ETH zürich

Felix Friedrich, Malte Schwerhoff

Computer Science

Course at D-MATH / D-PHYS at ETH Zurich

Autumn 2019

Welcome

to the Course Informatik

at the MATH/PHYS departement of ETH Zürich.

Place and time:

Tuesday 13:15 - 15:00, ML D28, ML E12.
Pause 14:00 - 14:15, slight shift possible.

Course web page

`http://lec.inf.ethz.ch/ifmp`

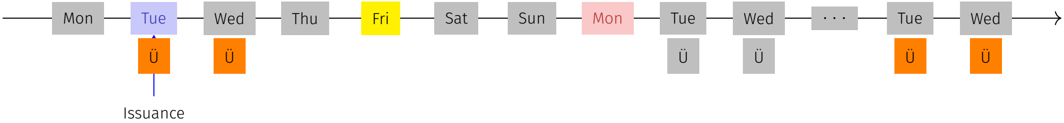
Team

chef assistant	Vytautas Astrauskas	
assistants	Benjamin Rothenberger	Charlotte Franke
	Claire Dick	David Sommer
	Edoardo Mazzoni	Eliza Wszola
	Enis Ulqinaku	Gaspard Zoss
	Janet Greutmann	Jannik Kochert
	Kevin Kaiwen Zhang	Manuel Mekkattu
	Moritz Schneider	Orhan Saeedi
	Raul Rao	Reza Sefidgar
	Sammy Christen	Tanja Kaister
	Tobias Klenze	Viera Klasovita
lecturers	Dr. Malte Schwerhoff / Dr. Felix Friedrich	

Registration for Exercise Sessions

- Registration via web page
- Registration already open

Procedure



- Exercises availabe at lectures
- Preliminary discussion in the following exercise session (on the same/next day)
- StudyCenter (`studycenter.ethz.ch`)
- Solution must be submitted at latest one day before the next lecture (23:59h)
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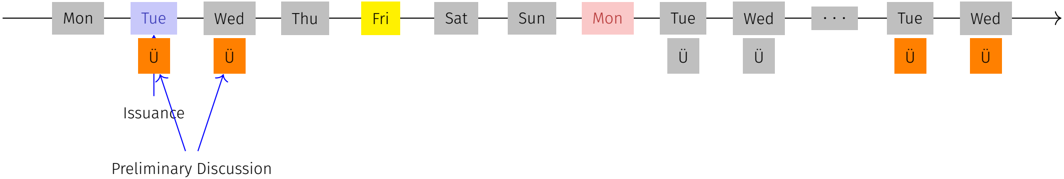

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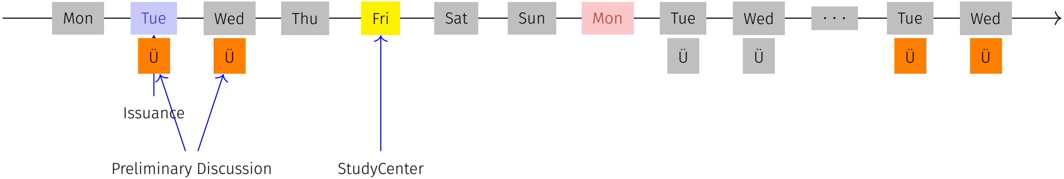

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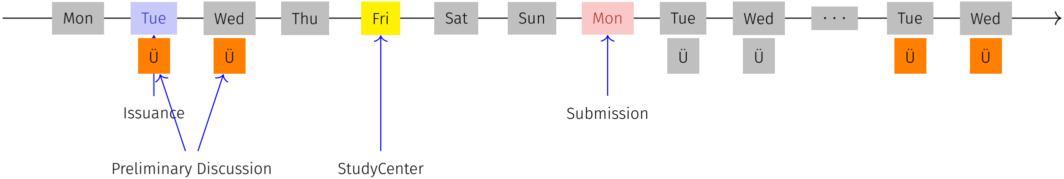

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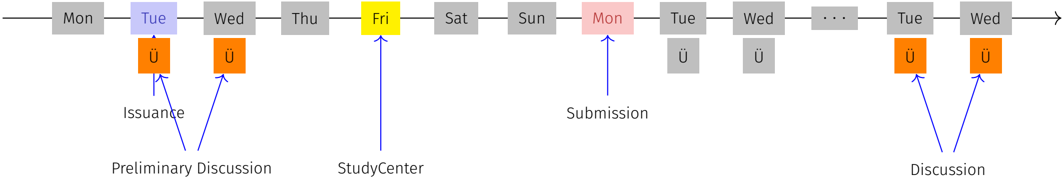

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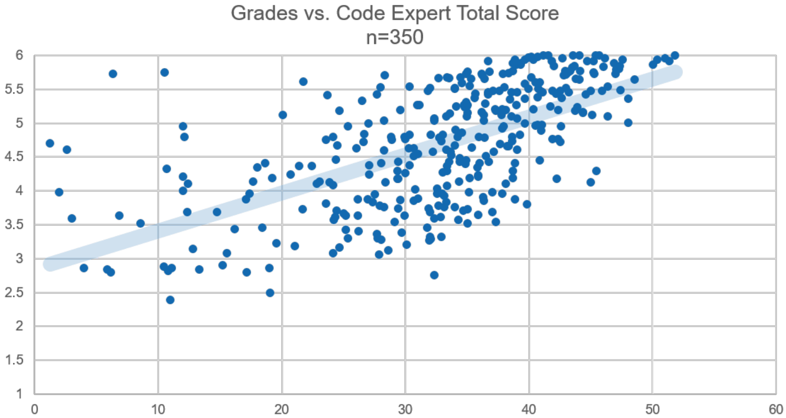

Exams

The exam (in examination period 2018) will cover

- Lectures content (lectures, handouts)
- Exercise content (exercise sessions, exercises).

Exams

Written exam.
We will test your practical skills (programming skills) and theoretical knowledge (background knowledge, systematics).

Offer (VVZ)

- During the semester we offer weekly programming exercises that are graded. Points achieved will be taken as a bonus to the exam.
- The bonus is proportional to the score achieved in specially marked bonus tasks, where a full score equals a bonus of 0.25. The admission to specially marked bonus depends on the successful completion of other exercises. The achieved mark bonus expires as soon as the lecture is given anew.

Offer (Concretely)

- 3 bonus exercises in total; 2/3 of the points suffice for the exam bonus of 0.25 marks
- You can, e.g. fully solve 2 bonus exercises, or solve 3 bonus exercises to 66% each, or ...
- Bonus exercises must be unlocked ($\rightarrow$ experience points) by successfully completing the weekly exercises
- It is again not necessary to solve all weekly exercises completely in order to unlock a bonus exercise
- Details: course website, exercise sessions, online exercise system (Code Expert)

Academic integrity

Rule

You submit solutions that you have written yourself and that you have understood.

We check this (partially automatically) and reserve our rights to invite you to interviews.

Should you be invited to an interview: don't panic. Primary we presume your innocence and want to know if you understood what you have submitted.

1. Introduction

Computer Science: Definition and History, Algorithms, Turing Machine, Higher Level Programming Languages, Tools, The first C++Program and its Syntactic and Semantic Ingredients

What is Computer Science?

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- The science of **systematic processing of informations**,...

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- The science of **systematic processing of informations**,…
- …particularly the automatic processing using digital computers.

(Wikipedia, according to “Duden Informatik”)

Computer Science vs. Computers

Computer science is not about machines, in the same way that astronomy is not about telescopes.

Mike Fellows, US Computer Scientist (1991)

Computer Science vs. Computers

- Computer science is also concerned with the development of fast computers and networks…

Computer Science vs. Computers

- Computer science is also concerned with the development of fast computers and networks…
- …but not as an end in itself but for the **systematic processing of informations**.

Computer Science $\neq$ Computer Literacy

Computer literacy: *user knowledge*

- Handling a computer
- Working with computer programs for text processing, email, presentations …

Computer Science $\neq$ Computer Literacy

Computer Science *Fundamental knowledge*

- How does a computer work?
- How do you write a computer program?

Back from the past: This course

- Systematic problem solving with algorithms and the programming language C++.
- Hence: **not only**
  but also programming course.

Algorithm: Fundamental in Computer Science

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- Instructions to solve a problem step by step

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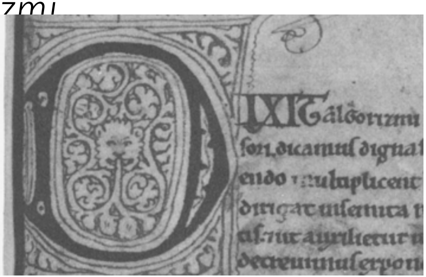
- according to *Muhammed al-Chwarizmi*, author of an arabic computation textbook (about 825)



DIXIT algorizmi
ſori, dicamuſ digna
endo multiplicent
dirigat uiſ
aſ. aurilietur
decreuimuſ
"Dixit algorizmi..." (Latin translation)

Oldest Nontrivial Algorithm

Euclidean algorithm (from the *elements* from Euklid, 3. century B.C.)

- Input: integers $a > 0$, $b > 0$
- Output: gcd of a und b

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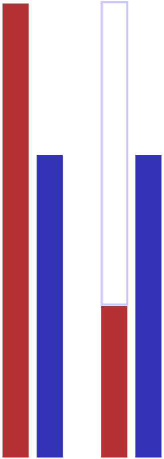

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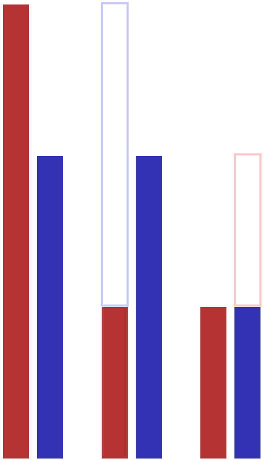

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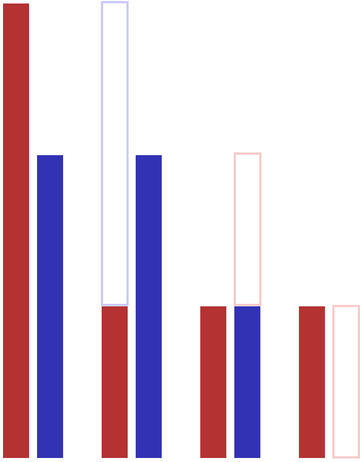

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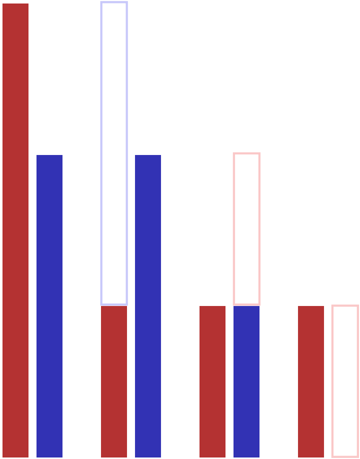

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```
While $b \neq 0$
        If $a > b$ then
                $a \leftarrow a - b$
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Result: $a$.
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Algorithms: 3 Levels of Abstractions

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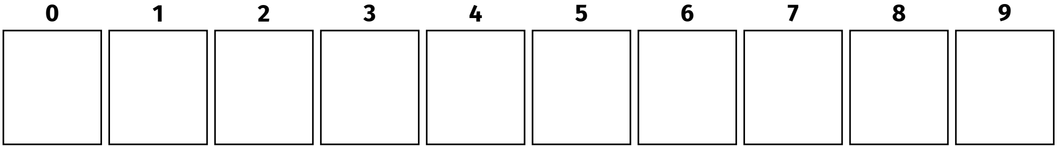
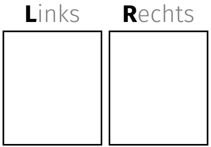
Euclid: Core idea and pseudo code shown, implementation yet missing

Euklid in the Box

Speicher





Register

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Speicher

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
[**8**] → **L**	[**9**] → **R**	**L** = 0? *stop*	**R** > **L**? *springe* *zu* **6**	**L** − **R** → [**8**]	*springe* *zu* 0	**R** − **L** → [**9**]	*springe* *zu* 0		

$\underbrace{\text{0 – 7}}_{\text{Programmcode}}$

Programmcode

Links	**R**echts

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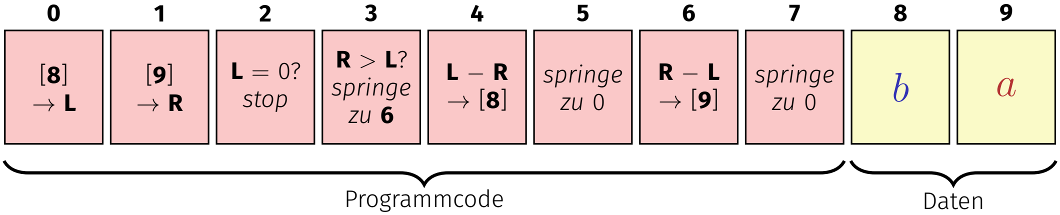

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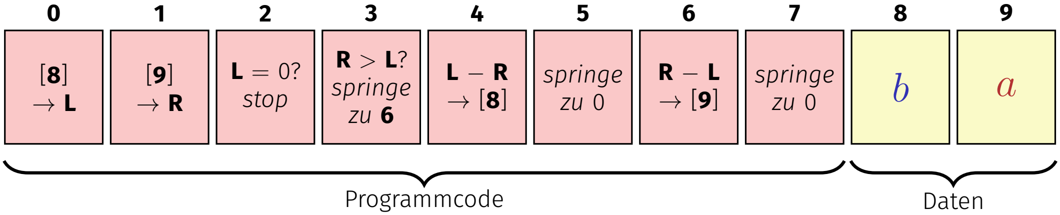

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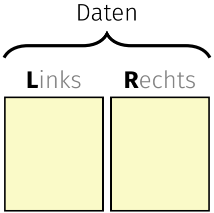

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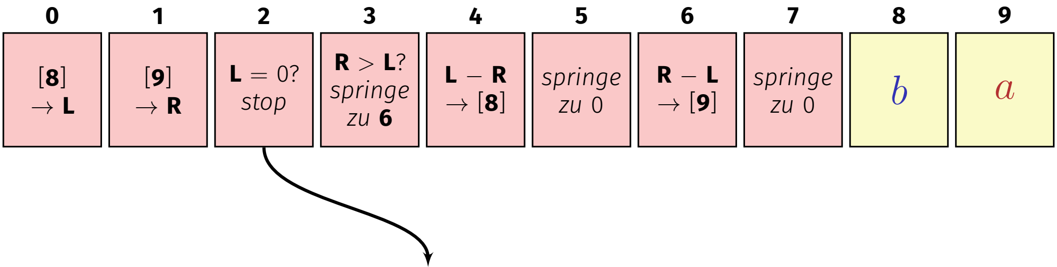

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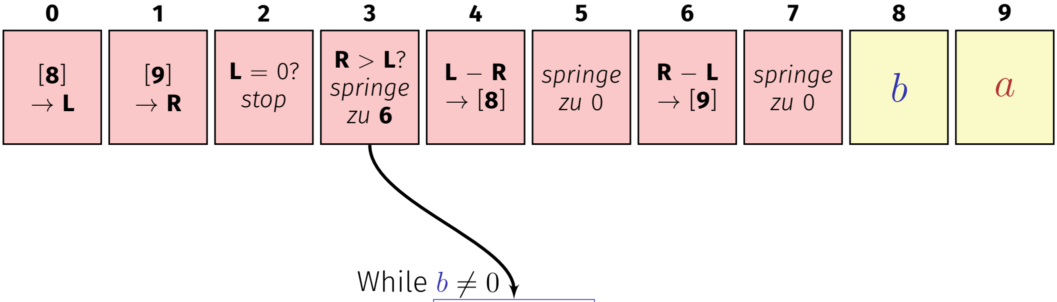

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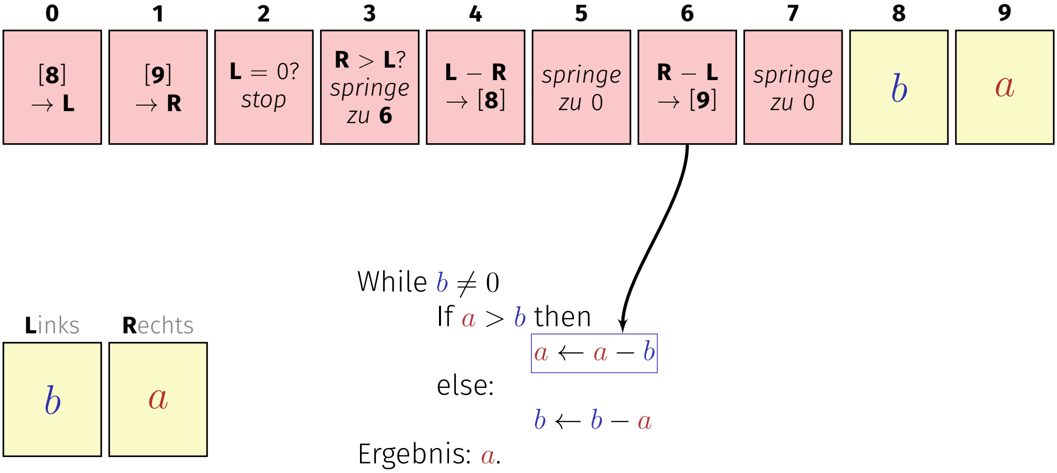

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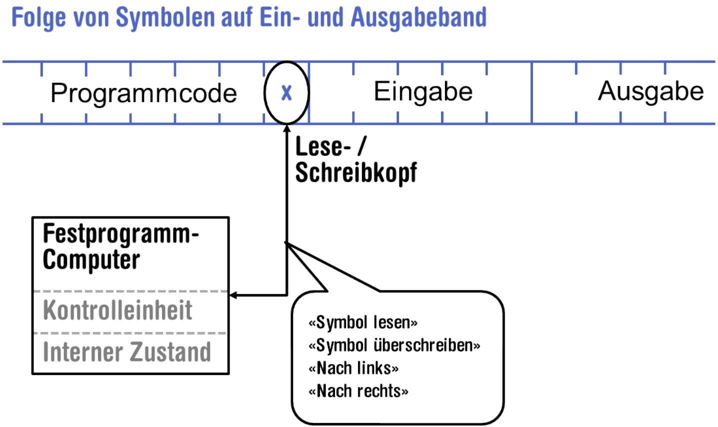
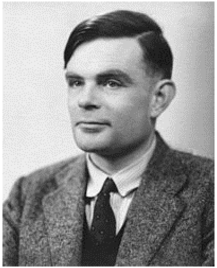
Computers – Concept

A bright idea: universal Turing machine (Alan Turing, 1936)

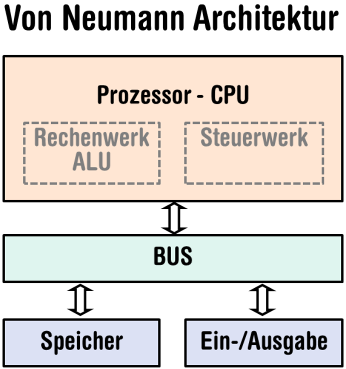




Alan Turing

Computer – Implementation

- Z1 – Konrad Zuse (1938)
- ENIAC – John Von Neumann (1945)



Konrad Zuse

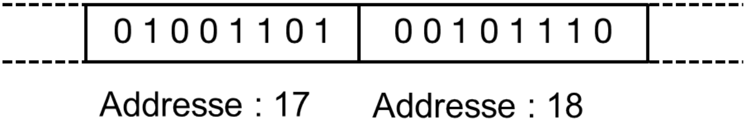
John von Neumann

Memory for data *and* program

- Sequence of bits from $\{0, 1\}$.
- Program state: value of all bits.
- Aggregation of bits to memory cells (often: 8 Bits = 1 Byte)

Memory for data *and* program

- Every memory cell has an address.
- Random access: access time to the memory cell is (nearly) independent of its address.



Programming

- With a **programming language** we issue commands to a computer such that it does exactly what we want.
- The sequence of instructions is the **(computer) program**

The Harvard Computers, human computers, ca.1890

Computing speed

In the time, on average, that the sound takes to travel from from my mouth to you …

[1] Uniprocessor computer at 1 GHz.

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In the time, on average, that the sound takes to travel from from my mouth to you …

a contemporary desktop PC can process more than 100

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Computing speed

In the time, on average, that the sound takes to travel from from my mouth to you …

30 m ≙ more than 100.000.000 instructions

a contemporary desktop PC can process more than 100 millions instructions [1]

[1] Uniprocessor computer at 1 GHz.

Why programming?

- Do I study computer science or what …

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- . . .

> *Mathematics used to be the lingua franca of the natural sciences on all universities. Today this is computer science.*

Lino Guzzella, president of ETH Zurich 2015-2018, NZZ Online, 1.9.2017

((BTW: Lino Guzzella is not a computer scientist, he is a mechanical engineer and prof. for thermotronics ☺)

This is why programming!

- Any understanding of modern technology requires knowledge about the fundamental operating principles of a computer.
- Programming (with the computer as a tool) is evolving a cultural technique like reading and writing (using the tools paper and pencil)

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- Programming (with the computer as a tool) is evolving a cultural technique like reading and writing (using the tools paper and pencil)
- Programming is *the* interface between engineering and computer science – the interdisciplinary area is growing constantly.
- Programming is fun (and is useful)!

Programming Languages

- The language that the computer can understand (machine language) is very primitive.
- Simple operations have to be subdivided into (extremely) many single steps
- The machine language varies between computers.

Higher Programming Languages

can be represented as program text that

- can be *understood* by humans
- is *independent* of the computer model

  → Abstraction!

Why C++?

Other popular programming languages: Java, C#, Python, Javascript, Swift, Kotlin, Go, … …

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General consensus:

- „The” programming language for systems programming: C
- C has a fundamental weakness: missing (type) safety

Why C++?

Over the years, C++'s greatest strength and its greatest weakness has been its C-Compatibility – B. Stroustrup

Syntax and Semantics

- Like our language, programs have to be formed according to certain rules.
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  - **Semantics**: interpretation rules for connected symbols.

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- Like our language, programs have to be formed according to certain rules.
  - **Syntax**: Connection rules for elementary symbols (characters)
  - **Semantics**: interpretation rules for connected symbols.
- Corresponding rules for a computer program are simpler but also more strict because computers are relatively stupid.

Deutsch vs. C++

Deutsch

Alleen sind nicht gefährlich, Rasen ist gefährlich!
(Wikipedia: Mehrdeutigkeit)

C++

```cpp
// computation
int b = a * a; // $b = a^2$
b = b * b;     // $b = a^4$
```

Syntax and Semantics of $\mathrm{C}{+}{+}$

Syntax:

- When is a text a $C{+}{+}$ *program*?
- I.e. is it *grammatically* correct?
- → Can be checked by a computer

Semantics:

- What does a program *mean*?
- Which algorithm does a program *implement*?
- → Requires human understanding

Programming Tools

- **Editor:** Program to modify, edit and store C++program texts
- **Compiler:** program to translate a program text into machine language

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- **Editor:** Program to modify, edit and store C++program texts
- **Compiler:** program to translate a program text into machine language
- **Computer:** machine to execute machine language programs
- **Operating System:** program to organize all procedures such as file handling, editor-, compiler- and program execution.

The first C++ program

```cpp
// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

Most important ingredients... Statements

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}
```

Do something (read in a)!

Most important ingredients... Expressions

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```

← Compute a value (a^2)!

“Accessories:” Comments

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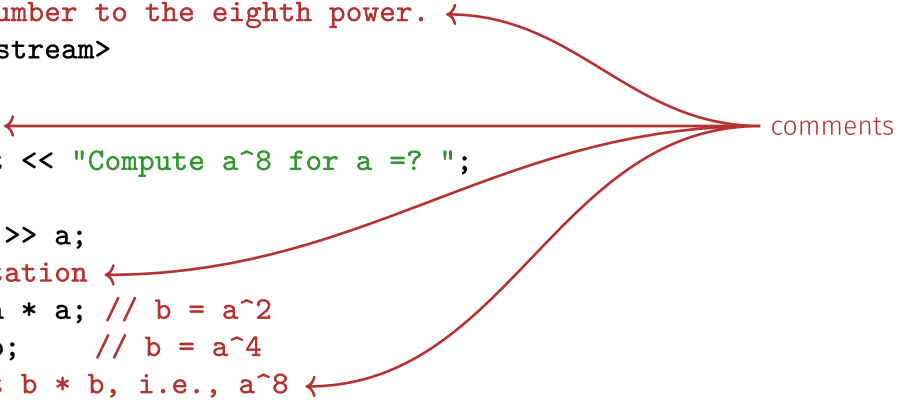
“Accessories:” Comments



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    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

Comments and Layout

The compiler does not care...

```
#include <iostream>
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int a; std::cin >> a; int b = a * a; b = b * b;
std::cout << a << "^8 = " << b*b << "\n";return 0;}
```

Comments and Layout

The compiler does not care...

```
#include <iostream>
int main(){std::cout << "Compute a^8 for a =? ";
int a; std::cin >> a; int b = a * a; b = b * b;
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```

... but we do!

“Accessories:” Include and Main Function

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“Accessories:” Include and Main Function

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    std::cout << a << "^8 = " << b * b << "\n";
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include directive

“Accessories:” Include and Main Function

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}
```

declaration of the main function

Statements: Do something!

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expression statements

Statements: Do something!

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    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

return statement

Statements – Effects

```
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4

    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

effect: output of the string `Compute` .

Effect: input of a number stored in a

Effect: saving the computed value of $a \cdot a$ into b

Effect: saving the computed value of $b \cdot b$ into b

Effect: return the value 0

Effect: output of the value of a and the c

Statements – Variable Definitions

```
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

declaration statement

type names

Variables

- represent (varying) values
- have
  - **name**
  - **type**
  - **value**
  - **address**

Variables

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- have
  - **name**
  - **type**
  - **value**
  - **address**

`int a;` defines a variable with

- name: **`a`**
- type: **`int`**
- value: (initially) undefined
- Address: determined by compiler

Expressions: compute a value!

Expressions

- represent *Computations*

Expressions: compute a value!

Expressions

- represent *Computations*
- are either **primary** (**`b`**)

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- are either **primary** (`b`)
- or **composed** (`b*b`)...

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- have a type and a value

Expressions: compute a value!

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- represent *Computations*
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- or **composed** (`b*b`)...
- ...from different expressions, using **operators**
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Analogy: building blocks

Expressions

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// input
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int b = a * a;  // b = a^2
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// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";

return 0;
```

Expressions

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;
```

variable name, primary expression (+ name and address)

```
// computation
int b = a * a;  // b = a^2
b = b * b;      // b = a^4
```

variable name, primary expression (+ name and address)

```
// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";

return 0;
```

literal, primary expression

Expressions

composite expression

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
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```

composite expression

```
// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";

return 0;
```

Expressions   Building Blocks

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;
```

← Two times composed expression

```
// output b * b, i.e., a^8
std::cout << a << "^8 = " << b * b << ".\ n";

return
```

↑ Four times composed expression

Literals

- represent constant values
- have a fixed **type** and **value**
- are "syntactical values"

- **`0`** has type **`int`**, value 0.
- **`1.2e5`** has type **`double`**, value $1.2 \cdot 10^5$.

L-Values and R-Values

```
// input
std::cout << "Compute a^8 for a =? ";
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std::cin >> a;

// computation
int b = a * a;  // b = a^2
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std::cout << a<< "^8 = " << b * b << ".\ n";
return 0;
```

L-value (expression + address)

L-value (expression + address)

R-Value (expression that is not an L-value)

L-Values and R-Values

R-Value

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std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
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```

R-Value

```
// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";
return 0;
```

L-Values and R-Values

L-Wert (“Left of the assignment operator”)

- Expression with **address**
- **Value** is the content at the memory location according to the type of the expression.

L-Values and R-Values

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- Expression with **address**
- **Value** is the content at the memory location according to the type of the expression.
- L-Value can change its value (e.g. via assignment)

Example: variable name

L-Values and R-Values

R-Wert ("**R**ight of the assignment operator")

- Expression that is no L-value

Example: literal 0

L-Values and R-Values

R-Wert ("**R**ight of the assignment operator")

- Expression that is no L-value
- Any L-Value can be used as R-Value (but not the other way round)

Example: literal `0`

L-Values and R-Values

R-Wert ("Right of the assignment operator")

- Expression that is no L-value
- Any L-Value can be used as R-Value (but not the other way round) Every E-Bike can be used as normal bike, but not the other way round

Example: literal 0

L-Values and R-Values

R-Wert (“**R**ight of the assignment operator”)

- Expression that is no L-value
- Any L-Value can be used as R-Value (but not the other way round)
- An R-Value *cannot change* its value

Example: literal 0

Operators and Operands   Building Blocks

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// input
std::cout << "Compute a^8 for a =? ";
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std::cin >> a;

// computation
int b = a * a;  // b = a^2
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// output b * b, i.e., a^8
std::cout << a << "^8 = " << b * b << "\n";
return 0;
```

Operators and Operands

left operand (output stream)

output operator

right operand (string)

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```

right operand (variable name)

input operator

left operand (input stream)

Operators and Operands

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std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
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```

assignment operator

```
// ou                   ^8
std::cout << a << "^8 = " << b * b << "\n";
return 0;
```

multiplication operator

2. Integers

Evaluation of Arithmetic Expressions, Associativity and Precedence, Arithmetic Operators, Domain of Types **`int`**, **`unsigned int`**

Example: power8.cpp

```
int a; // Input
int r; // Result

std::cout << "Compute a^8 for a = ?";
std::cin >> a;

r = a * a; // r = a^2
r = r * r; // r = a^4

std::cout << "a^8 = " << r*r << ’\n’;
```

Celsius to Fahrenheit

```
// Program: fahrenheit.cpp
// Convert temperatures from Celsius to Fahrenheit.
#include <iostream>

int main() {
  // Input
  std::cout << "Temperature in degrees Celsius =? ";
  int celsius;
  std::cin >> celsius;

  // Computation and output
  std::cout << celsius << " degrees Celsius are "
            << 9 * celsius / 5 + 32  << " degrees Fahrenheit.\n";
  return 0;
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```

9 * celsius / 5 + 32

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- Arithmetic expression,

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- Arithmetic expression,
- three literals, one variable, three operator symbols

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How to put the expression in parentheses?

Precedence

Multiplication/Division before Addition/Subtraction

```
9 * celsius / 5 + 32
```

bedeutet

```
(9 * celsius / 5) + 32
```

Precedence

Rule 1: precedence

Multiplicative operators (**`*`**, **`/`**, **`%`**) have a higher precedence ("bind more strongly") than additive operators (**`+`**, **`-`**)

Associativity

From left to right

```
9 * celsius / 5 + 32
```

bedeutet

```
((9 * celsius) / 5) + 32
```

Associativity

Rule 2: Associativity

Arithmetic operators (`*`, `/`, `%`, `+`, `-`) are left associative: operators of same precedence evaluate from left to right

Arity

Sign

```
-3 - 4
```

means

```
(-3) - 4
```

Arity

Rule 3: Arity

Unary operators **+**, **−** first, then binary operators **+**, **−**.

Parentheses

Any expression can be put in parentheses by means of

- associativities
- precedences
- arities

of the operands in an unambiguous way.

Expression Trees

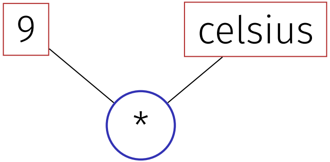
Parentheses yield the expression tree

```
9 * celsius / 5 + 32
```

Expression Trees

Parentheses yield the expression tree

```
(9 * celsius) / 5 + 32
```



Expression Trees

Parentheses yield the expression tree

```
((9 * celsius) / 5) + 32
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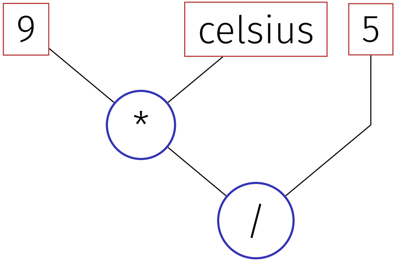

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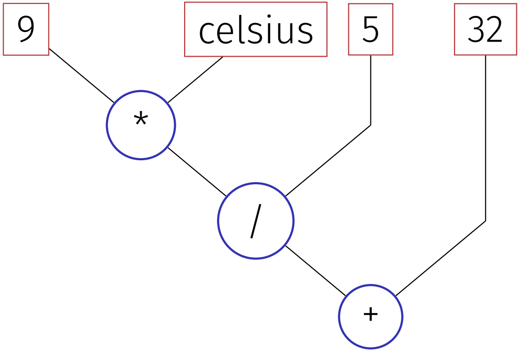
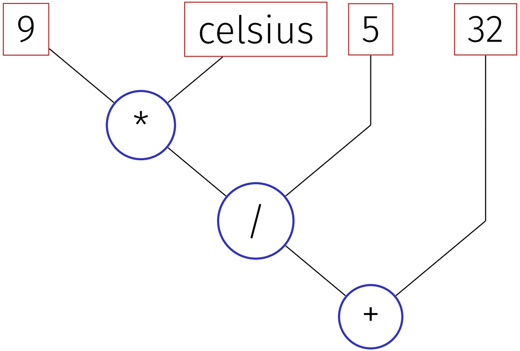


Evaluation Order

"From top to bottom" in the expression tree

`9 * celsius / 5 + 32`



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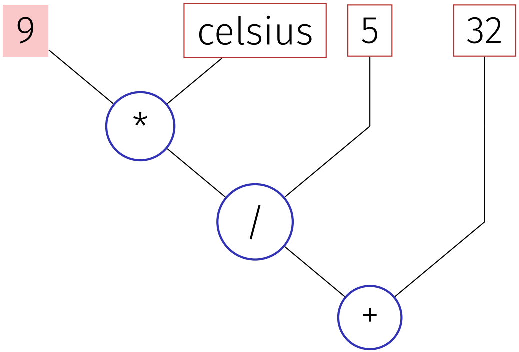

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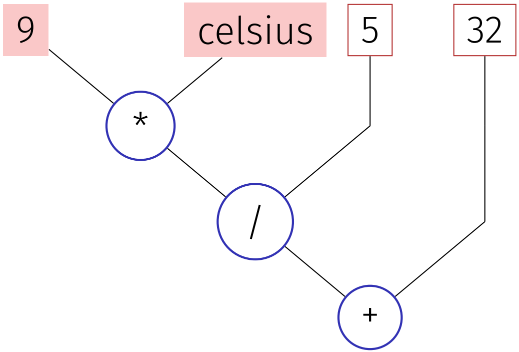

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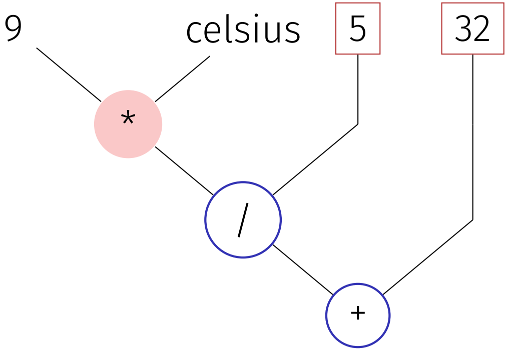

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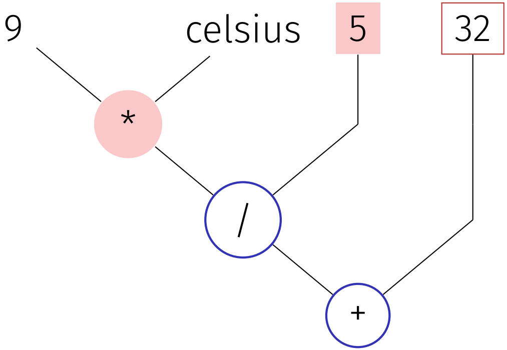

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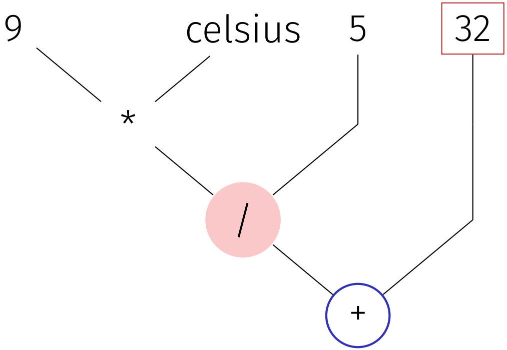

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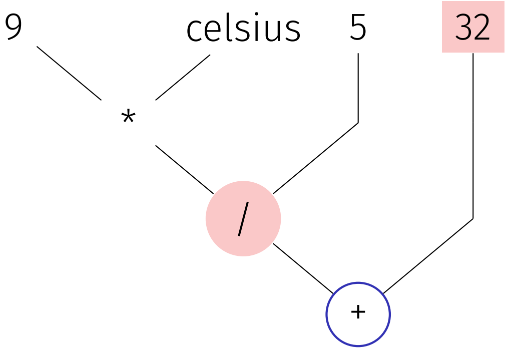

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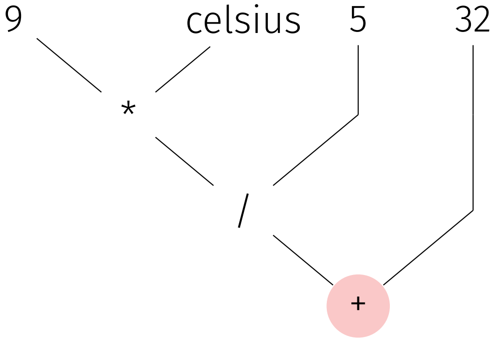

Evaluation Order

Order is not determined uniquely:

```
9 * celsius / 5 + 32
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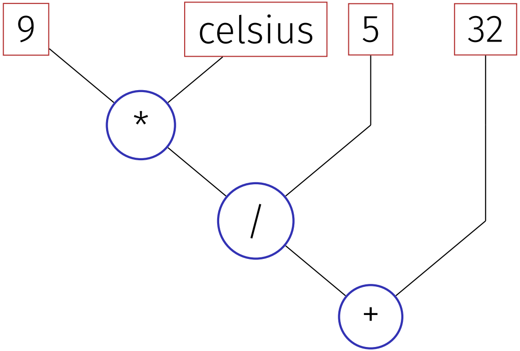

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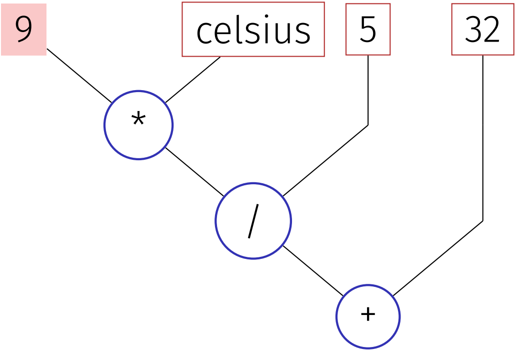

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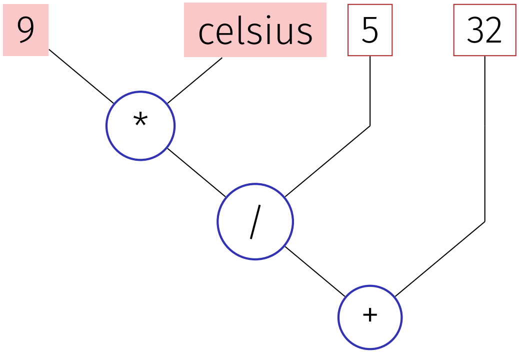

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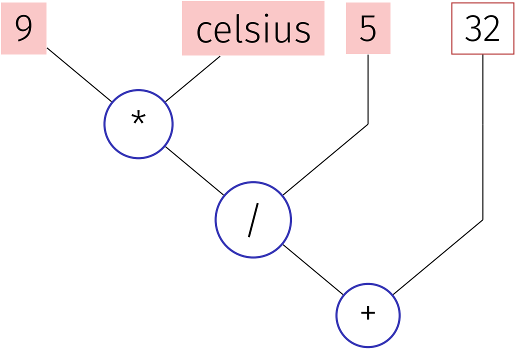

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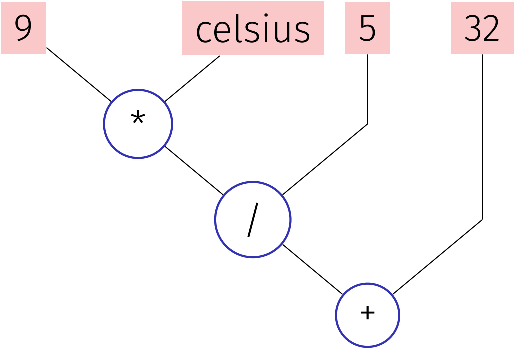

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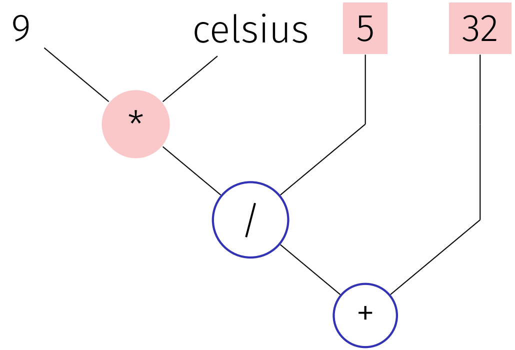
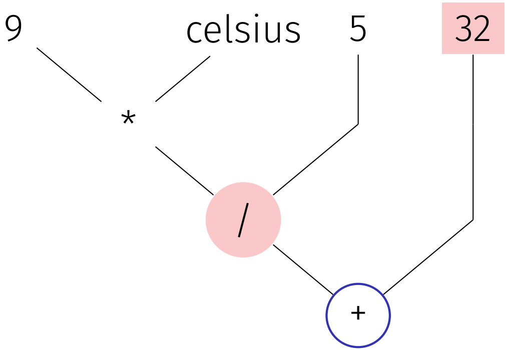


Evaluation Order

Order is not determined uniquely:

```
9 * celsius / 5 + 32
```



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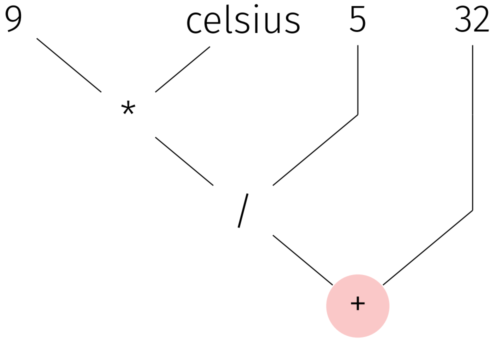
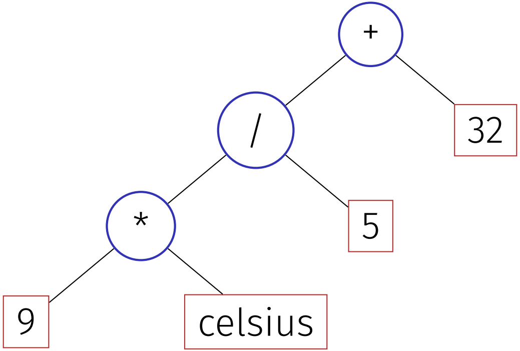


Expression Trees – Notation

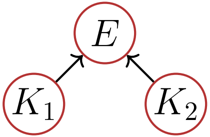
Common notation: root on top

```
9 * celsius / 5 + 32
```



Evaluation Order – more formally

Valid order: any node is evaluated **after** its children



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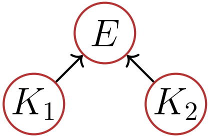

C++: the valid order to be used is not defined.

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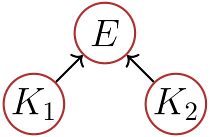

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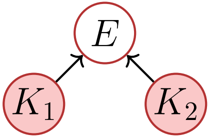

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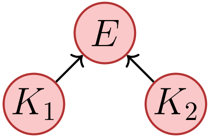
Evaluation Order – more formally

Valid order: any node is evaluated **after** its children



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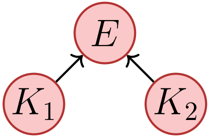
Evaluation Order – more formally

Valid order: any node is evaluated **after** its children

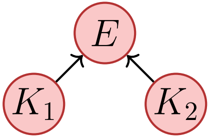


C++: the valid order to be used is not defined.

- "Good expression": any valid evaluation order leads to the same result.

Evaluation Order – more formally

Valid order: any node is evaluated **after** its children



C++: the valid order to be used is not defined.

- "Good expression": any valid evaluation order leads to the same result.
- Example for a “bad expression”: `a*(a=2)`

Evaluation order

Guideline

Avoid modifying variables that are used in the same expression more than once.

Arithmetic operations

	Symbol	Arity	Precedence	Associativity
Unary +	+	1	16	right
Negation	-	1	16	right
Multiplication	*	2	14	left
Division	/	2	14	left
Modulo	%	2	14	links
Addition	+	2	13	left
Subtraction	-	2	13	left

Interlude: Assignment expression – in more detail

- Already known: **`a = b`** means Assignment of **`b`** (R-value) to **`a`** (L-value). Returns: L-value.

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- Answer: assignment is right-associative

Interlude: Assignment expression – in more detail

- Already known: `a = b` means Assignment of `b` (R-value) to `a` (L-value). Returns: L-value.
- What does `a = b = c` mean?
- Answer: assignment is right-associative

`a = b = c` $\Longleftrightarrow$ `a = (b = c)`

Multiple assignment: `a = b = 0` $\Longrightarrow$ `b=0; a=0`

Division

- Operator `/` implements integer division

  `5 / 2` has value 2

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- In **`fahrenheit.cpp`**

  ```
  9 * celsius / 5 + 32
  ```

  ```
  15 degrees Celsius are 59 degrees Fahrenheit
  ```

Division

- Operator / implements integer division

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- Operator / implements integer division

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- In **`fahrenheit.cpp`**

  ```
  9 * celsius / 5 + 32
  ```

  ```
  15 degrees Celsius are 59 degrees Fahrenheit
  ```

- Mathematically equivalent...

