24. Subtyping, Inheritance and Polymorphism

Expression Trees, Separation of Concerns and Modularisation, Type Hierarchies, Virtual Functions, Dynamic Binding, Code Reuse, Concepts of Object-Oriented Programming

Goal: Represent arithmetic expressions, e.g.

2 + 3 * 2

Goal: Represent arithmetic expressions, e.g.

2 + 3 * 2

Arithmetic expressions form a *tree structure*



Goal: Represent arithmetic expressions, e.g.

2 + 3 * 2

Arithmetic expressions form a *tree structure*



Expression trees comprise *different* nodes:

Goal: Represent arithmetic expressions, e.g.

2 + 3 * 2

Arithmetic expressions form a *tree structure*



■ Expression trees comprise different nodes: literals (e.g. 2), binary operators (e.g. +), unary operators (e.g. √), function applications (e.g. cos), etc.

Disadvantages

Implemented via *a single* node type:



Observation: **tnode** is the "sum" of all required nodes (constants, addition, \ldots) \Rightarrow memory wastage, inelegant

Disadvantages

Observation: tnode is the "sum" of all required nodes -

Observation: **tnode** is the "sum" of all required nodes – and every function must "dissect" this "sum", e.g.:

```
double eval(const tnode* n) {
  if (n->op == '=') return n->val; // n is a constant
  double l = 0;
  if (n->left) l = eval(n->left); // n is not a unary operator
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l+r; // n is an addition node
    case '*': return l*r; // ...
  ...
```

Observation: **tnode** is the "sum" of all required nodes – and every function must "dissect" this "sum", e.g.:

```
double eval(const tnode* n) {
  if (n->op == '=') return n->val; // n is a constant
  double l = 0;
  if (n->left) l = eval(n->left); // n is not a unary operator
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l+r; // n is an addition node
    case '*': return l*r; // ...
   ...
```

 \Rightarrow Complex, and therefore error-prone

```
struct tnode {
   char op;
   double val;
   tnode* left;
   tnode* right;
   ...
};
```

```
double eval(const tnode* n) {
  if (n->op == '=') return n->val;
  double l = 0;
  if (n->left) l = eval(n->left);
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l+r;
    case '*': return l*r;
    ...
```

This code isn't *modular* – we'll change that today!

1. Subtyping

Type hierarchy: Exp represents general expressions, Literal etc. are concrete expression



1. Subtyping

- Type hierarchy: Exp represents general expressions, Literal etc. are concrete expression
- Every Literal etc. also is an Exp (subtype relation)



1. Subtyping

- Type hierarchy: Exp represents general expressions, Literal etc. are concrete expression
- Every Literal etc. also is an Exp (subtype relation)



That's why a Literal etc. can be used everywhere, where an Exp is expected:

Exp* e = new Literal(132);

2. Polymorphism and Dynamic Dispatch

■ A variable of *static* type **Exp** can "host" expressions of different *dynamic* types:

Exp* e = new Literal(2); // e is the literal 2
e = new Addition(e, e); // e is the addition 2 + 2

2. Polymorphism and Dynamic Dispatch

A variable of static type Exp can "host" expressions of different dynamic types:

Exp* e = new Literal(2); // e is the literal 2

e = new Addition(e, e); // e is the addition 2 + 2

Executed are the member functions of the *dynamic* type:

```
Exp* e = new Literal(2);
std::cout << e->eval(); // 2
e = new Addition(e, e);
std::cout << e->eval(); // 4
```

3. Inheritance

 Certain functionality is shared among type hierarchy members



3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (Addition, Times):
 - 1 + size(left operand) + size(right operand)



3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (Addition, Times):

 $1 + size(left \ operand) + size(right \ operand)$

⇒ Implement functionality once, and let subtypes *inherit* it



Advantages

 Subtyping, inheritance and dynamic binding enable modularisation through spezialisation



Exp* e = new Literal(2); std::cout << e->eval();

e = new Addition(e, e); std::cout << e->eval();

Advantages

- Subtyping, inheritance and dynamic binding enable modularisation through spezialisation
- Inheritance enables sharing common code across modules
 ⇒ avoid code duplication



