3. Logical Values

Boolean Functions; the Type `bool`; logical and relational operators; shortcut evaluation

Our Goal

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Behavior depends on the value of a Boolean expression

Boolean Values in Mathematics

Boolean expressions can take on one of two values:

0 or *1*

- *0* corresponds to *“false”*
- *1* corresponds to *“true”*

The Type `bool` in C++

- represents *logical values*
- Literals `false` and `true`
- Domain {*false*, *true*}

```
bool b = true; // Variable with value true
```

Relational Operators

```
a < b     (smaller than)
a >= b    (greater than)
a == b    (equals)
a != b    (not equal)
```

arithmetic type $\times$ arithmetic type $\rightarrow$ `bool`

R-value $\times$ R-value $\rightarrow$ R-value

Table of Relational Operators

	Symbol	Arity	Precedence	Associativity
smaller	`<`	2	11	left
greater	`>`	2	11	left
smaller equal	`<=`	2	11	left
greater equal	`>=`	2	11	left
equal	`==`	2	10	left
unequal	`!=`	2	10	left

arithmetic type $\times$ arithmetic type $\rightarrow$ `bool`

R-value $\times$ R-value $\rightarrow$ R-value

Boolean Functions in Mathematics

- Boolean function

$$f : \{0,1\}^2 \rightarrow \{0,1\}$$

- 0 corresponds to "false".
- 1 corresponds to "true".

$\mathrm{AND}(x, y)$ $\quad x \wedge y$

- "logical And"

$$f : \{0,1\}^2 \rightarrow \{0,1\}$$

- 0 corresponds to "false".
- 1 corresponds to "true".

x	y	$\mathrm{AND}(x,y)$
0	0	0
0	1	0
1	0	0
1	1	1

Logical Operator &&

`a && b` (logical and)

bool × bool → bool

R-value × R-value → R-value

```
int n = −1;
int p = 3;
bool b = (n < 0) && (0 < p); // b = true
```


$\mathrm{OR}(x, y)$ $x \vee y$

- "logical Or"

$$f : \{0,1\}^2 \to \{0,1\}$$

- 0 corresponds to "false".
- 1 corresponds to "true".

x	y	$\mathrm{OR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	1

Logical Operator ||

`a || b` (logical or)

bool × bool → bool

R-value × R-value → R-value

```
int n = 1;
int p = 0;
bool b = (n < 0) || (0 < p); // b = false
```


$\mathrm{NOT}(x)$ $\neg x$

- "logical Not"

$$f : \{0,1\} \to \{0,1\}$$

- 0 corresponds to "false".
- 1corresponds to "true".

x	$\mathrm{NOT}(x)$
0	1
1	0

Logical Operator !

`!b` (logical not)

`bool` → `bool`

R-value → R-value

```
int n = 1;
bool b = !(n < 0); // b = true
```


Precedences

```
!b && a
   ⇕
(!b) && a

a && b || c && d
        ⇕
(a && b) || (c && d)

a || b   &&   c || d
        ⇕
a || (b && c) || d
```


Table of Logical Operators

	Symbol	Arity	Precedence	Associativity
Logical and (AND)	&&	2	6	left
Logical or (OR)	||	2	5	left
Logical not (NOT)	!	1	16	right

Precedences

The unary logical operator !

binds more strongly than

binary arithmetic operators. These

bind more strongly than

relational operators,

and these bind more strongly than

binary logical operators.

```
7 + x < y && y != 3 * z || ! b
7 + x < y && y != 3 * z || (!b)
```

Completeness

- AND, OR and NOT are the boolean functions available in C++.
- Any other *binary* boolean function can be generated from them.

x	y	$\text{XOR}(x,y)$
0	0	0
0	1	1
1	0	1
1	1	0

Completeness: $\text{XOR}(x,y)$ $x \oplus y$

$$\text{XOR}(x,y) = \text{AND}(\text{OR}(x,y), \text{NOT}(\text{AND}(x,y))).$$

$$x \oplus y = (x \vee y) \wedge \neg(x \wedge y).$$

```
(x || y) && !(x && y)
```


Completeness Proof

- Identify binary boolean functions with their characteristic vector.