  ```
  9 / 5 * celsius + 32
  ```

Division

- Operator / implements integer division

`5 / 2` has value 2

- In **`fahrenheit.cpp`**

```
9 * celsius / 5 + 32

15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent...

```
1 * celsius + 32
```

Division

- Operator / implements integer division

`5 / 2` has value 2

- In **`fahrenheit.cpp`**

```
9 * celsius / 5 + 32
```

```
15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent…

```
15 + 32
```

Division

- Operator / implements integer division

  `5 / 2` has value 2

- In **`fahrenheit.cpp`**

```
9 * celsius / 5 + 32

15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent...

```
47
```

Division

- Operator / implements integer division

  `5 / 2` has value 2

- In **`fahrenheit.cpp`**

  ```
  9 * celsius / 5 + 32
  ```

  ```
  15 degrees Celsius are 59 degrees Fahrenheit
  ```

- Mathematically equivalent...but not in C++!

  ```
  9 / 5 * celsius + 32
  ```

  ```
  15 degrees Celsius are  47 degrees Fahrenheit
  ```

Loss of Precision

Guideline

- Watch out for potential loss of precision
- Postpone operations with potential loss of precision to avoid “error escalation”

Division and Modulo

- Modulo-operator computes the rest of the integer division

  `5 / 2` has value 2, `5 % 2` has value 1.

- It holds that

```
(-a)/b == -(a/b)
```

Division and Modulo

- Modulo-operator computes the rest of the integer division

  `5 / 2` has value 2, `5 % 2` has value 1.

- It holds that

  ```
  (-a)/b == -(a/b)
  ```

- It also holds:

  `(a / b) * b + a % b` has the value of `a`.

Division and Modulo

- Modulo-operator computes the rest of the integer division

  `5 / 2` has value 2, `5 % 2` has value 1.

- It holds that

  ```
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  ```

- It also holds:

  `(a / b) * b + a % b` has the value of `a`.

- From the above one can conclude the results of division and modulo with negative numbers

Increment and decrement

- Increment / Decrement a number by one is a frequent operation
- works like this for an L-value:

```
expr = expr + 1.
```

Increment and decrement

```
expr = expr + 1.
```

Disadvantages

- relatively long

Increment and decrement

```
expr = expr + 1.
```

Disadvantages

- relatively long
- `expr` is evaluated twice
  - Later: L-valued expressions whose evaluation is “expensive”

Increment and decrement

```
expr = expr + 1.
```

Disadvantages

- relatively long
- `expr` is evaluated twice
  - Later: L-valued expressions whose evaluation is “expensive”
  - `expr` could have an effect (but should not, cf. guideline)

In-/Decrement Operators

Post-Increment

```
expr++
```

Value of **`expr`** is increased by one, the **old** value of **`expr`** is returned (as R-value)

In-/Decrement Operators

Pre-increment

```
++expr
```

Value of **`expr`** is increased by one, the **new** value of **`expr`** is returned (as L-value)

In-/Decrement Operators

Post-Dekrement

```
expr--
```

Value of **`expr`** is decreased by one, the **old** value of **`expr`** is returned (as R-value)

In-/Decrement Operators

Prä-Dekrement

```
--expr
```

Value of **`expr`** is increased by one, the **new** value of **`expr`** is returned (as L-value)

In-/Decrement Operators

```
int a = 7;
std::cout << ++a << "\n";
std::cout << a++ << "\n";
std::cout << a << "\n";
```

In-/Decrement Operators

```
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n";
std::cout << a << "\n";
```

In-/Decrement Operators

```
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n"; // 8
std::cout << a << "\n";
```

In-/Decrement Operators

```cpp
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n"; // 8
std::cout << a << "\n"; // 9
```

Arithmetic Assignments

```
a += b
```

$$\Leftrightarrow$$

```
a = a + b
```

Arithmetic Assignments

```
a += b
```

$$\Leftrightarrow$$

```
a = a + b
```

analogously for `-`, `*`, `/` and `%`

Binary Number Representations

Binary representation (Bits from $\{0, 1\}$)

$$b_n b_{n-1} \dots b_1 b_0$$

corresponds to the number $b_n \cdot 2^n + \dots + b_1 \cdot 2^1 + b_0 \cdot 2^0$

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```
101011
```

Binary Number Representations

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101011 corresponds to **32+8+2+1**.

Binary Number Representations

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$$b_n b_{n-1} \ldots b_1 b_0$$

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`101011` corresponds to **43**.

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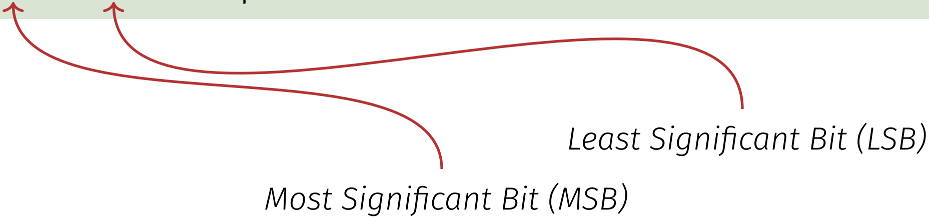
`101011` corresponds to **43**.



Least Significant Bit (LSB)

Most Significant Bit (MSB)

Hexadecimal Numbers

Numbers with base 16

$$h_n h_{n-1} \ldots h_1 h_0$$

corresponds to the number

$$h_n \cdot 16^n + \cdots + h_1 \cdot 16 + h_0.$$

notation in C++: prefix `0x`

`0xff` corresponds to **255**.

Hex Nibbles

hex	bin	dec
0	0000	0
1	**0001**	**1**
2	**0010**	**2**
3	0011	3
4	**0100**	**4**
5	0101	5
6	0110	6
7	0111	7
8	**1000**	**8**
9	1001	9
a	1010	10
b	1011	11
c	1100	12
d	1101	13
e	1110	14
f	1111	15

Why Hexadecimal Numbers?

- A Hex-Nibble requires exactly 4 bits.

Why Hexadecimal Numbers?

- A Hex-Nibble requires exactly 4 bits.
- “compact representation of binary numbers”

Example: Hex-Colors

$$\#\underbrace{00}_{r}\underbrace{FF}_{g}\underbrace{00}_{b}$$

Example: Hex-Colors

$$\#\underbrace{FF}_{r}\underbrace{FF}_{g}\underbrace{00}_{b}$$

Example: Hex-Colors

$$\#\underbrace{80}_{r}\underbrace{80}_{g}\underbrace{80}_{b}$$

Example: Hex-Colors

$$\#\underbrace{\mathrm{FF}}_{r}\underbrace{\mathrm{00}}_{g}\underbrace{\mathrm{50}}_{b}$$

Domain of Type `int`

```
// Output the smallest and the largest value of type int.
#include <iostream>
#include <limits>

int main() {
  std::cout << "Minimum int value is "
           << std::numeric_limits<int>::min() << ".\n"
           << "Maximum int value is "
           << std::numeric_limits<int>::max() << ".\n";
  return 0;
}
```

Domain of Type `int`

```cpp
// Output the smallest and the largest value of type int.
#include <iostream>
#include <limits>

int main() {
  std::cout << "Minimum int value is "
            << std::numeric_limits<int>::min() << ".\n"
            << "Maximum int value is "
            << std::numeric_limits<int>::max() << ".\n";
  return 0;
}
```

```
Minimum int value is -2147483648.
Maximum int value is 2147483647.
```

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}
```

```
Minimum int value is -2147483648.
Maximum int value is 2147483647.
```

Where do these numbers come from?

Domain of the Type `int`

- Representation with B bits. Domain

$$\{-2^{B-1}, \ldots, -1, 0, 1, \ldots, 2^{B-1} - 2, 2^{B-1} - 1\}$$

Domain of the Type `int`

- Representation with B bits. Domain

$$\{-2^{B-1}, \ldots, -1, 0, 1, \ldots, 2^{B-1}-2, 2^{B-1}-1\}$$

- On most platforms $B = 32$

Domain of the Type `int`

- Representation with B bits. Domain

$$\{-2^{B-1},\dots,-1,0,1,\dots,2^{B-1}-2,2^{B-1}-1\}$$

- For the type **`int`** $\mathrm{C}{++}$ guarantees $B \geq 16$

Over- and Underflow

- Arithmetic operations (`+,-,*`) can lead to numbers outside the valid domain.
- Results can be incorrect!

  `power8.cpp`: $15^8 = -1732076671$

- There is **no error message!**

The Type `unsigned int`

- Domain

$$\{0, 1, \ldots, 2^B - 1\}$$

- All arithmetic operations exist also for **`unsigned int`**.
- Literals: **`1u, 17u`** …

Mixed Expressions

- Operators can have operands of different type (e.g. `int` and `unsigned int`).

```
17 + 17u
```

- Such mixed expressions are of the “more general” type `unsigned int`.
- `int`-operands are **converted** to `unsigned int`.

Conversion

`int` Value	Sign	`unsigned int` Value
x	≥ 0	x
x	< 0	$x + 2^B$

Conversion

`int` Value	Sign	`unsigned int` Value
x	≥ 0	x
x	< 0	$x + 2^B$

Due to a clever representation (two's complement), no addition is internally needed

Signed Numbers

Note: the remaining slides on signed numbers, computing with binary numbers, and the two’s complement, are *not* relevant for the exam

Signed Number Representation

- (Hopefully) clear by now: binary number representation without sign, e.g.

$$[b_{31}b_{30}\dots b_0]_u \quad \widehat{=} \quad b_{31}\cdot 2^{31} + b_{30}\cdot 2^{30} + \cdots + b_0$$

- Looking for a consistent solution

> The representation with sign should coincide with the unsigned solution as much as possible. Positive numbers should arithmetically be treated equal in both systems.

Computing with Binary Numbers (4 digits)

Simple Addition

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array} \qquad \begin{array}{r} 0010 \\ +0011 \\ \hline 0101_2 \end{array} \quad = 5_{10}$$

Computing with Binary Numbers (4 digits)

Simple Subtraction

$$\begin{array}{r}
5 \\
-3 \\
\hline
2
\end{array}
\qquad
\begin{array}{r}
0101 \\
-0011 \\
\hline
0010_2
\end{array}
\quad = 2_{10}$$

Computing with Binary Numbers (4 digits)

Addition with Overflow

$$\begin{array}{r}7\\+10\\\hline 17\end{array}\qquad\begin{array}{r}0111\\+1010\\\hline (1)0001_2\end{array}\quad = 1_{10} (= 17 \bmod 16)$$

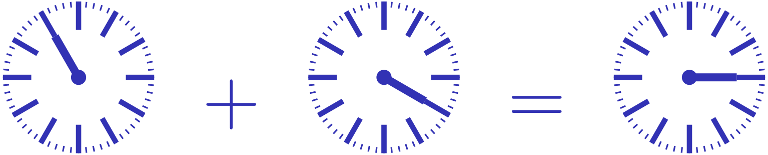
Computing with Binary Numbers (4 digits)

Subtraction with underflow

$$\begin{array}{rcrl}
5 & & 0101 & \\
+(-10) & & 1010 & \\
\hline
-5 & & (\ldots 11)1011_2 & = 11_{10} (= -5 \bmod 16)
\end{array}$$

Why this works

Modulo arithmetics: Compute on a circle[3]



$11 \equiv 23 \equiv -1 \equiv \ldots \pmod{12}$

$4 \equiv 16 \equiv \ldots \pmod{12}$

$3 \equiv 15 \equiv \ldots \pmod{12}$

[3] The arithmetics also work with decimal numbers (and for multiplication).

Negative Numbers (3 Digits)

	a	$-a$
0	000	
1	001	
2	010	
3	011	
4	100	
5	101	
6	110	
7	111	

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001		
2	010		
3	011		
4	100		
5	101		
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Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	**111**	-1
2	010		
3	011		
4	100		
5	101		
6	110		
7	**111**		

Negative Numbers (3 Digits)

	a	$-a$	
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2	010	**110**	-2
3	011		
4	100		
5	101		
6	**110**		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	**101**	-3
4	100		
5	**101**		
6	110		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	101	-3
4	**100**	**100**	-4
5	101		
6	110		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	101	-3
4	100	100	-4
5	101		
6	110		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	101	-3
4	100	100	-4
5	101		
6	110		
7	111		

The most significant bit decides about the sign *and* it contributes to the value.

3. Logical Values

Boolean Functions; the Type **`bool`**; logical and relational operators; shortcut evaluation

Our Goal

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

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Behavior depends on the value of a *Boolean expression*

Our Goal

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Behavior depends on the value of a **Boolean expression**

Boolean Values in Mathematics

Boolean expressions can take on one of two values:

0 or **1**

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Boolean expressions can take on one of two values:

0 or **1**

- **0** corresponds to **“false”**
- **1** corresponds to **“true”**

The Type `bool` in C++

- represents **logical values**

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- Literals `false` and `true`

The Type `bool` in C++

- represents **logical values**
- Literals **`false`** and **`true`**
- Domain $\{$***false***, ***true***$\}$

```
bool b = true; // Variable with value true
```

Relational Operators

`a < b` (smaller than)

arithmetic type × arithmetic type → `bool`

R-value × R-value → R-value

Relational Operators

`a < b` (smaller than)

```
bool b = (1 < 3); // b =
```

Relational Operators

`a < b` (smaller than)

```
bool b = (1 < 3); // b = true
```

Relational Operators

`a >= b` (greater than)

```
int a = 0;
bool b = (a >= 3); // b =
```

Relational Operators

`a >= b` (greater than)

```
int a = 0;
bool b = (a >= 3); // b = false
```

Relational Operators

`a == b` (equals)

```
int a = 4;
bool b = (a % 3 == 1); // b =
```

Relational Operators

`a == b` (equals)

```
int a = 4;
bool b = (a % 3 == 1); // b = true
```

Relational Operators

`a != b` (not equal)

```
int a = 1;
bool b = (a != 2*a-1); // b =
```

Relational Operators

a != b (not equal)

```
int a = 1;
bool b = (a != 2*a-1); // b = false
```

Boolean Functions in Mathematics

- Boolean function

$$f : \{0,1\}^2 \to \{0,1\}$$

- 0 corresponds to “false”.
- 1 corresponds to “true”.

$\mathrm{AND}(x, y)$ $\qquad x \wedge y$

- “logical And”

$$f : \{0,1\}^2 \to \{0,1\}$$

- 0 corresponds to “false”.
- 1 corresponds to “true”.

x	y	$\mathrm{AND}(x, y)$
0	0	0
0	1	0
1	0	0
1	1	1

Logical Operator &&

`a && b` (logical and)

```
int n = -1;
int p = 3;
bool b = (n < 0) && (0 < p); //
```

Logical Operator `&&`

`a && b` (logical and)

```
int n = -1;
int p = 3;
bool b = (n < 0) && (0 < p); // b = true
```

$\mathrm{OR}(x, y)$ $x \vee y$

- "logical Or"

$$f : \{0,1\}^2 \to \{0,1\}$$

- 0 corresponds to "false".
- 1 corresponds to "true".

x	y	$\mathrm{OR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	1

Logical Operator ||

`a || b` (logical or)

```
int n = 1;
int p = 0;
bool b = (n < 0) || (0 < p); //
```

Logical Operator ||

`a || b` (logical or)

```
int n = 1;
int p = 0;
bool b = (n < 0) || (0 < p); // b = false
```

NOT(x) $\neg x$

- “logical Not”

$$f : \{0,1\} \to \{0,1\}$$

x	NOT(x)
0	1
1	0

- 0 corresponds to “false”.
- 1corresponds to “true”.

Logical Operator `!`

`!b` (logical not)

`bool` → `bool`

R-value → R-value

Logical Operator !

`!b` (logical not)

```
int n = 1;
bool b = !(n < 0); //
```

Logical Operator !

`!b` (logical not)

```
int n = 1;
bool b = !(n < 0); // b = true
```

Precedences

```
!b && a
```

Precedences

```
 !b && a
```

$$\Updownarrow$$

```
(!b) && a
```

Precedences

```
a && b || c && d
```

Precedences

```
a && b || c && d
```

$$\Updownarrow$$

```
(a && b) || (c && d)
```

Precedences

```
a || b   &&   c || d
```

Precedences

```
a || b   &&   c || d
         ⇕
a || (b && c) || d
```

Precedences

```
7 + x < y && y != 3 * z || ! b
```

Precedences

The unary logical operator `!`
binds more strongly than

```
7 + x < y && y != 3 * z || (!b)
```

Precedences

The unary logical operator !

binds more strongly than

binary arithmetic operators. These

bind more strongly than

```
(7 + x) < y && y != (3 * z) || (!b)
```

Precedences

The unary logical operator `!`

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binary arithmetic operators. These

bind more strongly than

relational operators,

and these bind more strongly than

```
((7 + x) < y) && (y != (3 * z)) || (!b)
```

Precedences

The unary logical operator `!`
binds more strongly than
binary arithmetic operators. These
bind more strongly than
relational operators,
and these bind more strongly than
binary logical operators.

```
((7 + x) < y) && (y != (3 * z)) || (!b)
```

Some parentheses on the previous slides were actually redundant.

Completeness

- AND, OR and NOT are the boolean functions available in C++.

Completeness: $\mathrm{XOR}(x, y)$ $x \oplus y$

- AND, OR and NOT are the boolean functions available in $\mathrm{C{+}{+}}$.
- Any other *binary* boolean function can be generated from them.

x	y	$\mathrm{XOR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	0

Completeness: $\mathrm{XOR}(x, y)$ $x \oplus y$

$$\mathrm{XOR}(x, y) = \mathrm{AND}(\mathrm{OR}(x, y), \mathrm{NOT}(\mathrm{AND}(x, y))).$$

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$$x \oplus y = (x \vee y) \wedge \neg(x \wedge y).$$

Completeness: $\mathrm{XOR}(x, y)$ $x \oplus y$

$$\mathrm{XOR}(x, y) = \mathrm{AND}(\mathrm{OR}(x, y), \mathrm{NOT}(\mathrm{AND}(x, y))).$$

$$x \oplus y = (x \vee y) \wedge \neg(x \wedge y).$$

```
(x || y) && !(x && y)
```

Completeness Proof

- Identify binary boolean functions with their characteristic vector.

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x	y	$\mathrm{XOR}(x, y)$
0	0	0
0	1	1
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characteristic vector: 0110

Completeness Proof

- Identify binary boolean functions with their characteristic vector.

x	y	$\mathrm{XOR}(x,y)$
0	0	0
0	1	1
1	0	1
1	1	0

characteristic vector: 0110

$$\mathrm{XOR} = f_{0110}$$

Completeness Proof

- Step 1: generate the *fundamental* functions f_{0001}, f_{0010}, f_{0100}, f_{1000}

$$\begin{aligned}
f_{0001} &= \mathrm{AND}(x, y) \\
f_{0010} &= \mathrm{AND}(x, \mathrm{NOT}(y)) \\
f_{0100} &= \mathrm{AND}(y, \mathrm{NOT}(x)) \\
f_{1000} &= \mathrm{NOT}(\mathrm{OR}(x, y))
\end{aligned}$$

Completeness Proof

- Step 2: generate all functions by applying logical or

$$f_{1101} = \mathrm{OR}(f_{1000}, \mathrm{OR}(f_{0100}, f_{0001}))$$

Completeness Proof

- Step 2: generate all functions by applying logical or

$$f_{1101} = \mathrm{OR}(f_{1000}, \mathrm{OR}(f_{0100}, f_{0001}))$$

- Step 3: generate f_{0000}

$$f_{0000} = 0.$$

`bool` vs **`int`**: Conversion

- **`bool`** can be used whenever **`int`** is expected

`bool` vs `int`: Conversion

- **`bool`** can be used whenever **`int`** is expected

`bool`	→	**`int`**
true	→	1
false	→	0

`bool` vs `int`: Conversion

- **`bool`** can be used whenever **`int`** is expected – and vice versa.

`bool`	→	**`int`**
true	→	1
false	→	0

`int`	→	**`bool`**
≠0	→	*true*
0	→	*false*

bool vs **int**: Conversion

- **bool** can be used whenever **int** is expected – and vice versa.

bool	→	**int**
true	→	1
false	→	0

int	→	**bool**
≠0	→	*true*
0	→	*false*

```
bool b = 3; // b=tru
```

`bool` vs `int`: Conversion

- **`bool`** can be used whenever **`int`** is expected – and vice versa.
- Many existing programs use **`int`** instead of **`bool`**

  This is bad style originating from the language C .

`bool`	→	`int`
true	→	1
false	→	0

`int`	→	`bool`
≠0	→	*true*
0	→	*false*

```
bool b = 3; // b=tru
```

DeMorgan Rules

- `!(a && b) == (!a || !b)`

DeMorgan Rules

- `!(a && b) == (!a || !b)`

! (rich *and* beautiful) == (poor *or* ugly)

DeMorgan Rules

- `!(a && b) == (!a || !b)`
- `!(a || b) == (!a && !b)`

`!` (rich *and* beautiful) == (poor *or* ugly)

Application: either … or (XOR)

```
(x || y)      && !(x && y)
```

Application: either ... or (XOR)

```
(x || y)     && !(x && y)
```

x or y, and not both

Application: either … or (XOR)

```
(x || y)      && !(x && y)
```

x or y, and not both

```
(x || y)      && (!x || !y)
```

Application: either … or (XOR)

`(x || y)     && !(x && y)`	x or y, and not both
`(x || y)     && (!x || !y)`	x or y, and one of them not

Application: either ... or (XOR)

`(x || y)      && !(x && y)`	x or y, and not both
`(x || y)      && (!x || !y)`	x or y, and one of them not
`!(!x && !y)  && !(x && y)`	

Application: either … or (XOR)

`(x || y)    && !(x && y)`	x or y, and not both
`(x || y)    && (!x || !y)`	x or y, and one of them not
`!(!x && !y)  && !(x && y)`	not none and not both

Application: either … or (XOR)

`(x || y)       && !(x && y)`	x or y, and not both
`(x || y)       && (!x || !y)`	x or y, and one of them not
`!(!x && !y)   && !(x && y)`	not none and not both
`!(!x && !y || x && y)`	

Application: either … or (XOR)

`(x || y)      && !(x && y)` x or y, and not both

`(x || y)      && (!x || !y)` x or y, and one of them not

`!(!x && !y)   && !(x && y)` not none and not both

`!(!x && !y || x && y)` not: both or none

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

Short circuit Evaluation

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x has value 6 ⇒

```
x != 0 && z / x > y
```

Short circuit Evaluation

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true && z / x > y
```

Short circuit Evaluation

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- Logical operators **&&** and **||** evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
x != 0 && z / x > y
```

Short circuit Evaluation

- Logical operators **&&** and **||** evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
false && z / x > y
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
false
```

Short circuit Evaluation

- Logical operators **&&** and **||** evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
x != 0 && z / x > y
```

⇒ No division by 0

4. Defensive Programming

Constants and Assertions

Sources of Errors

- Errors that the compiler can find:
  syntactical and some semantical errors

Sources of Errors

- Errors that the compiler can find: syntactical and some semantical errors
- Errors that the compiler cannot find: runtime errors (always semantical)

The Compiler as Your Friend: Constants

Constants

- are variables with immutable value

```
const int speed_of_light = 299792458;
```

- Usage: `const` before the definition

The Compiler as Your Friend: Constants

Constants

- are variables with immutable value

  ```
  const int speed_of_light = 299792458;
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The Compiler as Your Friend: Constants

Constants

- are variables with immutable value

```
const int speed_of_light = 299792458;
```

- Usage: **`const`** before the definition

The Compiler as Your Friend: Constants

- Compiler checks that the **`const`**-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise :*“value does not change”*

The Compiler as Your Friend: Constants

- Compiler checks that the **`const`**-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise :*“value does not change”*

The Compiler as Your Friend: Constants

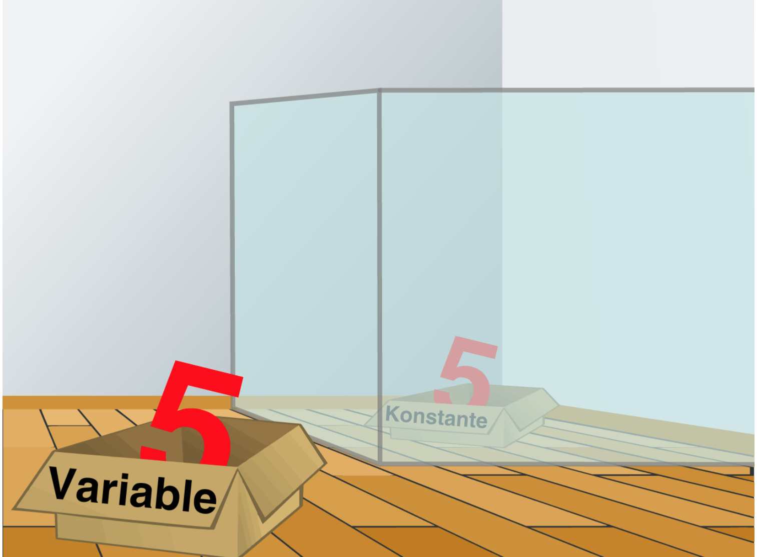
- Compiler checks that the `const`-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise :“*value does not change*”

Constants: Variables behind Glass



The `const`-guideline

const-guideline

For *each variable*, think about whether it will change its value in the lifetime of a program. If not, use the keyword `const` in order to make the variable a constant.

A program that adheres to this guideline is called `const`-correct.

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior

Avoid Sources of Bugs

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≫ It’s not a bug, it’s a feature! ≪

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior
2. Check at many places in the code if the program is still on track

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior
2. Check at many places in the code if the program is still on track
3. Question the (seemingly) obvious, there could be a typo in the code

Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression **`expr`** is false

Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression **`expr`** is false
- requires **`#include <cassert>`**

Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression **`expr`** is false
- requires **`#include <cassert>`**
- can be switched off (potential performance gain)

Assertions for the $gcd(x, y)$

Check if the program is on track …

```
// Input x and y
std::cout << "x =? ";
std::cin >> x;
std::cout << "y =? ";
std::cin >> y;
```

Input arguments for calculation

```
// Check validity of inputs
assert(x > 0 && y > 0);

... // Compute gcd(x,y), store result in variable a
```

Assertions for the $gcd(x, y)$

Check if the program is on track ...

```
// Input x and y
std::cout << "x =? ";
std::cin >> x;
std::cout << "y =? ";
std::cin >> y;

// Check validity of inputs
assert(x > 0 && y > 0);
```

Precondition for the ongoing computation

```
... // Compute gcd(x,y), store result in variable a
```

Assertions for the $gcd(x, y)$

... and question the obvious! ...

```
...
assert(x > 0 && y > 0);
```

Precondition for the ongoing computation

```
... // Compute gcd(x,y), store result in variable a

assert (a >= 1);
assert (x % a == 0 && y % a == 0);
for (int i = a+1; i <= x && i <= y; ++i)
  assert(!(x % i == 0 && y % i == 0));
```

Assertions for the $gcd(x, y)$

... and question the obvious! ...

```
...
assert(x > 0 && y > 0);

... // Compute gcd(x,y), store result in variable a

assert (a >= 1);
assert (x % a == 0 && y % a == 0);
for (int i = a+1; i <= x && i <= y; ++i)
  assert(!(x % i == 0 && y % i == 0));
```

Properties of the gcd

Switch off Assertions

```
#define NDEBUG // To ignore assertions
#include<cassert>

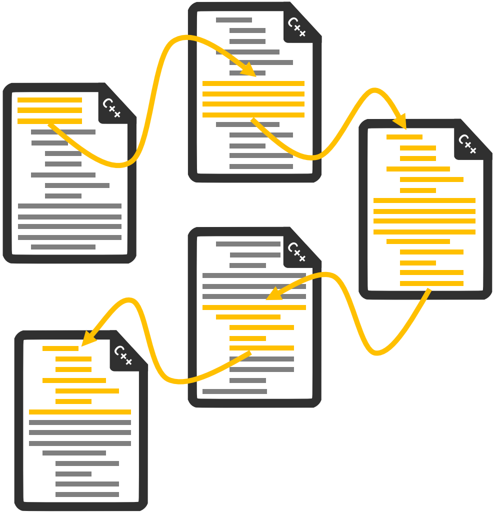
...
assert(x > 0 && y > 0); // Ignored

... // Compute gcd(x,y), store result in variable a

assert(a >= 1); // Ignored
...
```

Fail-Fast with Assertions

- Real software: many C++ files, complex control flow



Fail-Fast with Assertions

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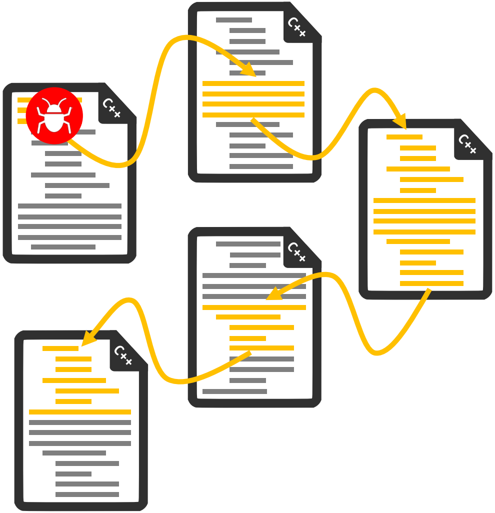
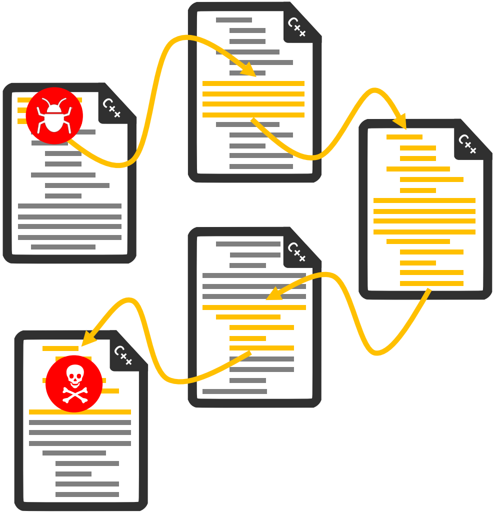


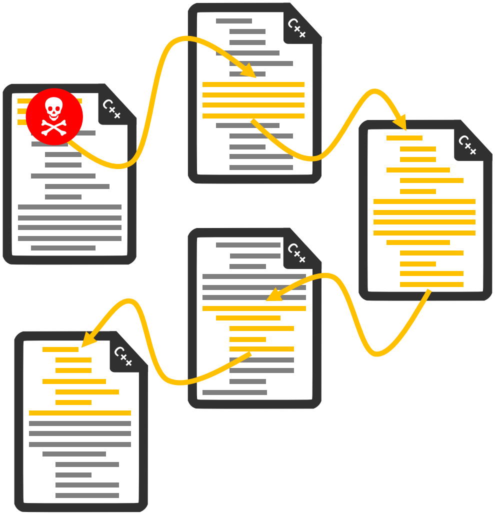
Fail-Fast with Assertions

- Real software: many C++ files, complex control flow
- Errors surface late(r) → impedes error localisation



Fail-Fast with Assertions

- Real software: many C++ files, complex control flow
- Errors surface late(r) → impedes error localisation
- Assertions: Detect errors early



5. Control Structures I

Selection Statements, Iteration Statements, Termination, Blocks

Control Flow

- Up to now: *linear* (from top to bottom)
- Interesting programs require “branches” and “jumps”

```
// Project Hangman
...
while (game_not_over) {
  ...
  if (word.contains(guess)) {
    ...
  } else {
    ...
  }
}
...
```

Selection Statements

implement branches

- **`if`** statement
- **`if-else`** statement

`if`-Statement

`if` (*condition*)
    statement

if-Statement

if (*condition*)
statement

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

`if`-Statement

`if` (*condition*)
    statement

If *condition* is true then *statement* is executed

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

`if`-Statement

`if` (*condition*)
    statement

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

If *condition* is true then *statement* is executed

- *statement*: arbitrary statement (*body* of the `if`-Statement)
- *condition*: convertible to `bool`

if-else-statement

```
if ( condition )
    statement1
else
    statement2
```

if-else-statement

```
if ( condition )
    statement1
else
    statement2
```

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

if-else-statement

```
if ( condition )
    statement1
else
    statement2
```

If *condition* is true then *statement1* is executed, otherwise *statement2* is executed.

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

if-else-statement

```
if ( condition )
    statement1
else
    statement2
```

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

If *condition* is true then *statement1* is executed, otherwise *statement2* is executed.

- *condition*: convertible to **bool**.
- *statement1*: *body* of the **if**-branch
- *statement2*: *body* of the **else**-branch

Layout!

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Layout!

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Indentation

Indentation

Iteration Statements

implement loops

- **`for`**-statement
- **`while`**-statement
- **`do`**-statement

Compute $1 + 2 + ... + n$

```
// input
std::cout << "Compute the sum 1+...+n for n=?";
unsigned int n;
std::cin >> n;

// computation of sum_{i=1}^n i
unsigned int s = 0;
for (unsigned int i = 1; i <= n; ++i)
    s += i;

// output
std::cout << "1+...+" << n << " = " << s << ".\n";
```

Compute $1 + 2 + ... + n$

```
// input
std::cout << "Compute the sum 1+...+n for n=?";
unsigned int n;
std::cin >> n;

// computation of sum_{i=1}^n i
unsigned int s = 0;
for (unsigned int i = 1; i <= n; ++i)
    s += i;

// output
std::cout << "1+...+" << n << " = " << s << ".\n";
```

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i	s

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i	s
i==1	

for-Statement Example

```
for ( unsigned int i=1; i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	`i <= 2`?	

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
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i		s
`i==1`	wahr	

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for ( unsigned int i=1;  i <= n ; ++i )
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i		s
`i==1`	wahr	`s == 1`

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
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```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2		

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
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Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	wahr	`s == 1`
`i==2`	`i <= 2`?	

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	wahr	

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	wahr	s == 3

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	wahr	`s == 1`
`i==2`	wahr	`s == 3`
`i==3`		

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	wahr	s == 3
i==3	i <= 2?	

for-Statement Example

```
for ( unsigned int i=1; i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	wahr	`s == 1`
`i==2`	wahr	`s == 3`
`i==3`	falsch	

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	wahr	`s == 1`
`i==2`	wahr	`s == 3`
`i==3`	falsch	

`s == 3`

`for`-Statement: Syntax

`for` (*init statement* `;` *condition* `;` *expression*)

    body statement

`for`-Statement: Syntax

```
for (init statement; condition; expression)
    body statement
```

- *init statement*: expression statement, declaration statement, null statement

`for`-Statement: Syntax

`for` (*init statement* **`;`** *condition* **`;`** *expression*)

body statement

- *init statement*: expression statement, declaration statement, null statement
- *condition*: convertible to **`bool`**

`for`-Statement: Syntax

`for` (*init statement* `;` *condition* `;` *expression*)
    body statement

- *init statement*: expression statement, declaration statement, null statement
- *condition*: convertible to `bool`
- *expression*: any expression

`for`-Statement: Syntax

```
for (init statement; condition; expression)
    body statement
```

- *init statement*: expression statement, declaration statement, null statement
- *condition*: convertible to **`bool`**
- *expression*: any expression
- *body statement*: any statement (*body* of the for-statement)

`for`-Statement: Termination

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

Here and in most cases:

- *expression* changes its value that appears in *condition* .

for-Statement: Termination

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

Here and in most cases:

- After a finite number of iterations *condition* becomes false: **Termination**

Infinite Loops

- Infinite loops are easy to generate:

```
for ( ; ; ) ;
```

  - Die *empty condition* is true.
  - Die *empty expression* has no effect.
  - Die *null statement* has no effect.

