10. Functions II

Pre- and Postconditions Stepwise Refinement, Scope, Libraries and Standard Functions

Pre- and Postconditions

- characterize (as complete as possible) what a function does
- document the function for users and programmers (we or other people)
- make programs more readable: we do not have to understand *how* the function works
- are ignored by the compiler
- Pre and postconditions render statements about the correctness of a program possible – provided they are correct.

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

0^e is undefined for $e < 0$

```
// PRE: e >= 0 || b != 0.0
```


Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Here only value, no effect.

```
// POST: return value is b^e
```

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Funktion **pow**: works for all numbers $b \neq 0$

Pre- and Postconditions

- We do not make a statement about what happens if the precondition does not hold.
- C++-standard-slang: "Undefined behavior".

Function **pow**: division by 0

Pre- and Postconditions

- pre-condition should be as *weak* as possible (largest possible domain)
- post-condition should be as *strong* as possible (most detailed information)

White Lies...

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect:

- Overflow if e or b are too large
- b^e potentially not representable as a double (holes in the value range!)

White Lies are Allowed

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

The exact pre- and postconditions are platform-dependent and often complicated. We abstract away and provide the mathematical conditions. ⇒ compromise between formal correctness and lax practice.

Checking Preconditions...

- Preconditions are only comments.
- How can we ensure that they hold when the function is called?

...with assertions

```
#include <cassert>
...
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e) {
    assert (e >= 0 || b != 0);
    double result = 1.0;
    ...
}
```


Postconditions with Asserts

- The result of "complex" computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

```
// PRE: the discriminant p*p/4 − q is nonnegative
// POST: returns larger root of the polynomial x^2 + p x + q
double root(double p, double q)
{
    assert(p*p/4 >= q); // precondition
    double x1 = − p/2 + sqrt(p*p/4 − q);
    assert(equals(x1*x1+p*x1+q,0)); // postcondition
    return x1;
}
```

Exceptions

- Assertions are a rough tool; if an assertions fails, the program is halted in a unrecoverable way.
- C++provides more elegant means (exceptions) in order to deal with such failures depending on the situation and potentially without halting the program
- Failsafe programs should only halt in emergency situations and therefore should work with exceptions. For this course, however, this goes too far.

Stepwise Refinement

- A simple *technique* to solve complex problems

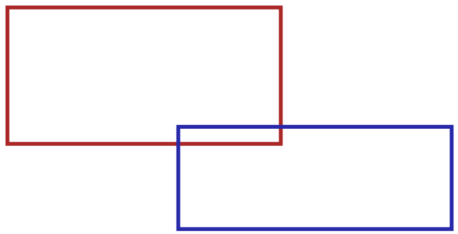
©Niklaus Wirth. Program development by stepwise refinement. Commun. ACM 14, 4, 1971

Stepwise Refinement

- Solve the problem step by step. Start with a coarse solution on a high level of abstraction (only comments and abstract function calls)
- At each step, comments are replaced by program text, and functions are implemented (using the same principle again)
- The refinement also refers to the development of data representation (more about this later).
- If the refinement is realized as far as possible by functions, then partial solutions emerge that might be used for other problems.
- Stepwise refinement supports (but does not replace) the structural understanding of a problem.

Example Problem

Find out if two rectangles intersect!

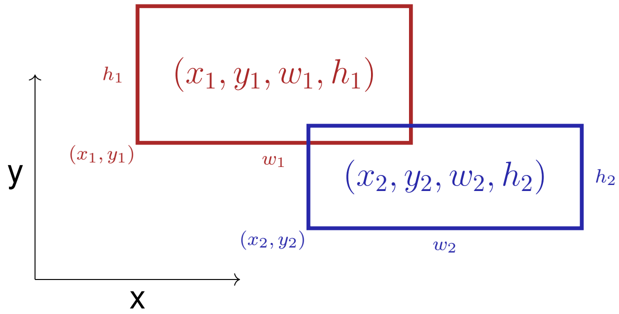


Coarse Solution

(include directives omitted)

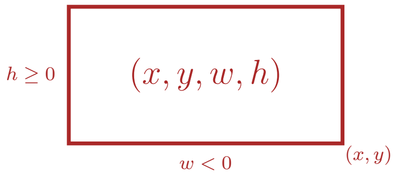
```
int main()
{
    // input rectangles

    // intersection?

    // output solution

    return 0;
}
```


Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

Width w and height h may be negative.