Exp* e = n	lew	Literal(2);
std::cout	<<	e->eval();

e = new Addition(e, e); std::cout << e->eval();





Note: Today, we focus on the new concepts (subtyping, ...) and ignore the orthogonal aspect of encapsulation (class, private vs. public member variables)







BinExp is a subclass¹ of Exp
Exp is the superclass² of BinExp
BinExp inherits from Exp



BinExp is a subclass¹ of Exp
Exp is the superclass² of BinExp
BinExp inherits from Exp
BinExp publicly inherits from Exp (public), that's why BinExp is a subtype of Exp



BinExp is a subclass¹ of Exp
Exp is the superclass² of BinExp
BinExp inherits from Exp
BinExp publicly inherits from Exp (public), that's why BinExp is a subtype of Exp
Analogously: Times and BinExp



■ **BinExp** is a subclass¹ of **Exp Exp** is the superclass² of **BinExp BinExp** inherits from **Exp BinExp** *publicly* inherits from **Exp** (**public**), that's why **BinExp** is a subtype of Exp Analogously: Times and BinExp ■ Subtype relation is transitive: **Times** is also a subtype of Exp

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Exp {
   virtual int size() const = 0;
   virtual double eval() const = 0;
};   Activates dynamic dispatch
```





```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {
  double val;
  Literal(double v);
  int size() const;
  double eval() const;
};
```

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {  Literal inherits from Exp...
double val;
Literal(double v);
int size() const;
double eval() const;
};
```

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {  Literal inherits from Exp ...
double val;
Literal(double v);
int size() const;  ... but is otherwise just a regular class
double eval() const;
};
```

Literal: Implementation

Literal::Literal(double v): val(v) {}
Literal::Literal(double v): val(v) {}

```
int Literal::size() const {
  return 1;
}
```

Literal::Literal(double v): val(v) {}

```
int Literal::size() const {
  return 1;
}
```

```
double Literal::eval() const {
  return this->val;
}
```

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

Literal* lit = new Literal(5);

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

Literal* lit = new Literal(5);
Exp* e = lit; // OK: Literal is a subtype of Exp

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

Literal* lit = new Literal(5);
Exp* e = lit; // OK: Literal is a subtype of Exp

But not vice versa:

Exp* e = ... Literal* lit = e; // ERROR: Exp is not a subtype of Literal

```
struct Exp {
    ...
    virtual double eval();
};
double Literal::eval() {
    return this->val;
}
```

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

```
struct Exp {
    ...
    virtual double eval();
};
double Literal::eval() {
    return this->val;
}
```

 virtual member function: the dynamic (here: Literal) type determines the member function to be executed
 ⇒ dynamic binding

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

```
struct Exp {
    ...
    virtual double eval();
};
```

```
double Literal::eval() {
  return this->val;
}
```

```
■ virtual member function: the
dynamic (here: Literal) type
determines the member function to
be executed
⇒ dynamic binding
```

```
    Without Virtual the static type (hier:
Exp) determines which function is
executed
```

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

```
struct Exp {
```

. . .

```
virtual double eval();
};
```

```
double Literal::eval() {
  return this->val;
}
```

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

- virtual member function: the dynamic (here: Literal) type determines the member function to be executed ⇒ dynamic binding
- Without Virtual the static type (hier: Exp) determines which function is executed

```
We won't go into further details
```

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
   ...
};
```

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
   ...
};
```

struct Times : public Exp {
 Exp* left; // left operand
 Exp* right; // right operand
 ...
};

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

struct Times : public Exp {
 Exp* left; // left operand
 Exp* right; // right operand
 ...
};

```
struct Addition : public Exp {
   Exp* left; // left operand
   Exp* right; // right operand
   ...
};
```

```
struct Addition : public Exp {
   Exp* left; // left operand
   Exp* right; // right operand
   ...
};
int Addition::size() const {
```

+ right->size();

return 1 + left->size()

}