Infinite Loops

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```
for ( ; ; ) ;
```

  - Die *empty condition* is true.
  - Die *empty expression* has no effect.
  - Die *null statement* has no effect.

- ... but can in general not be automatically detected.

```
for (init; cond; expr) stmt;
```

Halting Problem

Undecidability of the Halting Problem

There is no $\mathrm{C}{+}{+}$ program that can determine for each $\mathrm{C}{+}{+}$-Program P and each input I if the program P terminates with the input I.

[4] Alan Turing, 1936. Theoretical questions of this kind were the main motivation for Alan Turing to construct a computing machine.

Halting Problem

Undecidability of the Halting Problem

There is no C++ program that can determine for each C++-Program P and each input I if the program P terminates with the input I.

This means that the correctness of programs can in general *not* be automatically checked.[4]

[4] Alan Turing, 1936. Theoretical questions of this kind were the main motivation for Alan Turing to construct a computing machine.

Example: Prime Number Test

Def.: a natural number $n \geq 2$ is a prime number, if no $d \in \{2, \ldots, n-1\}$ divides n .

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```
unsigned int d;
for (d=2; n%d != 0; ++d);
```

Example: Prime Number Test

Def.: a natural number $n \geq 2$ is a prime number, if no $d \in \{2, \ldots, n-1\}$ divides n .

A loop that can test this:

```
unsigned int d;
for (d=2; n%d != 0; ++d);
```

(body is the null statement)

Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value **d=2**, then plus 1 in every iteration (**++d**)

Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value `d=2`, then plus 1 in every iteration (`++d`)
- Exit: `n%d != 0` evaluates to `false` as soon as a divisor is found — at the latest, once `d == n`

Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value **`d=2`**, then plus 1 in every iteration (**`++d`**)
- Exit: **`n%d != 0`** evaluates to **`false`** as soon as a divisor is found — at the latest, once **`d == n`**
- Progress guarantees that the exit condition will be reached

Example: Correctness

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

Every potential divisor `2 <= d <= n` will be tested. If the loop terminates with `d == n` then and only then is `n` prime.

Blocks

- Blocks group a number of statements to a new statement

```
{statement1 statement2 ... statementN}
```

Blocks

- Blocks group a number of statements to a new statement

- Example: body of the main function

```
int main() {
     ...
}
```

Blocks

- Blocks group a number of statements to a new statement

- Example: loop body

```
for (unsigned int i = 1; i <= n; ++i) {
    s += i;
    std::cout << "partial sum is " << s << "\n";
}
```

Blocks

- Blocks group a number of statements to a new statement

- Beispiel: if / else

```
if (d < n) // d is a divisor of n in {2,...,n-1}
    std::cout << n << " = " << d << " * " << n / d << ".\n";
else {
    assert (d == n);
    std::cout << n << " is prime.\n";
}
```

6. Control Statements II

Visibility, Local Variables, While Statement, Do Statement, Jump Statements

Visibility

Declaration in a block is not *visible* outside of the block.

```
int main()
{
    {
        int i = 2;
    }
    std::cout << i; // Error:  undeclared name
    return 0;
}
```

main block

block

„Blickrichtung“

Potential Scope

in the block

```
{
    ...
    int i = 2;
    ...
}
```

in function body

```
int main() {
    ...
    int i = 2;
    ...
    return 0;
}
```

in control statement

```
for (int i = 0; i < 10; ++i) {s += i; ... }
```

Potential Scope

in the block

```
{
    ...
    int i = 2;
    ...
}
```

scope

in function body

```
int main() {
    ...
    int i = 2;
    ...
    return 0;
}
```

scope

in control statement

```
for (int i = 0; i < 10; ++i) {s += i; ... }
```

scope

Scope

```
int main()
{
   int i = 2;
   for (int i = 0; i < 5; ++i)
      // outputs 0,1,2,3,4
      std::cout << i;
    // outputs 2
    std::cout << i;
   return 0;
}
```

Potential Scope

```
int main()
{
   int i = 2;
   for (int i = 0; i < 5; ++i)
      // outputs 0,1,2,3,4
      std::cout << i;
    // outputs 2
    std::cout << i;
   return 0;
}
```

Real Scope

```
int main()
{
   int i = 2;
   for (int i = 0; i < 5; ++i)
       // outputs 0,1,2,3,4
       std::cout << i;
    // outputs 2
    std::cout << i;
   return 0;
}
```

Local Variables

```
int main()
{
    int i = 5;
    for (int j = 0; j < 5; ++j) {
        std::cout << ++i; // outputs
        int k = 2;
        std::cout << --k; // outputs
    }
}
```

Local Variables

```
int main()
{
    int i = 5;
    for (int j = 0; j < 5; ++j) {
        std::cout << ++i; // outputs 6, 7, 8, 9, 10
        int k = 2;
        std::cout << --k; // outputs 1, 1, 1, 1, 1
    }
}
```

Local Variables

```
int main()
{
    int i = 5;
    for (int j = 0; j < 5; ++j) {
        std::cout << ++i; // outputs
        int k = 2;
        std::cout << --k; // outputs
    }
}
```

Local variables (declaration in a block) have *automatic storage duration.*

`while` Statement

```
while (condition)
  statement
```

`while` Statement

```
while (condition)
  statement
```

is equivalent to

```
for (; condition; )
  statement
```

Example: The Collatz-Sequence $(n \in \mathbb{N})$

- $n_0 = n$
- $n_i = \begin{cases} \frac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1} + 1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

The Collatz-Sequence

- $n_0 = n$
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n=5: 5

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n=5: 5, 16

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n=5: 5, 16, 8

The Collatz-Sequence

- $n_0 = n$
- $n_i = \begin{cases} \dfrac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1} + 1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

n=5: 5, 16, 8, 4

The Collatz-Sequence

- $n_0 = n$
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n=5: 5, 16, 8, 4, 2

The Collatz-Sequence

- $n_0 = n$
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n=5: 5, 16, 8, 4, 2, 1

The Collatz-Sequence

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n=5: 5, 16, 8, 4, 2, 1, 4

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n=5: 5, 16, 8, 4, 2, 1, 4, 2

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n=5: 5, 16, 8, 4, 2, 1, 4, 2, 1

The Collatz-Sequence

- $n_0 = n$
- $n_i = \begin{cases} \dfrac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1} + 1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

n=5: 5, 16, 8, 4, 2, 1, 4, 2, 1, ... (repetition at 1)

do Statement

```
do
  statement
while (condition);
```

do Statement

```
do
  statement
while (condition);
```

is equivalent to

```
statement
while (condition)
  statement
```

`break` and `continue` in practice

- Advantage: Can avoid nested **`if-else`**blocks (or complex disjunctions)

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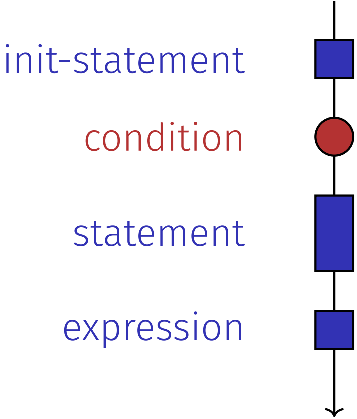
`break` and `continue` in practice

- Advantage: Can avoid nested **`if-else`**blocks (or complex disjunctions)
- But they result in additional jumps and thus potentially complicate the control flow
- Their use is thus controversial, and should be carefully considered

Control Flow `for`

`for` (*init statement* *condition* **;** *expression*)
    statement



init-statement

condition

statement

expression

Control Flow `for`

`for` (*init statement* *condition* **;** *expression*)
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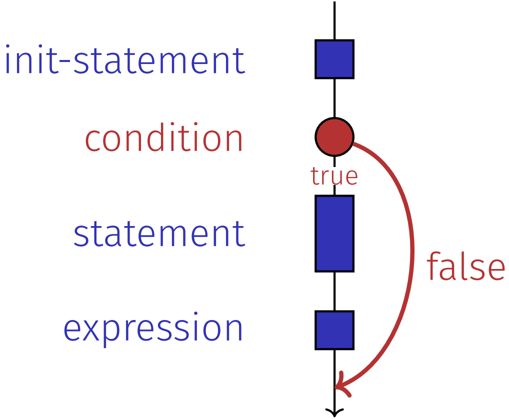


init-statement

condition

true

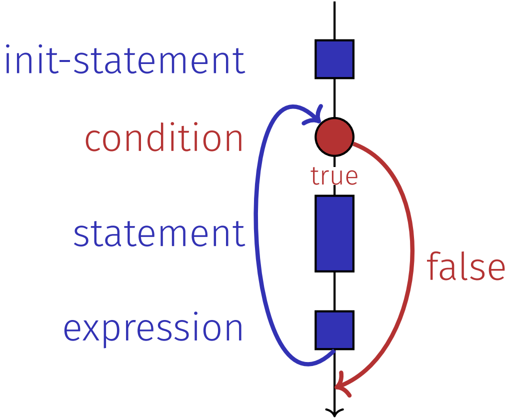
statement

false

expression

Control Flow `for`

`for` (*init statement*   *condition* **;** *expression*)
    statement



init-statement

condition

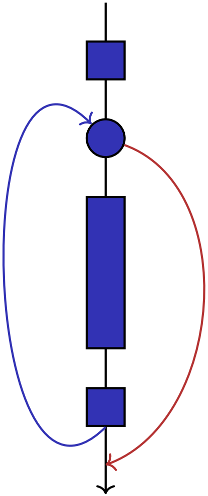
true

statement

false

expression

Control Flow `break` and `continue` in `for`



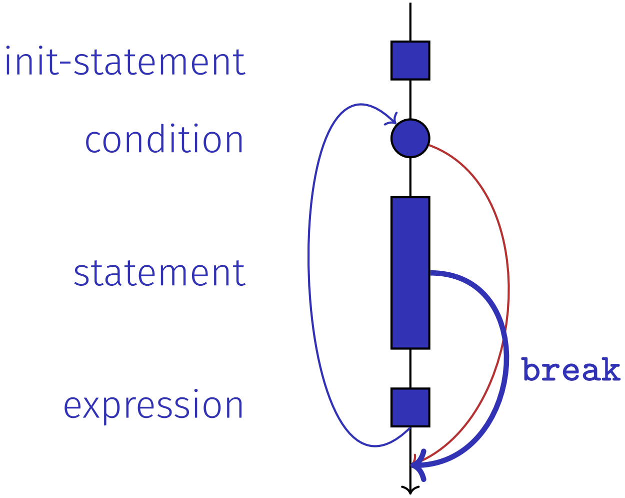
init-statement

condition

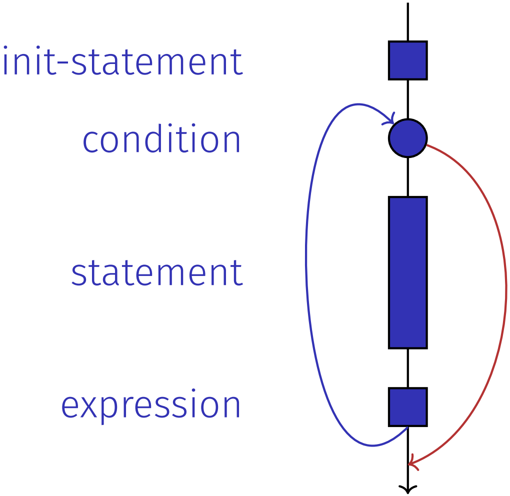
statement

expression

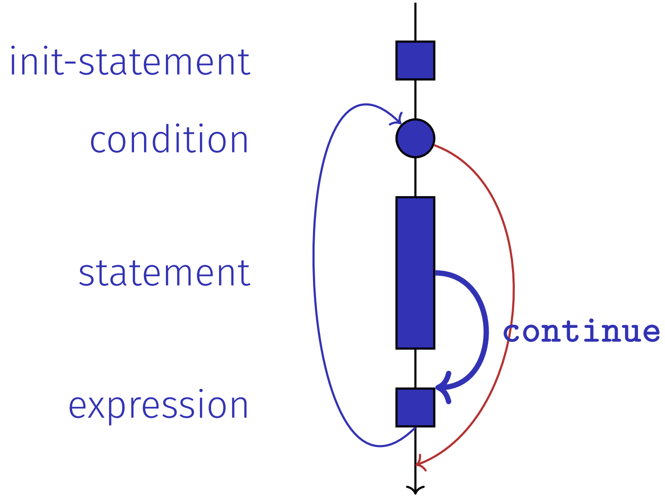
Control Flow *break* and continue in for



Control Flow `break` and `continue` in for



Control Flow `break` and *`continue`* in for



Control Flow: the Good old Times?

Observation

Actually, we only need **`if`** and jumps to arbitrary places in the program (**`goto`**).

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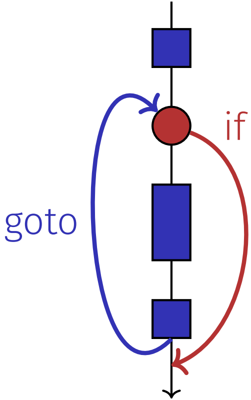

Control Flow: the Good old Times?

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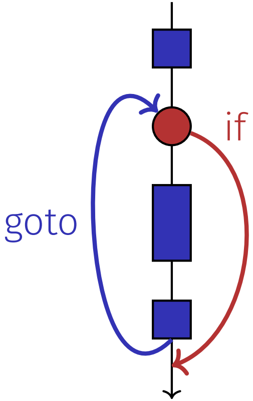
Actually, we only need **`if`** and jumps to arbitrary places in the program (**`goto`**).

Languages based on them:

- Machine Language



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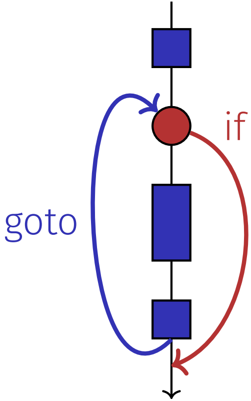
Actually, we only need **`if`** and jumps to arbitrary places in the program (**`goto`**).

Languages based on them:

- Machine Language
- Assembler (“higher” machine language)



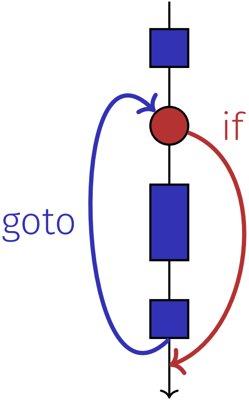
Control Flow: the Good old Times?

Observation

Actually, we only need **`if`** and jumps to arbitrary places in the program (**`goto`**).

Languages based on them:

- Machine Language
- Assembler ("higher" machine language)
- BASIC, the first programming language for the general public (1964)



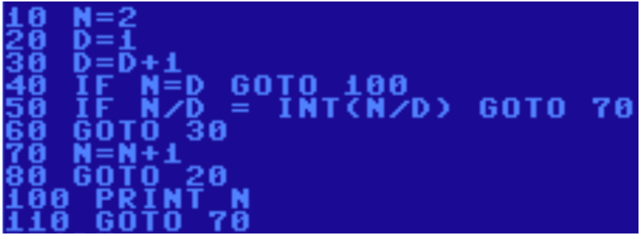
BASIC and home computers...

...allowed a whole generation of young adults to program.

Home-Computer Commodore C64 (1982)

Spaghetti-Code with goto

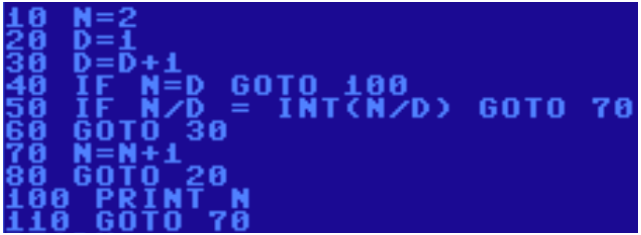
Output of of ??????????? using the programming language BASIC:



```
10  N=2
20  D=1
30  D=D+1
40  IF  N=D  GOTO  100
50  IF  N/D  =  INT(N/D)  GOTO  70
60  GOTO  30
70  N=N+1
80  GOTO  20
100  PRINT  N
110  GOTO  70
```

Spaghetti-Code with goto

Output of all prime numbers using the programming language BASIC:



```
10  N=2
20  D=1
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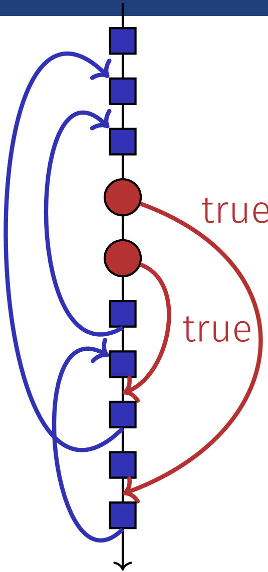
40  IF  N=D  GOTO  100
50  IF  N/D  =  INT(N/D)  GOTO  70
60  GOTO  30
70  N=N+1
80  GOTO  20
100 PRINT  N
110 GOTO  70
```



The “right” Iteration Statement

Goals: readability, conciseness, in particular

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- few statements

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- simple expressions

The “right” Iteration Statement

Goals: readability, conciseness, in particular

- few statements
- few lines of code
- simple control flow
- simple expressions

Often not all goals can be achieved simultaneously.

Odd Numbers in $\{0, \ldots, 100\}$

First (correct) attempt:

```
for (unsigned int i = 0; i < 100; ++i) {
    if (i % 2 == 0)
        continue;
    std::cout << i << "\n";
}
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, **less** lines:

```
for (unsigned int i = 0; i < 100; ++i) {
    if (i % 2 != 0)
        std::cout << i << "\n";
}
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, **simpler** control flow:

```
for (unsigned int i = 1; i < 100; i += 2)
    std::cout << i << "\n";
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, **simpler** control flow:

```
for (unsigned int i = 1; i < 100; i += 2)
    std::cout << i << "\n";
```

This is the “right” iteration statement

Outputting Grades

1. Functional requirement:

$$\begin{aligned} 6 &\rightarrow \texttt{"Excellent ... You passed!"} \\ 5, 4 &\rightarrow \texttt{"You passed!"} \\ 3 &\rightarrow \texttt{"Close, but ... You failed!"} \\ 2, 1 &\rightarrow \texttt{"You failed!"} \\ \mathit{otherwise} &\rightarrow \texttt{"Error!"} \end{aligned}$$

Outputting Grades

1. Functional requirement:

$$\begin{aligned} 6 &\rightarrow \texttt{"Excellent ... You passed!"} \\ 5, 4 &\rightarrow \texttt{"You passed!"} \\ 3 &\rightarrow \texttt{"Close, but ... You failed!"} \\ 2, 1 &\rightarrow \texttt{"You failed!"} \\ \textit{otherwise} &\rightarrow \texttt{"Error!"} \end{aligned}$$

2. Moreover: Avoid duplication of text and code

Outputting Grades with `if` Statements

```
int grade;
...
if (grade == 6) std::cout << "Excellent ... ";
if (4 <= grade && grade <= 6) {
    std::cout << "You passed!";
} else if (1 <= grade && grade < 4) {
    if (grade == 3) std::cout << "Close, but ... ";
    std::cout << "You failed!";
} else std::cout << "Error!";
```

Outputting Grades with `if` Statements

```
int grade;
...
if (grade == 6) std::cout << "Excellent ... ";
if (4 <= grade && grade <= 6) {
    std::cout << "You passed!";
} else if (1 <= grade && grade < 4) {
    if (grade == 3) std::cout << "Close, but ... ";
    std::cout << "You failed!";
} else std::cout << "Error!";
```

Disadvantage: Control flow – and thus program behaviour – not quite obvious

Outputting Grades with `switch` Statement

```
switch (grade) {
  case 6: std::cout << "Excellent ... ";
  case 5:
  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

Outputting Grades with `switch` Statement

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switch (grade) {
  case 6: std::cout << "Excellent ... ";
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  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

Jump to matching **`case`**

Outputting Grades with `switch` Statement

```
switch (grade) {
  case 6: std::cout << "Excellent ... ";
  case 5:
  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

Fall-through

Outputting Grades with **`switch`** Statement

```
switch (grade) {
  case 6: std::cout << "Excellent ... ";
  case 5:
  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

Fall-through

Exit **`switch`**

Outputting Grades with `switch` Statement

```
switch (grade) {
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Outputting Grades with `switch` Statement

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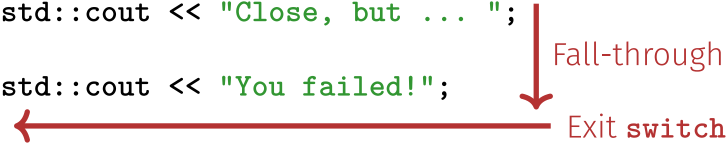
```



Fall-through

Exit `switch`

Outputting Grades with `switch` Statement

```
switch (grade) {
  case 6: std::cout << "Excellent ... ";
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  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

← In all other cases

Outputting Grades with switch Statement

```
switch (grade) {
  case 6: std::cout << "Excellent ... ";
  case 5:
  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

Advantage: Control flow clearly recognisable

The switch-Statement

```
switch (expression)
  statement
```

- *expression*: Expression, convertible to integral type
- *statement* : arbitrary statemet, in which **case** and **default**-lables are permitted, **break** has a special meaning.

The `switch`-Statement

```
switch (expression)
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```

- *expression*: Expression, convertible to integral type
- *statement* : arbitrary statemet, in which **`case`** and **`default`**-lables are permitted, **`break`** has a special meaning.
- Use of fall-through property is controversial and should be carefully considered (corresponding compiler warning can be enabled)

7. Floating-point Numbers I

Types **`float`** and **`double`**; Mixed Expressions and Conversion; Holes in the Value Range

“Proper” Calculation

```
// Input
std::cout << "Temperature in degrees Celsius =? ";
int celsius;
std::cin >> celsius;

// Computation and output
std::cout << celsius << " degrees Celsius are "
         << 9 * celsius / 5 + 32 << " degrees Fahrenheit.\\n";
```

```
28 degrees Celsius are 82 degrees Fahrenheit.
```

“Proper” Calculation

```
// Input
std::cout << "Temperature in degrees Celsius =? ";
int celsius;
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```

```
28 degrees Celsius are 82 degrees Fahrenheit.
```

richtig wäre 82.4

“Proper” Calculation

```
// Input
std::cout << "Temperature in degrees Celsius =? ";
float celsius; // Enable fractional numbers
std::cin >> celsius;

// Computation and output
std::cout << celsius << " degrees Celsius are "
         << 9 * celsius / 5 + 32 << " degrees Fahrenheit.\\n";
```

```
28 degrees Celsius are 82.4 degrees Fahrenheit.
```

Fixed-point numbers

- fixed number of integer places (e.g. 7)
- fixed number of decimal places (e.g. 3)

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```
    82.4 = 0000082.400
```

Fixed-point numbers

- fixed number of integer places (e.g. 7)
- fixed number of decimal places (e.g. 3)

```
   82.4 = 0000082.400
```

Disadvantages

- Value range is getting *even* smaller than for integers.

Fixed-point numbers

- fixed number of integer places (e.g. 7)
- fixed number of decimal places (e.g. 3)

`0.0824 = 0000000.082` ⟵ third place truncated

Disadvantages

- Representability depends on the position of the decimal point.

Floating-point numbers

- Observation: same number, different representations with varying “efficiency”, e.g.

  $$\begin{aligned} 0.0824 &= 0.00824 \cdot 10^{1} &&= 0.824 \cdot 10^{-1} \\ &= 8.24 \cdot 10^{-2} &&= 824 \cdot 10^{-4} \end{aligned}$$

  Number of *significant digits* remains constant

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  Number of *significant digits* remains constant

- Floating-point number representation thus:
  - Fixed number of significant places (e.g. 10),
  - Plus position of the decimal point via exponent
  - Number is $\mathit{Mantissa} \times 10^{\mathit{Exponent}}$

Types `float` and `double`

- are the fundamental C++ types for floating point numbers
- approximate the field of real numbers $(\mathbb{R}, +, \times)$ from mathematics

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  - **`float`**: approx. 7 digits, exponent up to ± 38
  - **`double`**: approx. 15 digits, exponent up to ± 308

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- approximate the field of real numbers $(\mathbb{R}, +, \times)$ from mathematics
- have a big value range, sufficient for many applications:
  - **`float`**: approx. 7 digits, exponent up to ± 38
  - **`double`**: approx. 15 digits, exponent up to ± 308
- are fast on most computers (hardware support)

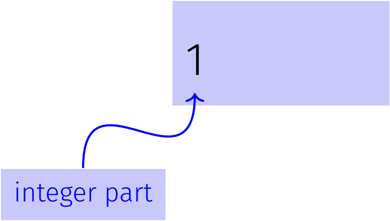
Arithmetic Operators

Analogous to **`int`**, but ...

- Division operator **`/`** models a “proper” division (real-valued, not integer)
- No modulo operator, i.e. no **`%`**

Literals

are different from integers



1

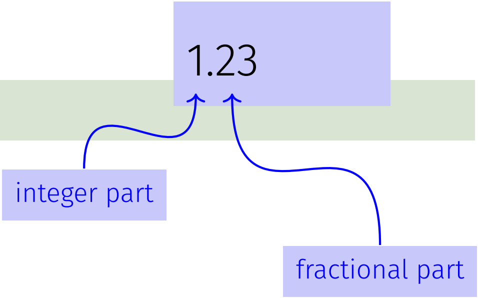
integer part

Literals

are different from integers by providing

- decimal point

  `1.0` : type **`double`**, value 1



1.23

integer part

fractional part

Literals

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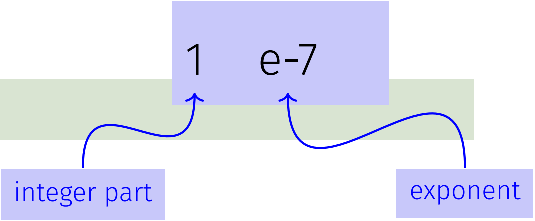


1 e-7

integer part

exponent

- or exponent.

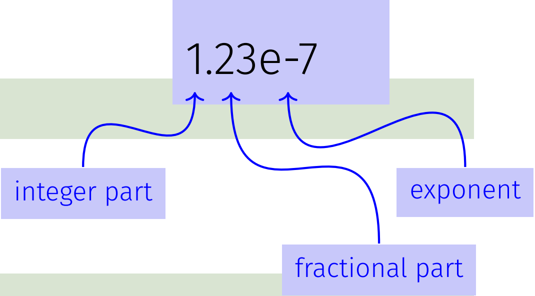
  1e3 : type **double**, value 1000

Literals

are different from integers by providing

- decimal point

`1.0` : type **`double`**, value 1



1.23e-7

integer part

exponent

fractional part

- and / or exponent.

`1e3` : type **`double`**, value 1000

`1.23e-7` : type **`double`**, value $1.23 \cdot 10^{-7}$

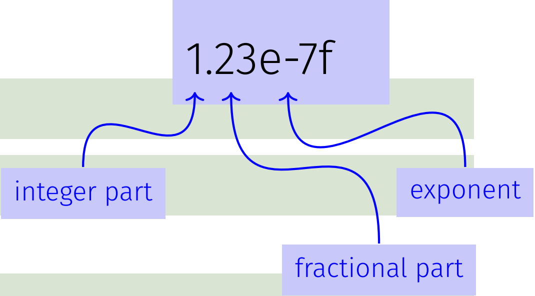
Literals

are different from integers by providing

- decimal point

  `1.0` : type **`double`**, value 1

  `1.27f` : type **`float`**, value 1.27


1.23e-7f

integer part

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exponent

- and / or exponent.

  `1e3` : type **`double`**, value 1000

  `1.23e-7` : type **`double`**, value $1.23 \cdot 10^{-7}$

  `1.23e-7f` : type **`float`**, value $1.23 \cdot 10^{-7}$

Computing with `float`: Example

Approximating the Euler-Number

$$e = \sum_{i=0}^{\infty} \frac{1}{i!} \approx 2.71828\ldots$$

using the first 10 terms.

Computing with `float`: Euler Number

```
std::cout << "Approximating the Euler number... \n";

// values for i-th iteration, initialized for i = 0
float t = 1.0f; // term 1/i!
float e = 1.0f; // i-th approximation of e

// iteration 1, ..., n
for (unsigned int i = 1; i < 10; ++i) {
    t /= i;    // 1/(i-1)! -> 1/i!
    e += t;
    std::cout << "Value after term " << i << ": "
             << e << "\n";
}
```

Computing with `float`: Euler Number

```
Value after term 1: 2
Value after term 2: 2.5
Value after term 3: 2.66667
Value after term 4: 2.70833
Value after term 5: 2.71667
Value after term 6: 2.71806
Value after term 7: 2.71825
Value after term 8: 2.71828
Value after term 9: 2.71828
```

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.

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```
9 * celsius / 5  + 32
```

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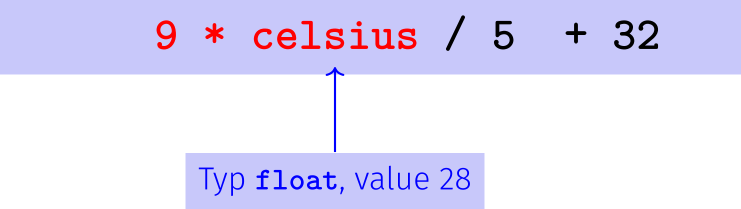

```
9 * celsius / 5  + 32
```

Typ **`float`**, value 28

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.
- In mixed expressions integers are converted to floating point numbers.

```
9 * 28.0f / 5  + 32
```

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.
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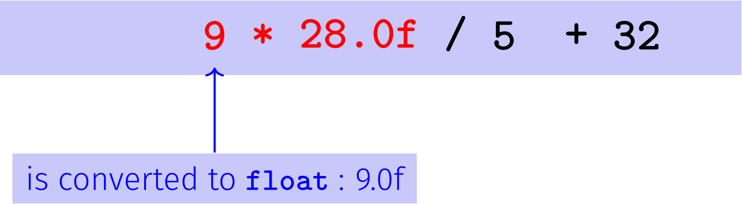


```
9 * 28.0f / 5  + 32
```

is converted to **`float`** : 9.0f

Mixed Expressions, Conversion

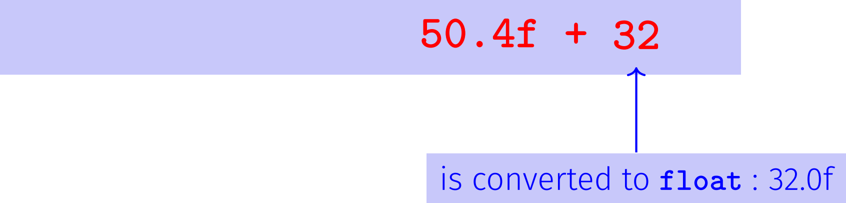
- Floating point numbers are more general than integers.
- In mixed expressions integers are converted to floating point numbers.

```
252.0f / 5  + 32
```

is converted to **`float`** : 5.0f

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.
- In mixed expressions integers are converted to floating point numbers.



```
50.4f + 32
```

is converted to **`float`** : 32.0f

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.
- In mixed expressions integers are converted to floating point numbers.

```
82.4f
```

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;

float n2;
std::cout << "Second number =? ";
std::cin >> n2;

float d;
std::cout << "Their difference =? ";
std::cin >> d;

std::cout << "Computed difference - input difference = "
         << n1 - n2 - d << "\n";
```

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;
```

input 1.5

```
float n2;
std::cout << "Second number =? ";
std::cin >> n2;
```

input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.5

```
std::cout << "Computed difference - input difference = "
         << n1 - n2 - d << "\n";
```

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;
```

input 1.5

```
float n2;
std::cout << "Second number =? ";
std::cin >> n2;
```

input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.5

```
std::cout << "Computed difference - input difference = "
          << n1 - n2 - d << "\n";
```

output 0

Holes in the value range

```
float n1;
std::cout << "First number  =? ";      input 1.1
std::cin >> n1;

float n2;
std::cout << "Second number =? ";      input 1.0
std::cin >> n2;

float d;
std::cout << "Their difference =? ";   input 0.1
std::cin >> d;

std::cout << "Computed difference - input difference = "
         << n1 - n2 - d << "\n";
```

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;
```

input 1.1

```
float n2;
std::cout << "Second number =? ";
std::cin >> n2;
```

input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.1

```
std::cout << "Computed difference - input difference = "
          << n1 - n2 - d << "\n";
```

output 2.23517e-8

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;
```

input 1.1

```
float n2;
std::cout << "Second number =? ";
std::cin >> n2;
```

input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.1

```
std::cout << "Computed difference - input difference = "
         << n1 - n2 - d << "\n";
```

output 2.23517e-8

What is going on here?

Value range

Integer Types:

- Over- and Underflow relatively frequent, but ...
- the value range is contiguous (no holes): $\mathbb{Z}$ is “discrete”.

Value range

Integer Types:

- Over- and Underflow relatively frequent, but ...
- the value range is contiguous (no holes): $\mathbb{Z}$ is “discrete”.

Floating point types:

- Overflow and Underflow seldom, but ...
- there are holes: $\mathbb{R}$ is “continuous”.

8. Floating-point Numbers II

Floating-point Number Systems; IEEE Standard; Limits of Floating-point Arithmetics; Floating-point Guidelines; Harmonic Numbers

Floating-point Number Systems

A Floating-point number system is defined by the four natural numbers:

- $\beta \geq 2$, the base,
- $p \geq 1$, the precision (number of places),
- $e_{\min}$, the smallest possible exponent,
- $e_{\max}$, the largest possible exponent.

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- $e_{\max}$, the largest possible exponent.

Notation:

$$F(\beta, p, e_{\min}, e_{\max})$$

Floating-point number Systems

$F(\beta, p, e_{\min}, e_{\max})$ contains the numbers

$$\pm \sum_{i=0}^{p-1} d_i \beta^{-i} \cdot \beta^e,$$

$$d_i \in \{0, \ldots, \beta - 1\}, \quad e \in \{e_{\min}, \ldots, e_{\max}\}.$$

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$$d_i \in \{0, \ldots, \beta - 1\}, \quad e \in \{e_{\min}, \ldots, e_{\max}\}.$$

represented in base β:

$$\pm\, d_{0\bullet} d_1 \ldots d_{p-1} \times \beta^e,$$

Floating-point Number Systems

Representations of the decimal number 0.1 (with $\beta = 10$):

$$1.0 \cdot 10^{-1}, \quad 0.1 \cdot 10^{0}, \quad 0.01 \cdot 10^{1}, \quad \ldots$$

Different representations due to choice of exponent

Normalized representation

Normalized number:

$$\pm\, d_0{\bullet}d_1 \ldots d_{p-1} \times \beta^e, \qquad d_0 \neq 0$$

Remark 1

The normalized representation is unique and therefore preferred.

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Normalized representation

Normalized number:

$$\pm\, d_0 {\bullet} d_1 \dots d_{p-1} \times \beta^e, \qquad d_0 \neq 0$$

Remark 2

The number 0, as well as all numbers smaller than $\beta^{e_{\min}}$, have no normalized representation (we will come back to this later)

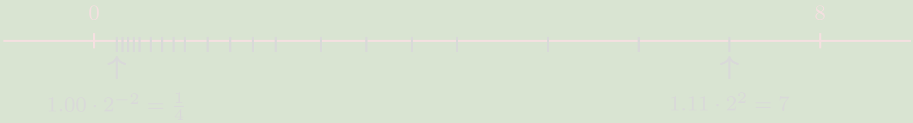
Set of Normalized Numbers

$$F^*(\beta, p, e_{\min}, e_{\max})$$

Normalized Representation

Example $F^*(2,3,-2,2)$ (only positive numbers)

$d_{0\bullet}d_1d_2$	$e=-2$	$e=-1$	$e=0$	$e=1$	$e=2$
1.00_2	0.25	0.5	1	2	4
1.01_2	0.3125	0.625	1.25	2.5	5
1.10_2	0.375	0.75	1.5	3	6
1.11_2	0.4375	0.875	1.75	3.5	7



Normalized Representation

Example $F^*(2, 3, -2, 2)$ (only positive numbers)

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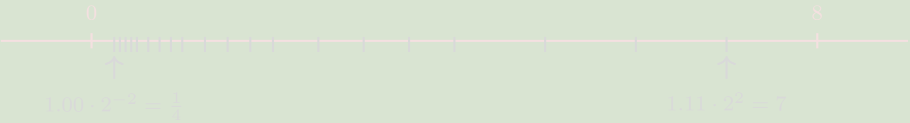

Normalized Representation

Example $F^*(2, 3, -2, 2)$ (only positive numbers)

$d_0 \bullet d_1 d_2$	$e = -2$	$e = -1$	$e = 0$	$e = 1$	$e = 2$
1.00_2	0.25	0.5	1	2	4
1.01_2	0.3125	0.625	1.25	2.5	5
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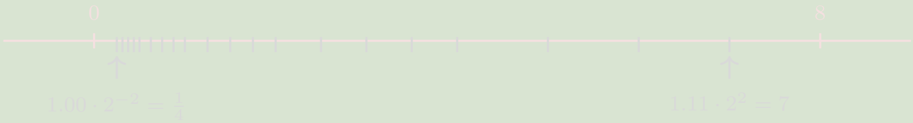

Normalized Representation

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1.00_2	0.25	0.5	1	2	4
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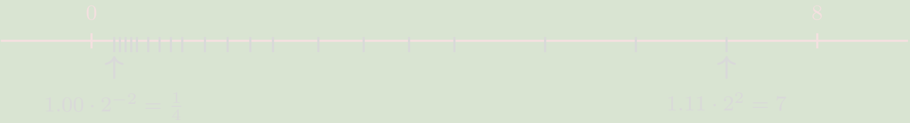

Normalized Representation

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1.00_2	0.25	0.5	1	2	4
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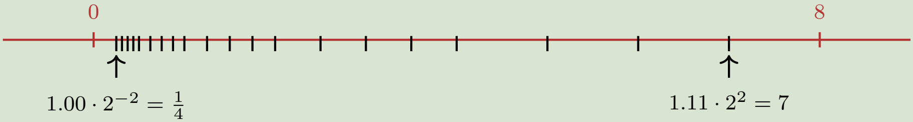

$1.00 \cdot 2^{-2} = \frac{1}{4}$

$1.11 \cdot 2^{2} = 7$

Binary and Decimal Systems

- Internally the computer computes with $\beta = 2$ (binary system)

Binary and Decimal Systems

- Internally the computer computes with $\beta = 2$ (binary system)
- Literals and inputs have $\beta = 10$ (decimal system)

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$x = \sum_{i=0}^{\infty} b_i 2^{-i}$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$x = b_0{\bullet}b_1b_2b_3\ldots$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$\begin{aligned} x &= b_0 \bullet b_1 b_2 b_3 \ldots \\ &= b_0 + 0 \bullet b_1 b_2 b_3 \ldots \end{aligned}$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$\begin{aligned} x &= b_0{}_\bullet b_1 b_2 b_3 \dots \\ &= b_0 + 0_\bullet b_1 b_2 b_3 \dots \\ &\Longrightarrow \end{aligned}$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$\begin{aligned} x &= b_0{\bullet}b_1b_2b_3\ldots \\ &= b_0 + 0{\bullet}b_1b_2b_3\ldots \\ &\Longrightarrow \\ (x - b_0) &= 0{\bullet}b_1b_2b_3b_4\ldots \end{aligned}$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$
\begin{aligned}
x &= b_0{\bullet}b_1b_2b_3\ldots \\
&= b_0 + 0{\bullet}b_1b_2b_3\ldots \\
&\Longrightarrow \\
2\cdot(x-b_0) &= b_1{\bullet}b_2b_3b_4\ldots
\end{aligned}
$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$\begin{aligned} x &= b_0{}_\bullet b_1 b_2 b_3 \ldots \\ &= b_0 + 0_\bullet b_1 b_2 b_3 \ldots \\ &\Longrightarrow \\ 2 \cdot (x - b_0) &= b_1{}_\bullet b_2 b_3 b_4 \ldots \end{aligned}$$

Conversion $(0 < x < 2)$

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$$
\begin{aligned}
x &= b_0{\bullet}b_1b_2b_3\ldots \\
&= b_0 + 0{\bullet}b_1b_2b_3\ldots \\
&\Longrightarrow \\
2\cdot(x-b_0) &= b_1{\bullet}b_2b_3b_4\ldots
\end{aligned}
$$

