15. Recursion 2

Building a Calculator, Formal Grammars, Extended Backus Naur Form (EBNF), Parsing Expressions

Motivation: Calculator

Goal: we build a command line calculator

Example

Input: `3 + 5`
Output: `8`
Input: `3 / 5`
Output: `0.6`
Input: `3 + 5 * 20`
Output: `103`
Input: `(3 + 5) * 20`
Output: `160`
Input: `-(3 + 5) + 20`
Output: `12`

- binary Operators +, -, ∗, / and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses
- unary operator -

Naive Attempt (without Parentheses)

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != '=') {
    double rval;
    std::cin >> rval;

    if (op == '+')
        lval += rval;
    else if (op == '∗')
        lval ∗= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 2 + 3 ∗ 3 =
Result 15
```

Analyzing the Problem

Example

Input:

$$\underbrace{13 + 4 * (15 - 7 * 3)}_{\text{Needs to be stored such that evaluation can be performed}} =$$

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

We will store symbols elegantly using recursion.

We need a new formal tool (that is independent of $\mathrm{C}{+}{+}$).

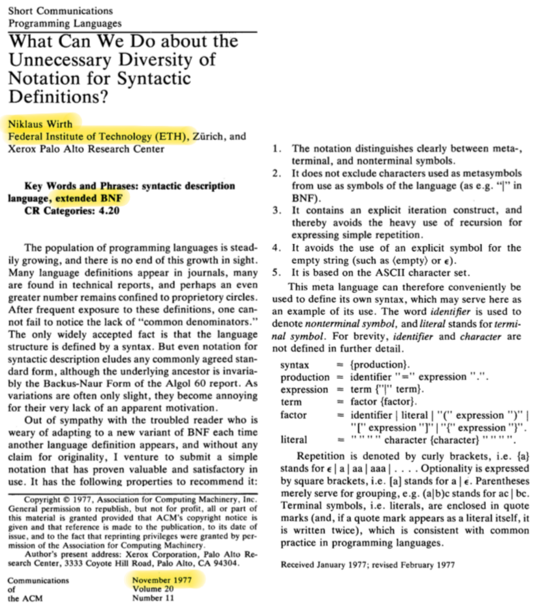
Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

A formal grammar defines which strings are valid.

To describe the formal grammar, we use:

Extended Backus Naur Form (EBNF)



Short Communications
Programming Languages

What Can We Do about the Unnecessary Diversity of Notation for Syntactic Definitions?

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The population of programming languages is steadily growing, and there is no end of this growth in sight. Many language definitions appear in journals, many are found in technical reports, and perhaps an even greater number remains confined to proprietory circles. After frequent exposure to these definitions, one cannot fail to notice the lack of “common denominators.” The only widely accepted fact is that the language structure is defined by a syntax. But even notation for syntactic description eludes any commonly agreed standard form, although the underlying ancestor is invariably the Backus-Naur Form of the Algol 60 report. As variations are often only slight, they become annoying for their very lack of an apparent motivation.

Out of sympathy with the troubled reader who is weary of adapting to a new variant of BNF each time another language definition appears, and without any claim for originality, I venture to submit a simple notation that has proven valuable and satisfactory in use. It has the following properties to recommend it:

1. The notation distinguishes clearly between meta-, terminal, and nonterminal symbols.
2. It does not exclude characters used as metasymbols from use as symbols of the language (as e.g. “|” in BNF).
3. It contains an explicit iteration construct, and thereby avoids the heavy use of recursion for expressing simple repetition.
4. It avoids the use of an explicit symbol for the empty string (such as ⟨empty⟩ or ϵ).
5. It is based on the ASCII character set.

This meta language can therefore conveniently be used to define its own syntax, which may serve here as an example of its use. The word *identifier* is used to denote *nonterminal symbol*, and *literal* stands for *terminal symbol*. For brevity, *identifier* and *character* are not defined in further detail.