```
struct Addition : public Exp {
   Exp* left; // left operand
   Exp* right; // right operand
   ...
};
int Addition::size() const {
```

+ right->size();

return 1 + left->size()

}

struct Times : public Exp { Exp* left; // left operand Exp* right; // right operand . . . }; int Times::size() const { return 1 + left->size() + right->size(); }



🤬 Code duplication

Extracting Commonalities ...: BinExp

```
struct BinExp : public Exp {
  Exp* left;
  Exp* right;
  BinExp(Exp* 1, Exp* r);
  int size() const;
};
```

BinExp::BinExp(Exp* 1, Exp* r): left(1), right(r) {}

Extracting Commonalities ...: BinExp

```
struct BinExp : public Exp {
  Exp* left;
  Exp* right;
  BinExp(Exp* 1, Exp* r);
  int size() const;
};
```

BinExp::BinExp(Exp* 1, Exp* r): left(1), right(r) {}

```
int BinExp::size() const {
  return 1 + this->left->size() + this->right->size();
}
```

Note: BinExp does not implement eval and is therefore also an abstract class, just like Exp

```
struct Addition : public BinExp {
  Addition(Exp* 1, Exp* r);
  double eval() const;
};
```

```
Addition(Exp* 1, Exp* r);
double eval() const;
};
```

Addition inherits member vari-(size) from BinExp

```
struct Addition : public BinExp {
  Addition(Exp* 1, Exp* r);
  double eval() const;
};
```

Addition::Addition(Exp* 1, Exp* r): BinExp(1, r) {}

Calling the super constructor (constructor of BinExp) initialises the member variables left and right

```
struct Addition : public BinExp {
  Addition(Exp* 1, Exp* r);
  double eval() const;
};
```

Addition::Addition(Exp* 1, Exp* r): BinExp(1, r) {}

```
double Addition::eval() const {
  return
    this->left->eval() +
    this->right->eval();
}
```

... Inheriting Commonalities: Times

```
struct Times : public BinExp {
  Times(Exp* 1, Exp* r);
  double eval() const;
};
```

Times::Times(Exp* 1, Exp* r): BinExp(1, r) {}