```
for (int b_0; x != 0; x = 2 * (x - b_0)) {
  b_0 = (x >= 1);
  std::cout << b_0;
}
```

Example (binary)

$$\begin{aligned} x &= 1_\bullet 01011 \\ &= 1 + 0_\bullet 01011 \\ &\Longrightarrow \\ 2 \cdot (x - 1) &= 0_\bullet 1011 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 1{\bullet}01011 \\ &= 1 + 0{\bullet}01011 \\ &\Longrightarrow \\ 2 \cdot (x - 1) &= 0{\bullet}1011 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 0{\bullet}1011 \\ &= 0 + 0{\bullet}1011 \\ &\Longrightarrow \\ 2 \cdot (x - 0) &= 1{\bullet}011 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 0_{\bullet}1011 \\ &= 0 + 0_{\bullet}1011 \\ &\Longrightarrow \\ 2 \cdot (x - 0) &= 1_{\bullet}011 \end{aligned}$$

Example (binary)

$$
\begin{aligned}
x &= \textcolor{red}{1}_{\bullet}011 \\
&= \textcolor{red}{1} + 0_{\bullet}011 \\
&\Longrightarrow \\
2 \cdot (x - \textcolor{red}{1}) &= 0_{\bullet}11
\end{aligned}
$$

Example (binary)

$$\begin{aligned} x &= 1_{\bullet}011 \\ &= 1 + 0_{\bullet}011 \\ &\Longrightarrow \\ 2 \cdot (x-1) &= 0_{\bullet}11 \end{aligned}$$

Example (binary)

$$
\begin{aligned}
x &= 0{\bullet}11 \\
&= 0 + 0{\bullet}11 \\
&\implies \\
2 \cdot (x - 0) &= 1{\bullet}1
\end{aligned}
$$

Example (binary)

$$\begin{aligned} x &= 0_\bullet 11 \\ &= 0 + 0_\bullet 11 \\ &\Longrightarrow \\ 2 \cdot (x - 0) &= 1_\bullet 1 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 1{\bullet}1 \\ &= 1 + 0{\bullet}1 \\ &\Longrightarrow \\ 2 \cdot (x - 1) &= 1 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 1_\bullet 1 \\ &= 1 + 0_\bullet 1 \\ &\implies \\ 2 \cdot (x - 1) &= 1 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 1 \\ &= 1 + 0 \\ &\implies \\ 2 \cdot (x - 1) &= 0 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 1 \\ &= 1 + 0 \\ &\implies \\ 2 \cdot (x - 1) &= \color{blue}{0} \end{aligned}$$

Binary representation of 1.1_{10}

x	b_i	$x - b_i$	$2(x - b_i)$
1.1	$b_0 = 1$		

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x	b_i	$x - b_i$	$2(x - b_i)$
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0.2	$b_1 = 0$	0.2	0.4

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1.2	$b_5 = 1$		

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0.8	$b_3 = 0$	0.8	1.6
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1.2	$b_5 = 1$	0.2	0.4

$\Rightarrow$ $1.0\overline{0011}$, periodic, *not* finite

Binary Number Representations of 1.1 and 0.1

- are not finite ⇒ conversion errors

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- `1.1f` und `0.1f`: *Approximations* of 1.1 and 0.1

Binary Number Representations of 1.1 and 0.1

- are not finite ⇒ conversion errors
- `1.1f` und `0.1f`: *Approximations* of 1.1 and 0.1
- In `diff.cpp`: $1.1 - 1.0 \neq 0.1$

Binary Number Representations of 1.1 and 0.1

on my computer:

$$\begin{aligned} \texttt{1.1} &= \underline{1.1000000000000000}888178\ldots \\ \texttt{1.1f} &= \underline{1.1000000}238418\ldots \end{aligned}$$

Computing with Floating-point Numbers

is nearly as simple as with integers.

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 1.011 \cdot 2^{-1} \end{array}$$

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 1.011 \cdot 2^{-1} \end{array}$$

1. adjust exponents by denormalizing one number

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \checkmark \end{array}$$

1. adjust exponents by denormalizing one number

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline \end{array}$$

2. binary addition of the significands

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline = & 100.101 \cdot 2^{-2} \checkmark \end{array}$$

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Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline & = 100.101 \cdot 2^{-2} \end{array}$$

3. renormalize

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline & = 1.00101 \cdot 2^{0} \checkmark \end{array}$$

3. renormalize

Computing with Floating-point Numbers

Example ($\beta = 2, p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline & = 1.00101 \cdot 2^{0} \end{array}$$

4. round to p significant places, if necessary

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

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4. round to p significant places, if necessary

The IEEE Standard 754

defines floating-point number systems and their rounding behavior and is used nearly everywhere

- Single precision (**`float`**) numbers:

  $F^*(2, 24, -126, 127)$ (32 bit)

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- Double precision (**`double`**) numbers:

  $F^*(2, 53, -1022, 1023)$ (64 bit)

The IEEE Standard 754

defines floating-point number systems and their rounding behavior and is used nearly everywhere

- Single precision (**`float`**) numbers:

  $F^*(2, 24, -126, 127)$ (32 bit) plus $0, \infty, \ldots$

- Double precision (**`double`**) numbers:

  $F^*(2, 53, -1022, 1023)$ (64 bit) plus $0, \infty, \ldots$

- All arithmetic operations round the *exact* result to the next representable number

Example: 32-bit Representation of a Floating Point Number

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

$\pm$ Exponent Mantisse

$$\pm \begin{matrix} 2^{-126}, \ldots, 2^{127} \\ 0, \infty, \ldots \end{matrix}$$

$$\begin{matrix} 1.00000000000000000000000 \\ \cdots \\ 1.11111111111111111111111 \end{matrix}$$

Floating-point Rules

Rule 1

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Do not test rounded floating-point numbers for equality.

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Do not test rounded floating-point numbers for equality.

```
for (float i = 0.1; i != 1.0; i += 0.1)
    std::cout << i << "\n";
```

Floating-point Rules Rule 1

Rule 1

Do not test rounded floating-point numbers for equality.

```
for (float i = 0.1; i != 1.0; i += 0.1)
    std::cout << i << "\n";
```

endless loop because i never becomes exactly 1

Floating-point Rules

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Do not add two numbers of very different orders of magnitude!

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$$\begin{array}{r} 1.000 \cdot 2^5 \\ +1.000 \cdot 2^0 \end{array}$$

Floating-point Rules

Rule 2

Do not add two numbers of very different orders of magnitude!

$$\begin{aligned} & 1.000 \cdot 2^5 \\ & +1.000 \cdot 2^0 \\ = & \ 1.00001 \cdot 2^5 \end{aligned}$$

Rule 2

Do not add two numbers of very different orders of magnitude!

$$
\begin{aligned}
&1.000 \cdot 2^5 \\
&+1.000 \cdot 2^0 \\
= {}&1.00001 \cdot 2^5 \\
\text{“}{=}\text{”} {}&1.000 \cdot 2^5 \quad \text{(Rounding on 4 places)}
\end{aligned}
$$

Floating-point Rules

Rule 2

Do not add two numbers of very different orders of magnitude!

$$
\begin{aligned}
&1.000 \cdot 2^5 \\
&+1.000 \cdot 2^0 \\
&= 1.00001 \cdot 2^5 \\
\text{“}{=}\text{”}\ &1.000 \cdot 2^5 \quad \text{(Rounding on 4 places)}
\end{aligned}
$$

Addition of 1 does not have any effect!

Harmonic Numbers Rule 2

- The n-the harmonic number is

$$H_n = \sum_{i=1}^{n} \frac{1}{i}$$

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Harmonic Numbers Rule 2

- The n-the harmonic number is

$$H_n = \sum_{i=1}^{n} \frac{1}{i} \approx \ln n.$$

- This sum can be computed in forward or backward direction, which is mathematically clearly equivalent

Harmonic Numbers Rule 2

```
std::cout << "Compute H_n for n =? ";
unsigned int n;
std::cin >> n;

float fs = 0;
for (unsigned int i = 1; i <= n; ++i)
    fs += 1.0f / i;
std::cout << "Forward sum = " << fs << "\n";

float bs = 0;
for (unsigned int i = n; i >= 1; --i)
    bs += 1.0f / i;
std::cout << "Backward sum = " << bs << "\n";
```

Harmonic Numbers Rule 2

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float bs = 0;
for (unsigned int i = n; i >= 1; --i)
    bs += 1.0f / i;
std::cout << "Backward sum = " << bs << "\n";
```

Input: **10000000**

forwards: **15.4037**

backwards: **16.686**

Harmonic Numbers Rule 2

```
std::cout << "Compute H_n for n =? ";
unsigned int n;
std::cin >> n;

float fs = 0;
for (unsigned int i = 1; i <= n; ++i)
    fs += 1.0f / i;
std::cout << "Forward sum = " << fs << "\n";

float bs = 0;
for (unsigned int i = n; i >= 1; --i)
    bs += 1.0f / i;
std::cout << "Backward sum = " << bs << "\n";
```

Input: **100000000**

forwards: **15.4037**

backwards: **18.8079**

Harmonic Numbers Rule 2

Observation:

- The forward sum stops growing at some point and is “really” wrong.

Harmonic Numbers Rule 2

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- The backward sum approximates H_n well.

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- For $1 + 1/2 + 1/3 + \cdots$, later terms are too small to actually contribute

Harmonic Numbers   Rule 2

Observation:

- The forward sum stops growing at some point and is “really” wrong.
- The backward sum approximates H_n well.

Explanation:

- For $1 + 1/2 + 1/3 + \cdots$, later terms are too small to actually contribute
- Problem similar to $2^5 + 1$ “$=$” 2^5

Floating-point Guidelines Rule 3

Rule 4

Do not subtract two numbers with a very similar value.

Cancellation problems, cf. lecture notes.

Literature

David Goldberg: What Every Computer Scientist Should Know About Floating-Point Arithmetic (1991)

Randy Glasbergen, 1996

9. Functions I

Defining and Calling Functions, Evaluation of Function Calls, the Type **`void`**

Computing Powers

```
double a;
int n;
std::cin >> a; // Eingabe a
std::cin >> n; // Eingabe n

double result = 1.0;
if (n < 0) { // a^n = (1/a)^(-n)
  a = 1.0/a;
  n = -n;
}
for (int i = 0; i < n; ++i)
  result *= a;

std::cout << a << "^" << n << " = " << result << ".\n";
```

Computing Powers

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int n;
std::cin >> a; // Eingabe a
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double result = 1.0;
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  result *= a;

std::cout << a << "^" << n << " = " << result << ".\n";
```

Computing Powers

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double a;
int n;
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std::cin >> n; // Eingabe n
```

```
double result = 1.0;
if (n < 0) { // a^n = (1/a)^(-n)
  a = 1.0/a;
  n = -n;
}
for (int i = 0; i < n; ++i)
  result *= a;
```

"Funktion pow"

```
std::cout << a << "^" << n << " = " << pow(a,n) << ".\n";
```

Function to Compute Powers

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Function to Compute Powers

```
double pow(double b, int e){...}
```

Function to Compute Powers

```
// Prog: callpow.cpp
// Define and call a function for computing powers.
#include <iostream>

  double pow(double b, int e){...}

int main()
{
  std::cout << pow( 2.0, -2) << "\n"; // outputs 0.25
  std::cout << pow( 1.5, 2) << "\n"; // outputs 2.25
  std::cout << pow(-2.0, 9) << "\n"; // outputs -512

  return 0;
}
```

Function Definitions

T fname (T_1 pname$_1$, T_2 pname$_2$, …, T_N pname$_N$)
block

function name

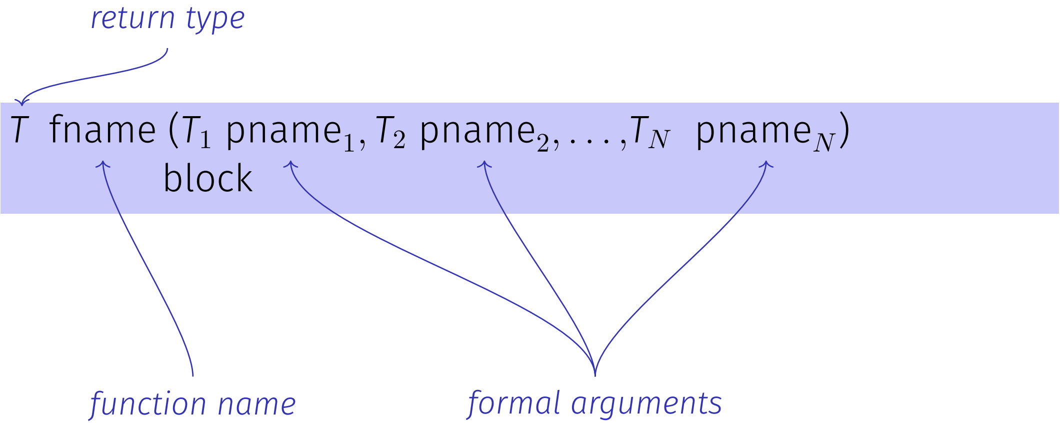
Function Definitions

return type

$$T \;\; \text{fname}\; (T_1 \; \text{pname}_1, T_2 \; \text{pname}_2, \ldots, T_N \;\; \text{pname}_N)$$
$$\text{block}$$

function name

Function Definitions



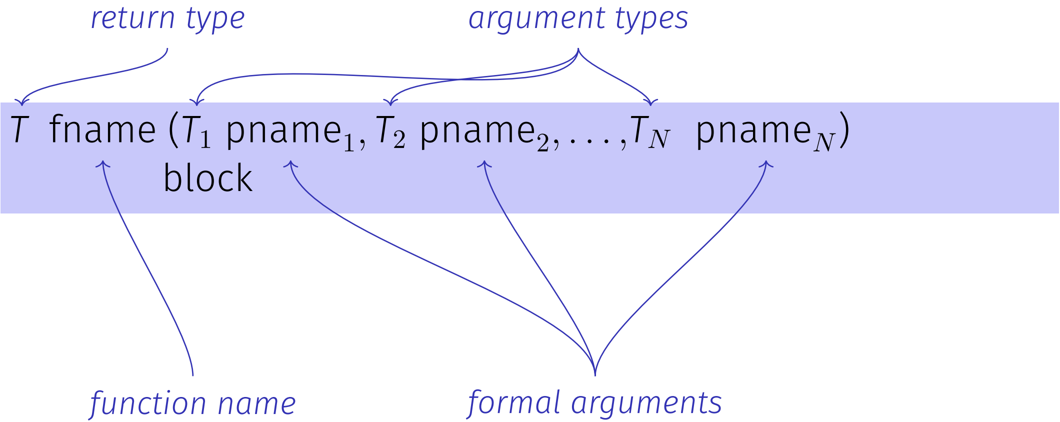
return type

T fname $(T_1$ pname$_1$, T_2 pname$_2, \ldots, T_N$ pname$_N)$
block

function name

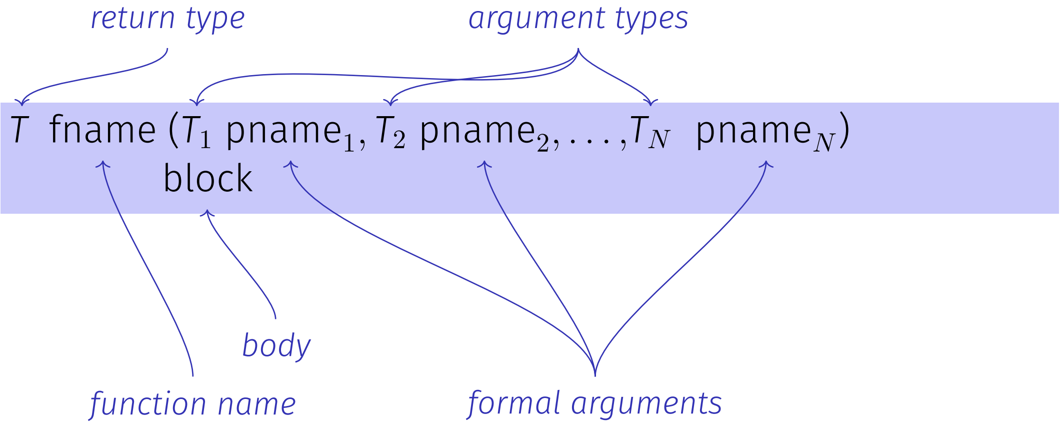
formal arguments

Function Definitions



T fname (T_1 pname$_1$, T_2 pname$_2$, . . . , T_N pname$_N$)
block

Function Definitions



return type

argument types

T fname $(T_1$ pname$_1$, T_2 pname$_2$, $\ldots$, T_N pname$_N)$
block

body

function name

formal arguments

Xor

```
// post: returns l XOR r
bool Xor(bool l, bool r)
{
    return l && !r || !l && r;
}
```

Harmonic

```
// PRE: n >= 0
// POST: returns nth harmonic number
//       computed with backward sum
float Harmonic(int n)
{
    float res = 0;
    for (unsigned int i = n; i >= 1; --i)
        res += 1.0f / i;
    return res;
}
```

min

```
// POST: returns the minimum of a and b
int min(int a, int b)
{
    if (a<b)
        return a;
    else
        return b;
}
```

Function Calls

fname (*expression*$_1$, *expression*$_2$, $\ldots$, *expression*$_N$)

- All call arguments must be convertible to the respective formal argument types.

Function Calls

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fname (*expression*$_1$, *expression*$_2$, $\ldots$, *expression*$_N$)

- All call arguments must be convertible to the respective formal argument types.
- The function call is an expression of the return type of the function.

Example: **`pow(a,n)`**: Expression of type **`double`**

Function Calls

For the types we know up to this point it holds that:

- Call arguments are R-values
  ↪ *call-by-value* (also *pass-by-value*), more on this soon
- The function call is an R-value.

Function Calls

For the types we know up to this point it holds that:

- Call arguments are R-values
  $\hookrightarrow$ *call-by-value* (also *pass-by-value*), more on this soon
- The function call is an R-value.

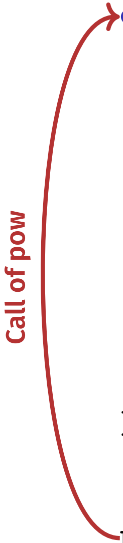
$$\textit{fname}\text{: R-value} \times \text{R-value} \times \cdots \times \text{R-value} \longrightarrow \text{R-value}$$

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

Evaluation Function Call



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    assert (e >= 0 || b != 0);
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        b = 1.0/b;
        e = -e;
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    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}


...
pow (2.0, -2)
```

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        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

b=2.0,e=-2

Evaluation Function Call

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    assert (e >= 0 || b != 0);
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    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

b=2.0,e=-2

// ok

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}


...
pow (2.0, -2)
```

result=1.0

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

e == -2

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

b=0.5

Evaluation Function Call

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double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

e=2

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=0

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
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    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=0

result=0.5

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=1

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=1

result=0.25

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=2

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

result=0.25

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

result=0.25

Return

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

Return

value: 0.25

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

value: 0.25

Scope of Formal Arguments

```
int main(){
    double b = 2.0;
    int e = -2;
    double z = pow(b, e);

    std::cout << z; // 0.25
    std::cout << b; // 2
    std::cout << e; // -2
    return 0;
}
```

Scope of Formal Arguments

```
double pow(double b, int e){
    double r = 1.0;
    if (e<0) {
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        r * = b;
    return r;
}
```

```
int main(){
    double b = 2.0;
    int e = -2;
    double z = pow(b, e);

    std::cout << z; // 0.25
    std::cout << b; // 2
    std::cout << e; // -2
    return 0;
}
```

Scope of Formal Arguments

```
double pow(double b, int e){
    double r = 1.0;
    if (e<0) {
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        r * = b;
    return r;
}
```

```
int main(){
    double b = 2.0;
    int e = -2;
    double z = pow(b, e);

    std::cout << z; // 0.25
    std::cout << b; // 2
    std::cout << e; // -2
    return 0;
}
```

Not the formal arguments **b** and **e** of pow but the variables defined here locally in the body of **main**

The type `void`

```
// POST: "(i, j)" has been written to standard output
???? print_pair(int i, int j) {
    std::cout << "(" << i << ", " << j << ")\n";
}

int main() {
    print_pair(3,4); // outputs (3, 4)
    return 0;
}
```

The type `void`

```cpp
// POST: "(i, j)" has been written to standard output
void print_pair(int i, int j) {
    std::cout << "(" << i << ", " << j << ")\n";
}

int main() {
    print_pair(3,4); // outputs (3, 4)
    return 0;
}
```

The type `void`

- Fundamental type with empty value range

The type `void`

- Fundamental type with empty value range
- Usage as a return type for functions that do *only* provide an effect

`void`-Functions

- do not require **`return`**.
- execution ends when the end of the function body is reached or if
- **`return;`** is reached

Functions and return

Wrong:

```
bool compare(float x, float y) {
  float delta = x - y;
  if (delta*delta < 0.001f) return true;
}
```

Functions and `return`

The behavior of a function with non-**`void`** return type is **undefined** if the end of the function body is reached without a **`return`** statement.

Wrong:

```
bool compare(float x, float y) {
  float delta = x - y;
  if (delta*delta < 0.001f) return true;
}
```

Here the value of `compare(10,20)` is undefined.

Functions and `return`

The behavior of a function with non-**`void`** return type is **undefined** if the end of the function body is reached without a **`return`** statement.

Better:

```
bool compare(float x, float y) {
   float delta = x - y;
   if (delta*delta < 0.001f)
    return true;
   else
    return false;
}
```

All execution paths reach a **`return`**

Functions and `return`

The behavior of a function with non-**`void`** return type is **undefined** if the end of the function body is reached without a **`return`** statement.

Even better and simpler

```
bool compare(float x, float y) {
   float delta = x - y;
   return delta*delta < 0.001f;
 }
```

10. Functions II

Pre- and Postconditions Stepwise Refinement, Scope, Libraries and Standard Functions

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

0^e is undefined for $e < 0$

```
// PRE: e >= 0 || b != 0.0
```

Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Here only value, no effect.

```
// POST: return value is b^e
```

Pre- and Postconditions

- should be correct:

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Funktion **`pow`**: works for all numbers $b \neq 0$

White Lies. . .

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect

White Lies. . .

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect:

- Overflow if e or b are too large
- b^e potentially not representable as a double (holes in the value range!)

White Lies are Allowed

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

Mathematical conditions as a compromise between formal correctness and lax practice

Checking Preconditions…

- Preconditions are only comments.

Checking Preconditions...

- Preconditions are only comments.
- How can we ensure that they hold when the function is called?

...with assertions

```
#include <cassert>
...
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e) {
    assert (e >= 0 || b != 0);
    double result = 1.0;
    ...
}
```

Postconditions with Asserts

- The result of “complex” computations is often easy to check.

Postconditions with Asserts

- The result of “complex” computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

Postconditions with Asserts

- The result of “complex” computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

```
// PRE: the discriminant p*p/4 - q is nonnegative
// POST: returns larger root of the polynomial x^2 + p x + q
double root(double p, double q)
{
    assert(p*p/4 >= q); // precondition
    double x1 = - p/2 + sqrt(p*p/4 - q);
    assert(equals(x1*x1+p*x1+q,0)); // postcondition
    return x1;
}
```

Stepwise Refinement

A simple *technique* to solve complex problems

Niklaus Wirth. Program development by stepwise refinement. Commun. ACM 14, 4, 1971

Education

P. Wegner
Editor

Program Development by Stepwise Refinement

Niklaus Wirth
Eidgenössische Technische Hochschule
Zürich, Switzerland

The creative activity of programming—to be distinguished from coding—is usually taught by examples serving to exhibit certain techniques. It is here considered as a sequence of design decisions concerning the decomposition of tasks into subtasks and of data into data structures. The process of successive refinement of specifications is illustrated by a short but nontrivial example, from which a number of conclusions are drawn regarding the art and the instruction of programming.

Key Words and Phrases: education in programming, programming techniques, stepwise program construction

CR Categories: 1.50, 4.0

1. Introduction

Programming is usually taught by examples. Experience shows that the success of a programming course critically depends on the choice of these examples. Unfortunately, they are too often selected with the prime intent to demonstrate what a computer can do. Instead, a main criterion for selection should be their suitability to exhibit certain widely applicable *techniques*. Furthermore, examples of programs are commonly presented as finished "products" followed by explanations of their purpose and their linguistic details. But active programming consists of the design of *new* programs, rather than contemplation of old programs. As a consequence of these teaching methods, the student obtains the impression that programming consists mainly of mastering a language (with all the peculiarities and intricacies so abundant in modern PL's) and relying on one's intuition to somehow transform ideas into finished programs. Clearly, programming courses should teach methods of design and construction, and the selected examples should be such that a gradual *development* can be nicely demonstrated.

This paper deals with a single example chosen with these two purposes in mind. Some well-known techniques are briefly demonstrated and motivated (strategy of preselection, stepwise construction of trial solutions, introduction of auxiliary data, recursion), and the program is gradually developed in a sequence of *refinement steps*.

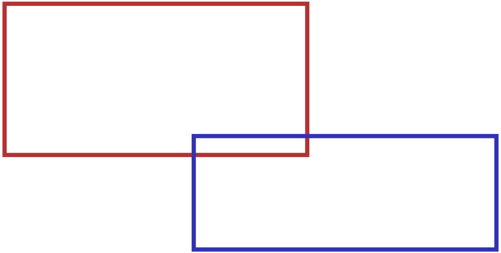
In each step, one or several instructions of the given program are decomposed into more detailed instructions. This successive decomposition or refinement of specifications terminates when all instructions are expressed in terms of an underlying computer or programming language, and must therefore be guided by the facilities available on that computer or language. The result of the execution of a program is expressed in terms of data, and it may be necessary to introduce further data for communication between the obtained subtasks or instructions. As tasks are refined, so the data may have to be refined, decomposed, or structured, and it is natural to *refine program and data specifications in parallel*.

Every refinement step implies some design decisions. It is important that these decision be made explicit, and that the programmer be aware of the underlying criteria and of the existence of alternative solutions. The possible solutions to a given problem emerge as the leaves of a tree, each node representing a point of deliberation and decision. Subtrees may be considered as *families of solutions* with certain common characteristics and structures. The notion of such a tree may be particularly helpful in the situation of changing purpose and environment to which a program may sometime have to be adapted.

A guideline in the process of stepwise refinement should be the principle to decompose decisions as much as possible, to untangle aspects which are only seemingly interdependent, and to defer those decisions which concern details of representation as long as possible. This

Example Problem

Find out if two rectangles intersect!



Top-Down Approach

- Formulate a coarse solution using
  - comments
  - ficticious functions

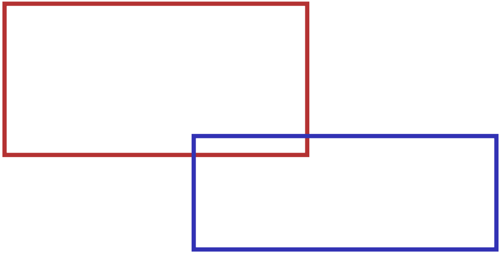
Top-Down Approach

- Formulate a coarse solution using
  - comments
  - ficticious functions
- Repeated refinement:
  - comments ⟶ program text
  - ficticious functions ⟶ function definitions

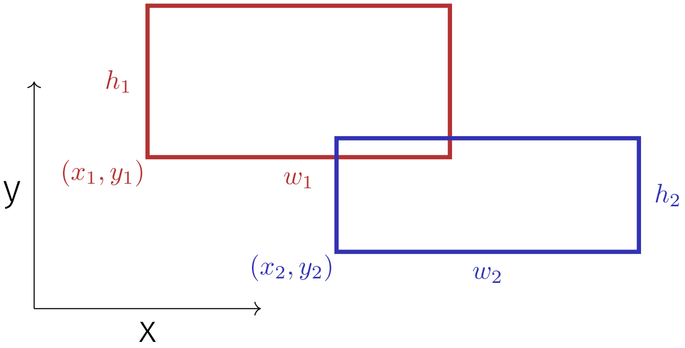
Coarse Solution

```
int main()
{
    // input rectangles

    // intersection?

    // output solution

    return 0;
}
```

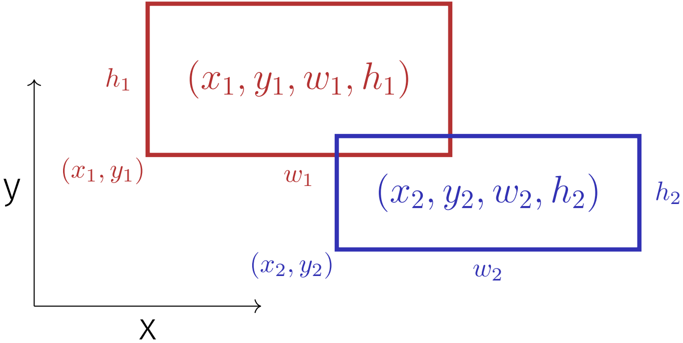
Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

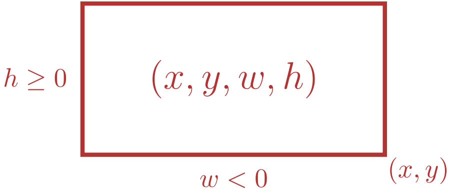


Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

Width w and height h may be negative.



Refinement 1: Input Rectangles

```cpp
int main()
{
    std::cout << "Enter two rectangles [x y w h each] \n";
    int x1, y1, w1, h1;
    std::cin >> x1 >> y1 >> w1 >> h1;
    int x2, y2, w2, h2;
    std::cin >> x2 >> y2 >> w2 >> h2;

    // intersection?

    // output solution

    return 0;
}
```

Refinement 2: Intersection? and Output

```
int main()
{
    input rectangles ✓

    bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);

    if (clash)
        std::cout << "intersection!\n";
    else
        std::cout << "no intersection!\n";

    return 0;
}
```

Refinement 3: Intersection Function...

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}

int main() {
    input rectangles ✓

    intersection? ✓

    output solution ✓

    return 0;
}
```

Refinement 3: Intersection Function...

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                         int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

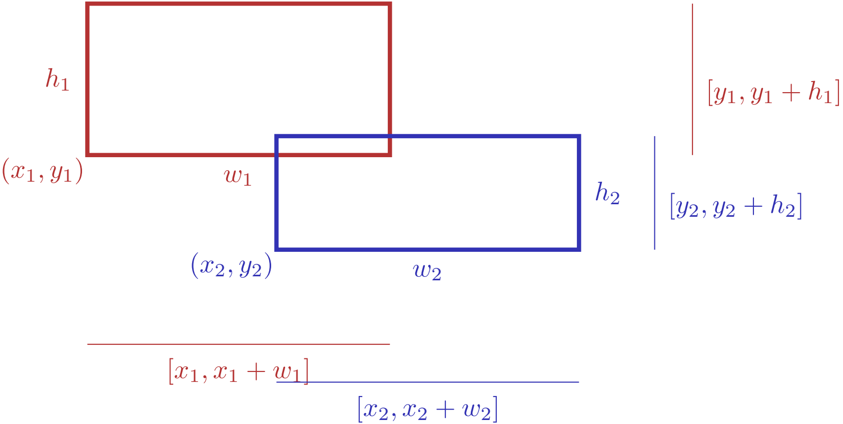
Function main ✓

Refinement 3: ...with PRE and POST

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles,
//      where w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1) and
//      (x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                         int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

Refinement 4: Interval Intersection

Two rectangles intersect if and only if their x and y-intervals intersect.



Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//       w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2); ✓
}
```

Refinement 4: Interval Intersections

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return false; // todo
}
```

Function rectangles_intersect ✓

Function main ✓

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2);
}
```

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2); ✓
}
```

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

Function intervals_intersect ✓

Function rectangles_intersect ✓

Function main ✓

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

already exists in the standard library

Function intervals_intersect ✓

Function rectangles_intersect ✓

Function main ✓

Back to Intervals

```
// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return std::max(a1, b1) >= std::min(a2, b2)
       && std::min(a1, b1) <= std::max(a2, b2); ✓
}
```

Look what we have achieved step by step!

```
#include <iostream>
#include <algorithm>

// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//     with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
  return std::max(a1, b1) >= std::min(a2, b2)
      && std::min(a1, b1) <= std::max(a2, b2);
}

// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

```
int main ()
{
  std::cout << "Enter two rectangles [x y w h each]\n";
  int x1, y1, w1, h1;
  std::cin >> x1 >> y1 >> w1 >> h1;
  int x2, y2, w2, h2;
  std::cin >> x2 >> y2 >> w2 >> h2;
  bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);
  if (clash)
    std::cout << "intersection!\n";
  else
    std::cout << "no intersection!\n";
  return 0;
}
```

Result

- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

  `rectangles_intersect`

Result

- Clean solution of the problem
- Useful functions have been implemented

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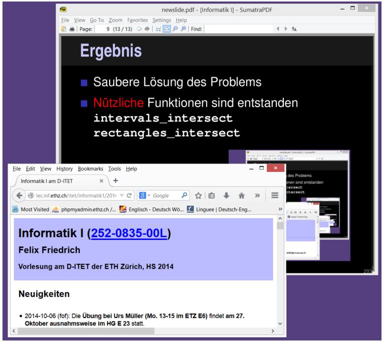
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Ergebnis

- Saubere Lösung des Problems
- Nützliche Funktionen sind entstanden

  `intervals_intersect`

  `rectangles_intersect`

Informatik I (252-0835-00L)

Felix Friedrich

Vorlesung am D-ITET der ETH Zürich, HS 2014

Neuigkeiten

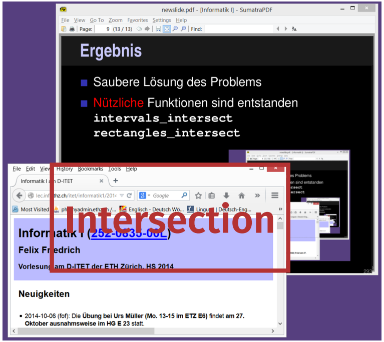
- 2014-10-06 (fof): Die **Übung bei Urs Müller (Mo. 13-15 im ETZ E6)** findet am 27. Oktober ausnahmsweise im **HG E 23** statt.

Result

- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

  `rectangles_intersect`



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Ergebnis

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  intervals_intersect
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Intersection

Where can a Function be Used?

```
#include <iostream>

int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

Scope of a Function

- is the part of the program where a function can be called

Scope of a Function

- is the part of the program where a function can be called

Extension by **declaration** of a function: like the definition but without `{...}`.

```
double pow(double b, int e);
```

This does not work...

```
#include <iostream>


int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

…but this works!

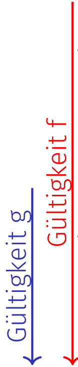
```
#include <iostream>
int f(int i); // Gueltigkeitsbereich von f ab hier

int main()
{
    std::cout << f(1);
    return 0;
}

int f(int i)
{
    return i;
}
```

Forward Declarations, why?

Functions that mutually call each other:

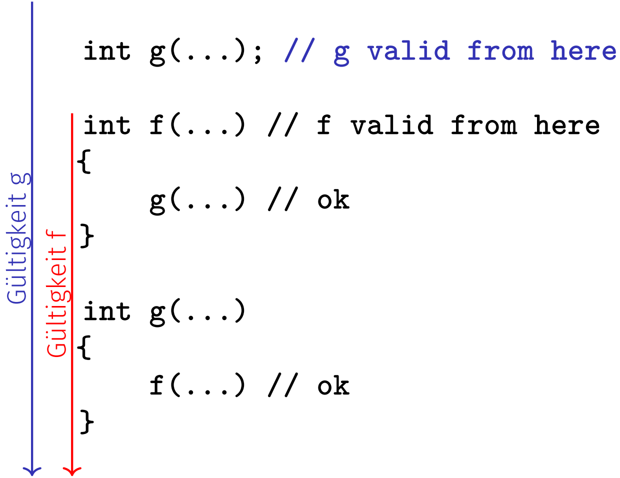


```
int f(...) // f valid from here
{
    g(...) // g undeclared
}

int g(...) // g valid from here!
{
    f(...) // ok
}
```

Forward Declarations, why?

Functions that mutually call each other:



```
int g(...); // g valid from here

int f(...) // f valid from here
{
    g(...) // ok
}

int g(...)
{
    f(...) // ok
}
```

Reusability

- Functions such as **`rectangles_intersect`** and **`pow`** are useful in many programs.

Reusability

- Functions such as **`rectangles_intersect`** and **`pow`** are useful in many programs.
- “Solution”: copy-and-paste the source code

Level 1: Outsource the Function

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Level 1: Outsource the Function

`double pow(double b, int e);` in **separate file** `mymath.cpp`

Level 1: Include the Function

```cpp
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"

int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

Level 1: Include the Function

```
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"

int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

← in working directory

Disadvantage of Including

- **`#include`** copies the file (**`mymath.cpp`**) into the main program (**`callpow2.cpp`**).