```
syntax      = {production}.
production  = identifier "=" expression ".".
expression  = term {"|" term}.
term        = factor {factor}.
factor      = identifier | literal | "(" expression ")" |
              "[" expression "]" | "{" expression "}".
literal     = """" character {character} """".
```

Repetition is denoted by curly brackets, i.e. {a} stands for ϵ | a | aa | aaa | . . . . Optionality is expressed by square brackets, i.e. [a] stands for a | ϵ. Parentheses merely serve for grouping, e.g. (a|b)c stands for ac | bc. Terminal symbols, i.e. literals, are enclosed in quote marks (and, if a quote mark appears as a literal itself, it is written twice), which is consistent with common practice in programming languages.

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Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (Expression)
  −Number, −(Expression)
- Factor ∗ Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term − Term, ...

Factor

Term

Expression

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses or
- a negated factor.

```
factor     = number
           | "(" expression ")"
           | "−" factor.
```

non-terminal symbol

terminal symbol

alternative

The EBNF for Expressions

A term is

- factor,
- factor ∗ factor, factor / factor,
- factor ∗ factor ∗ factor, factor / factor ∗ factor, ...
- ...

```
term = factor {"∗" factor | "/" factor }.
```

optional repetition

The EBNF for Expressions

```
factor     = number
           | "(" expression ")"
           | "−" factor.

term       = factor { "∗" factor | "/" factor }.

expression = term { "+" term |"−" term }.
```

Numbers

An *integer comprises at least one digit, followed by an arbitrary number of digits.*

```
number  =  digit { digit }.
digit   =  '0' | '1' | '2' | ... |'9'.
```

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.
- **Parser:** A program for parsing.
- **Useful:** From the EBNF we can (nearly) automatically generate a parser

Construct a Parser

- Rules become functions
- Alternatives and options become `if`–statements.
- Nonterminial symbols on the right hand side become function calls
- Optional repetitions become `while`–statements

Rules (except `number`)

```
factor     = number
           | "(" expression ")"
           | "−" factor.

term       = factor { "∗" factor | "/" factor }.


expression = term { "+" term |"−" term }.
```

Functions (Parser)

Expression is read from an input stream.

```
// POST: returns true if and only if is = factor ...
//       and in this case extracts factor from is
bool factor (std::istream& is);

// POST: returns true if and only if is = term ...,
//       and in this case extracts all factors from is
bool term (std::istream& is);

// POST: returns true if and only if is = expression ...,
//       and in this case extracts all terms from is
bool expression (std::istream& is);
```

Functions (Parser with Evaluation)

Expression is read from an input stream.

```
// POST: extracts a factor from is
//       and returns its value
double factor (std::istream& is);

// POST: extracts a term from is
//       and returns its value
double term (std::istream& is);

// POST: extracts an expression from is
//       and returns its value
double expression (std::istream& is);
```

One Character Lookahead...

...to find the right alternative.

```
// POST: leading whitespace characters are extracted
//       from input, and the first non−whitespace character
//       input returned (0 if there input no such character)
char lookahead (std::istream& input)
{
  input >> std::ws;          // skip whitespaces
  if (input.eof())
    return 0;              // end of stream
  else
    return input.peek();     // next character in input
}
```

Cherry-Picking

...to extract the desired character.

```
// POST: if ch matches the next lookahead then consume it and return true
//       otherwise return false
bool consume (std::istream& input, char c)
{
  if (lookahead (input) == c) {
    input >> c;
    return true;
  } else
    return false;
}
```

Evaluating Factors

```
double factor (std::istream& input)
{
  double value;
  if (consume (input, '(')) {
    value = expression (input);       // "(" expression
    consume (input, ')');             // ")"
  } else if (consume (input, '−'))
    value = −factor (input);          // − factor
  else
    value = number(input);            /
  return value;
}
```

```
factor = "(" expression ")"
       | "−" factor
       | number.
```

Evaluating Terms

```
double term (std::istream& input)
{
  double value = factor (input);          // factor
  while (true) {
    if (consume (input, '*'))
      value *= factor (input);            // "*" factor
    else if (consume (input, '/'))
      value /= factor (input);            // "/" factor
    else
      return value;
  }
}
```

term = factor { "*" factor | "/" factor }.

Evaluating Expressions

