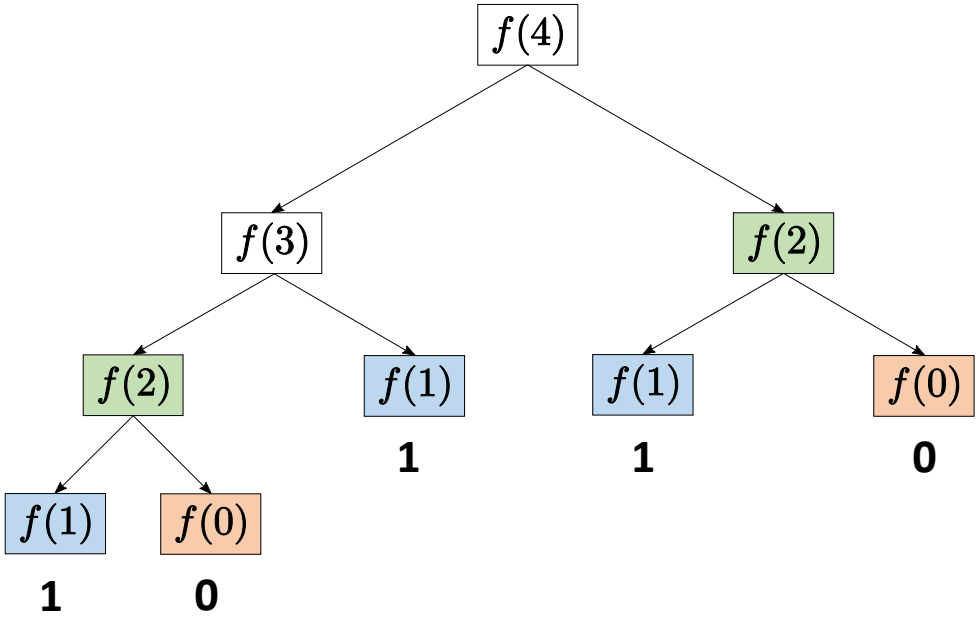
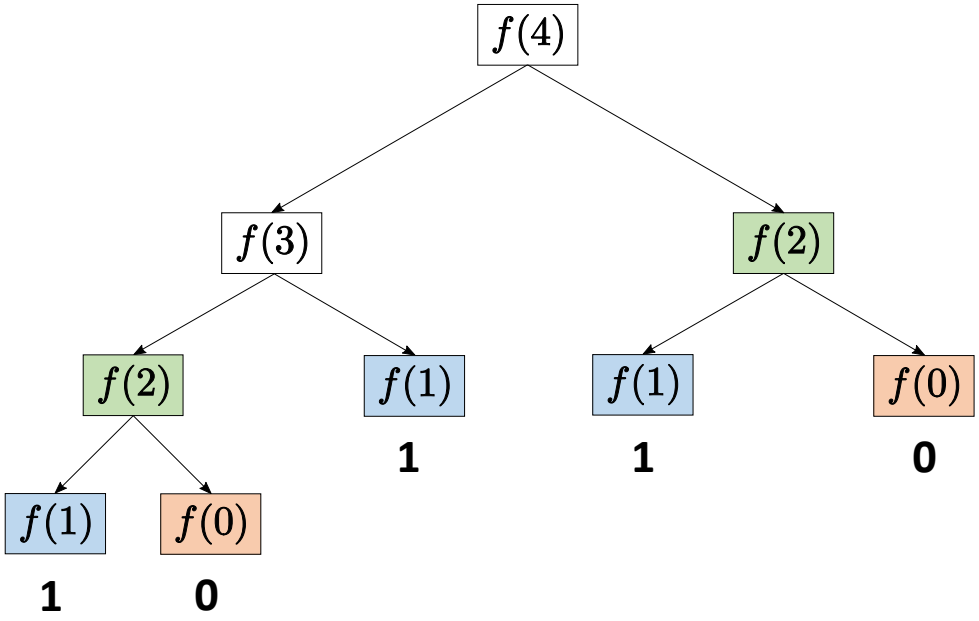