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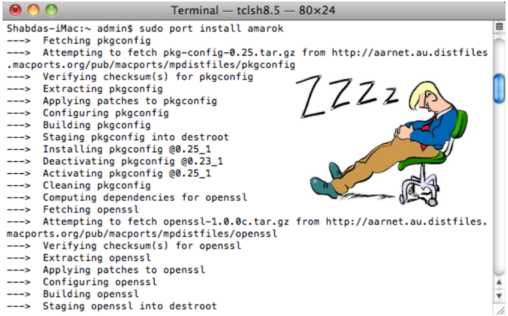
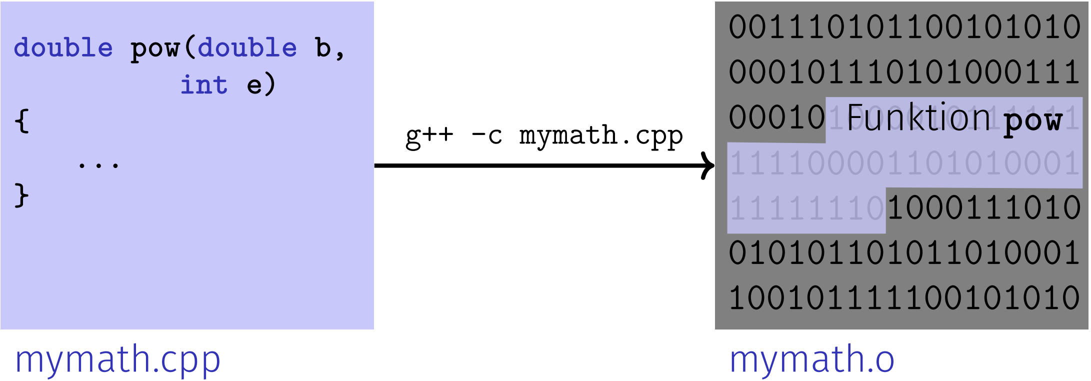
- **`#include`** copies the file (**`mymath.cpp`**) into the main program (**`callpow2.cpp`**).
- The compiler has to (re)compile the function definition for each program



```
Shabdas-iMac:~ admin$ sudo port install amarok
--->  Fetching pkgconfig
--->  Attempting to fetch pkg-config-0.25.tar.gz from http://aarnet.au.distfiles
.macports.org/pub/macports/mpdistfiles/pkgconfig
--->  Verifying checksum(s) for pkgconfig
--->  Extracting pkgconfig
--->  Applying patches to pkgconfig
--->  Configuring pkgconfig
--->  Building pkgconfig
--->  Staging pkgconfig into destroot
--->  Installing pkgconfig @0.25_1
--->  Deactivating pkgconfig @0.23_1
--->  Activating pkgconfig @0.25_1
--->  Cleaning pkgconfig
--->  Computing dependencies for openssl
--->  Fetching openssl
--->  Attempting to fetch openssl-1.0.0c.tar.gz from http://aarnet.au.distfiles.
macports.org/pub/macports/mpdistfiles/openssl
--->  Verifying checksum(s) for openssl
--->  Extracting openssl
--->  Applying patches to openssl
--->  Configuring openssl
--->  Building openssl
--->  Staging openssl into destroot
```

Level 2: Separate Compilation



```
double pow(double b,
           int e)
{
    ...
}
```

mymath.cpp

`g++ -c mymath.cpp`

```
001110101100101010
000101110101000111
00010 Funktion pow
111100001101010001
111111101000111010
010101101011010001
100101111100101010
```

mymath.o

Level 2: Separate Compilation

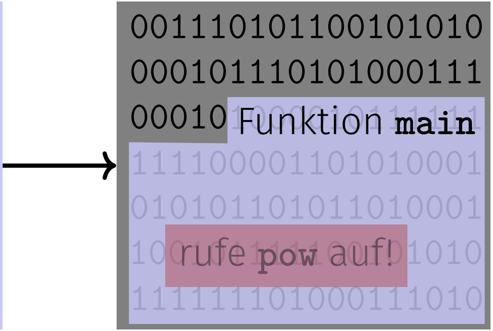
```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e);
```

mymath.h

Level 2: Separate Compilation

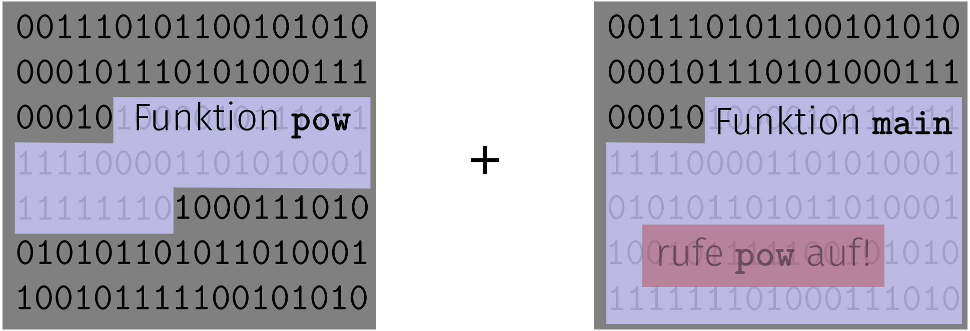
```
#include <iostream>
#include "mymath.h"
int main()
{
  std::cout << pow(2,-2) << "\n";
  return 0;
}
```

callpow3.cpp



callpow3.o

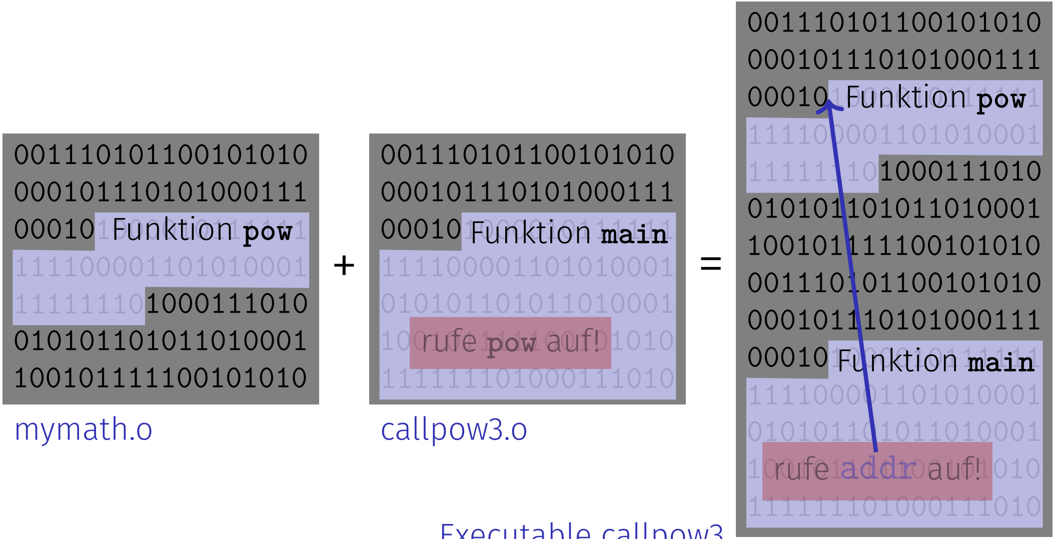
The linker unites...



mymath.o

callpow3.o

… what belongs together



001110101100101010
000101110101000111
00010 Funktion **pow**
111100001101010001
1111110 1000111010
010101101011010001
100101111100101010

mymath.o

+

001110101100101010
000101110101000111
00010 Funktion **main**
111100001101010001
010101101011010001
10 rufe **pow** auf! 1010
111111101000111010

callpow3.o

=

001110101100101010
000101110101000111
00010 Funktion **pow**
111100001101010001
1111110 1000111010
010101101011010001
100101111100101010
001110101100101010
000101110101000111
00010 Funktion **main**
111100001101010001
010101101011010001
10 rufe addr auf! 010
111111101000111010

Executable callpow3

Availability of Source Code?

Observation

`mymath.cpp` (source code) is not required any more when the **`mymath.o`** (object code) is available.

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Many vendors of libraries do not provide source code.

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`mymath.cpp` (source code) is not required any more when the **`mymath.o`** (object code) is available.

Many vendors of libraries do not provide source code.
Header files then provide the *only* readable informations.

Open-Source Software

- Source code is generally available.

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- Only this allows the continued development of code by users and dedicated “hackers”.

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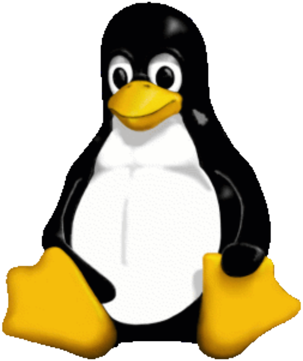
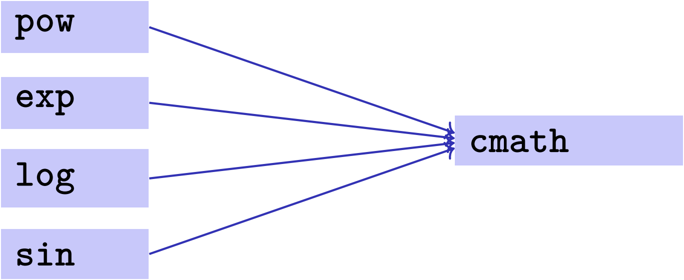
- Source code is generally available.



Libraries

- Logical grouping of similar functions



Name Spaces...

```
// cmath
namespace std {

  double pow(double b, int e);

  ....
  double exp(double x);
  ...
}
```

…Avoid Name Conflicts

```
#include <cmath>
#include "mymath.h"

int main()
{
    double x = std::pow(2.0, -2); // <cmath>
    double y = pow(2.0, -2); // mymath.h
}
```

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with **`std::pow`**);
- lead to interesting and efficient programs in a simple way;

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with **`std::pow`**);
- lead to interesting and efficient programs in a simple way;
- guarantee a quality standard that cannot easily be achieved with code written from scratch.

Example: Prime Number Test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, n-1\}$ dividing n .

```
unsigned int d;
for (d=2; n % d != 0; ++d);
```

Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

- This works because `std::sqrt` rounds to the next representable `double` number (IEEE Standard 754).

Functions Should be More Capable! Swap ?

```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2);
}
```

Functions Should be More Capable! Swap ?

```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // fail!  ☹
}
```

Functions Should be More Capable! Swap ?

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2);
}
```

Functions Should be More Capable! Swap ?

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // ok!  ☺
}
```

Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.

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- Not a new concept specific to functions, but rather a new class of types

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- Not a new concept specific to functions, but rather a new class of types

Reference types (e.g. `int&`)

11. Reference Types

Reference Types: Definition and Initialization, Pass By Value, Pass by Reference, Temporary Objects, Const-References

Swap!

```
// POST: values of x and y have been exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}

int main() {
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a == 1 && b == 2); // ok!  ☺
}
```

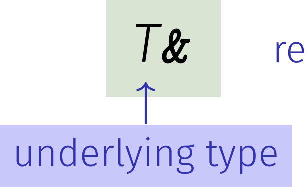
Reference Types

- We can make functions change the values of the call arguments

Reference Types

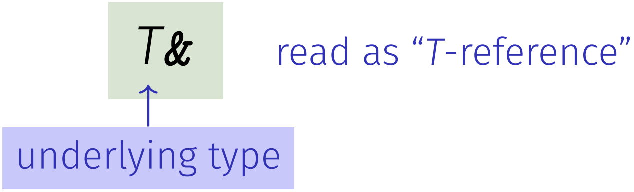
- We can make functions change the values of the call arguments
- not a function-specific concept, but a new class of types: *reference types*

Reference Types: Definition



read as “T-reference”

Reference Types: Definition



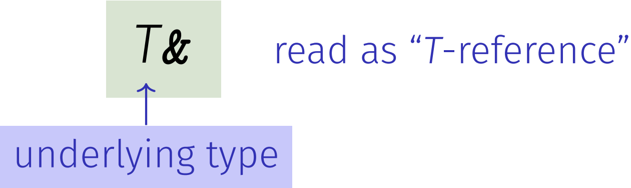
$T\&$

read as "T-reference"

underlying type

- $T\&$ has the same range of values and functionality as T …

Reference Types: Definition



T&

read as "*T*-reference"

underlying type

- *T*& has the same range of values and functionality as *T* …
- … but initialization and assignment work differently

Anakin Skywalker alias Darth Vader

Anakin Skywalker alias Darth Vader

```cpp
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; // Alias
darth_vader = 22;

std::cout << anakin_skywalker;
```

Anakin Skywalker alias Darth Vader

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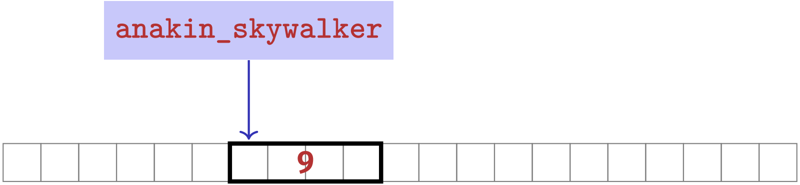

Anakin Skywalker alias Darth Vader

```
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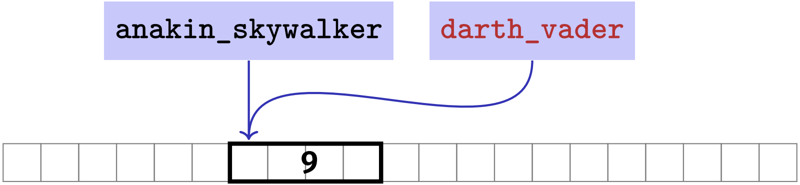
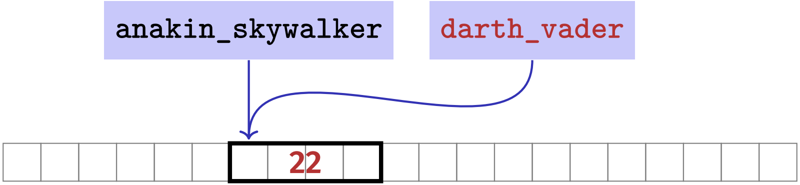
```



Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; // Alias
darth_vader = 22;

std::cout << anakin_skywalker;
```

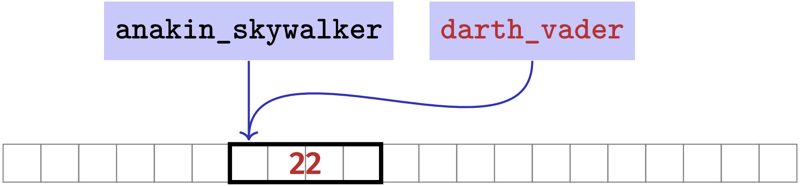


Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; // Alias
darth_vader = 22;

std::cout << anakin_skywalker;
```

assignment to the L-value behind the alias



anakin_skywalker

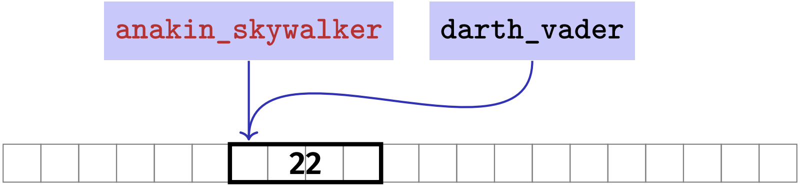
darth_vader

22

Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; // Alias
darth_vader = 22;

std::cout << anakin_skywalker; // 22
```



Reference Types: Intialization and Assignment

```
int& darth_vader = anakin_skywalker;
```

- A variable of reference type (a *reference*) must be initialized with an L-Value

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- The variable becomes an *alias* of the L-value (a different name for the referenced object)

Reference Types: Intialization and Assignment

```
int& darth_vader = anakin_skywalker;
darth_vader = 22; // effect: anakin_skywalker = 22
```

- A variable of reference type (a *reference*) must be initialized with an L-Value
- The variable becomes an *alias* of the L-value (a different name for the referenced object)
- Assignment to the reference updates the object *behind* the alias

Reference Types: Implementation

Internally, a value of type *T***&** is represented by the address of an object of type *T*.

```
int& j; // Error: j must be an alias of something
```

Reference Types: Implementation

Internally, a value of type *T&* is represented by the address of an object of type *T*.

```
int& j; // Error: j must be an alias of something

int& k = 5; // Error: literal 5 has no address
```

Pass by Reference

```
void increment (int& i) {
  ++i;
}
...
int j = 5;
increment (j);
std::cout << j;
```

Pass by Reference

```
void increment (int& i) {
  ++i;
}
...
int j = 5;
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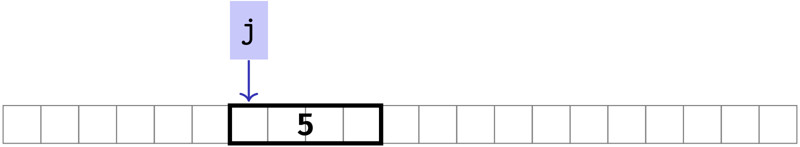
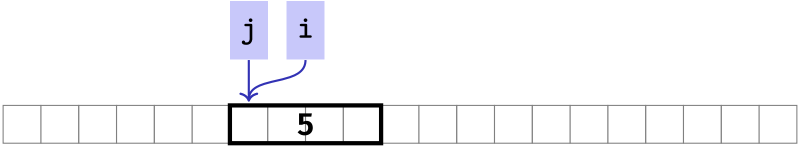
std::cout << j;
```



Pass by Reference

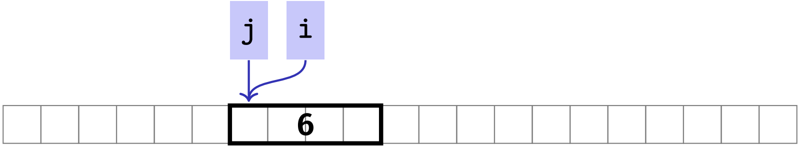
```
void increment (int& i){
  ++i;
}
...
int j = 5;
increment (j);
std::cout << j;
```

initialization of the formal arguments: **i** becomes an alias of call argument **j**



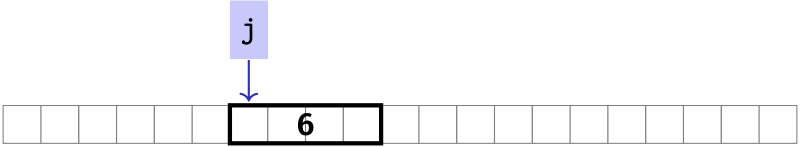
Pass by Reference

```
void increment (int& i) {
  ++i;
}
...
int j = 5;
increment (j);
std::cout << j;
```



Pass by Reference

```
void increment (int& i) {
  ++i;
}
...
int j = 5;
increment (j);
std::cout << j; // 6
```



Pass by Reference

Formal argument *is of* reference type:

⇒ *Pass by Reference*

Formal argument is (internally) initialized with the **address** of the call argument (L-value) and thus becomes an **alias**.

Pass by Value

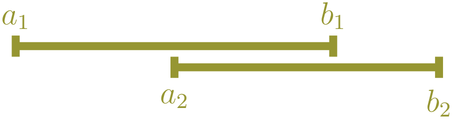
Formal argument *is not of* reference type:

⇒ *Pass by Value*

Formal argument is initialized with the *value* of the actual parameter (R-Value) and thus becomes a *copy*.

References in the Context of intervals intersect

```
// PRE:  [a1, b1], [a2, b2] are (generalized) intervals,
// POST: returns true if [a1, b1], [a2, b2] intersect, in which case
//       [l, h] contains the intersection of [a1, b1], [a2, b2]
bool intervals_intersect(int& l, int& h,
                         int a1, int b1, int a2, int b2) {
  sort(a1, b1);
  sort(a2, b2);
  l = std::max(a1, a2); // Assignments
  h = std::min(b1, b2); // via references
  return l <= h;
}
...
int lo = 0; int hi = 0;
if (intervals_intersect(lo, hi, 0, 2, 1, 3)) // Initialization
   std::cout << "[" << lo << "," << hi << "]" << "\n"; // [1,2]
```



References in the Context of intervals intersect

```
// POST: a <= b
void sort(int& a, int& b) {
   if (a > b)
     std::swap(a, b); // Initialization ("passing through" a, b
}

bool intervals_intersect(int& l, int& h,
                         int a1, int b1, int a2, int b2) {
  sort(a1, b1); // Initialization
  sort(a2, b2); // Initialization
  l = std::max(a1, a2);
  h = std::min(b1, b2);
  return l <= h;
}
```

Return by Reference

- Even the return type of a function can be a reference type: *Return by Reference*

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```
int& inc(int& i) {
  return ++i;
}
```

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```

- call **`inc(x)`**, for some **`int`** variable **`x`**, has exactly the semantics of the pre-increment **`++x`**

Return by Reference

- Even the return type of a function can be a reference type: *Return by Reference*

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int& inc(int& i) {
  return ++i;
}
```

- call **`inc(x)`**, for some **`int`** variable **`x`**, has exactly the semantics of the pre-increment **`++x`**
- Function call *itself* now is an L-value

Return by Reference

- Even the return type of a function can be a reference type: *Return by Reference*

```
int& inc(int& i) {
  return ++i;
}
```

- call **inc(x)**, for some **int** variable **x**, has exactly the semantics of the pre-increment **++x**
- Function call *itself* now is an L-value
- Thus possible: **inc(inc(x))** or **++(inc(x))**

Temporary Objects

What is wrong here?

```
int& foo(int i) {
  return i;
}
```

Temporary Objects

What is wrong here?

```
int& foo(int i) {
  return i;
}
```

```
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

Temporary Objects

What is wrong here?

```
int& foo(int i) {
  return i;
}
```

```
                                           // main()

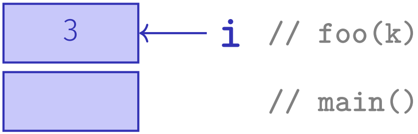
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

Temporary Objects

What is wrong here?

value of the actual parameter is pushed onto the *call stack*

```
int& foo(int i) {
  return i;
}
```



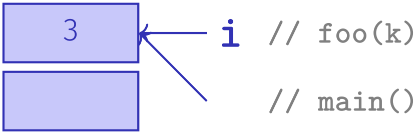
```
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

Temporary Objects

What is wrong here?

i is returned as reference

```
int& foo(int i) {
  return i;
}
```



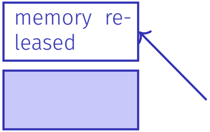
```
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

Temporary Objects

What is wrong here?

…and disappears from the stack

```
int& foo(int i) {
  return i;
}
```



```
// main()
```

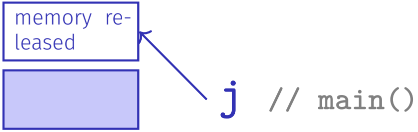
```
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

Temporary Objects

What is wrong here?

j becomes alias to released memory

```
int& foo(int i) {
  return i;
}
```



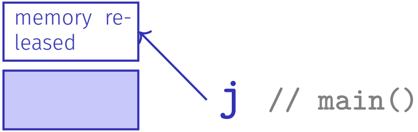
```
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

Temporary Objects

What is wrong here?

Accessing `j` is undefined behaviour!

```
int& foo(int i) {
  return i;
}
```



memory released

j // main()

```
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

The Reference Guidline

Reference Guideline

When a reference is created, the object referred to must "stay alive" at least as long as the reference.

Const-References

- have type **`const`** *T* **`&`**
- type can be interpreted as “(**`const`** *T*) **`&`**”
- can be initialized with R-Values (compiler generates a temporary object with sufficient lifetime)

Const-References

- have type **`const`** *T* **`&`**
- type can be interpreted as "(**`const`** *T*) **`&`**"
- can be initialized with R-Values (compiler generates a temporary object with sufficient lifetime)

```
const T& r = lvalue;
```

`r` is initialized with the address of *lvalue* (efficient)

Const-References

- have type **`const`** *T* **`&`**
- type can be interpreted as “(**`const`** *T*) **`&`**”
- can be initialized with R-Values (compiler generates a temporary object with sufficient lifetime)

```
const T& r = rvalue;
```

`r` is initialized with the address of a temporary object with the value of the *rvalue* (pragmatic)

What exactly does Constant Mean?

Consider L-value of type **`const`** *T*. **Case: 1** *T is no* reference type.

⇒ Then the *L-value is a constant*

```
const int n = 5;
int& a = n;
a = 6;
```

What exactly does Constant Mean?

Consider L-value of type **`const`** *T*. **Case: 1** *T is no* reference type.

⇒ Then the *L-value is a constant*

```
const int n = 5;
int& a = n; // Compiler error: const-qualification discarded
a = 6;
```

The compiler detects our *cheating attempt*

What exactly does Constant Mean?

Consider L-value of type **`const`** *T*. **Case 2:** *T is* reference type.

⇒ Then the *L-value is a read-only alias* which cannot be used to change the *underlying* L-value.

What exactly does Constant Mean?

Consider L-value of type **`const`** *T*. **Case 2:** *T is* reference type.

⇒ Then the *L-value is a read-only alias* which cannot be used to change the *underlying* L-value.

```
int n = 5;

const int& r = n; // r is read-only alias of n
r = 6;            // Compiler error: read-only reference
```

What exactly does Constant Mean?

Consider L-value of type **`const`** *T*. **Case 2:** *T is* reference type.

⇒ Then the *L-value is a read-only alias* which cannot be used to change the *underlying* L-value.

```
int n = 5;

const int& r = n; // r is read-only alias of n
r = 6;            // Compiler error: read-only reference

int& rw = n;      // rw is read-write alias
rw = 6;           // OK
```

When to use `const` *T*&?

```
void f_1(T& arg);
```

```
void f_2(const T& arg);
```

- Argument types are references; call arguments are thus not copied, which is efficient
- But only **`f_2`** “promises” to not modify the argument

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Rule

If possible, declare function argument types as `const` *T*& (*pass by read-only reference*) : efficient *and* safe.

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Rule

If possible, declare function argument types as **`const`** T& (*pass by read-only reference*) : efficient *and* safe.

Typically doesn’t pay off for fundamental types (**`int`**, **`double`**, ...). Types with a larger memory footprint will be introduced later in this course.

12. Vectors I

Vector Types, Sieve of Erathostenes, Memory Layout, Iteration

Vectors: Motivation

- Now we can iterate over numbers

```
for (int i=0; i<n ; ++i) {...}
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- ... but not yet over data!

Vectors: Motivation

- Now we can iterate over numbers

```
for (int i=0; i<n ; ++i) {...}
```

- ... but not yet over data!
- Vectors store *homogeneous* data.

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

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Cross out all real factors of 2 ...

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... and go to the next number

Vectors: a first Application

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cross out all real factors of 3 ...

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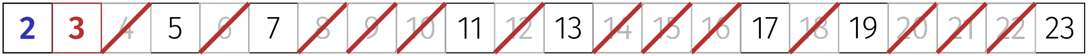

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at the end of the crossing out process, only prime numbers remain.

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~	**11**	~~12~~	**13**	~~14~~	~~15~~	~~16~~	**17**	~~18~~	**19**	~~20~~	~~21~~	~~22~~	**23**

- Question: how do we cross out numbers?

Vectors: a first Application

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- Question: how do we cross out numbers?
- Answer: with a *vector*.

Erathostenes with Vectors: Initialization

```
...

#include <vector>

...

std::vector<bool> crossed_out(n, false);
```

Initialization with `n` elements
initial value `false`.

element type in triangular brackets

Erathostenes with Vectors: Computation

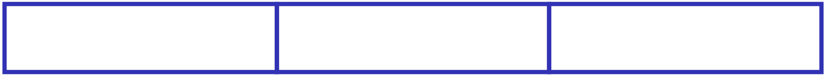
```
for (unsigned int i = 2; i < crossed_out.size(); ++i)
 if (!crossed_out[i]) { // i is prime
  std::cout << i << " ";

  // cross out all proper multiples of i
  for (unsigned int m = 2*i; m < crossed_out.size(); m += i)
   crossed_out[m] = true;
 }
```

Memory Layout of a Vector

A vector occupies a *contiguous* memory area

Example: a vector with 3 elements of type **`T`**



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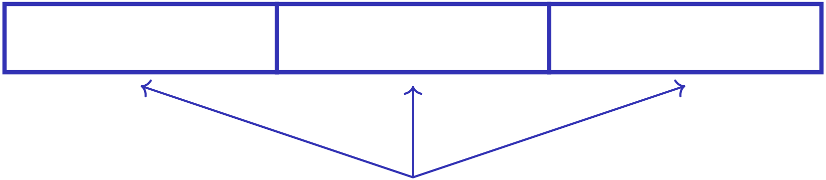

Memory segments for a value of type **`T`** each

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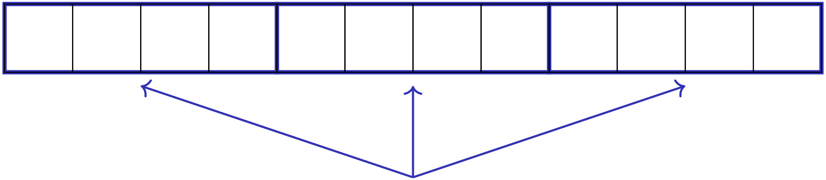


Memory segments for a value of type **`T`** each
(**`T`** occupies e.g. 4 bytes)

Random Access

Given

- vector **`vec`** with **`T`** elements
- **`int`** expression **`exp`** with value $i \geq 0$

Random Access

Given

- vector **`vec`** with **`T`** elements
- **`int`** expression **`exp`** with value $i \geq 0$

Then the expression

```
vec[exp]
```

- is an *L-value* of type **`T`**

Random Access

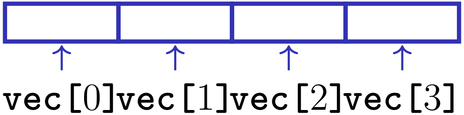
Given

- vector **`vec`** with **`T`** elements
- **`int`** expression **`exp`** with value $i \geq 0$

Then the expression

`vec[exp]`

- is an *L-value* of type **`T`**
- that refers to the ith element **`vec`** (counting from 0!)



Random Access

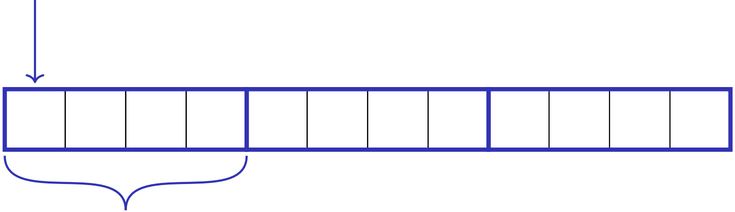
```
vec[exp]
```

- The value i of **`exp`** is called *index*
- **`[]`** is the *index operator* (also *subscript operator*)

Random Access

Random access is very efficient:

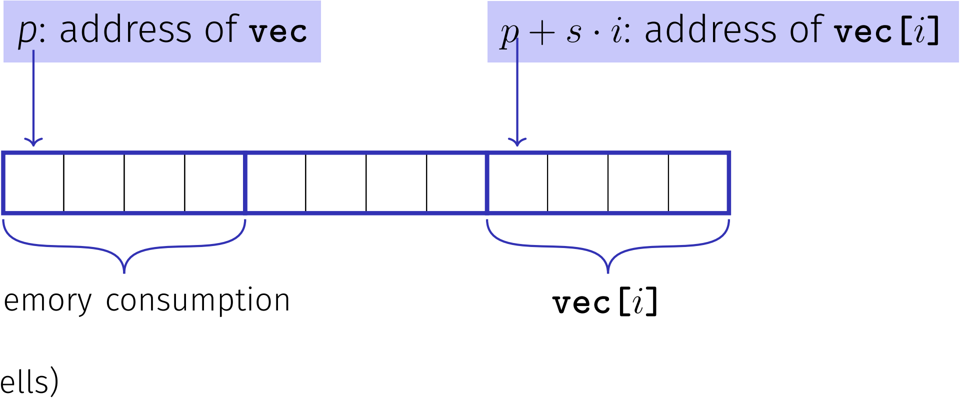
p: address of **`vec`**, i.e. address of the first memory cell



s: memory consumption of **`T`** (in cells)

Random Access

Random access is very efficient:



p: address of **vec**

$p + s \cdot i$: address of **vec**[i]

s: memory consumption of **T** (in cells)

vec[i]

Vector Initialization

- **`std::vector<int> vec(5);`**
  The five elements of **`vec`** are intialized with zeros)

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- **`std::vector<int> vec{4, 3, 5, 2, 1};`**
  the vector is initialized with an *initialization list*
- **`std::vector<int> vec;`**
  An initially empty vector is initialized

Attention

Accessing elements outside the valid bounds of a vector leads to *undefined behavior*

```
std::vector vec(10);
for (unsigned int i = 0; i <= 10; ++i)
  vec[i] = 30;
```

Attention

Accessing elements outside the valid bounds of a vector leads to *undefined behavior*

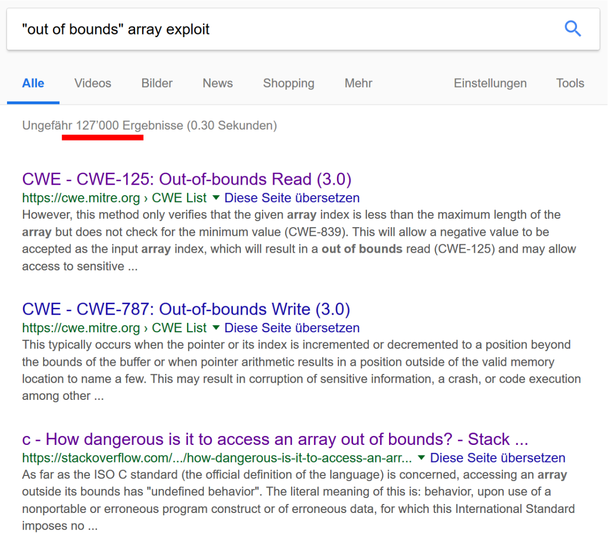
```
std::vector vec(10);
for (unsigned int i = 0; i <= 10; ++i)
  vec[i] = 30; // Runtime error: accessing vec[10]
```

Attention

Bound Checks

When using a subscript operator on a vector, it is the sole *responsibility of the programmer* to check the validity of element accesses.

Consequences of illegal index accesses



"out of bounds" array exploit

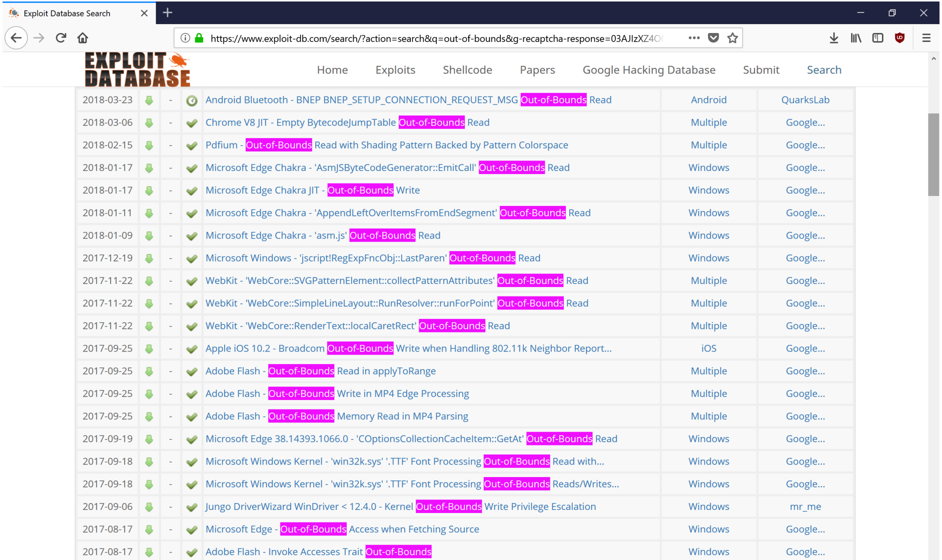
Ungefähr 127’000 Ergebnisse (0.30 Sekunden)

CWE - CWE-125: Out-of-bounds Read (3.0)
https://cwe.mitre.org › CWE List ▾ Diese Seite übersetzen
However, this method only verifies that the given **array** index is less than the maximum length of the **array** but does not check for the minimum value (CWE-839). This will allow a negative value to be accepted as the input **array** index, which will result in a **out of bounds** read (CWE-125) and may allow access to sensitive ...

CWE - CWE-787: Out-of-bounds Write (3.0)
https://cwe.mitre.org › CWE List ▾ Diese Seite übersetzen
This typically occurs when the pointer or its index is incremented or decremented to a position beyond the bounds of the buffer or when pointer arithmetic results in a position outside of the valid memory location to name a few. This may result in corruption of sensitive information, a crash, or code execution among other ...

c - How dangerous is it to access an array out of bounds? - Stack ...
https://stackoverflow.com/.../how-dangerous-is-it-to-access-an-arr... ▾ Diese Seite übersetzen
As far as the ISO C standard (the official definition of the language) is concerned, accessing an **array** outside its bounds has "undefined behavior". The literal meaning of this is: behavior, upon use of a nonportable or erroneous program construct or of erroneous data, for which this International Standard imposes no ...

Consequences of illegal index accesses



2018-03-23	⬇	-	○	Android Bluetooth - BNEP BNEP_SETUP_CONNECTION_REQUEST_MSG Out-of-Bounds Read	Android	QuarksLab
2018-03-06	⬇	-	✓	Chrome V8 JIT - Empty BytecodeJumpTable Out-of-Bounds Read	Multiple	Google...
2018-02-15	⬇	-	✓	Pdfium - Out-of-Bounds Read with Shading Pattern Backed by Pattern Colorspace	Multiple	Google...
2018-01-17	⬇	-	✓	Microsoft Edge Chakra - 'AsmJSByteCodeGenerator::EmitCall' Out-of-Bounds Read	Windows	Google...
2018-01-17	⬇	-	✓	Microsoft Edge Chakra JIT - Out-of-Bounds Write	Windows	Google...
2018-01-11	⬇	-	✓	Microsoft Edge Chakra - 'AppendLeftOverItemsFromEndSegment' Out-of-Bounds Read	Windows	Google...
2018-01-09	⬇	-	✓	Microsoft Edge Chakra - 'asm.js' Out-of-Bounds Read	Windows	Google...
2017-12-19	⬇	-	✓	Microsoft Windows - 'jscript!RegExpFncObj::LastParen' Out-of-Bounds Read	Windows	Google...
2017-11-22	⬇	-	✓	WebKit - 'WebCore::SVGPatternElement::collectPatternAttributes' Out-of-Bounds Read	Multiple	Google...
2017-11-22	⬇	-	✓	WebKit - 'WebCore::SimpleLineLayout::RunResolver::runForPoint' Out-of-Bounds Read	Multiple	Google...
2017-11-22	⬇	-	✓	WebKit - 'WebCore::RenderText::localCaretRect' Out-of-Bounds Read	Multiple	Google...
2017-09-25	⬇	-	✓	Apple iOS 10.2 - Broadcom Out-of-Bounds Write when Handling 802.11k Neighbor Report...	iOS	Google...
2017-09-25	⬇	-	✓	Adobe Flash - Out-of-Bounds Read in applyToRange	Multiple	Google...
2017-09-25	⬇	-	✓	Adobe Flash - Out-of-Bounds Write in MP4 Edge Processing	Multiple	Google...
2017-09-25	⬇	-	✓	Adobe Flash - Out-of-Bounds Memory Read in MP4 Parsing	Multiple	Google...
2017-09-19	⬇	-	✓	Microsoft Edge 38.14393.1066.0 - 'COptionsCollectionCacheItem::GetAt' Out-of-Bounds Read	Windows	Google...
2017-09-18	⬇	-	✓	Microsoft Windows Kernel - 'win32k.sys' '.TTF' Font Processing Out-of-Bounds Read with...	Windows	Google...
2017-09-18	⬇	-	✓	Microsoft Windows Kernel - 'win32k.sys' '.TTF' Font Processing Out-of-Bounds Reads/Writes...	Windows	Google...
2017-09-06	⬇	-	✓	Jungo DriverWizard WinDriver < 12.4.0 - Kernel Out-of-Bounds Write Privilege Escalation	Windows	mr_me
2017-08-17	⬇	-	✓	Microsoft Edge - Out-of-Bounds Access when Fetching Source	Windows	Google...
2017-08-17	⬇	-	✓	Adobe Flash - Invoke Accesses Trait Out-of-Bounds	Windows	Google...

Vectors Offer Great Functionality

Here a few example functions, additional follow later in the course.

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```
std::vector<int> v(10);
std::cout << v.at(10);
  // Access with index check → runtime error
  // Ideal for homework
```

Vectors Offer Great Functionality

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```
std::vector<int> v(10);
std::cout << v.at(10);
  // Access with index check → runtime error
  // Ideal for homework
```

```
v.push_back(-1); // -1 is appended (added at end)
std::cout << v.size(); // outputs 11
std::cout << v.at(10); // outputs -1
```

13. Characters and Texts I

Characters and Texts, ASCII, UTF-8, Caesar Code

Characters and Texts

- We have seen texts before:

$$\texttt{std::cout << }\underbrace{\texttt{"Prime numbers in \{2,...,999\}:\textbackslash n"}}_{\text{String-Literal}}\texttt{;}$$

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- can we really work with texts?

Characters and Texts

- We have seen texts before:

```
std::cout << "Prime numbers in {2,...,999}:\n";
```

String-Literal

- can we really work with texts? Yes!

Character: Value of the fundamental type **`char`**

Text: **`std::string`** ≈ vector of **`char`** elements

The type **`char`** (“character”)

Represents printable characters (e.g. **`'a'`**) and *control characters* (e.g. **`'\n'`**)

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char c = ’a’;
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Declares and initialises variable **`c`** of type **`char`** with value **`’a’`**

The type **char** (“character”)

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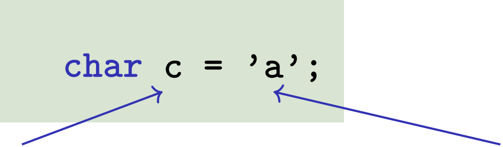

```
char c = ’a’;
```

Declares and initialises variable **c** of type **char** with value **’a’**

literal of type **char**

The type **`char`** (“character”)

Is formally an integer type

- values convertible to **`int`** / **`unsigned int`**

The type **`char`** (“character”)

Is formally an integer type

- values convertible to **`int`** / **`unsigned int`**
- values typically occupy 8 Bit

domain:
$\{-128, \ldots, 127\}$ or $\{0, \ldots, 255\}$

The ASCII-Code

- Defines concrete conversion rules **`char`** $\longrightarrow$ **`(unsigned) int`**

Zeichen $\longrightarrow \{0, \ldots, 127\}$

`’A’, ’B’, ... , ’Z’` $\longrightarrow 65, 66, ..., 90$
`’a’, ’b’, ... , ’z’` $\longrightarrow 97, 98, ..., 122$
`’0’, ’1’, ... , ’9’` $\longrightarrow 48, 49, ..., 57$

The ASCII-Code

- Defines concrete conversion rules **`char`** $\longrightarrow$ **`(unsigned) int`**

Zeichen $\longrightarrow \{0, \ldots, 127\}$

`’A’, ’B’, ... , ’Z’` $\longrightarrow 65, 66, ..., 90$

`’a’, ’b’, ... , ’z’` $\longrightarrow 97, 98, ..., 122$

`’0’, ’1’, ... , ’9’` $\longrightarrow 48, 49, ..., 57$

- Is supported on all common computer systems

The ASCII-Code

- Defines concrete conversion rules **`char`** ⟶ **`(unsigned) int`**

Zeichen $\longrightarrow \{0, \ldots, 127\}$

`’A’, ’B’, ... , ’Z’` $\longrightarrow 65, 66, ..., 90$

`’a’, ’b’, ... , ’z’` $\longrightarrow 97, 98, ..., 122$

`’0’, ’1’, ... , ’9’` $\longrightarrow 48, 49, ..., 57$

- Is supported on all common computer systems
- Enables arithmetic over characters