```
double Times::eval() const {
  return
    this->left->eval() *
    this->right->eval();
}
```

Observation: Additon::eval() and Times::eval() are very similar and could also be unified. However, this would require the concept of *functional programming*, which is outside the scope of this course.

Further Expressions and Operations

■ Further expressions, as classes derived from **Exp**, are possible, e.g. -, /, √, cos, log

Further Expressions and Operations

- Further expressions, as classes derived from **Exp**, are possible, e.g. -, /, √, cos, log
- A former bonus exercise (included in today's lecture examples on Code Expert) illustrates possibilities: variables, trigonometric functions, parsing, pretty-printing, numeric simplifications, symbolic derivations,

• • •

Mission: Monolithic ightarrow Modular \checkmark



And there is so much more ...

Not shown/discussed:

- Private inheritance (class B : public A)
- Subtyping and polymorphism without pointers
- Non-virtuell member functions and static dispatch (virtual double eval())
- Overriding inherited member functions and invoking overridden implementations
- Multiple inheritance

Object-Oriented Programming

In the last 3rd of the course, several concepts of *object-oriented programming* were introduced, that are briefly summarised on the upcoming slides.

Encapsulation (weeks 10-13):

- Hide the implementation details of types (private section) from users
- Definition of an interface (public area) for accessing values and functionality in a controlled way
- Enables ensuring invariants, and the modification of implementations without affecting user code

Object-Oriented Programming

Subtyping (week 14):

- Type hierarchies, with super- and subtypes, can be created to model relationships between more abstract and more specialised entities
- A subtype supports at least the functionality that its supertype supports typically more, though, i.e. a subtype extends the interface (public section) of its supertype
- That's why supertypes can be used anywhere, where subtypes are required ...
- ...and functions that can operate on more abstract type (supertypes) can also operate on more specialised types (subtypes)
- The streams introduced in week 7 form such a type hierarchy: ostream is the abstract supertyp, ofstream etc. are specialised subtypes

Object-Oriented Programming

Polymorphism and dynamic binding (week 14):

- A pointer of static typ T_1 can, at runtime, point to objects of (dynamic) type T_2 , if T_2 is a subtype of T_1
- When a virtual member function is invoked from such a pointer, the dynamic type determines which function is invoked
- I.e.: despite having the same static type, a different behaviour can be observed when accessing the common interface (member functions) of such pointers
- In combination with subtyping, this enables adding further concrete types (streams, expressions, ...) to an existing system, without having to modify the latter

Inheritance (week 14):

- Derived classes inherit the functionality, i.e. the implementation of member functions, of their parent classes
- This enables sharing common code and thereby avoids code duplication
- An inherited implementation can be overridden, which allows derived classes to behave differently than their parent classes (not shown in this course)

25. Conclusion

Name the most important key words to each chapter. Checklist: "does every notion make some sense for me?"

- 🔘 motivating example for each chapter
- \bigcirc concepts that do not depend from the implementation (language)
- \bigcirc language (C++): all that depends on the chosen language
- (E) examples from the lectures

Kapitelüberblick

- 1. Introduction
- 2. Integers
- 3. Booleans
- 4. Defensive Programming
- **5**./6. Control Statements
- **7./8.** Floating Point Numbers
- 9./10. Functions
- 11. Reference Types
- 12./13. Vectors and Strings
- 14./15. Recursion
- 16. Structs and Overloading
- 17. Classes
- 18./19. Dynamic Datastructures
- 20. Containers, Iterators and Algorithms
- 21. Dynamic Datatypes and Memory Management
- 22. Subtyping, Polymorphism and Inheritance

1. Introduction

M

Euclidean algorithm

- algorithm, Turing machine, programming languages, compilation, syntax and semantics
 - values and effects, fundamental types, literals, variables
- include directive #include <iostream>
 - main function int main(){...}
 - comments, layout // Kommentar
 - types, variables, L-value a , R-value a+b
 - expression statement b=b*b; , declaration statement int a;, return statement return 0;

2. Integers

M

(E)

- Celsius to Fahrenheit
- associativity and precedence, arity
 - expression trees, evaluation order
 - arithmetic operators
 - binary representation, hexadecimal numbers
 - signed numbers, twos complement
- 🕛 🛛 🔳 arithmetic operators 9 * celsius / 5 + 32
 - increment / decrement expr++
 - arithmetic assignment expr1 += expr2
 - \blacksquare conversion int $\leftrightarrow \texttt{unsigned}$ int
 - Celsius to Fahrenheit, equivalent resistance

3. Booleans

🔘 🛛 🗉 Boolean functions, completeness