# The Problem

- Problem: Same computation multiple times
- Gets worse as `n` increases :-(



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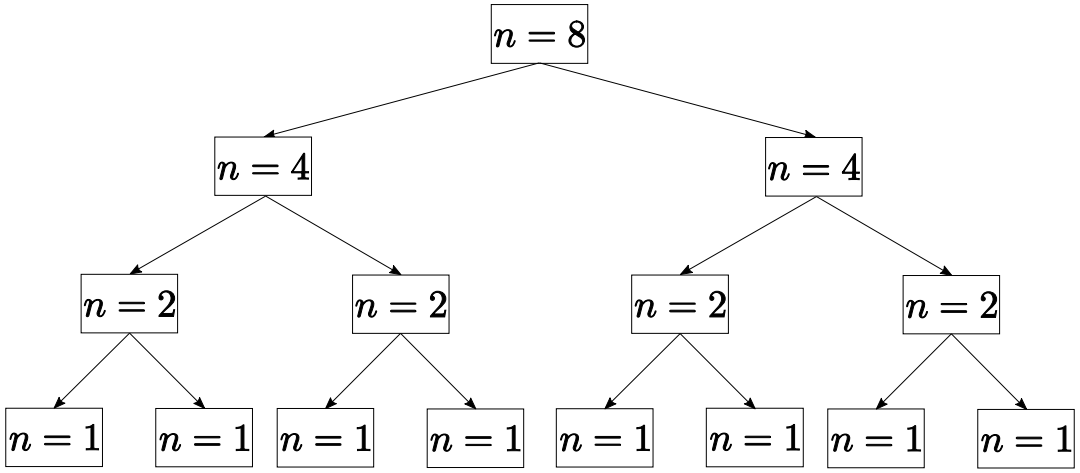

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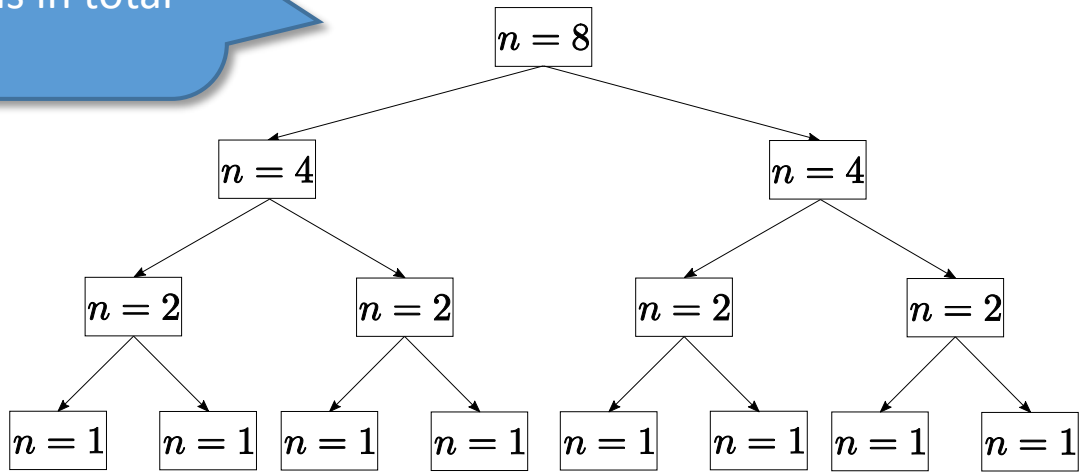
**2n - 1**

recursive calls in total



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**Number of Recursive Calls**

| n | fib | fnc |
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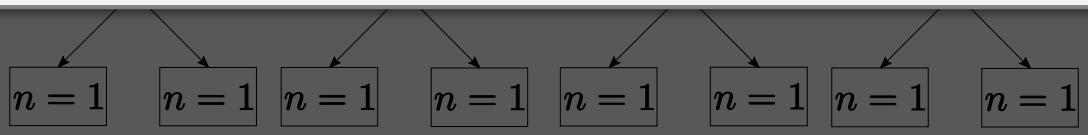

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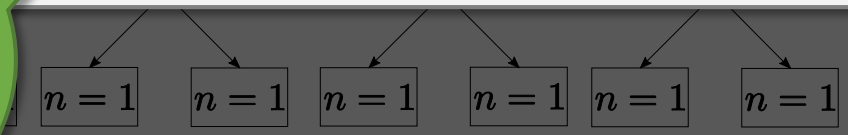


**Mindblowing difference!**

# The Problem

- Reason: $n/2$ falls much faster than $n-1$ and $n-2$
  - $n/2$ → sub-tree of height: $\log_2(n)$
  - $n-1, n-2$ → sub-tree of height: $n-1, n-2$