```
for (char c = ’a’; c <= ’z’; ++c)
  std::cout << c; // abcdefghijklmnopqrstuvwxyz
```

Extension of ASCII: Unicode, UTF-8

- Internationalization of Software ⇒ large character sets required. Thus common today:
  - Character set *Unicode*: 150 scripts, ca. 137,000 characters
  - encoding standard *UTF-8*: mapping characters ↔ numbers

Extension of ASCII: Unicode, UTF-8

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- Internationalization of Software ⇒ large character sets required. Thus common today:
  - Character set *Unicode*: 150 scripts, ca. 137,000 characters
  - encoding standard *UTF-8*: mapping characters ↔ numbers
- UTF-8 is a *variable-width encoding*: Commonly used characters (e.g. Latin alphabet) require only one byte, other characters up to four
- Length of a character’s byte sequence is encoded via bit patterns

Useable Bits	Bit patterns
7	`0xxxxxxx`
11	`110xxxxx 10xxxxxx`

Some Unicode characters in UTF-8

Symbol	Codierung (jeweils 16 Bit)
ئ	11101111 10101111 10111001

Some Unicode characters in UTF-8

Symbol	Codierung (jeweils 16 Bit)
ﯹ	11101111 10101111 10111001
☠	11100010 10011000 10100000
☃	11100010 10011000 10000011
☙	11100010 10011000 10011001

Some Unicode characters in UTF-8

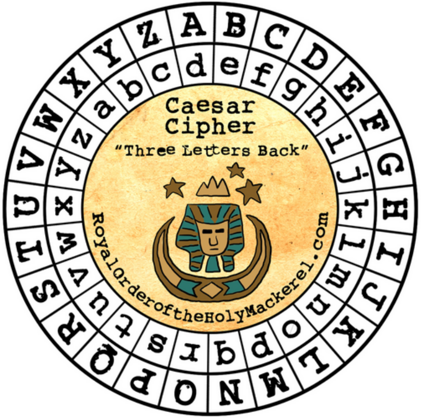
Symbol	Codierung (jeweils 16 Bit)
ﯹ	11101111 10101111 10111001
☠	11100010 10011000 10100000
☃	11100010 10011000 10000011
☙	11100010 10011000 10011001
A	01000001

P.S.: Search for **`apple "unicode of death"`** P.S.: Unicode & UTF-8 are not relevant for the exam

Caesar-Code

Replace every printable character in a text by its pre-pre-predecessor.

' '	(32)	→	'|'	(124)
'!'	(33)	→	'}'	(125)
		...		
'D'	(68)	→	'A'	(65)
'E'	(69)	→	'B'	(66)
		...		
∼	(126)	→	'{'	(123)



Caesar-Code: shift-Function

```
// PRE:  divisor > 0
// POST: return the remainder of dividend / divisor
//       with 0 <= result < divisor
int mod(int dividend, int divisor);

// POST: if c is one of the 95 printable ASCII characters, c is
//       cyclically shifted s printable characters to the right
char shift(char c, int s) {
    if (c >= 32 && c <= 126) { // c is printable
      c = 32 + mod(c - 32 + s,95);
    }

    return c;
}
```

Caesar-Code: shift-Function

```
// PRE:  divisor > 0
// POST: return the remainder of dividend / divisor
//       with 0 <= result < divisor
int mod(int dividend, int divisor);

// POST: if c is one of the 95 printable ASCII characters, c is
//       cyclically shifted s printable characters to the right
char shift(char c, int s) {
    if (c >= 32 && c <= 126) { // c is printable
      c = 32 + mod(c - 32 + s,95);
    }

    return c;
}
```

"**- 32**" transforms interval $[32, 126]$ to $[0, 94]$
"**mod(**x**, 95)**" computes $x \bmod 95$ in $[0, 94]$

Caesar-Code: caesar-Function

```
// POST: Each character read from std::cin was shifted cyclically
//       by s characters and afterwards written to std::cout
void caesar(int s) {
  std::cin >> std::noskipws; // #include <ios>

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

Spaces and newline characters shall *not* be ignored

Caesar-Code: caesar-Function

```
// POST: Each character read from std::cin was shifted cyclically
//       by s characters and afterwards written to std::cout
void caesar(int s) {
  std::cin >> std::noskipws; // #include <ios>

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

Conversion to **bool**: returns *false* if and only if the input is empty

Caesar-Code: caesar-Function

```
// POST: Each character read from std::cin was shifted cyclically
//       by s characters and afterwards written to std::cout
void caesar(int s) {
  std::cin >> std::noskipws; // #include <ios>

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

Shift printable characters by **s**

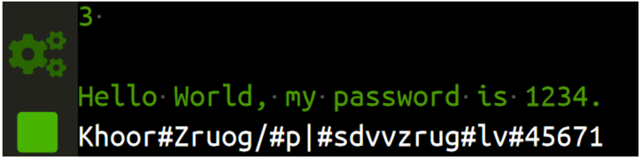
Caesar-Code: Main Program

```
int main() {
  int s;
  std::cin >> s;

  // Shift input by s
  caesar(s);

  return 0;
}
```

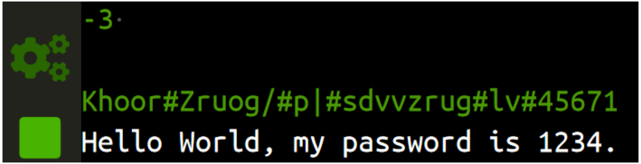
Encode: shift by n (here: 3)



```
3·

Hello·World,·my·password·is·1234.
Khoor#Zruog/#p|#sdvvzrug#lv#45671
```

Encode: shift by $-n$ (here: -3)



```
-3·

Khoor#Zruog/#p|#sdvvzrug#lv#45671
Hello World, my password is 1234.
```

Caesar-Code: Generalisation

```cpp
void caesar(int s) {
  std::cin >> std::noskipws;

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

- Currently only from `std::cin` to `std::cout`

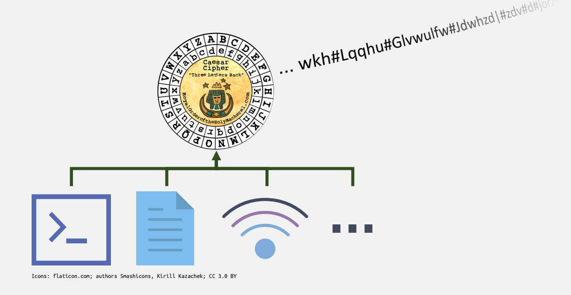
Caesar-Code: Generalisation

```
void caesar(int s) {
  std::cin >> std::noskipws;

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

- Currently only from `std::cin` to `std::cout`

- Better: from arbitrary character source (console, file, ...) to arbitrary character sink (console, ...)



Icons: flaticon.com; authors Smashicons, Kirill Kazachek; CC 3.0 BY

14. Characters and Texts II

Caesar Code with Streams, Text as Strings, String Operations

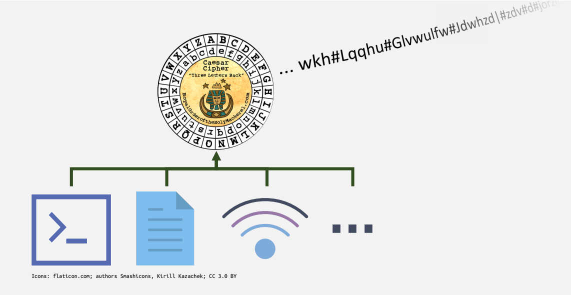
Caesar-Code: Generalisation

```
void caesar(int s) {
  std::cin >> std::noskipws;

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

- Currently only from `std::cin` to `std::cout`

- Better: from arbitrary character source (console, file, ...) to arbitrary character sink (console, ...)


... wkh#Lqqhu#Glvwulfw#Jdwhzd|#zdv#d#jorz
Caesar Cipher
"Three Letters Back"
RoyalOrderoftheHolyMackerel.com
...

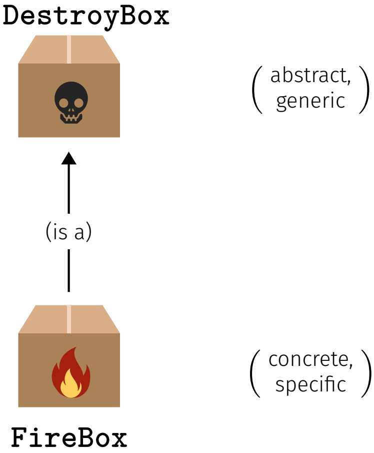
Icons: flaticon.com; authors Smashicons, Kirill Kazachek; CC 3.0 BY

Interlude: Abstract vs. Concrete Types

`DestroyBox`

$\left(\begin{array}{c} \text{abstract,} \\ \text{generic} \end{array} \right)$

Interlude: Abstract vs. Concrete Types



`DestroyBox`

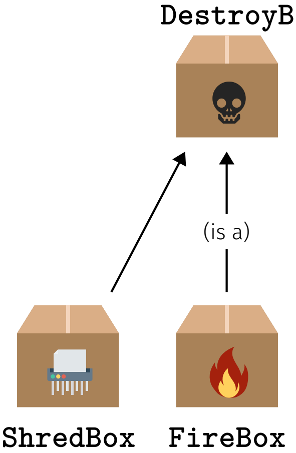
(abstract, generic)

(is a)

`FireBox`

(concrete, specific)

Interlude: Abstract vs. Concrete Types



DestroyBox

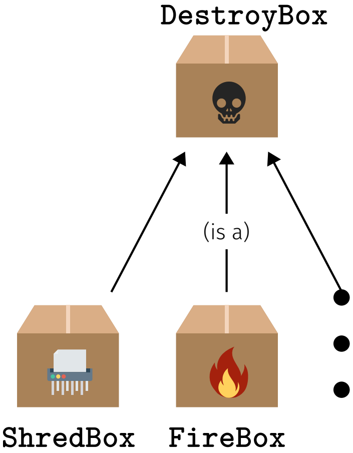
(is a)

ShredBox FireBox

(abstract, generic)

(concrete, specific)

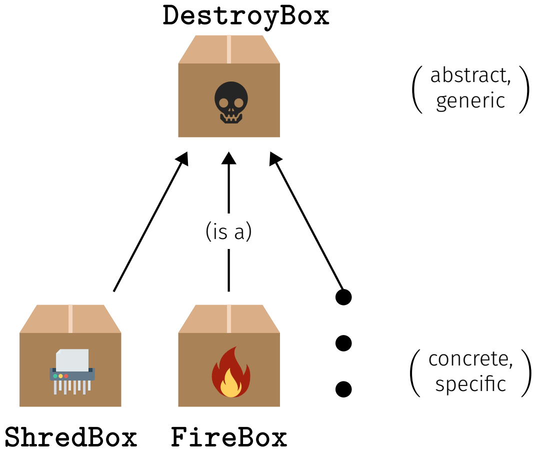
Interlude: Abstract vs. Concrete Types



(abstract, generic)

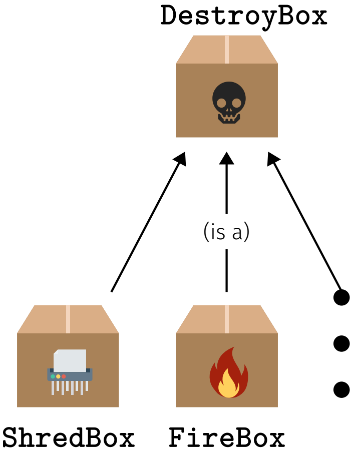
(concrete, specific)

Interlude: Abstract vs. Concrete Types



```
void move_house(DestroyBox& db) {
  // any destroy box will do
  db.dispose(old_ikea_couch);
  db.dispose(cheap_wine);
  ...
}
```

Interlude: Abstract vs. Concrete Types



(abstract, generic)

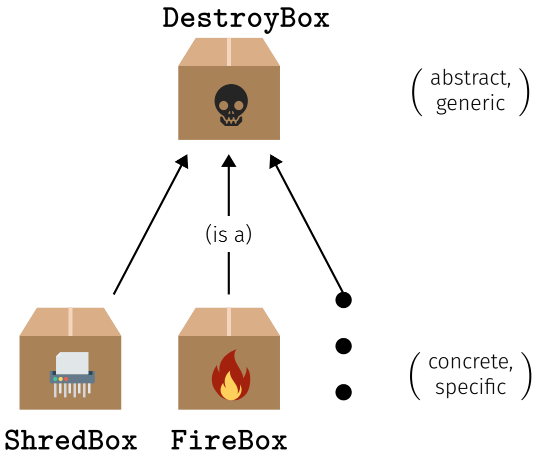
```
void move_house(DestroyBox& db) {
  // any destroy box will do
  db.dispose(old_ikea_couch);
  db.dispose(cheap_wine);
  ...
}
```

(concrete, specific)

```
FireBox fb(5000°C);
move_house(fb);
```

```
ShredBox sb;
move_house(sb);
```

Abstract and Concrete Character Streams



DestroyBox

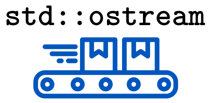
(abstract, generic)

(is a)

ShredBox

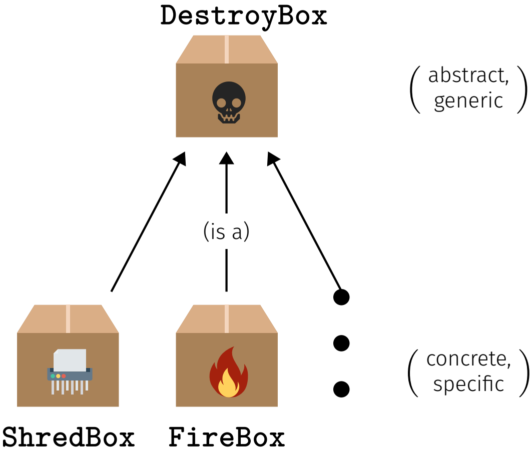
FireBox

(concrete, specific)



`std::ostream`

Abstract and Concrete Character Streams



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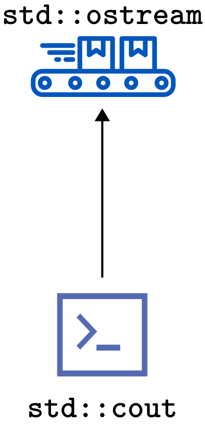
(abstract, generic)

(is a)

ShredBox

FireBox

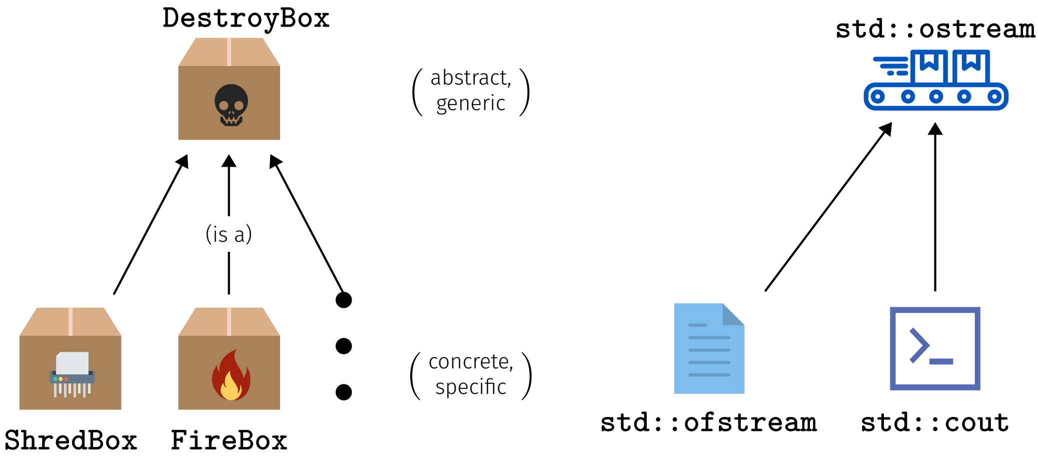
(concrete, specific)



std::ostream

std::cout

Abstract and Concrete Character Streams



DestroyBox

(abstract, generic)

(is a)

ShredBox

FireBox

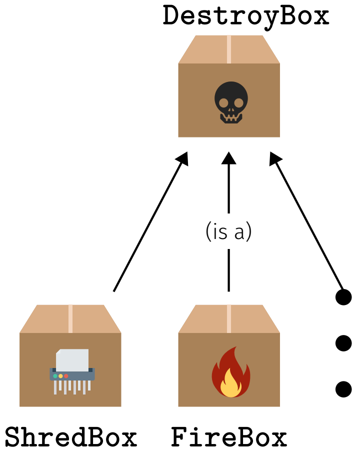
(concrete, specific)

std::ostream

std::ofstream

std::cout

Abstract and Concrete Character Streams



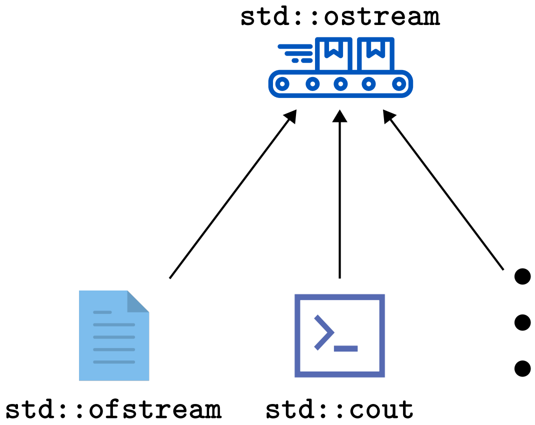
DestroyBox

(is a)

ShredBox FireBox

(abstract, generic)

(concrete, specific)



std::ostream

std::ofstream std::cout

Caesar-Code: Generalisation

```
void caesar(std::istream& in,
            std::ostream& out,
            int s) {

  in >> std::noskipws;

  char next;
  while (in >> next) {
    out << shift(next, s);
  }
}
```

- `std::istream`/`std::ostream` is an *abstract input/output stream* of `char`s

Caesar-Code: Generalisation

```
void caesar(std::istream& in,
            std::ostream& out,
            int s) {

  in >> std::noskipws;

  char next;
  while (in >> next) {
    out << shift(next, s);
  }
}
```

- **std::istream**/**std::ostream** is an *abstract input/output stream* of **char**s
- Function is called with *concrete* streams, e.g.:
  - Console: **std::cin**/**cout**
  - Files: **std::ifstream**/**ofstream**

Caesar-Code: Generalisation, Example 1

```
#include <iostream>
...

// in void main():
caesar(std::cin, std::cout, s);
```

Calling the generalised **caesar** function: from **std::cin** to **std::cout**

Caesar-Code: Generalisation, Example 2

```
#include <iostream>
#include <fstream>
...

// in void main():
std::string to_file_name = ...; // Name of file to write to
std::ofstream to(to_file_name); // Output file stream

caesar(std::cin, to, s);
```

Calling the generalised **`caesar`** function: from **`std::cin`** to file

Caesar-Code: Generalisation, Example 3

```
#include <iostream>
#include <fstream>
...

// in void main():
std::string from_file_name = ...; // Name of file to read from
std::string to_file_name = ...; // Name of file to write to
std::ifstream from(from_file_name); // Input file stream
std::ofstream to(to_file_name); // Output file stream

caesar(from, to, s);
```

Calling the generalised **caesar** function: from file to file

Streams: Final Words

Note: You only need to be able to *use* streams

- *User knowledge*, on the level of the previous slides, suffices for exercises and exam
- I.e. you do not need to know how streams work internally
- At the end of this course, you'll hear how you can define *abstract*, and corresponding *concrete*, types yourself

Texts

- Text “**`to be or not to be`**” could be represented as **`vector<char>`**

Texts

- Text "**`to be or not to be`**" could be represented as **`vector<char>`**
- Texts are ubiquitous, however, and thus have their own typ in the standard library: **`std::string`**
- Requires **`#include <string>`**

Using `std::string`

- Declaration, and initialisation with a literal:

```
std::string text = "Essen ist fertig!"
```

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```
std::string text(n, ’a’)
```

Using `std::string`

- Declaration, and initialisation with a literal:

```
std::string text = "Essen ist fertig!"
```

- Initialise with variable length:

```
std::string text(n, 'a')
```

- Comparing texts:

```
if (text1 == text2) ...
```

Using std::string

- Querying size:

```
for (unsigned int i = 0; i < text.size(); ++i) ...
```

Using `std::string`

- Querying size:

```
for (unsigned int i = 0; i < text.size(); ++i) ...
```

- Reading single characters:

```
if (text[0] == ’a’) ... // or text.at(0)
```

Using std::string

- Querying size:

```
for (unsigned int i = 0; i < text.size(); ++i) ...
```

- Reading single characters:

```
if (text[0] == ’a’) ... // or text.at(0)
```

- Writing single characters:

```
text[0] = ’b’; // or text.at(0)
```

Using `std::string`

- Concatenate strings:

```
text = ":-";
text += ")";
assert(text == ":-)");
```

- Many more operations; if interested, see `https://en.cppreference.com/w/cpp/string`

15. Vectors II

Multidimensional Vector/Vectors of Vectors, Shortest Paths, Vectors as Function Arguments

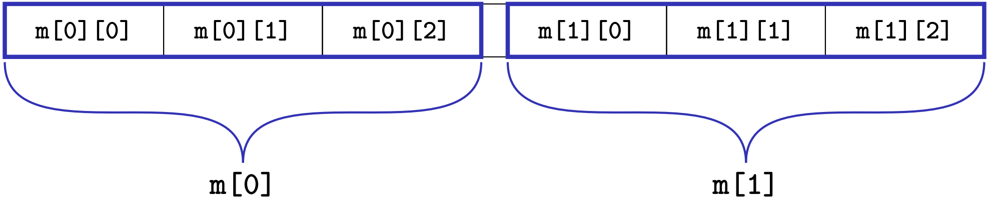
Multidimensional Vectors

- For storing multidimensional structures such as tables, matrices, . . .
- . . . *vectors of vectors* can be used:

```
std::vector<std::vector<int>> m; // An empty matrix
```

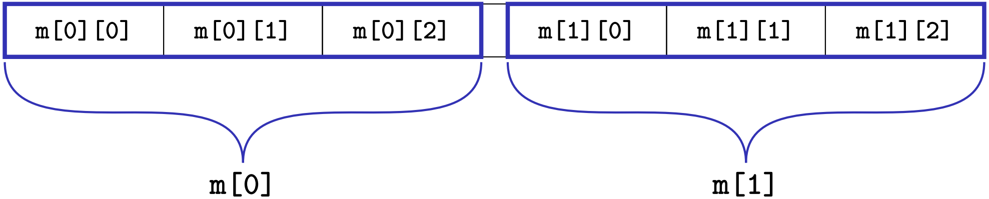
Multidimensional Vectors

In memory: flat



Multidimensional Vectors

In memory: flat



in our head: matrix

columns

rows

	0	1	2
0	m[0][0]	m[0][1]	m[0][2]
1	m[1][0]	m[1][1]	m[1][2]

Multidimensional Vectors: Initialisation

Using initialisation lists:

```
// A 3-by-5 matrix
std::vector<std::vector<std::string>> m = {
  {"ZH", "BE", "LU", "BS", "GE"},
  {"FR", "VD", "VS", "NE", "JU"},
  {"AR", "AI", "OW", "IW", "ZG"}
};

assert(m[1][2] == "VS");
```

Multidimensional Vectors: Initialisation

Fill to specific size:

```
unsigned int a = ...;
unsigned int b = ...;

// An a-by-b matrix with all ones
std::vector<std::vector<int>>
  m(a, std::vector<int>(b, 1));
```

Multidimensional Vectors: Initialisation

Fill to specific size:

```
unsigned int a = ...;
unsigned int b = ...;

// An a-by-b matrix with all ones
std::vector<std::vector<int>>
  m(a, std::vector<int>(b, 1));
```

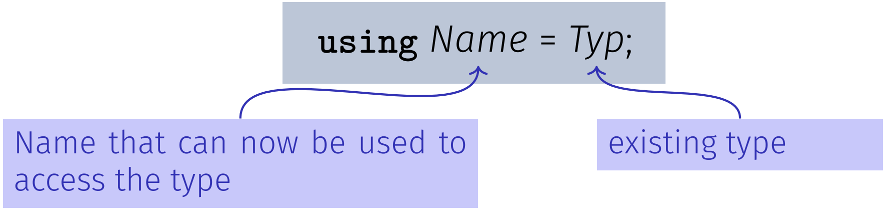
(Many further ways of initialising a vector exist)

Multidimensional Vectors and Type Aliases

- Also possible: vectors of vectors of vectors of ...:
  `std::vector<std::vector<std::vector<...>>>`
- Type names can obviously become looooooong

Multidimensional Vectors and Type Aliases

- Also possible: vectors of vectors of vectors of ...:
  `std::vector<std::vector<std::vector<...>>>`
- Type names can obviously become looooooong
- The declaration of a *type alias* helps here:



`using` *Name* = *Typ*;

Name that can now be used to access the type

existing type

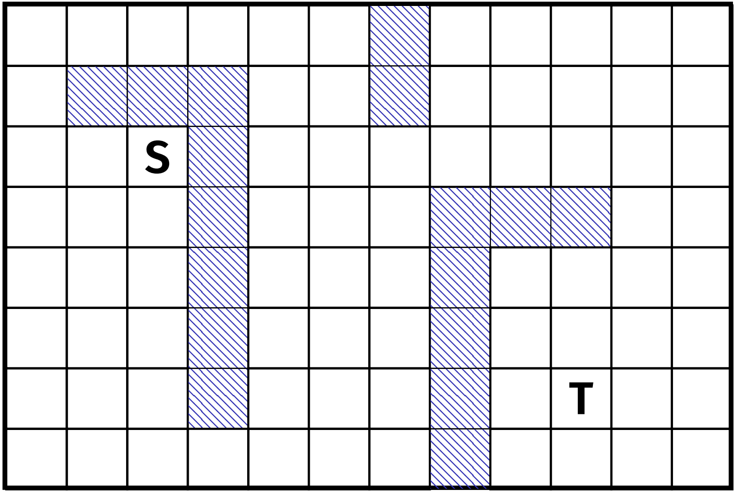
Type Aliases: Example

```
#include <iostream>
#include <vector>
using imatrix = std::vector<std::vector<int>>;

// POST: Matrix ’m’ was output to stream ’out’
void print(const imatrix& m, std::ostream& out);

int main() {
  imatrix m = ...;
  print(m, std::cout);
}
```

Recall: **`const`** reference for enfficiency (no copy) and safety (immutable)

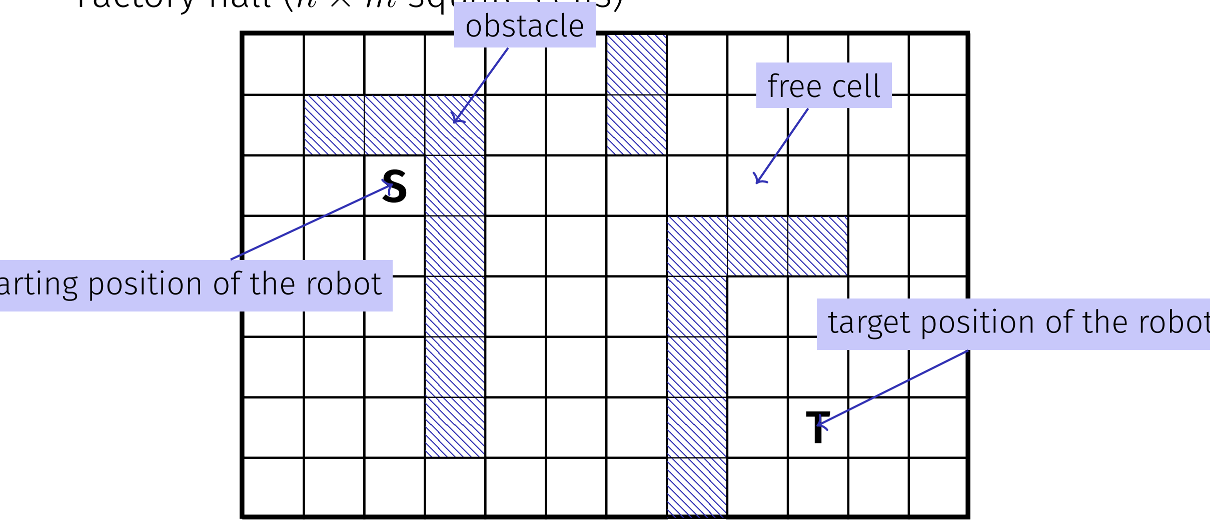
Application: Shortest Paths

Factory hall ($n \times m$ square cells)



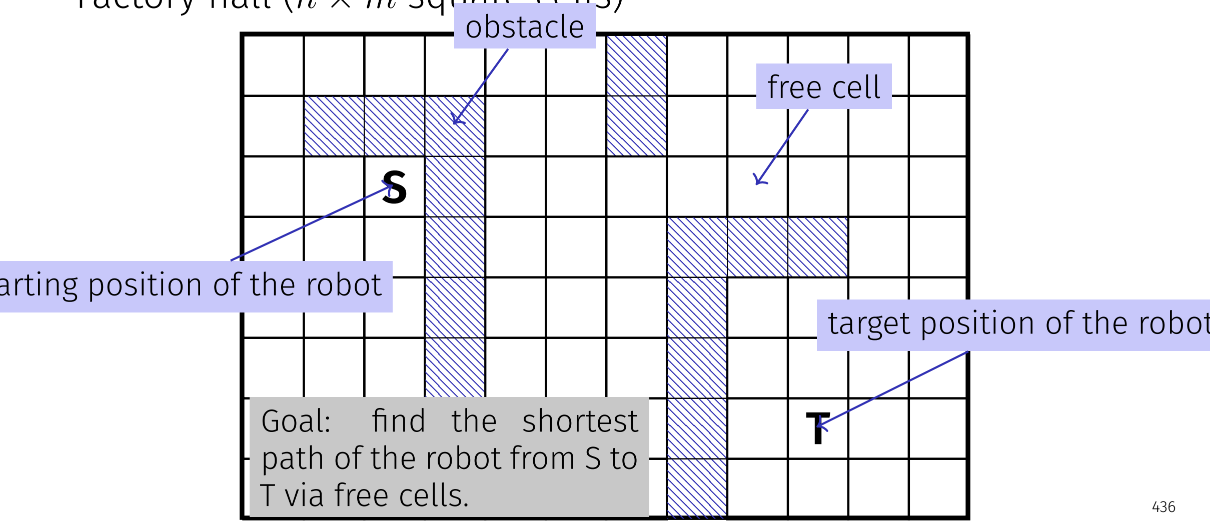
Application: Shortest Paths

Factory hall ($n \times m$ square cells)



Application: Shortest Paths

Factory hall ($n \times m$ square cells)



obstacle

free cell

S

arting position of the robot

target position of the robot

T

Goal: find the shortest path of the robot from S to T via free cells.

This problem appears to be different

Find the *lengths* of the shortest paths to *all* possible targets.

4	5	6	7	8	9	▨	15	16	17	18	19
3	▨	▨	▨	9	10	▨	14	15	16	17	18
2	1	0	▨	10	11	12	13	14	15	16	17
3	2	1	▨	11	12	13	▨	▨	▨	17	18
4	3	2	▨	10	11	12	▨	20	19	18	19
5	4	3	▨	9	10	11	▨	21	20	19	20
6	5	4	▨	8	9	10	▨	22	21	20	21
7	6	5	6	7	8	9	▨	23	22	21	22

This problem appears to be different

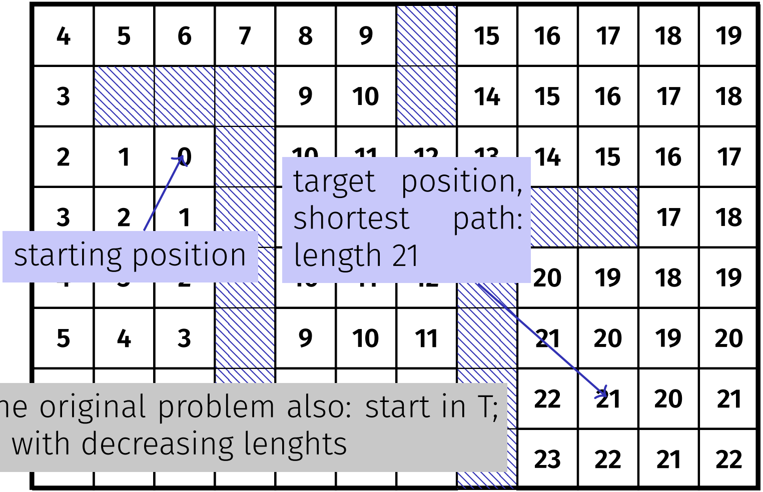
Find the *lengths* of the shortest paths to *all* possible targets.

4	5	6	7	8	9	▨	15	16	17	18	19
3	▨	▨	▨	9	10	▨	14	15	16	17	18
2	1	0	▨	10	11	12	13	14	15	16	17
3	2	1	▨	11	12	13	▨	▨	▨	17	18
4	3	2	▨	10	11	12	▨	20	19	18	19
5	4	3	▨	9	10	11	▨	21	20	19	20
			▨				▨	22	21	20	21
							▨	23	22	21	22

This solves the original problem also: start in T; follow a path with decreasing lenghts

This problem appears to be different

Find the *lengths* of the shortest paths to *all* possible targets.



4	5	6	7	8	9	▨	15	16	17	18	19
3	▨	▨	▨	9	10	▨	14	15	16	17	18
2	1	0	▨	10	11	12	13	14	15	16	17
3	2	1	▨					▨	▨	17	18
			▨				▨	20	19	18	19
5	4	3	▨	9	10	11	▨	21	20	19	20
			▨				▨	22	21	20	21
							▨	23	22	21	22

starting position

target position, shortest path: length 21

This solves the original problem also: start in T; follow a path with decreasing lenghts

This problem appears to be different

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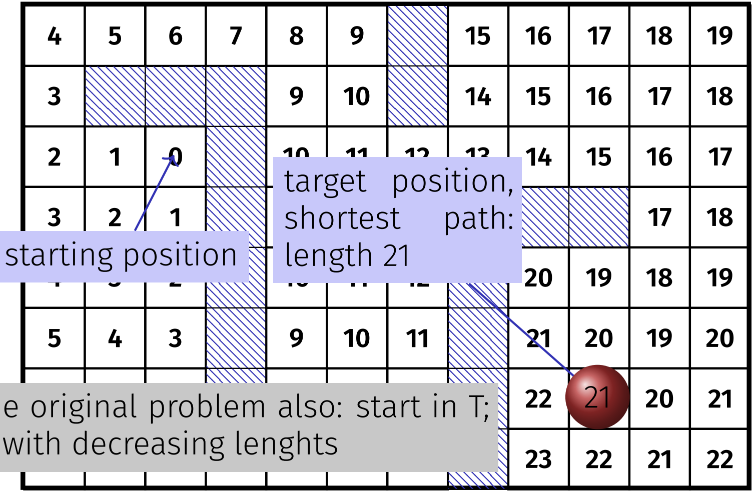

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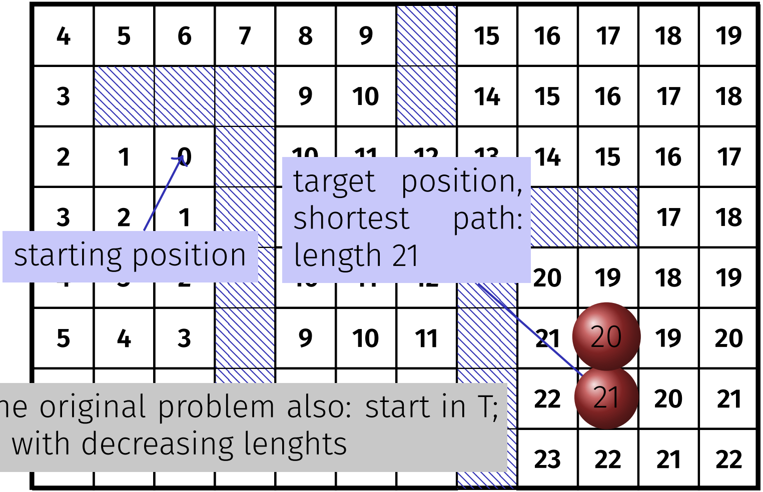
Find the *lengths* of the shortest paths to *all* possible targets.