x	y	$\text{XOR}(x,y)$
0	0	0
0	1	1
1	0	1
1	1	0

characteristic vector: 0110

$$\text{XOR} = f_{0110}$$

Completeness Proof

- Step 1: generate the *fundamental* functions f_{0001}, f_{0010}, f_{0100}, f_{1000}

$$f_{0001} = \text{AND}(x,y)$$
$$f_{0010} = \text{AND}(x, \text{NOT}(y))$$
$$f_{0100} = \text{AND}(y, \text{NOT}(x))$$
$$f_{1000} = \text{NOT}(\text{OR}(x,y))$$

Completeness Proof

- Step 2: generate all functions by applying logical or

$$f_{1101} = \mathrm{OR}(f_{1000}, \mathrm{OR}(f_{0100}, f_{0001}))$$

- Step 3: generate f_{0000}

$$f_{0000} = 0.$$

`bool` vs `int`: Conversion

- `bool` can be used whenever `int` is expected – and vice versa.
- Many existing programs use `int` instead of `bool`
  This is bad style originating from the language C .

`bool`	$\rightarrow$ `int`
true	$\rightarrow$ 1
false	$\rightarrow$ 0

`int`	$\rightarrow$ `bool`
$\neq$0	$\rightarrow$ *true*
0	$\rightarrow$ *false*

```
bool b = 3; // b=true
```


DeMorgan Rules

- `!(a && b) == (!a || !b)`
- `!(a || b) == (!a && !b)`

! (rich *and* beautiful) == (poor *or* ugly)

Application: either ... or (XOR)

`(x || y) && !(x && y)`	x or y, and not both
`(x || y) && (!x || !y)`	x or y, and one of them not
`!(!x && !y) && !(x && y)`	not none and not both
`!(!x && !y || x && y)`	not: both or none

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

```
x != 0 && z / x > y
```

$\Rightarrow$ No division by 0

4. Defensive Programming

Constants and Assertions

Sources of Errors

- Errors that the compiler can find:
  syntactical and some semantical errors
- Errors that the compiler cannot find:
  runtime errors (always semantical)

The Compiler as Your Friend: Constants

Constants

- are variables with immutable value

```
const int speed_of_light = 299792458;
```

- Usage: `const` before the definition

The Compiler as Your Friend: Constants

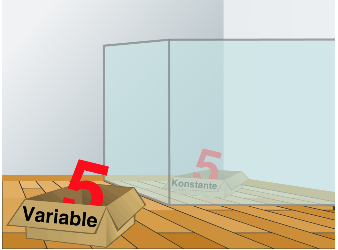
- Compiler checks that the `const`-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise :*"value does not change"*

Constants: Variables behind Glass



The `const`-guideline

`const`-guideline

For *each variable*, think about whether it will change its value in the lifetime of a program. If not, use the keyword `const` in order to make the variable a constant.

A program that adheres to this guideline is called `const`-correct.

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior
2. Check at many places in the code if the program is still on track
3. Question the (seemingly) obvious, there could be a typo in the code

Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression `expr` is false
- requires `#include <cassert>`
- can be switched off (potential performance gain)

Assertions for the $gcd(x, y)$

Check if the program is on track . . .

```
// Input x and y
std::cout << "x =? ";
std::cin >> x;
std::cout << "y =? ";
std::cin >> y;

// Check validity of inputs
assert(x > 0 && y > 0);

... // Compute gcd(x,y), store result in variable a
```

Input arguments for calculation

Precondition for the ongoing computation

Assertions for the $gcd(x, y)$

... and question the obvious! . . .

```
...
assert(x > 0 && y > 0);

... // Compute gcd(x,y), store result in variable a

assert (a >= 1);
assert (x % a == 0 && y % a == 0);
for (int i = a+1; i <= x && i <= y; ++i)
  assert(!(x % i == 0 && y % i == 0));
```

Precondition for the ongoing computation

Properties of the gcd

Switch off Assertions

```
#define NDEBUG // To ignore assertions
#include<cassert>

...
assert(x > 0 && y > 0); // Ignored

... // Compute gcd(x,y), store result in variable a

assert(a >= 1); // Ignored
...
```

Fail-Fast with Assertions

- Real software: many C++ files, complex control flow
- Errors surface late(r) → impedes error localisation
- Assertions: Detect errors early

5. Control Structures I

Selection Statements, Iteration Statements, Termination, Blocks

Control Flow

- Up to now: *linear* (from top to bottom)
- Interesting programs require "branches" and "jumps"

0	1	2	3	4	5	6	7	8	9
[8] → L	[9] → R	L = 0? *stop*	R > L? *springe zu* 6	L − R → [8]	*springe zu* 0	R − L → [9]	*springe zu* 0	b	a

Selection Statements

implement branches

- `if` statement
- `if-else` statement

if-Statement

if (*condition*)
    statement

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

If *condition* is true then *statement* is executed

- *statement*: arbitrary statement (*body* of the `if`-Statement)
- *condition*: convertible to `bool`

if-else-statement

if (*condition*)
    statement1
else
    statement2

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

If *condition* is true then *statement1* is executed, otherwise *statement2* is executed.