```
double expression (std::istream& input)
{
  double value = term (input);            // term
  while (true) {
    if (consume (input, '+'))
      value += term (input);              // "+" term
    else if (consume (input, '-'))
      value -= term (input);              // "-" term
    else
      return value;
  }
}
```

expression = term { "+" term | "-" term }.

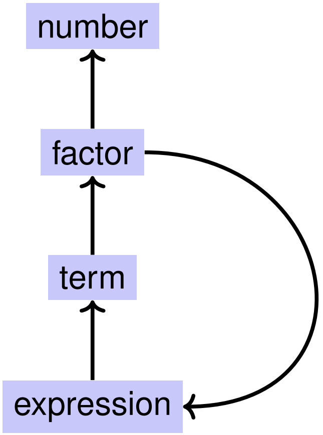
Digits ...

```
// POST: returns the digit that could be consumed from a stream
//       (0 if no digit available)
// digit = '0' | '1' | ... | '9'.
char digit(std::istream& input){
  char ch = input.peek(); // one symbol lookahead
  if (input.eof()) return 0; // nothing available on the stream
  if  (ch >= '0' && ch <= '9'){
    input >> ch; // consume
    return ch;
  }
  return 0;
}
```

... and Numbers

```
// POST: returns an unsigned integer consumed from the stream
// number = digit {digit}.
unsigned int number (std::istream& input){
  input >> std::skipws;// skip whitespaces before the first digit
  char ch = digit(input);
  input >> std::noskipws; // no whitespaces allowed within a number
  unsigned int num = 0;
  while(ch > 0){ // skip remaining digits
    num = num * 10 + ch - '0';
    ch = digit(input);
  }
  return num;
}
```

Recursion!



EBNF — and it works!

EBNF (`calculator.cpp`, Evaluation from left to right):

```
factor     = number
           | "(" expression ")"
           | "−" factor.

term       = factor { "∗" factor | "/" factor }.

expression = term { "+" term |"−" term }.
```

```
std::stringstream input ("1−2−3");
std::cout << expression (input) << "\n"; // −4
```

16. Structs

Rational Numbers, Struct Definition, Function- and Operator Overloading

Calculating with Rational Numbers

- Rational numbers ($\mathbb{Q}$) are of the form $\frac{n}{d}$ with n and d in $\mathbb{Z}$
- $\mathrm{C}{+}{+}$does not provide a built-in type for rational numbers

Goal

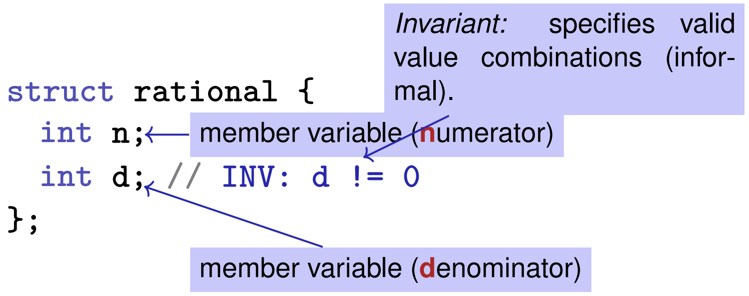
We build a $\mathrm{C}{+}{+}$-type for rational numbers ourselves! ☺

Vision

How it could (will) look like

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;
std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

A First Struct



```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

Invariant: specifies valid value combinations (informal).

member variable (numerator)

member variable (denominator)