4	5	6	7	8	9	▨	15	16	17	18	19
3	▨	▨	▨	9	10	▨	14	15	16	17	18
2	1	0	▨	10	11	12	13	14	15	16	17
3	2	1	▨	[illegible]	[illegible]	[illegible]	[illegible]	▨	▨	17	18
4	3	2	▨	10	11	12	▨	20	19	18	19
5	4	3	▨	9	10	11	▨	21	20	19	20
[illegible]	[illegible]	[illegible]	▨	[illegible]	[illegible]	[illegible]	▨	22	21	20	21
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	▨	23	22	21	22

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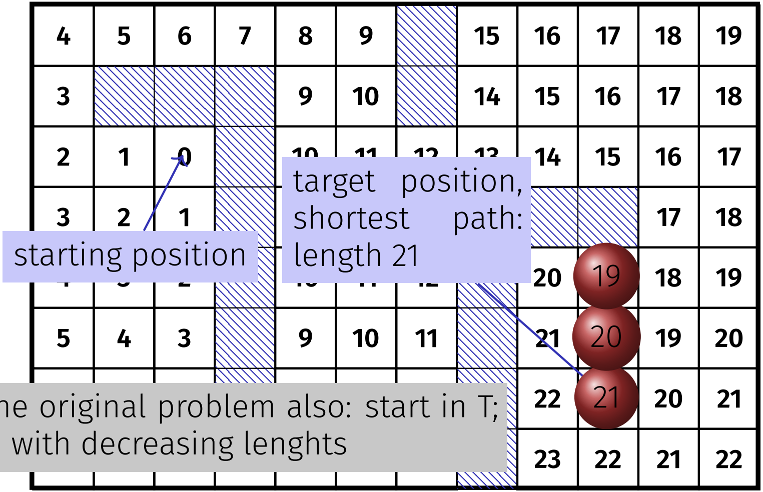

4	5	6	7	8	9	▨	15	16	17	18	19
3	▨	▨	▨	9	10	▨	14	15	16	17	18
2	1	0	▨	10	11	12	13	14	15	16	17
3	2	1	▨	[illegible]	[illegible]	[illegible]	[illegible]	▨	▨	17	18
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5	4	3	▨	9	10	11	▨	21	20	19	20
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3	2	1	▨					▨	▨	17	18
4	3	2	▨	10	11	12	▨	20	19	18	19
5	4	3	▨	9	10	11	▨	21	20	19	20
			▨				▨	22	21	20	21
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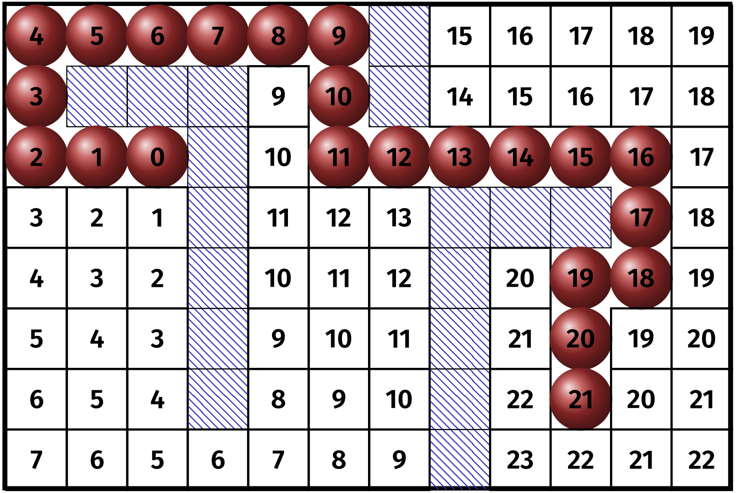
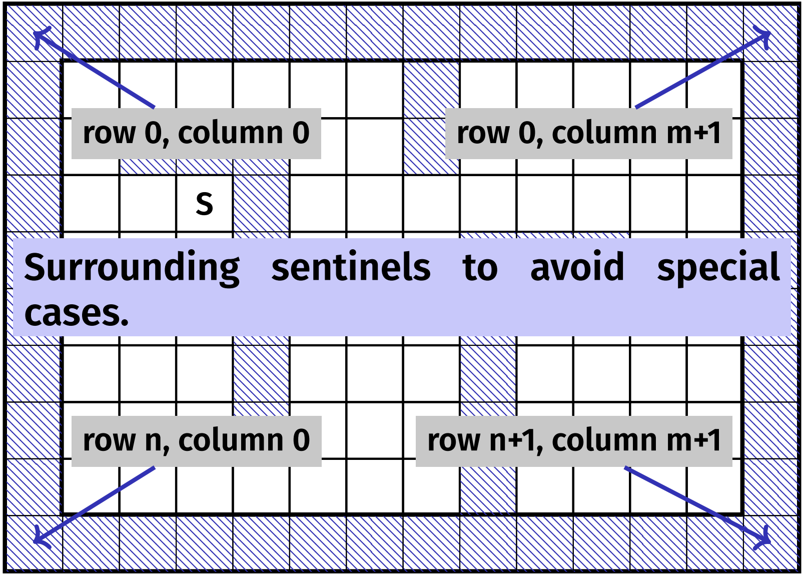


4	5	6	7	8	9	▨	15	16	17	18	19
3	▨	▨	▨	9	10	▨	14	15	16	17	18
2	1	0	▨	10	11	12	13	14	15	16	17
3	2	1	▨	11	12	13	▨	▨	▨	17	18
4	3	2	▨	10	11	12	▨	20	19	18	19
5	4	3	▨	9	10	11	▨	21	20	19	20
6	5	4	▨	8	9	10	▨	22	21	20	21
7	6	5	6	7	8	9	▨	23	22	21	22

Preparation: Sentinels



row 0, column 0

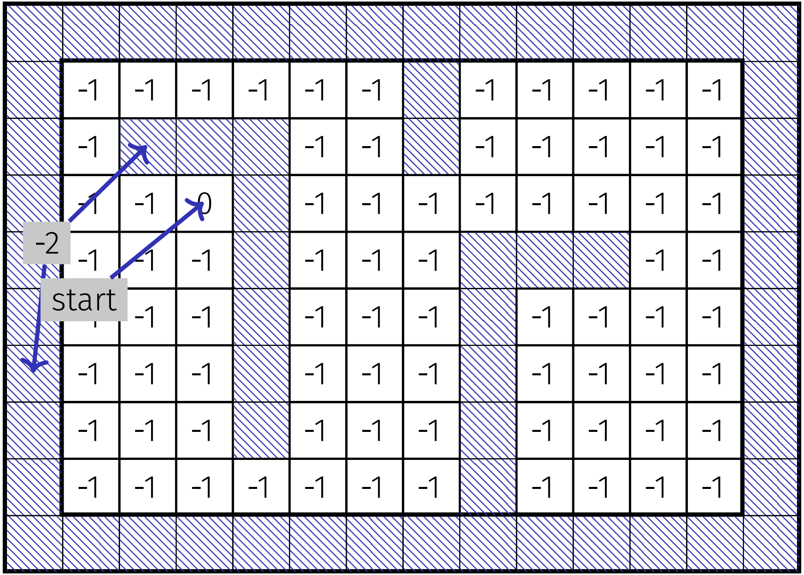
row 0, column m+1

Surrounding sentinels to avoid special cases.

row n, column 0

row n+1, column m+1

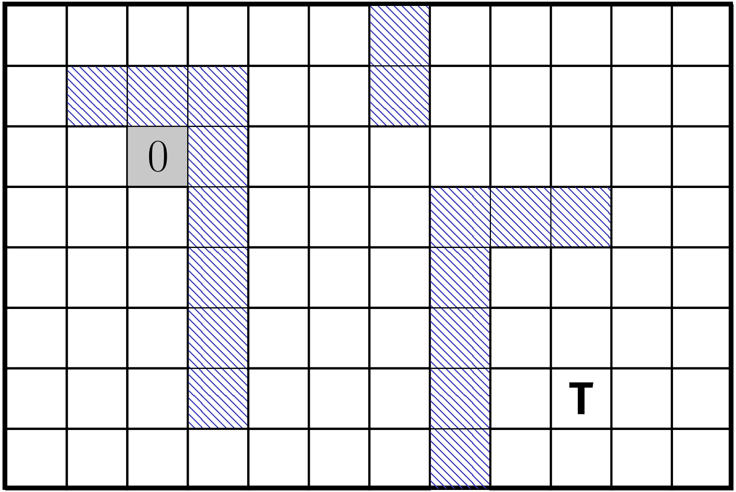
Preparation: Initial Marking



▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨
▨	-1	-1	-1	-1	-1	-1	▨	-1	-1	-1	-1	-1	▨
▨	-1	▨	▨	▨	-1	-1	▨	-1	-1	-1	-1	-1	▨
▨	-1	-1	0	▨	-1	-1	-1	-1	-1	-1	-1	-1	▨
▨	-1	-1	-1	▨	-1	-1	-1	▨	▨	▨	-1	-1	▨
▨	-1	-1	-1	▨	-1	-1	-1	▨	-1	-1	-1	-1	▨
▨	-1	-1	-1	▨	-1	-1	-1	▨	-1	-1	-1	-1	▨
▨	-1	-1	-1	▨	-1	-1	-1	▨	-1	-1	-1	-1	▨
▨	-1	-1	-1	-1	-1	-1	-1	▨	-1	-1	-1	-1	▨
▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨

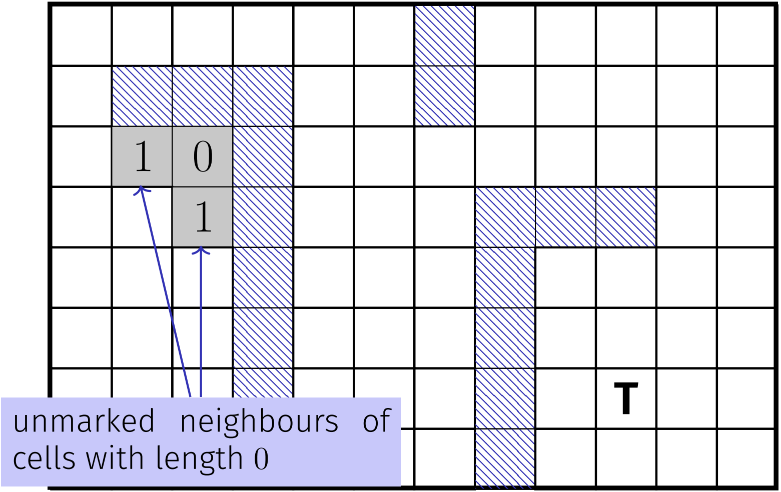
Mark all Cells with their Path Lengths

Step 0: all cells with path length 0



Mark all Cells with their Path Lengths

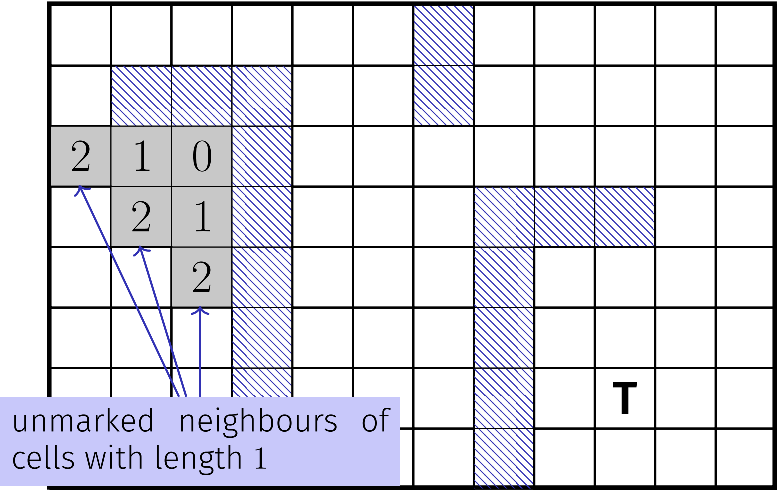
Step 1: all cells with path length 1



unmarked neighbours of cells with length 0

Mark all Cells with their Path Lengths

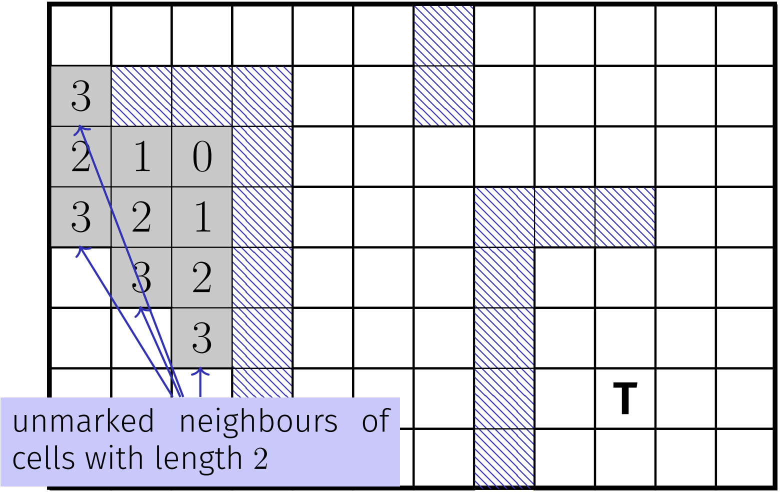
Step 2: all cells with path length 2



unmarked neighbours of cells with length 1

Mark all Cells with their Path Lengths

Step 3: all cells with path length 3



unmarked neighbours of cells with length 2

Main Loop

Find and mark all cells with path lengths $i = 1, 2, 3 \ldots$

```
for (int i=1;; ++i) {
  bool progress = false;
  for (int r=1; r<n+1; ++r)
    for (int c=1; c<m+1; ++c) {
     if (floor[r][c] != -1) continue;
     if (floor[r-1][c] == i-1 || floor[r+1][c] == i-1 ||
         floor[r][c-1] == i-1 || floor[r][c+1] == i-1 ) {
       floor[r][c] = i; // label cell with i
       progress = true;
     }
    }
  if (!progress) break;
}
```

Main Loop

Find and mark all cells with path lengths $i = 1, 2, 3...$

```
for (int i=1;; ++i) {
  bool progress = false;
  for (int r=1; r<n+1; ++r)
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         floor[r][c-1] == i-1 || floor[r][c+1] == i-1 ) {
       floor[r][c] = i; // label cell with i
       progress = true;
     }
    }
  if (!progress) break;
}
```

indicates if in sweep through all cells there was progress

Main Loop

Find and mark all cells with path lengths $i = 1, 2, 3 \ldots$

```
for (int i=1;; ++i) {
  bool progress = false;
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        floor[r][c] = i; // label cell with i
        progress = true;
      }
    }
  if (!progress) break;
}
```

sweep over all cells

Main Loop

Find and mark all cells with path lengths $i = 1, 2, 3...$

```
for (int i=1;; ++i) {
  bool progress = false;
  for (int r=1; r<n+1; ++r)
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       floor[r][c] = i; // label cell with i
       progress = true;
     }
    }
  if (!progress) break;
}
```

cell already marked or obstacle

Main Loop

Find and mark all cells with path lengths $i = 1, 2, 3 ...$

```
for (int i=1;; ++i) {
  bool progress = false;
  for (int r=1; r<n+1; ++r)
     for (int c=1; c<m+1; ++c) {
      if (floor[r][c] != -1) continue;
      if (floor[r-1][c] == i-1 || floor[r+1][c] == i-1 ||
          floor[r][c-1] == i-1 || floor[r][c+1] == i-1 ) {
        floor[r][c] = i; // label cell with i
        progress = true;
      }
    }
  if (!progress) break;
}
```

a neighbour has path length $i-1$. The sentinels guarantee that there are always 4 neighbours

Main Loop

Find and mark all cells with path lengths $i = 1, 2, 3...$

```
for (int i=1;; ++i) {
  bool progress = false;
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       floor[r][c] = i; // label cell with i
       progress = true;
     }
    }
  if (!progress) break;
}
```

← no progress, all reachable cells marked; done.

The Shortest Paths Program

- Algorithm: *Breadth-First Search* (Breadth-first vs. *depth-first* search is typically discussed in lectures on algorithms)

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The Shortest Paths Program

- Algorithm: *Breadth-First Search* (Breadth-first vs. *depth-first* search is typically discussed in lectures on algorithms)
- The program can become pretty slow because for each i all cells are traversed
- Improvement: for marking with i, traverse only the neighbours of the cells marked with $i-1$.
- Improvement: stop once the goal has been reached

16. Recursion 1

Mathematical Recursion, Termination, Call Stack, Examples, Recursion vs. Iteration, n-Queen Problem

Mathematical Recursion

- Many mathematical functions can be naturally defined *recursively*

Mathematical Recursion

- Many mathematical functions can be naturally defined *recursively*
- This means, the function appears in its own definition

$$n! = \begin{cases} 1, & \text{if } n \leq 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

Recursion in C++: In the same Way!

$$n! = \begin{cases} 1, & \text{if } n \leq 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

```
// POST: return value is n!
unsigned int fac(unsigned int n) {
 if (n <= 1)
   return 1;
  else
   return n * fac(n-1);
}
```

Infinite Recursion

- is as bad as an infinite loop ...

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- ...but even worse: it burns time *and* memory

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```
void f() {
  f() // f() → f() → ... → stack overflow
}
```

Infinite Recursion

- is as bad as an infinite loop …
- …but even worse: it burns time *and* memory

```
void f() {
  f() // f() → f() → ... → stack overflow
}
```

Ein Euro ist ein Euro.

Wim Duisenberg, erster Präsident der EZB

Recursive Functions: Termination

As with loops we need **guaranteed progress towards an exit condition ($\approx$ base case)**

Example `fac(n)`:

- Recursion ends if $n \leq 1$
- Recursive call with new argument $< n$
- Exit condition will thus be reached eventually

```
unsigned int fac(
    unsigned int n) {

  if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}
```

Recursive Functions: Evaluation

```
int fac(int n) {
  if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}

...
std::cout << fac(4);
```

Recursive Functions: Evaluation

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int fac(int n) {
  if (n <= 1)
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...
std::cout << fac(4);
```

`fac(4)`

Calling **`fac(4)`**

Recursive Functions: Evaluation

```
int fac(int n) {
  if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}

...
std::cout << fac(4);
```

`fac(4)` ⇝ `int n = 4`

Calling `fac(4)` ⇝ Initialisation of formal argument `n`

Recursive Functions: Evaluation

```
int fac(int n) {
  if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}

...
std::cout << fac(4);
```

`fac(4)` $\rightsquigarrow$ `int n = 4`

Evaluation of return expression

Recursive Functions: Evaluation

```
int fac(int n) {
  if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}

...
std::cout << fac(4);
```

`fac(4)` $\rightsquigarrow$ `int n = 4`

$\hookrightarrow$ `fac(n - 1)`

Recursive call with argument `n - 1` $= 4 - 1 = 3$

Recursive Functions: Evaluation

```
int fac(int n) {
  if (n <= 1)
    return 1;
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...
std::cout << fac(4);
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↪ `fac(n - 1)` ⇝ `int n = 3`

Initialisation of formal argument **`n`**

Recursive Functions: Evaluation

```
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  if (n <= 1)
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}

...
std::cout << fac(4);
```

`fac(4)` $\rightsquigarrow$ `int n = 4`

$\hookrightarrow$ `fac(n - 1)` $\rightsquigarrow$ `int n = 3`

$\vdots$

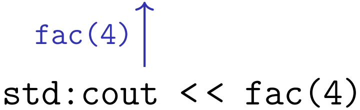
Every call of **`fac`** *operates on its own* **`n`**

The Call Stack

```
std:cout << fac(4)
```

The Call Stack

For each function call:



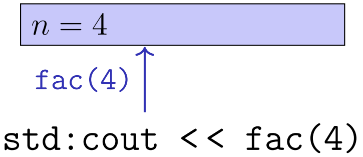
fac(4)

std:cout << fac(4)

The Call Stack

For each function call:

- push value of the call argument onto the stack



$n = 4$

`fac(4)`

`std:cout << fac(4)`

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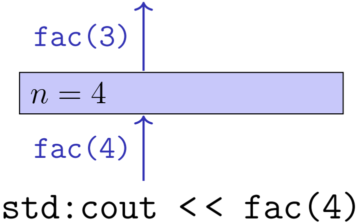

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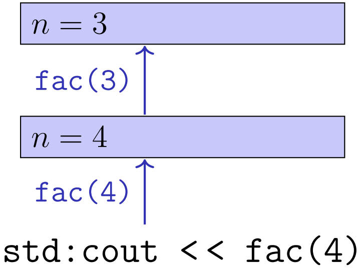

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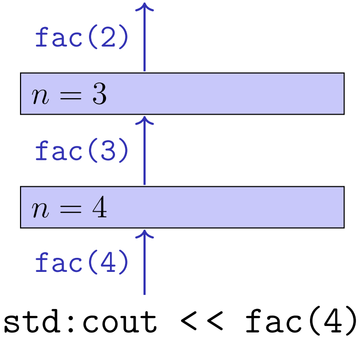

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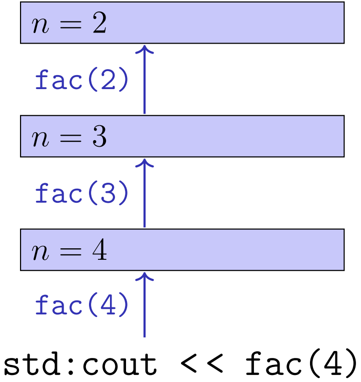


$n = 2$

fac(2)

$n = 3$

fac(3)

$n = 4$

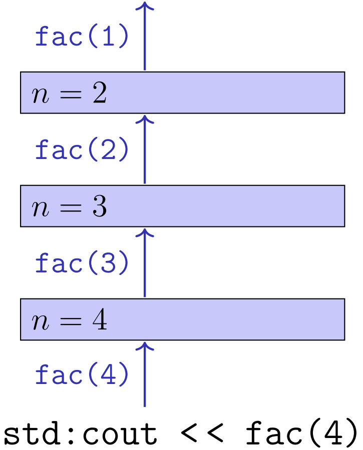
fac(4)

```
std:cout << fac(4)
```

The Call Stack

For each function call:

- push value of the call argument onto the stack



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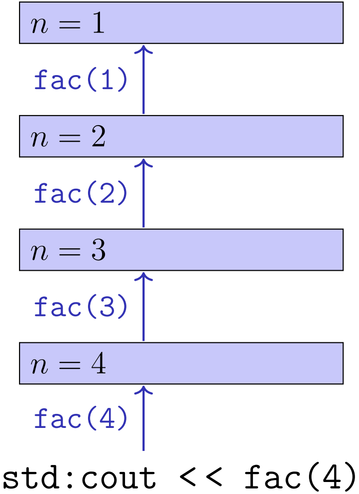
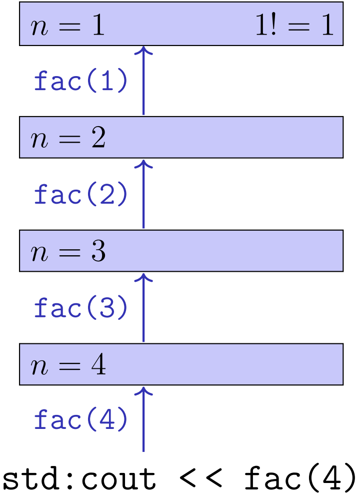


The Call Stack

For each function call:

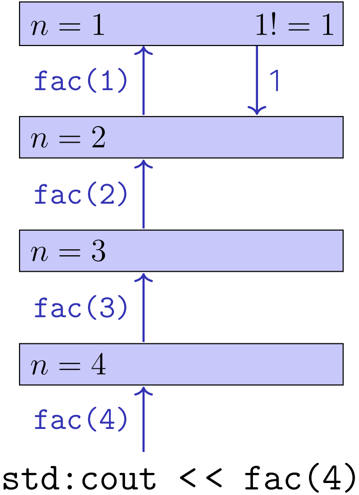
- push value of the call argument onto the stack
- always work with the top value



The Call Stack

For each function call:

- push value of the call argument onto the stack
- always work with the top value
- at the end of the call the top value is removed from the stack



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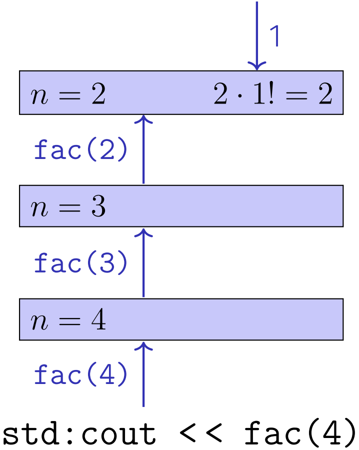

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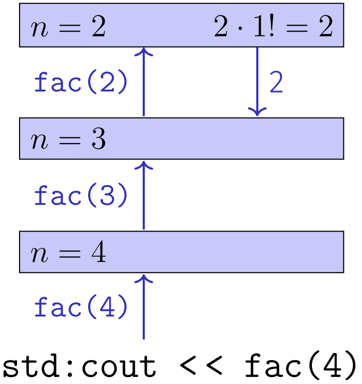

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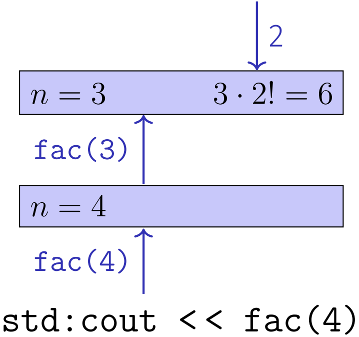

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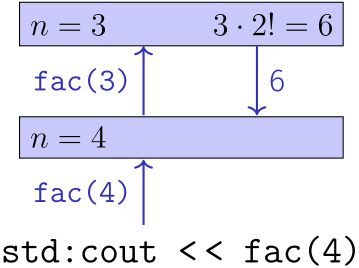

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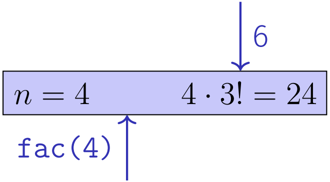
- push value of the call argument onto the stack
- always work with the top value
- at the end of the call the top value is removed from the stack



```
std:cout << fac(4)
```

The Call Stack

For each function call:

- push value of the call argument onto the stack
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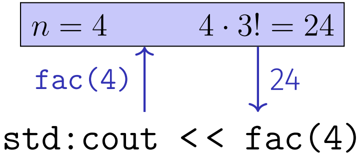

$n = 4$ $\quad 4 \cdot 3! = 24$

`fac(4)`

24

`std:cout << fac(4)`

The Call Stack

For each function call:

- push value of the call argument onto the stack
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```
            │24
            ↓
std:cout << fac(4)
```

Fibonacci Numbers

$$F_n := \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{if } n > 1 \end{cases}$$

Fibonacci Numbers

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$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots$$

Fibonacci Numbers in Zurich

Fibonacci Numbers in C++

$$F_n := \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{if } n > 1 \end{cases}$$

```cpp
unsigned int fib(unsigned int n) {
  if (n == 0) return 0;
  if (n == 1) return 1;
  return fib(n-1) + fib(n-2); // n > 1
}
```

Fibonacci Numbers in C++

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}
```

Fibonacci Numbers in C++

Laufzeit

`fib(50)` takes “forever” because it computes
F_{48} two times, F_{47} 3 times, F_{46} 5 times, F_{45} 8 times, F_{44} 13 times,
F_{43} 21 times … F_1 ca. 10^9 times (!)

```
unsigned int fib(unsigned int n) {
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Fast Fibonacci Numbers

Idea:

- Compute each Fibonacci number only once, in the order $F_0, F_1, F_2, \ldots, F_n$

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- Compute the next number as a sum of `a` and `b`

Can be implemented recursively and iteratively, the latter is easier/more direct

Fast Fibonacci Numbers in C++

```
unsigned int fib(unsigned int n) {
  if (n == 0) return 0;
  if (n == 1) return 1;

  unsigned int a = 0; // F_0
  unsigned int b = 1; // F_1

  for (unsigned int i = 2; i <= n; ++i) {
    unsigned int a_old = a; // $F_{i-2}$
    a = b; // a becomes $F_{i-1}$
    b += a_old; // b becomes $F_{i-1} + F_{i-2}$, i.e. $F_i$
  }

  return b;
}
```

$$(F_{i-2}, F_{i-1}) \longrightarrow (F_{i-1}, F_i)$$

Fast Fibonacci Numbers in C++

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  return b;
}
```

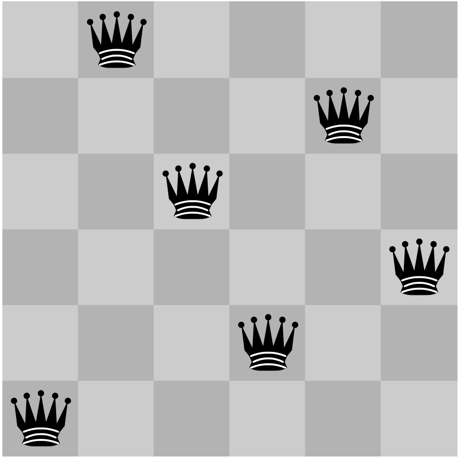
very fast, also for **`fib(50)`**

$$(F_{i-2}, F_{i-1}) \longrightarrow (F_{i-1}, F_i)$$

The Power of Recursion

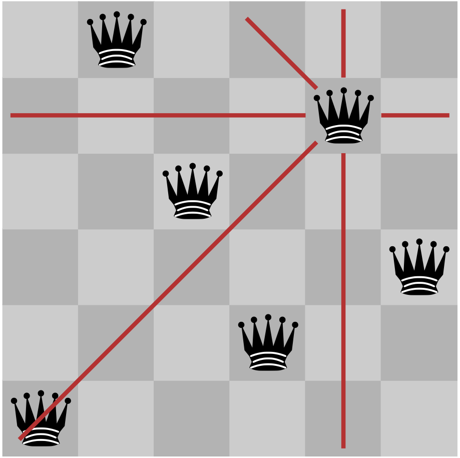
- Some problems appear to be hard to solve without recursion. With recursion they become significantly simpler.
- Examples: *The n-Queens-Problem*, The towers of Hanoi, Sudoku-Solver, Expression Parsers, Reversing In- or Output, Searching in Trees, Divide-And-Conquer (e.g. sorting) , …
- …and the 2. bonus exercise: Nonograms

The n-Queens Problem



- Provided is a $n timesn$ chessboard
- For example $n = 6$
- Question: is it possiblt to position n queens such that no two queens threaten each other?

The n-Queens Problem



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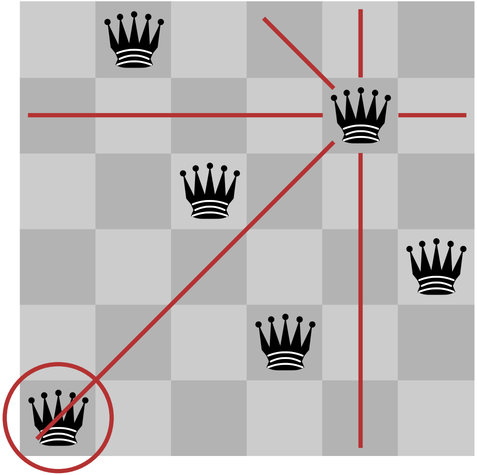

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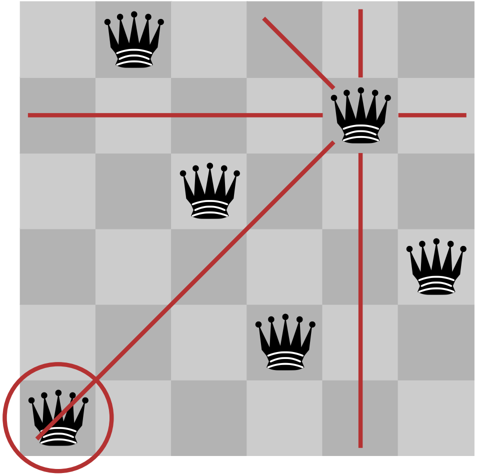

- Provided is a $n \ timesn$ chessboard
- For example $n = 6$
- Question: is it possiblt to position n queens such that no two queens threaten each other?
- If yes, how many solutions are there?

Solution?

- Try all possible placements?

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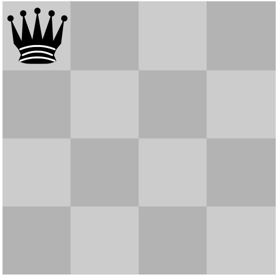
Solution?

- Try all possible placements?
- $\binom{n^2}{n}$ possibilities. Too many!
- Only ne queen per row: n^n possibilities. Better – but still too many.
- Idea: don't proceed with futile attempts, retract incorrect moves instead $\Rightarrow$ *Backtracking*

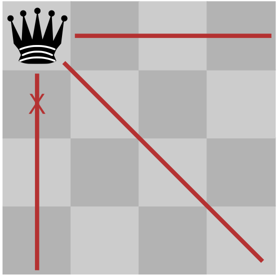
Solution with Backtracking



First Queen

queens
0
0
0
0

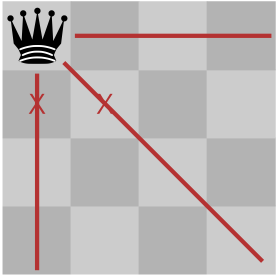
Solution with Backtracking



Forbidden Squares: no other queens may be here.

queens
0
0
0
0

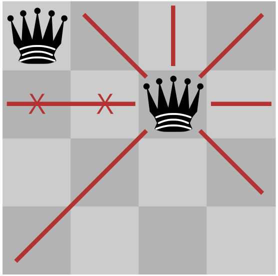
Solution with Backtracking



Forbidden Squares: no other queens may be here.

queens
0
1
0
0

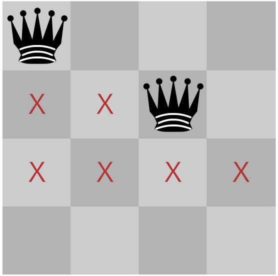
Solution with Backtracking



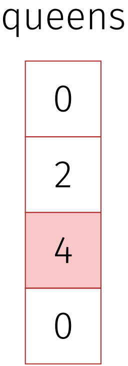
Second Queen in next row (no collision)

queens
0
2
0
0

Solution with Backtracking



All squares in next row forbiden. Track back !


queens

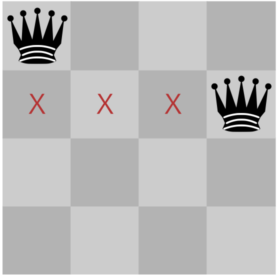
0

2

4

0

Solution with Backtracking

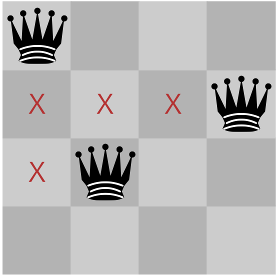


Move queen one step further and try again

queens

0
3
0
0

Solution with Backtracking

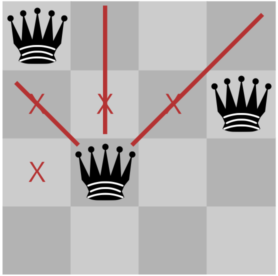


next row

queens

0
3
1
0

Solution with Backtracking

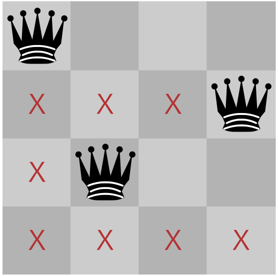


Ok (only previous queens have to be tested)

queens

0
3
1
0

Solution with Backtracking

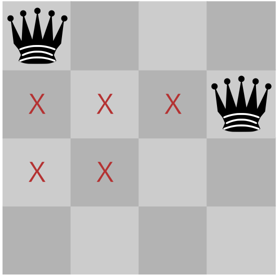


All squares of the next row forbidden. Track back.

queens

0
3
1
4

Solution with Backtracking

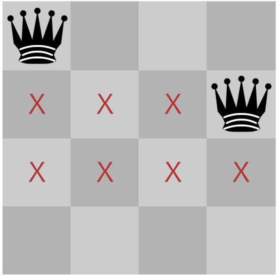


Continue in previous row.

queens

0
3
1
0

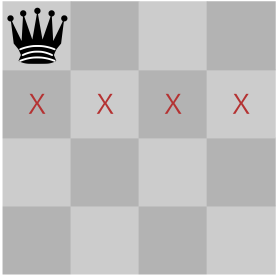
Solution with Backtracking



Remaining squares also forbidden. Track back!

queens
0
3
4
0

Solution with Backtracking

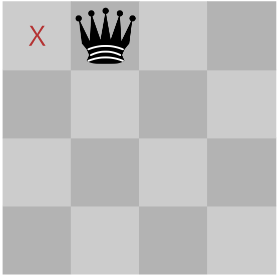


All squares of this row did not yield a solution. Track back!

queens

0
4
0
0

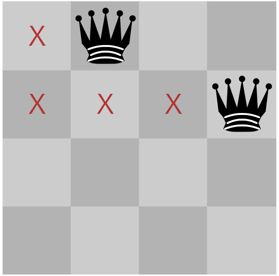
Solution with Backtracking



again advance queen by one square

queens
1
0
0
0

Solution with Backtracking

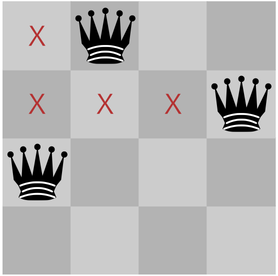


next row

queens

1
3
0
0

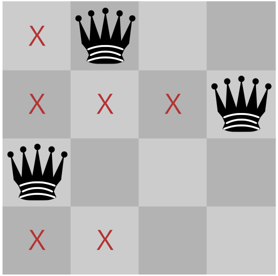
Solution with Backtracking



next row

queens
1
3
0
0

Solution with Backtracking

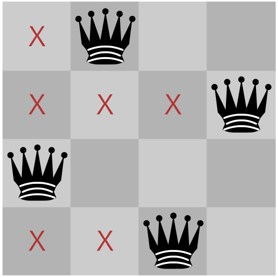


next row

queens

1
3
0
1

Solution with Backtracking



Found a solution

queens

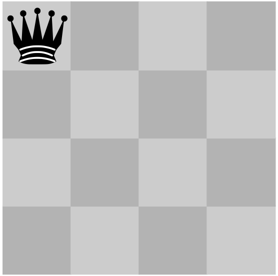
1

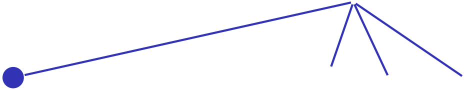
3

0

2

Search Strategy Visualized as a Tree





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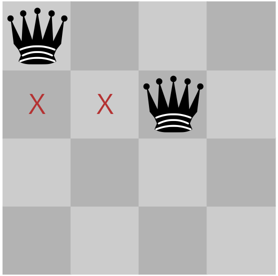

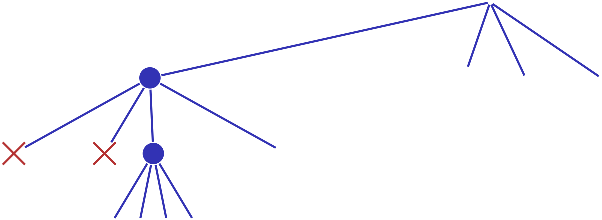

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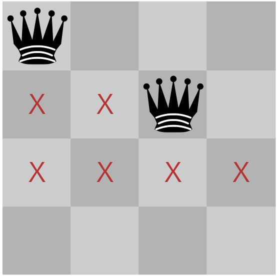

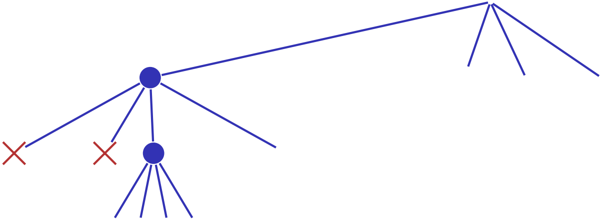

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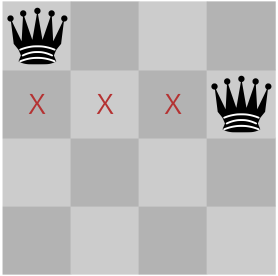

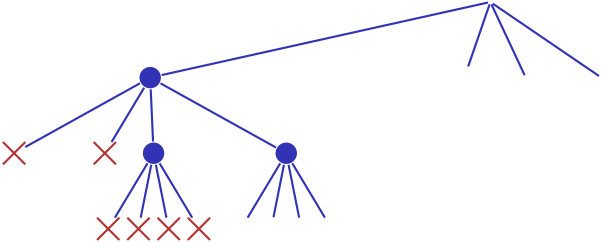

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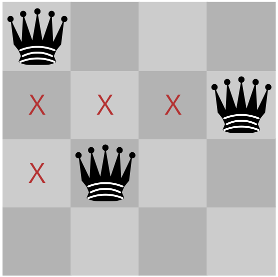

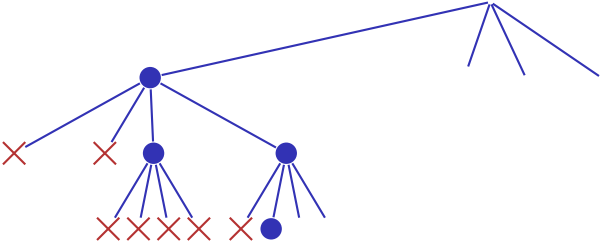

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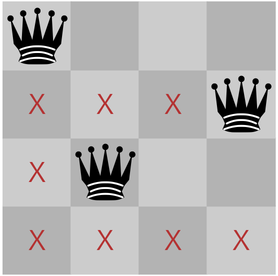

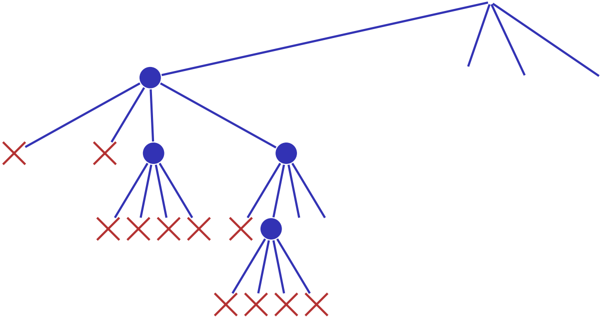

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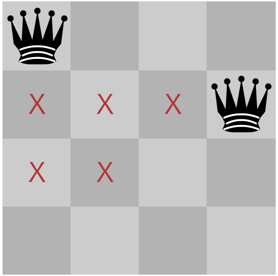

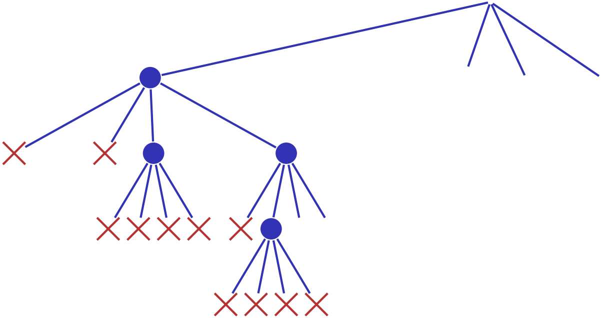

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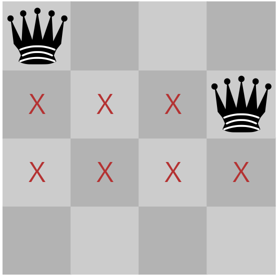

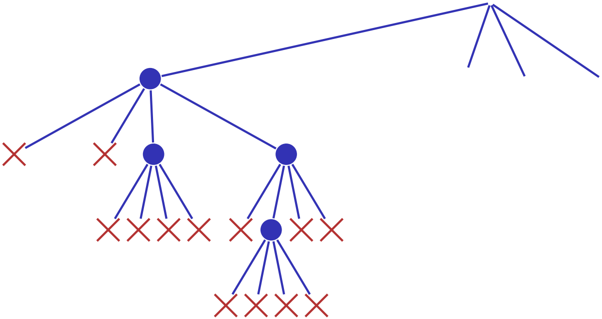

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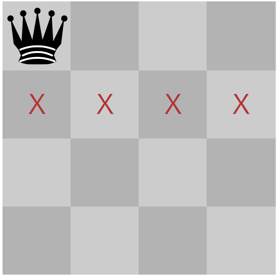

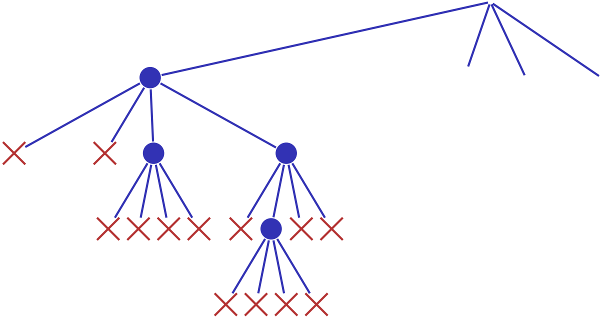

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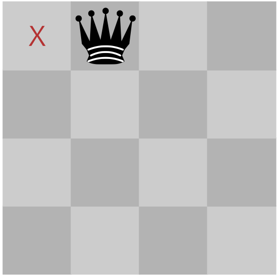

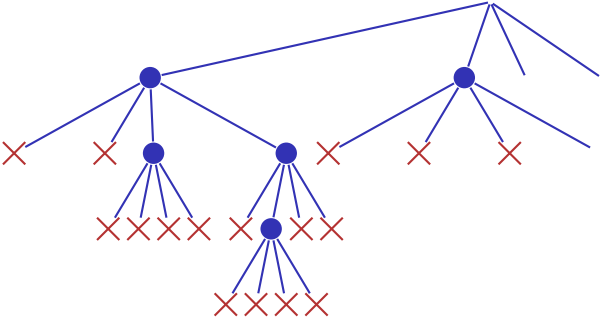

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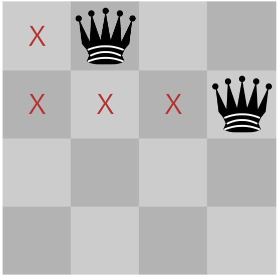

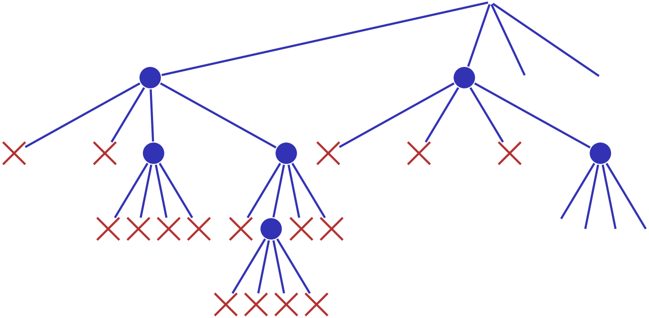

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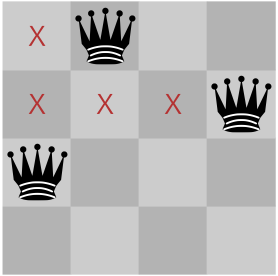
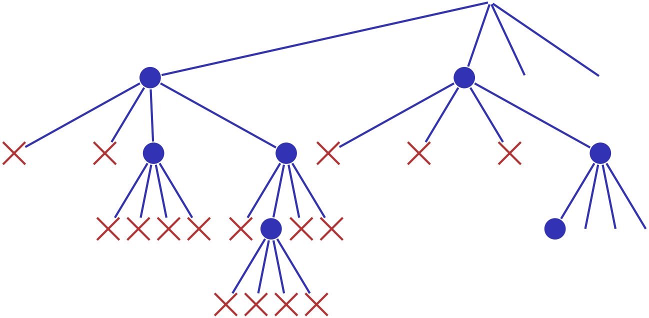
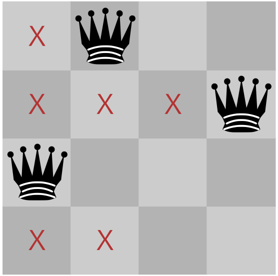
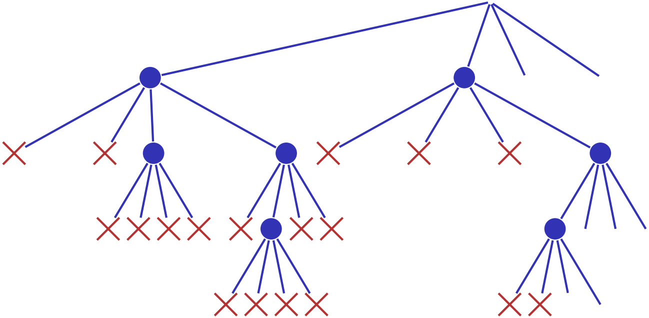


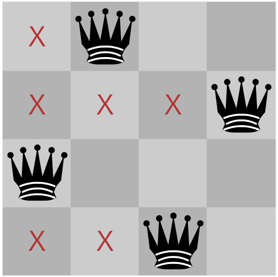


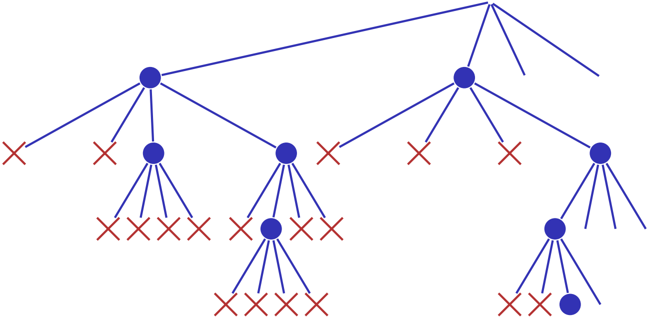
Search Strategy Visualized as a Tree





Search Strategy Visualized as a Tree





Check Queen