DeMorgan rules

🛈 🛛 🔳 the type bool

- logical operators a && !b
- relational operators x < y</p>
- precedences 7 + x < y && y != 3 * z
- short circuit evaluation x != 0 && z / x > y
- the assert-statement, #include <cassert>
- 🕒 🔳 Div-Mod identity.
4. Definsive Programming

Assertions and Constants

- The assert-statement, #include <cassert>
 const int speed of light=2999792458
- Assertions for the GCD

5./6. Control Statements

M

- linear control flow vs. interesting programs
- selection statements, iteration statements
 - (avoiding) endless loops, halting problem
 - Visibility and scopes, automatic memory
 - equivalence of iteration statement
- if statements if (a % 2 == 0) {..}
 - for statements for (unsigned int i = 1; i <= n; ++i) ...
 - while and do-statements while $(n > 1) \{...\}$
 - blocks and branches if (a < 0) continue;</p>
 - Switch statement switch(grade) {case 6: }
- sum computation (Gauss), prime number tests, Collatz sequence, Fibonacci numbers, calculator, output grades

7./8. Floating Point Numbers

🔘 🛛 🗉 correct computation: Celsius / Fahrenheit

- fixpoint vs. floating point
 - holes in the value range
 - compute using floating point numbers
 - floating point number systems, normalisation, IEEE standard 754
 - guidelines for computing with floating point numbers
- 🕨 🔳 types float, double
 - floating point literals 1.23e-7f
- 🕒 🛛 🗉 Celsius/Fahrenheit, Euler, Harmonic Numbers

9./10. Functions

(M)

Computation of Powers

- Encapsulation of Functionality
 - functions, formal arguments, arguments
 - scope, forward declarations
 - procedural programming, modularization, separate compilation
 - Stepwise Refinement
- declaration and definition of functions double pow(double b, int e){ ... }
 - function call pow (2.0, -2)
 - the type void
 - 🔹 🔳 powers, perfect numbers, minimum, calendar

11. Reference Types

🕚 🔳 Swap

- value- / reference- semantics, pass by value, pass by reference, return by reference
 - lifetime of objects / temporary objects
 - constants
- 🛈 🛛 🔳 reference type int& a
 - call by reference, return by reference int& increment (int& i)
 - const guideline, const references, reference guideline
- 🗉 🛛 🔳 swap, increment

12./13. Vectors and Strings

- Iterate over data: sieve of erathosthenes
- vectors, memory layout, random access
 - (missing) bound checks
 - vectors

M

Œ

- characters: ASCII, UTF8, texts, strings
- vector types std::vector<int> a {4,3,5,2,1};
 - characters and texts, the type char char c = 'a';, Konversion nach int
 - vectors of vectors
 - Streams std::istream, std::ostream
 - sieve of Erathosthenes, Caesar-code, shortest paths

14./15. Recursion

M

- recursive math. functions, the n-Queen problem, Lindenmayer systems, a command line calculator
- 🖸 🛛 🔹 recursion
 - call stack, memory of recursion
 - correctness, termination,
 - recursion vs. iteration
 - Backtracking, EBNF, formal grammars, parsing
- Interpretended in the second state of the s

16. Structs and Overloading

🔘 🔳 build your own rational number

heterogeneous data types

 \bigcirc

- function and operator overloading
- encapsulation of data
- Istruct definition struct rational {int n; int d;};
 - member access result.n = a.n * b.d + a.d * b.n;
 - initialization and assignment,
 - function overloading pow(2) vs. pow(3,3);, operator overloading
- Interpretended and the second state of the

17. Classes

🕚 🛛 🔳 rational numbers with encapsulation

- 🔘 🛛 🗉 Encapsulation, Construction, Member Functions
- Classes class rational { ... };
 - access control public: / private:
 - member functions int rational::denominator () const
 - The implicit argument of the member functions
- 🕒 🛛 🔳 finite rings, complex numbers

18./19. Dynamic Datastructures

Our own vector

(M)

🔘 🔹 linked list, allocation, deallocation, dynamic data type

🛈 🛛 The **new** statement

- pointer int* x;, Null-pointer nullptr.
- address and derference operator int *ip = &i; int j = *ip;
- pointer and const const int *a;

🕒 📲 linked list, stack

20. Containers, Iterators and Algorithms

🚳 🛛 🔳 vectors are containers

- iteration with pointers
 - containers and iterators
 - algorithms

 \bigcirc

(L)

- Iterators std::vector<int>::iterator
 - Algorithms of the standard library std::fill (a, a+5, 1);
 - implement an iterator
 - iterators and const
- 🗉 🛛 🔳 output a vector, a set

21. Dynamic Datatypes and Memory Management

М 🔳 Stack

(E)

- Expression Tree
- 🔘 🛛 📱 Guideline "dynamic memory"
 - Pointer sharing
 - Dynamic Datatype
 - Tree-Structure
 - new and delete
 - Destructor stack::~stack()
 - Copy-Constructor stack::stack(const stack& s)
 - Assignment operator stack& stack::operator=(const stack& s)
 - Rule of Three
 - Binary Search Tree

22. Subtyping, Polymorphism and Inheritance

🔘 🔹 extend and generalize expression trees

- 🖸 🛛 🛯 Subtyping
 - polymorphism and dynamic binding
 - Inheritance
- base class struct Exp{}
 - derived class struct BinExp: public Exp{}
 - abstract class struct Exp{virtual int size() const = 0...}
 - polymorphie virtual double eval()
- E expression node and extensions

The End

End of the Course