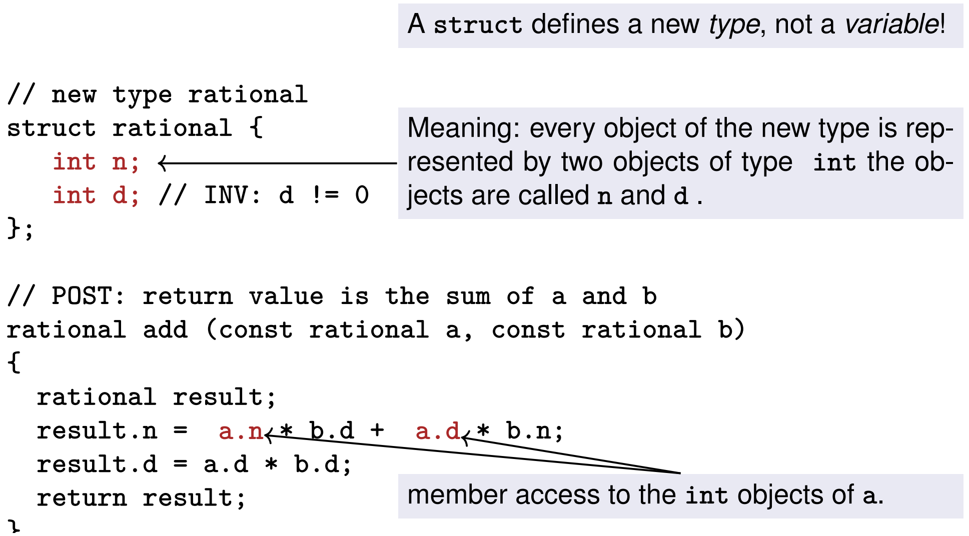
- `struct` defines a new *type*
- formal range of values: *cartesian product* of the value ranges of existing types
- real range of values: `rational` $\subsetneq$ `int` × `int`.

Accessing Member Variables

```
struct rational {
    int n;
    int d; // INV: d != 0
};

rational add (rational a, rational b){
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
```

$$\frac{r_n}{r_d} := \frac{a_n}{a_d} + \frac{b_n}{b_d} = \frac{a_n \cdot b_d + a_d \cdot b_n}{a_d \cdot b_d}$$

A First Struct: Functionality



A `struct` defines a new *type*, not a *variable*!

```
// new type rational
struct rational {
    int n;
    int d; // INV: d != 0
};

// POST: return value is the sum of a and b
rational add (const rational a, const rational b)
{
  rational result;
  result.n =  a.n * b.d +  a.d * b.n;
  result.d = a.d * b.d;
  return result;
}
```

Meaning: every object of the new type is represented by two objects of type `int` the objects are called `n` and `d` .

member access to the `int` objects of `a`.

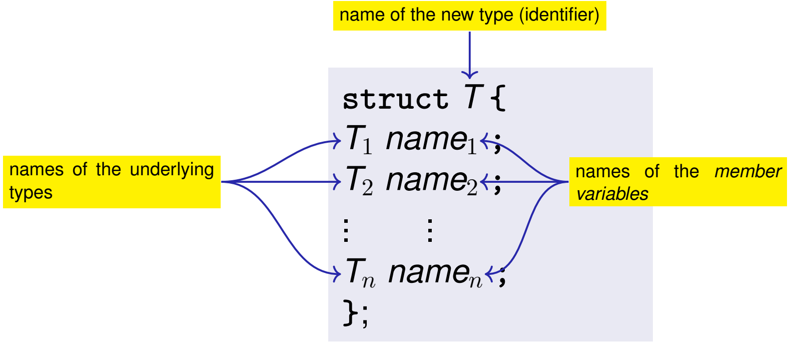
Input

```
// Input r
rational r;
std::cout << "Rational number r:\n";
std::cout << " numerator =? ";
std::cin >> r.n;
std::cout << " denominator =? ";
std::cin >> r.d;

// Input s the same way
rational s;
...
```

Vision comes within Reach ...

```
// computation
const rational t = add (r, s);

// output
std::cout << "Sum is " << t.n << "/" << t.d << ".\n";
```

Struct Definitions


name of the new type (identifier)
struct T {
T_1 *name*$_1$;
T_2 *name*$_2$;
$\vdots \quad \vdots$
T_n *name*$_n$;
};
names of the underlying types
names of the *member variables*

Range of Values of T: $T_1 \times T_2 \times ... \times T_n$

Struct Defintions: Examples

```
struct rational_vector_3 {
  rational x;
  rational y;
  rational z;
};
```