Refinement 1: Input Rectangles

```
int main()
{
    std::cout << "Enter two rectangles [x y w h each] \n";
    int x1, y1, w1, h1;
    std::cin >> x1 >> y1 >> w1 >> h1;
    int x2, y2, w2, h2;
    std::cin >> x2 >> y2 >> w2 >> h2;

    // intersection?

    // output solution

    return 0;
}
```

Refinement 2: Intersection? and Output

```
int main()
{
    input rectangles ✓

    bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);

    if (clash)
        std::cout << "intersection!\n";
    else
        std::cout << "no intersection!\n";

    return 0;
}
```


Refinement 3: Intersection Function. . .

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}

int main() {
    input rectangles ✓

    intersection? ✓

    output solution ✓

    return 0;
}
```


Refinement 3: Intersection Function. . .

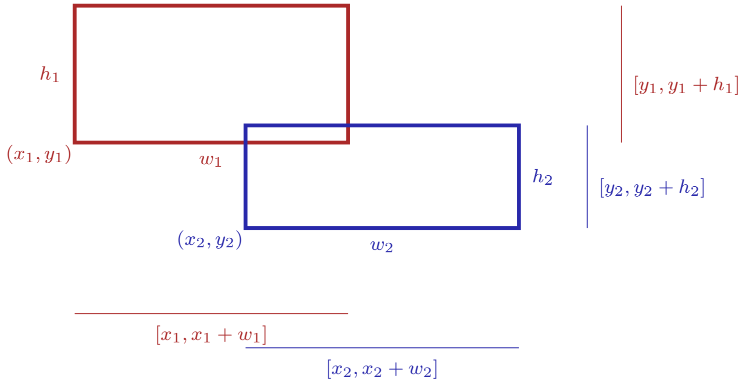
```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}

Function main ✓
```


Refinement 3: . . . with PRE and POST

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles,
//      where w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1) and
//      (x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

Refinement 4: Interval Intersection

Two rectangles intersect if and only if their x and y-intervals intersect.



Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2); ✓
}
```


Refinement 4: Interval Intersections

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//     with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return false; // todo
}
```

Function rectangles_intersect ✓

Function main ✓

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//     with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2); ✓
}
```

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

already exists in the standard library

Function `intervals_intersect` ✓

Function `rectangles_intersect` ✓

Function `main` ✓

Back to Intervals

```
// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return std::max(a1, b1) >= std::min(a2, b2)
        && std::min(a1, b1) <= std::max(a2, b2); ✓
}
```

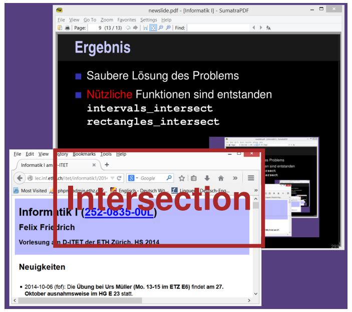

Look what we have achieved step by step!

```
#include <iostream>
#include <algorithm>

// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
  return std::max(a1, b1) >= std::min(a2, b2)
      && std::min(a1, b1) <= std::max(a2, b2);
}

// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

```
int main ()
{
  std::cout << "Enter two rectangles [x y w h each]\n";
  int x1, y1, w1, h1;
  std::cin >> x1 >> y1 >> w1 >> h1;
  int x2, y2, w2, h2;
  std::cin >> x2 >> y2 >> w2 >> h2;
  bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);
  if (clash)
    std::cout << "intersection!\n";
  else
    std::cout << "no intersection!\n";
  return 0;
}
```


Result

- Clean solution of the problem
- Useful functions have been implemented
  `intervals_intersect`
  `rectangles_intersect`



Where can a Function be Used?

```
#include <iostream>

int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

Scope of a Function

- is the part of the program where a function can be called
- is defined as the union of all scopes of its declarations (there can be more than one)

declaration of a function: like the definition but without `{...}`.

```
double pow(double b, int e);
```


This does not work...

```
#include <iostream>


int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

...but this works!