- *condition*: convertible to `bool`.
- *statement1*: *body* of the `if`-branch
- *statement2*: *body* of the `else`-branch

Layout!

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";      ← Indentation
else
    std::cout << "odd";       ← Indentation
```


Iteration Statements

implement “loops”

- `for`-statement
- `while`-statement
- `do`-statement

Compute $1 + 2 + ... + n$

```
// Program: sum_n.cpp
// Compute the sum of the first n natural numbers.

#include <iostream>

int main()
{
  // input
  std::cout << "Compute the sum 1+...+n for n =? ";
  unsigned int n;
  std::cin >> n;

  // computation of sum_{i=1}^n i
  unsigned int s = 0;
  for (unsigned int i = 1; i <= n; ++i) s += i;

  // output
  std::cout << "1+...+" << n << " = " << s << ".\n";
  return 0;
}
```


`for`-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	wahr	`s == 1`
`i==2`	wahr	`s == 3`
`i==3`	falsch	

`s == 3`

Gauß as a Child (1777 - 1855)

- As you probably know, there exists a more efficient way to compute the sum of the first n natural numbers. Here's a corresponding anecdote:
- Math-teacher wanted to keep the pupils busy with the following task:

Compute the sum of numbers from 1 to 100!

- Gauß finished after one minute.

The Solution of Gauß

- The requested number is

$$1 + 2 + 3 + \cdots + 98 + 99 + 100.$$

- This is half of

$$\begin{array}{rrrrrrrrr} & 1 & + & 2 & + \cdots + & 99 & + & 100 \\ + & 100 & + & 99 & + \cdots + & 2 & + & 1 \\ \hline = & 101 & + & 101 & + \cdots + & 101 & + & 101 \end{array}$$

- Answer: $100 \cdot 101/2 = 5050$

`for`-Statement: Syntax

```
for (init statement; condition; expression)
    body statement
```

- *init statement*: expression statement, declaration statement, null statement
- *condition*: convertible to `bool`
- *expression*: any expression
- *body statement*: any statement (*body* of the for-statement)

`for`-Statement: semantics

```
for ( init statement  condition ; expression )
    statement
```

- *init-statement* is executed
- *condition* is evaluated
  - `true`: Iteration starts
    statement is executed
    expression is executed
  - false: `for`-statement is ended.

`for`-Statement: Termination

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

Here and in most cases:

- *expression* changes its value that appears in *condition* .
- After a finite number of iterations *condition* becomes false: *Termination*

Infinite Loops

- Infinite loops are easy to generate:

```
for ( ; ; ) ;
```

  - Die *empty condition* is true.
  - Die *empty expression* has no effect.
  - Die *null statement* has no effect.

- ... but can in general not be automatically detected.

```
for (init cond; expr) stmt;
```

Halting Problem

Undecidability of the Halting Problem

There is no C++ program that can determine for each C++-Program P and each input I if the program P terminates with the input I.

This means that the correctness of programs can in general *not* be automatically checked.[4]

[4]Alan Turing, 1936. Theoretical questions of this kind were the main motivation for Alan Turing to construct a computing machine.

Example: Prime Number Test

Def.: a natural number $n \geq 2$ is a prime number, if no $d \in \{2, \ldots, n-1\}$ divides n .

A loop that can test this:

```
unsigned int d;
for (d=2; n%d != 0; ++d);
```


Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value `d=2`, then plus 1 in every iteration (`++d`)
- Exit: `n%d != 0` evaluates to `false` as soon as a divisor is found — at the latest, once `d == n`
- Progress guarantees that the exit condition will be reached

Example: Correctness

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

Every potential divisor `2 <= d <= n` will be tested. If the loop terminates with `d == n` then and only then is `n` prime.

Blocks

- Blocks group a number of statements to a new statement

{statement1 statement2 ... statementN}

- Example: body of the main function

```
int main() {
    ...
}
```

- Example: loop body

```
for (unsigned int i = 1; i <= n; ++i) {
    s += i;
    std::cout << "partial sum is " << s << "\n";
}
```