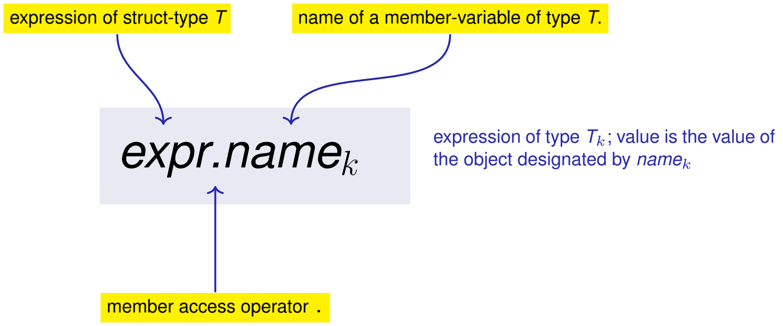
underlying types can be fundamental or user defined

Struct Definitions: Examples

```
struct extended_int {
  // represents value if is_positive==true
  // and −value otherwise
  unsigned int value;
  bool is_positive;
};
```

the underlying types can be different

Structs: Accessing Members



expression of struct-type *T*

name of a member-variable of type *T*.

$expr.name_k$

expression of type T_k; value is the value of the object designated by $name_k$

member access operator .

Structs: Initialization and Assignment

Default Initialization:

```
rational t;
```

- Member variables of `t` are default-initialized
- for member variables of fundamental types nothing happens (values remain undefined)

Structs: Initialization and Assignment

Initialization:

```
rational t = {5, 1};
```

- Member variables of `t` are initialized with the values of the list, according to the declaration order.

Structs: Initialization and Assignment

Assignment:

```
rational s;
...
rational t = s;
```

- The values of the member variables of `s` are assigned to the member variables of `t`.

Structs: Initialization and Assignment

```
t.n                .n
     = add (r, s)      ;
t.d                .d
```

Initialization:

```
rational t = add (r, s);
```

- `t` is initialized with the values of `add(r, s)`

Structs: Initialization and Assignment

Assignment:

```
rational t;
t = add (r, s);
```

- `t` is default-initialized
- The value of `add (r, s)` is assigned to `t`

Structs: Initialization and Assignment

`rational s;` ⟵ member variables are uninitialized

`rational t = {1,5};` ⟵ *member-wise* initialization: `t.n = 1, t.d = 5`

`rational u = t;` ⟵ member-wise copy

`t = u;` ⟵ member-wise copy

`rational v = add (u,t);` ⟵ member-wise copy

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators **==** and **!=** , not so for structs! Why?

- member-wise comparison does not make sense in general...
- ...otherwise we had, for example, $\frac{2}{3} \neq \frac{4}{6}$

Structs as Function Arguments

```
void increment(rational dest, const rational src)
{
    dest = add (dest, src); // modifies local copy only
}
```

Call by Value !

```
rational a;
rational b;
a.d = 1; a.n = 2;
b = a;
increment (b, a); // no effect!
std::cout << b.n << "/" << b.d; // 1 / 2
```

Structs as Function Arguments

```
void increment(rational & dest, const rational src)
{
    dest = add (dest, src);
}
```

Call by Reference

```
rational a;
rational b;
a.d = 1; a.n = 2;
b = a;
increment (b, a);
std::cout << b.n << "/" << b.d; // 2 / 2
```

User Defined Operators

Instead of

```
rational t = add(r, s);
```

we would rather like to write

```
rational t = r + s;
```

This can be done with *Operator Overloading*.

Overloading Functions

- Functions can be addressed by name in a scope
- It is even possible to declare and to defined several functions with the same name
- the “correct” version is chosen according to the *signature* of the function.