```
#include <iostream>
int f(int i); // Gueltigkeitsbereich von f ab hier

int main()
{
    std::cout << f(1);
    return 0;
}

int f(int i)
{
    return i;
}
```

Forward Declarations, why?

Functions that mutually call each other:

```
int g(...); // forward declaration

int f(...) // f valid from here
{
    g(...) // ok
}

int g(...)
{
    f(...) // ok
}
```

Gültigkeit g

Gültigkeit f

Reusability

- Functions such as `rectangles_intersect` and `pow` are useful in many programs.
- “Solution”: copy-and-paste the source code
- Main disadvantage: when the function definition needs to be adapted, we have to change *all* programs that make use of the function

Level 1: Outsource the Function

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```


Level 1: Include the Function

```
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"

int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

file in working directory

Disadvantage of Including

- `#include` copies the file (`mymath.cpp`) into the main program (`callpow2.cpp`).
- The compiler has to (re)compile the function definition for each program
- This can take long for many and large functions.

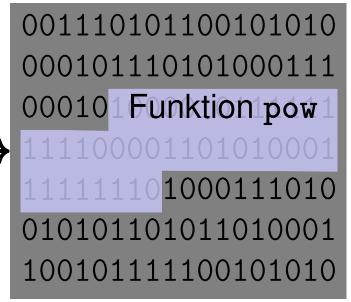
Level 2: Separate Compilation

of `mymath.cpp` independent of the main program:

```
double pow(double b,
           int e)
{
    ...
}
```

mymath.cpp

`g++ -c mymath.cpp`



mymath.o

Level 2: Separate Compilation

Declaration of all used symbols in so-called *header* file.

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e);
```

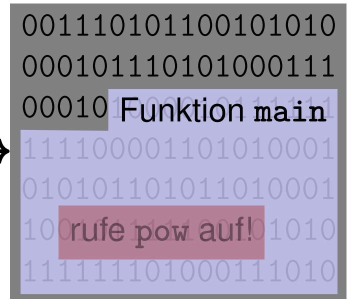
mymath.h

Level 2: Separate Compilation

of the main program, independent of `mymath.cpp`, if a *declaration* from `mymath` is included.

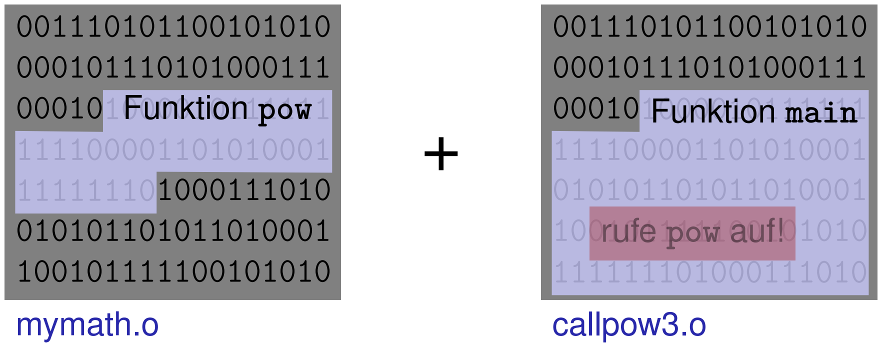
```
#include <iostream>
#include "mymath.h"
int main()
{
  std::cout << pow(2,-2) << "\n";
  return 0;
}
```

callpow3.cpp

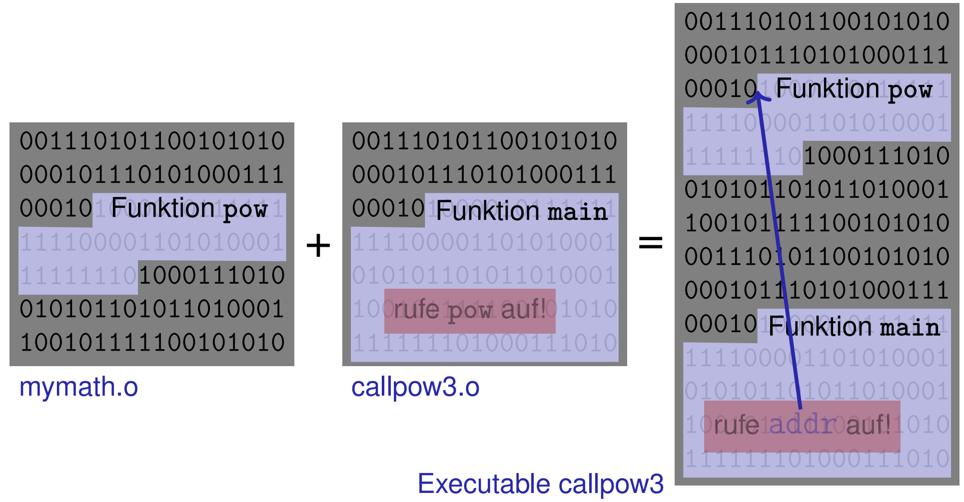


callpow3.o

The linker unites...



... what belongs together



Availability of Source Code?

Observation

`mymath.cpp` (source code) is not required any more when the `mymath.o` (object code) is available.

Many vendors of libraries do not provide source code.

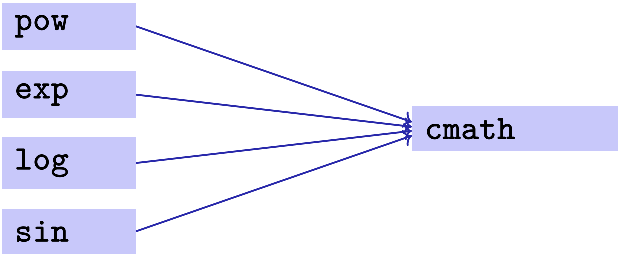
Header files then provide the *only* readable informations.

Open-Source Software

- Source code is generally available.
- Only this allows the continued development of code by users and dedicated “hackers”.
- Even in commercial domains, open-source software gains ground.
- Certain licenses force naming sources and open development. Example GPL (GNU Genereal Public License)
- Known open-source software: Linux (operating system), Firefox (browser), Thunderbird (email program)...

Libraries

- Logical grouping of similar functions



Name Spaces. . .