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }          // f1
int sq (int x) { ... }                // f2
int pow (int b, int e) { ... }        // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call (we do not go into details)

```
std::cout << sq (3);     // compiler chooses f2
std::cout << sq (1.414); // compiler chooses f1
std::cout << pow (2);    // compiler chooses f4
std::cout << pow (3,3);  // compiler chooses f3
```

Operator Overloading

- Operators are special functions and can be overloaded
- Name of the operator *op*:

```
operatorop
```

- we already know that, for example, `operator+` exists for different types

Adding `rational` Numbers – Before

```
// POST: return value is the sum of a and b
rational add (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = add (r, s);
```

Adding `rational` Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = r + s;
```

infix notation

Other Binary Operators for Rational Numbers

```
// POST: return value is difference of a and b
rational operator- (rational a, rational b);

// POST: return value is the product of a and b
rational operator* (rational a, rational b);

// POST: return value is the quotient of a and b
// PRE: b != 0
rational operator/ (rational a, rational b);
```

Unary Minus

has the same symbol as the binary minus but only one argument:

```
// POST: return value is -a
rational operator- (rational a)
{
    a.n = -a.n;
    return a;
}
```

Comparison Operators

are not built in for structs, but can be defined

```
// POST: returns true iff a == b
bool operator== (rational a, rational b)
{
    return a.n * b.d == a.d * b.n;
}
```

$$\frac{2}{3} = \frac{4}{6} \quad \checkmark$$

Arithmetic Assignment

We want to write

```
rational r;
r.n = 1; r.d = 2;                 // 1/2

rational s;
s.n = 1; s.d = 3;                 // 1/3

r += s;
std::cout << r.n << "/" << r.d;   // 5/6
```

Operator+= First Trial

```
rational operator+= (rational a, rational b)
{
    a.n = a.n ∗ b.d + a.d ∗ b.n;
    a.d ∗= b.d;
    return a;
}
```

does not work. Why?

- The expression `r += s` has the desired value, but because the arguments are R-values (call by value!) it does not have the desired effect of modifying `r`.
- The result of `r += s` is, against the convention of C++ no L-value.

Operator +=

```
rational& operator+= (rational& a, rational b)
{
    a.n = a.n ∗ b.d + a.d ∗ b.n;
    a.d ∗= b.d;
    return a;
}
```

this works

- The L-value `a` is increased by the value of `b` and returned as L-value

`r += s;` now has the desired effect.

In/Output Operators

can also be overloaded.

- Before:

```
std::cout << "Sum is "
          << t.n << "/" << t.d << "\n";
```

- After (desired):

```
std::cout << "Sum is "
          << t << "\n";
```

In/Output Operators

can be overloaded as well:

```
// POST: r has been written to out
std::ostream& operator<< (std::ostream& out,
                          rational r)
{
   return out << r.n << "/" << r.d;
}
```

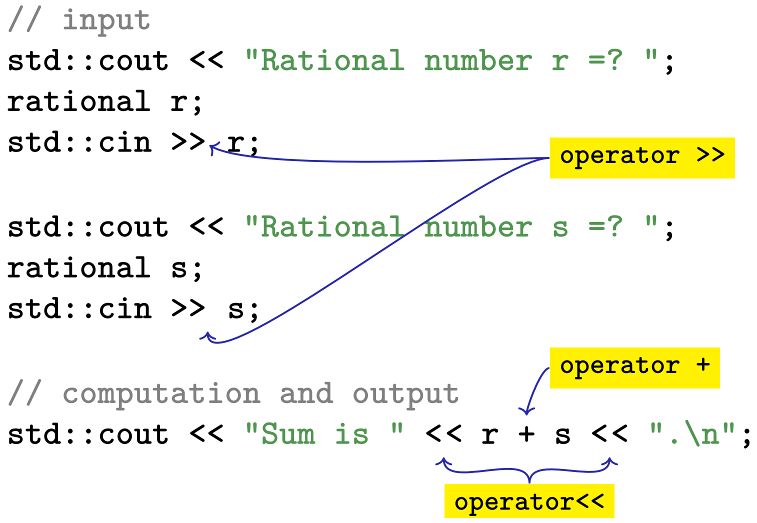
writes r to the output stream
and returns the stream as L-value.

Input

```
// PRE: in starts with a rational number
// of the form "n/d"
// POST: r has been read from in
std::istream& operator>> (std::istream& in,
                          rational& r){
   char c; // separating character '/'
   return in >> r.n >> c >> r.d;
}
```

reads r from the input stream
and returns the stream as L-value.

Goal Attained!



```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;

std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```