```
// cmath
namespace std {

  double pow(double b, int e);

  ....
  double exp(double x);
  ...
}
```


. . . Avoid Name Conflicts

```
#include <cmath>
#include "mymath.h"

int main()
{
    double x = std::pow(2.0, −2); // <cmath>
    double y = pow(2.0, −2); // mymath.h
}
```


Name Spaces / Compilation Units

In C++ the concept of separate compilation is *independent* of the concept of name spaces

In some other languages,e.g. Modula / Oberon (partially also for Java) the compilation unit can define a name space.

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with `std::pow`);
- lead to interesting and efficient programs in a simple way;
- guarantee a quality standard that cannot easily be achieved with code written from scratch.

Example: Prime Number Test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, n-1\}$ dividing n .

```
unsigned int d;
for (d=2; n % d != 0; ++d);
```


Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

- This works because `std::sqrt` rounds to the next representable `double` number (IEEE Standard 754).

Prime Number test with `sqrt`

```
// Test if a given natural number is prime.
#include <iostream>
#include <cassert>
#include <cmath>

int main ()
{
  // Input
  unsigned int n;
  std::cout << "Test if n>1 is prime for n =? ";
  std::cin >> n;
  assert (n > 1);

  // Computation: test possible divisors d up to sqrt(n)
  unsigned int bound = std::sqrt(n);
  unsigned int d;
  for (d = 2; d <= bound && n % d != 0; ++d);@

  // Output
  if (d <= bound)
    // d is a divisor of n in {2,...,[sqrt(n)]}
    std::cout << n << " = " << d << " * " << n / d << ".\n";
  else
    // no proper divisor found
    std::cout << n << " is prime.\n";

  return 0;
}
```

Functions Should be More Capable! Swap ?

```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // fail!
}
```


Functions Should be More Capable! Swap ?

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // ok!
}
```


Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.
- Not a new concept specific to functions, but rather a new class of types

Reference types (e.g. `int&`)