```
using Queens = std::vector<unsigned int>;

// post: returns if queen in the given row is valid, i.e.
//     does not share a common row, column or diagonal
//     with any of the queens on rows 0 to row-1
bool valid(const Queens& queens, unsigned int row) {
  unsigned int col = queens[row];
  for (unsigned int r = 0; r != row; ++r) {
    unsigned int c = queens[r];
    if (col == c || col - row == c - r || col + row == c + r)
      return false; // same column or diagonal
  }
  return true; // no shared column or diagonal
}
```

Recursion: Find a Solution

```
// pre: all queens from row 0 to row-1 are valid,
//      i.e. do not share any common row, column or diagonal
// post: returns if there is a valid position for queens on
//      row .. queens.size(). if true is returned then the
//      queens vector contains a valid configuration.
bool solve(Queens& queens, unsigned int row) {
  if (row == queens.size())
    return true;
  for (unsigned int col = 0; col != queens.size(); ++col) {
    queens[row] = col;
    if (valid(queens, row) && solve(queens,row+1))
       return true; // (else check next position)
  }
  return false; // no valid configuration found
}
```

Recursion: Count all Solutions

```
// pre: all queens from row 0 to row-1 are valid,
//  i.e. do not share any common row, column or diagonal
// post: returns the number of valid configurations of the
//  remaining queens on rows row ... queens.size()
int nSolutions(Queens& queens, unsigned int row) {
  if (row == queens.size())
    return 1;
  int count = 0;
  for (unsigned int col = 0; col != queens.size(); ++col) {
    queens[row] = col;
    if (valid(queens, row))
      count += nSolutions(queens,row+1);
  }
  return count;
}
```

Main Program

```
// pre: positions of the queens in vector queens
// post: output of the positions of the queens in a graphical way
void print(const Queens& queens);

int main() {
  int n;
  std::cin >> n;
  Queens queens(n);
  if (solve(queens,0)) {
    print(queens);
    std::cout << "# solutions:" << nSolutions(queens,0) << std::endl
  } else
    std::cout << "no solution" << std::endl;
  return 0;
}
```

17. Recursion 2

Building a Calculator, Formal Grammars, Extended Backus Naur Form (EBNF), Parsing Expressions

Motivation: Calculator

Input: `3 + 5`
Output: `8`

- binary Operators `+`, `-`, `*`, `/` and numbers

Motivation: Calculator

Input: `3 / 5`
Output: `0.6`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic

Motivation: Calculator

Input: `3 + 5 * 20`
Output: `103`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++

Motivation: Calculator

Input: `(3 + 5) * 20`
Output: `160`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses

Motivation: Calculator

Input: `-(3 + 5) + 20`
Output: `12`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses
- unary operator `-`

Naive Attempt (without Parentheses)

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != ’=’) {
    double rval;
    std::cin >> rval;

    if (op == ’+’)
        lval += rval;
    else if (op == ’*’)
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

Seems to work...

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != '=') {
    double rval;
    std::cin >> rval;

    if (op == '+')
        lval += rval;
    else if (op == '*')
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 1 * 2 * 3 * 4 =
Result 24
```

Oops, Multiplication first…

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != ’=’) {
    double rval;
    std::cin >> rval;

    if (op == ’+’)
        lval += rval;
    else if (op == ’*’)
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 2 + 3 * 3 =
Result 15
```

Analyzing the Problem

Input:

$$13 + \ldots$$

Analyzing the Problem

Input:

$$13 + 4 * \ldots$$

Analyzing the Problem

Input:

$$13 + 4 * (15 - \ldots$$

Analyzing the Problem

Input:

$$13 + 4 * (15 - 7 * \ldots$$

Analyzing the Problem

Input:

$$\underbrace{13 + 4 * (15 - 7 *}_{\text{Needs to be stored such that evaluation can be performed}} 3) =$$

Analyzing the Problem

Result:

$$13 + 4*(15 - 21)$$

Analyzing the Problem

Result:

$$13+4*(-6)$$

Analyzing the Problem

Result:

$$13 + (-24)$$

Analyzing the Problem

Result:

$$-11$$

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture is

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture is pretty

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture is pretty much

Analyzing the Problem

Expression:

$$13 + 4 * (15 - 7 * 3)$$

This lecture is pretty much recursive.

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

We will store symbols elegantly using recursion.

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

We will store symbols elegantly using recursion.

We need a new formal tool (that is independent of $\mathrm{C}{+}{+}$).

Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

A formal grammar defines which strings are valid.

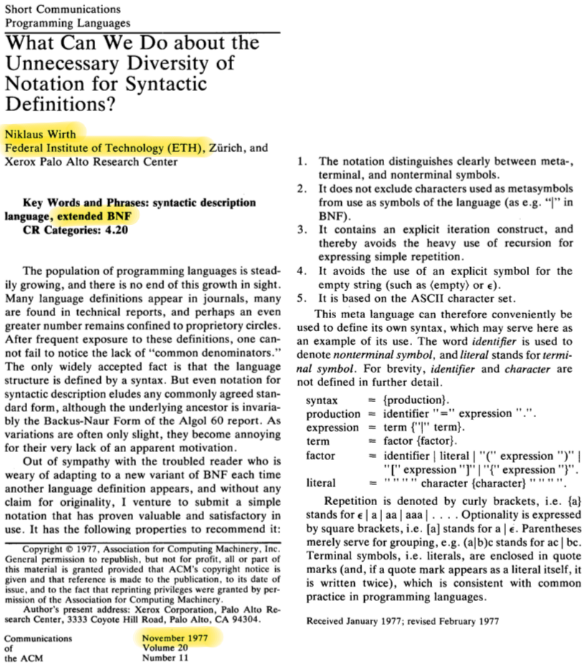
Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

A formal grammar defines which strings are valid.

To describe the formal grammar, we use:

Extended Backus Naur Form (EBNF)



What Can We Do about the Unnecessary Diversity of Notation for Syntactic Definitions?

Niklaus Wirth
Federal Institute of Technology (ETH), Zürich, and
Xerox Palo Alto Research Center

Key Words and Phrases: syntactic description language, extended BNF
CR Categories: 4.20

The population of programming languages is steadily growing, and there is no end of this growth in sight. Many language definitions appear in journals, many are found in technical reports, and perhaps an even greater number remains confined to proprietory circles. After frequent exposure to these definitions, one cannot fail to notice the lack of "common denominators." The only widely accepted fact is that the language structure is defined by a syntax. But even notation for syntactic description eludes any commonly agreed standard form, although the underlying ancestor is invariably the Backus-Naur Form of the Algol 60 report. As variations are often only slight, they become annoying for their very lack of an apparent motivation.

Out of sympathy with the troubled reader who is weary of adapting to a new variant of BNF each time another language definition appears, and without any claim for originality, I venture to submit a simple notation that has proven valuable and satisfactory in use. It has the following properties to recommend it:

1. The notation distinguishes clearly between meta-, terminal, and nonterminal symbols.
2. It does not exclude characters used as metasymbols from use as symbols of the language (as e.g. "|" in BNF).
3. It contains an explicit iteration construct, and thereby avoids the heavy use of recursion for expressing simple repetition.
4. It avoids the use of an explicit symbol for the empty string (such as ⟨empty⟩ or ϵ).
5. It is based on the ASCII character set.

This meta language can therefore conveniently be used to define its own syntax, which may serve here as an example of its use. The word *identifier* is used to denote *nonterminal symbol*, and *literal* stands for *terminal symbol*. For brevity, *identifier* and *character* are not defined in further detail.

```
syntax      = {production}.
production  = identifier "=" expression ".".
expression  = term {"|" term}.
term        = factor {factor}.
factor      = identifier | literal | "(" expression ")" |
              "[" expression "]" | "{" expression "}".
literal     = """" character {character} """".
```

Repetition is denoted by curly brackets, i.e. {a} stands for ϵ | a | aa | aaa | . . . . Optionality is expressed by square brackets, i.e. [a] stands for a | ϵ. Parentheses merely serve for grouping, e.g. (a|b)c stands for ac | bc. Terminal symbols, i.e. literals, are enclosed in quote marks (and, if a quote mark appears as a literal itself, it is written twice), which is consistent with common practice in programming languages.

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Author's present address: Xerox Corporation, Palo Alto Research Center, 3333 Coyote Hill Road, Palo Alto, CA 94304.

Number

An integer is a sequence of digits. A sequence of digits ist

Number

An integer is a sequence of digits. A sequence of digits ist

- a digit   2

Number

An integer is a sequence of digits. A sequence of digits ist

- a digit or 2
- a digit followed by a sequence of digits 2 0 1 9

Number

An integer is a sequence of digits. A sequence of digits ist

- a digit or

2

- a digit followed by a sequence of digits

2	0	1	9

```
unsigned_integer = digits .
digit   = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' .
digits  = digit  | digit  digits.
```

Number

An integer is a sequence of digits. A sequence of digits ist

- a digit or 2
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```
unsigned_integer = digits .
digit    = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' .
digits   = digit  | digit  digits .
```

alternative

terminal symbo

non-terminal symbol

Number (non-recursive)

An integer is a sequence of digits. A sequence of digits ist

- a digit, or 2
- a digit followed by an **arbitrary number of digits** 2 0 1 9

```
unsigned_integer = digits .
digit   = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' .
digits  = digit { digit }.
```

Number (non-recursive)

An integer is a sequence of digits. A sequence of digits ist

- a digit, or 2
- a digit followed by an **arbitrary number of digits** 2 0 1 9

```
unsigned_integer = digits .
digit  = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' .
digits = digit { digit } .
```

optional repetition

Expressions

```
-(3-(4-5))*(3+4*5)/6
```

What do we need in a grammar?

Expressions

```
-(3-(4-5))*(3+4*5)/6
```

What do we need in a grammar?

- Number

Expressions

$$-\underline{(}3-\underline{(}4-5\underline{)}\underline{)}*\underline{(}3+4*5\underline{)}/6$$

What do we need in a grammar?

- Number , (?)

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)

  -Number, -(?)

Expressions

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- ? * ?,   ? / ?,   ...

Expressions

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- ? * ?, ? / ?, ...
- ? - ?, ? + ?, ...

Expressions Multiplication/Division

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
-Number, -(?)
- ? * ?, ? / ?, ...
- ? - ?, ? + ?, ...

Factor

Expressions Multiplication/Division

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor,
  Factor / Factor , ...
- ? - ?, ? + ?, ...

Factor

Expressions Addition/Subtraction

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor,
  Factor / Factor , ...
- ? - ?, ? + ?, ...

Factor

Term

Expressions Addition/Subtraction

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- ? - ?, ? + ?, ...

Factor

Term

Expressions

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term,
  Term - Term, ...

Factor

Term

Expressions

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term,
  Term - Term, ...

Factor

Term

Expression

Expressions

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term - Term, ...

Factor

Term

Expression

Expressions

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (Expression)
  -Number, -(Expression)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term - Term, ...

Factor

Term

Expression

The EBNF for Expressions

A factor is

- a number,

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses

The EBNF for Expressions

A factor is

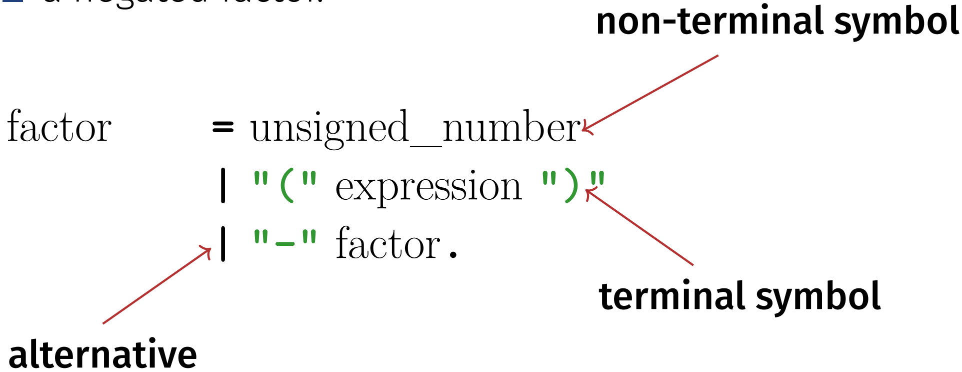
- a number,
- an expression in parentheses or
- a negated factor.

```
factor    = unsigned_number
          | "(" expression ")"
          | "-" factor.
```

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses or
- a negated factor.



non-terminal symbol

```
factor        = unsigned_number
              | "(" expression ")"
              | "-" factor .
```

terminal symbol

alternative

The EBNF for Expressions

```
factor    = unsigned_number
          | "(" expression ")"
          | "-" factor.
```

Implication: a factor starts with

- a digit, or
- with “(” , or
- with "-"”.

The EBNF for Expressions

A term is

- factor,

The EBNF for Expressions

A term is

- factor,
- factor * factor, factor / factor,

The EBNF for Expressions

A term is

- factor,
- factor * factor, factor / factor,
- factor * factor * factor, factor / factor * factor, ...
- ...

The EBNF for Expressions

A term is

- factor,
- factor ∗ factor, factor / factor,
- factor ∗ factor ∗ factor, factor / factor ∗ factor, ...
- ...

term = factor { "*" factor | "/" factor }.

The EBNF for Expressions

A term is

- factor,
- factor ∗ factor, factor / factor,
- factor ∗ factor ∗ factor, factor / factor ∗ factor, ...
- ...

term = factor { "*" factor | "/" factor }.

optional repetition

The EBNF for Expressions

```
factor     = unsigned_number
             | "(" expression ")"
             | "-" factor.

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term |"-" term }.
```

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.
- **Parser:** A program for parsing.

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.
- **Parser:** A program for parsing.
- **Useful:** From the EBNF we can automatically generate a parser:
    - Rules become functions
    - Alternatives and options become **`if`**–statements.
    - Nonterminial symbols on the right hand side become function calls
    - Optional repetitions become **`while`**–statements

Rules

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.



expression = term { "+" term | "-" term }.
```

Functions (Parser)

Expression is read from an input stream.

```
// POST: returns true if and only if in_stream = factor ...
//       and in this case extracts factor from in_stream
bool factor (std::istream& in_stream);

// POST: returns true if and only if in_stream = term ...,
//       and in this case extracts all factors from in_stream
bool term (std::istream& in_stream);

// POST: returns true if and only if in_stream = expression ...,
//       and in this case extracts all terms from in_stream
bool expression (std::istream& in_stream);
```

Functions (Parser with Evaluation)

Expression is read from an input stream.

```
// POST: extracts a factor from in_stream
//       and returns its value
double factor (std::istream& in_stream);

// POST: extracts a term from in_stream
//       and returns its value
double term (std::istream& in_stream);

// POST: extracts an expression from in_stream
//       and returns its value
double expression (std::istream& in_stream);
```

One Character Lookahead. . .

. . . to find the right alternative.

```
// POST: the next character at the stream is returned
//      without being consumed. returns 0 if stream ends.
char peek (std::istream& input){
  if (input.eof()) return 0; // end of stream
  return input.peek();  // next character in input
}

// POST: leading whitespace characters are extracted from input
//      and the first non-whitespace character on input returned
char lookahead (std::istream& input) {
  input >> std::ws;      // skip whitespaces
  return peek(input);
}
```

Parse numbers

```
bool isDigit(char ch){
  return ch >= '0' && ch <= '9';
}
// POST: returns an unsigned integer consumed from the stream
// number = digit {digit}.
unsigned int unsigned_number (std::istream& input){
  char ch = lookahead(input);
  assert(isDigit(ch));
  unsigned int num = 0;
  while(isDigit(ch) && input >> ch){ // read remaining digits
    num = num * 10 + ch - '0';
    ch = peek(input
  }
  return num;
}
```

```
unsigned_number =digit { digit }.
digit = '0'|'1'|'2'|'3'|'4'|'5'|'6'|'7'|'8
```

Cherry-Picking

…to extract the desired character.

```
// POST: if expected matches the next lookahead then consume it
//      and return true; return false otherwise
bool consume (std::istream& in_stream, char expected)
{
    if (lookahead(in_stream) == expected){
        in_stream >> expected; // consume one character
        return true;
    }
    return false;
}
```

Evaluating Factors

```
double factor (std::istream& in_stream)
{
  double value;
  if (consume(in_stream, ’(’)) {
    value = expression (in_stream);
    consume(in_stream, ’)’);
  } else if (consume(in_stream, ’-’)) {
    value = -factor (in_stream);
  } else {
    value = unsigned_number(in_stre
  }
  return value;
}
```

```
factor = "(" expression ")"
       | "-" factor
       | unsigned_number.
```

Evaluating Terms

```
double term (std::istream& in_stream)
{
  double value = factor (in_stream);
  while(true){
    if (consume(in_stream, ’*’))
      value *= factor(in_stream);
    else if (consume(in_stream, ’/’))
      value /= factor(in_stream)
    else
      return value;
  }
}
```

term = factor { "*" factor | "/" factor }.

Evaluating Expressions

```
double expression (std::istream& in_stream)
{
  double value = term(in_stream);
  while(true){
    if (consume(in_stream, ’+’))
      value += term (in_stream);
    else if (consume(in_stream, ’-’))
      value -= term(in_stream)
    else
      return value;
  }
}
```

expression = term { "+" term | "−" term }.

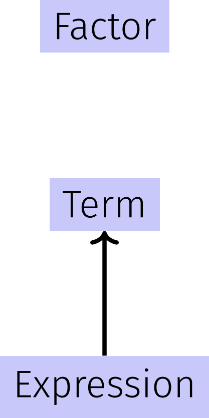
Recursion!

Factor

Term

Expression

Recursion!

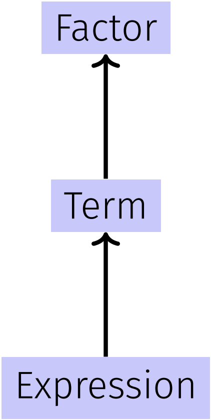


Factor

Term

Expression

Recursion!

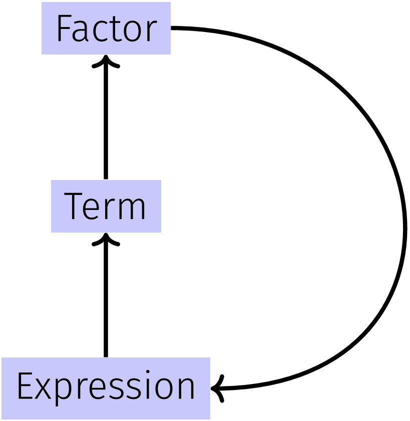


Factor

Term

Expression

Recursion!



EBNF — and it works!

EBNF (calculator.cpp, Evaluation from left to right):

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term |"-" term }.
```

```
std::stringstream input ("1-2-3");
std::cout << expression (input) << "\n"; // -4
```

18. Structs

Rational Numbers, Struct Definition

Calculating with Rational Numbers

- Rational numbers ($\mathbb{Q}$) are of the form $\frac{n}{d}$ with n and d in $\mathbb{Z}$
- $\mathrm{C{+}{+}}$does not provide a built-in type for rational numbers

Calculating with Rational Numbers

- Rational numbers ($\mathbb{Q}$) are of the form $\frac{n}{d}$ with n and d in $\mathbb{Z}$
- C++does not provide a built-in type for rational numbers

Goal

We build a C++-type for rational numbers ourselves! 🙂

Vision

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;
std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

A First Struct

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

A First Struct

member variable (**n**umerator)

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

member variable (**d**enominator)

A First Struct

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

member variable

member variable

- **`struct`** defines a new **type**

A First Struct

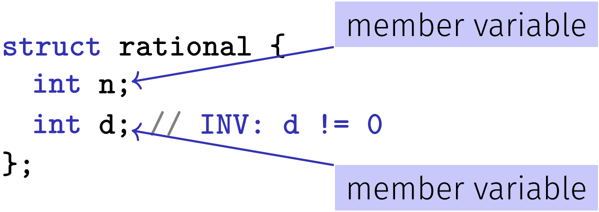
member variable

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

member variable

- **`struct`** defines a new **type**
- formal range of values: *cartesian product* of the value ranges of existing types

A First Struct



```
struct rational {
  int n;
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};
```

member variable

member variable

- **`struct`** defines a new **type**
- formal range of values: *cartesian product* of the value ranges of existing types
- real range of values: **`rational`** $\subsetneq$ **`int`** $\times$ **`int`**.

Accessing Member Variables

```
struct rational {
    int n;
    int d; // INV: d != 0
};

rational add (rational a, rational b){
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
```

$$\frac{r_n}{r_d} := \frac{a_n}{a_d} + \frac{b_n}{b_d} = \frac{a_n \cdot b_d + a_d \cdot b_n}{a_d \cdot b_d}$$

Input

```
// Input r
rational r;
std::cout << "Rational number r:\n";
std::cout << " numerator =? ";
std::cin >> r.n;
std::cout << " denominator =? ";
std::cin >> r.d;

// Input s the same way
rational s;
...
```

Vision comes within Reach ...

```
// computation
const rational t = add (r, s);

// output
std::cout << "Sum is " << t.n << "/" << t.d << ".\n";
```

Struct Defintions: Examples

```
struct rational_vector_3 {
  rational x;
  rational y;
  rational z;
};
```

underlying types can be fundamental or *user defined*

Struct Definitions: Examples

```
struct extended_int {
  // represents value if is_positive==true
  // and -value otherwise
  unsigned int value;
  bool is_positive;
};
```

the underlying types can be *different*

Structs: Initialization and Assignment

```
rational s;
```

← member variables are uninitialized

```
rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

member-wise initialization:
`t.n = 1, t.d = 5`

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;  ← member-wise copy

t = u;

rational v = add (u,t);
```

Structs: Initialization and Assignment

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rational s;

rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
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t = u; ← member-wise copy

Structs: Initialization and Assignment

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rational s;

rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

← member-wise copy

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

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For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

- member-wise comparison does not make sense in general...

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

- member-wise comparison does not make sense in general...
- ...otherwise we had, for example, $\frac{2}{3} \neq \frac{4}{6}$

User Defined Operators

Instead of

```
rational t = add(r, s);
```

we would rather like to write

```
rational t = r + s;
```

User Defined Operators

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```
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```

we would rather like to write

```
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```

This can be done with *Operator Overloading* (→ *next week*).

19. Classes

Overloading Functions and Operators, Encapsulation, Classes, Member Functions, Constructors

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }      // f1
int sq (int x) { ... }            // f2
int pow (int b, int e) { ... }    // f3
int pow (int e) { return pow (2,e); } // f4
```

Function Overloading

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```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);
```

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }       // f1
int sq (int x) { ... }             // f2
int pow (int b, int e) { ... }     // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);   // compiler chooses f2
std::cout << sq (1.414);
```

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }       // f1
int sq (int x) { ... }             // f2
int pow (int b, int e) { ... }     // f3
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```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);   // compiler chooses f2
std::cout << sq (1.414); // compiler chooses f1
std::cout << pow (2);
```

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }       // f1
int sq (int x) { ... }             // f2
int pow (int b, int e) { ... }     // f3
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- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);   // compiler chooses f2
std::cout << sq (1.414); // compiler chooses f1
std::cout << pow (2);  // compiler chooses f4
std::cout << pow (3,3);
```

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }      // f1
int sq (int x) { ... }            // f2
int pow (int b, int e) { ... }    // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);   // compiler chooses f2
std::cout << sq (1.414); // compiler chooses f1
std::cout << pow (2);  // compiler chooses f4
std::cout << pow (3,3); // compiler chooses f3
```

Operator Overloading

- Operators are special functions and can be overloaded
- Name of the operator *op*:

```
operatorop
```

Adding `rational` Numbers – Before

```
// POST: return value is the sum of a and b
rational add (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = add (r, s);
```

Adding `rational` Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = r + s;
```

Adding `rational` Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = r + s;
```

↑ infix notation

Adding `rational` Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = operator+ (r, s);
```

↑ equivalent but less handy: functional notation

Unary Minus

Only one argument:

```
// POST: return value is -a
rational operator- (rational a)
{
    a.n = -a.n;
    return a;
}
```

Comparison Operators

can be defined such that they do the right thing:

Comparison Operators

can be defined such that they do the right thing:

```
// POST: returns true iff a == b
bool operator== (rational a, rational b)
{
    return a.n * b.d == a.d * b.n;
}
```

Comparison Operators

can be defined such that they do the right thing:

```
// POST: returns true iff a == b
bool operator== (rational a, rational b)
{
    return a.n * b.d == a.d * b.n;
}
```

$$\frac{2}{3} = \frac{4}{6} \qquad \checkmark$$

Arithmetic Assignment

We want to write

```
rational r;
r.n = 1; r.d = 2;                      // 1/2

rational s;
s.n = 1; s.d = 3;                      // 1/3

r += s;
std::cout << r.n << "/" << r.d;        // 5/6
```

Operator +=

```
rational& operator+= (rational& a, rational b)
{
    a.n = a.n * b.d + a.d * b.n;
    a.d *= b.d;
    return a;
}
```

Operator +=

```
rational& operator+= (rational& a, rational b)
{
    a.n = a.n * b.d + a.d * b.n;
    a.d *= b.d;
    return a;
}
```

- The L-value **`a`** is increased by the value of **`b`** and returned as L-value

In/Output Operators

can also be overloaded.

- Before:

```
std::cout << "Sum is " << t.n << "/" << t.d << "\n";
```

- After (desired):

```
std::cout << "Sum is " << t << "\n";
```

In/Output Operators

can be overloaded as well:

```
// POST: r has been written to out
std::ostream& operator<< (std::ostream& out, rational r)
{
    return out << r.n << "/" << r.d;
}
```

In/Output Operators

can be overloaded as well:

```
// POST: r has been written to out
std::ostream& operator<< (std::ostream& out, rational r)
{
    return out << r.n << "/" << r.d;
}
```

writes **r** to the output stream
and returns the stream as L-value.

Input

```
// PRE: in starts with a rational number of the form "n/d"
// POST: r has been read from in
std::istream& operator>> (std::istream& in, rational& r){
    char c; // separating character ’/’
    return in >> r.n >> c >> r.d;
}
```

reads **r** from the input stream
and returns the stream as L-value.

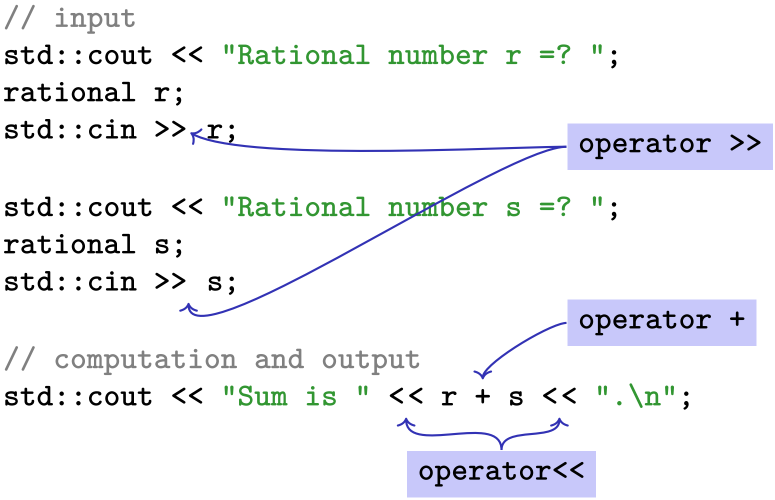
Goal Attained!

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;

std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

Goal Attained!



```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;

std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

operator >>

operator +

operator<<

A new Type with Functionality. . .

```
struct rational {
    int n;
    int d; // INV: d != 0
};

// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
```

. . .should be in a Library!

rational.h

- Definition of a struct **`rational`**
- Function declarations

rational.cpp

- arithmetic operators (**`operator+`**, **`operator+=`**, ...)
- relational operators (**`operator==`**, **`operator>`**, ...)
- in/output (**`operator >>`**, **`operator <<`**, ...)

Thought Experiment

The three core missions of ETH:

- research

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We found a startup: RAT PACK®!

Thought Experiment

The three core missions of ETH:

- research
- education
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We found a startup: RAT PACK®!

- Selling the `rational` library to customers
- ongoing development according to customer's demands

The Customer is Happy

> *"Buying RAT PACK® has been a game-changing move to put us on the forefront of cutting-edge technology in social media engineering."*
>
> B. Labla, CEO

The Customer is Happy

. . . and programs busily using `rational`.

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…and programs busily using `rational`.

- output as `double`-value $(\frac{3}{5} \rightarrow 0.6)$

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…and programs busily using `rational`.

- output as `double`-value $(\frac{3}{5} \rightarrow 0.6)$

```
// POST: double approximation of r
double to_double (rational r)
{
  double result = r.n;
  return result / r.d;
}
```

The Customer Wants More

“Can we have rational numbers with an extended value range?”

The Customer Wants More

“Can we have rational numbers with an extended value range?”

- Sure, no problem, e.g.:

```
~~struct rational {
  int n;
  int d;
};~~
```

⇒

```
struct rational {
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

New Version of RAT PACK®

It sucks, nothing works any more!

New Version of RAT PACK®

It sucks, nothing works any more!

- What is the problem?

New Version of RAT PACK®

- **It sucks, nothing works any more!**
  - What is the problem?
- **$-\frac{3}{5}$ is sometimes 0.6, this cannot be true!**

New Version of RAT PACK®

It sucks, nothing works any more!

- What is the problem?

$-\frac{3}{5}$ is sometimes 0.6, this cannot be true!

- That is your fault. Your conversion to `double` is the problem, our library is correct.

New Version of RAT PACK®

- **It sucks, nothing works any more!**
  - What is the problem?
- $-\frac{3}{5}$ **is sometimes** 0.6**, this cannot be true!**
  - That is your fault. Your conversion to `double` is the problem, our library is correct.
- **Up to now it worked, therefore the new version is to blame!**

Liability Discussion

```
// POST: double approximation of r
double to_double (rational r){
  double result = r.n;
  return result / r.d;
}
```

Liability Discussion

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struct rational {
  int n;
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…not correct using

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struct rational {
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

Liability Discussion

```
// POST: double approximation of r
double to_double (rational r){
  double result = r.n;
  return result / r.d;
}
```

`r.is_positive` and `result.is_positive` do not appear.

correct using…

```
struct rational {
  int n;
  int d;
};
```

…not correct using

```
struct rational {
  unsigned int n;
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We are to Blame!!

- Customer sees and uses our **representation** of rational numbers (initially `r.n, r.d`)

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⇒ RAT PACK® is history...

Idea of Encapsulation (Information Hiding)

- A type is uniquely defined by its *value range* and its *functionality*

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Idea of Encapsulation (Information Hiding)

- A type is uniquely defined by its *value range* and its *functionality*
- The *representation* should *not be visible*.
- ⇒ The customer is not provided with *representation* but with **functionality**!

```
str.length(),
v.push_back(1),...
```

Classes

- provide the concept for **encapsulation** in C++

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- are a variant of structs

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- provide the concept for **encapsulation** in C++
- are a variant of structs
- are provided in many *object oriented programming languages*

Encapsulation: public / private

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

is used instead of **struct** if anything at all shall be "hidden"

Encapsulation: `public` / `private`

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

is used instead of **`struct`** if anything at all shall be “hidden”

only difference

- **`struct`**: by default **nothing** is hidden
- **`class`** : by default **everything** is hidden

Encapsulation: public / private

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

Application Code

```
rational r;
r.n = 1;      // error: n is private
r.d = 2;      // error: d is private
int i = r.n; // error: n is private
```

Encapsulation: `public` / `private`

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class rational {
  int n;
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Good news: **`r.d = 0`** cannot happen any more by accident.

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class rational {
  int n;
  int d; // INV: d != 0
};
```

Good news: `r.d = 0` cannot happen any more by accident.

Bad news: the customer cannot do anything any more …

…and we can't, either. (no `operator+`,…)

Application Code

```
rational r;
r.n = 1;      // error: n is private
r.d = 2;      // error: d is private
int i = r.n; // error: n is private
```

Member Functions: Declaration

```
class rational {
public:
   // POST: return value is the numerator of this instance
   int numerator () const {
     return n;
   }
   // POST: return value is the denominator of this instance
   int denominator () const {
     return d;
   }
private:
   int n;
   int d; // INV: d!= 0
};
```

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public area

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public area

member function

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    int denominator () const {
      return d;
    }
private:
    int n;
    int d; // INV: d!= 0
};
```

public area

member function

member functions have access to private data

Member Functions: Call

```
// Definition des Typs
class rational {
    ...
};
...
// Variable des Typs
rational r;

int n = r.numerator();    // Zaehler
int d = r.denominator(); // Nenner
```

member access

Member Functions: Definition

```
// POST: returns numerator of this instance
int numerator () const
{
  return n;
}
```

Member Functions: Definition ???

```
// POST: returns numerator of this instance
int numerator ()  const
{
  return n;
}
```

Member Functions: Definition

```
// POST: returns numerator of this instance
int numerator () const
{
  return n;                              r.numerator()
}
```

- A member function is called for an expression of the class.

Member Functions: Definition

```
// POST: returns numerator of this instance
int numerator () const
{
   return n;
}
```

```
r.numerator()
```

- A member function is called for an expression of the class. in the function, **this** is the name of this *implicit argument*.

Member Functions: Definition

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- A member function is called for an expression of the class. in the function, **`this`** is the name of this *implicit argument.*
- *const* refers to the instance **`this`**

Member Functions: Definition

```
// POST: returns  numerator of this instance
int numerator () const
{
  return n;
}
```

`r.numerator()`

- A member function is called for an expression of the class. in the function, **`this`** is the name of this *implicit argument*.
- *const* refers to the instance **`this`**
- **`n`** is the shortcut for **`this->n`** (precise explanation of "**`->`**" next week)

`const` and Member Functions

```
class rational {
public:
  int numerator () const
  { return n; }
  void set_numerator (int N)
  { n = N;}
...
}
```

```
rational x;
x.set_numerator(10); // ok;
const rational y = x;
int n = y.numerator(); // ok;
y.set_numerator(10); // error;
```

The **`const`** at a member function is to promise that an instance cannot be changed via this function.

`const` items can only call **`const`** member functions.

Comparison

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return n;
    }
};

rational r;
...
std::cout << r.numerator();
```

Comparison

```
class rational {
   int n;
   ...
public:
   int numerator () const
   {
       return this->n;
   }
};

rational r;
...
std::cout << r.numerator();
```

Comparison

Roughly like this it were ...

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return this->n;
    }
};

rational r;
...
std::cout << r.numerator();
```

Comparison

Roughly like this it were ...

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return this->n;
    }
};

rational r;
...
std::cout << r.numerator();
```

... without member functions

```
struct bruch {
    int n;
    ...
};

int numerator (const bruch& dieser)
{
    return dieser.n;
}

bruch r;
..
std::cout << numerator(r);
```

Member-Definition: In-Class

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return n;
    }
    ....
};
```

- No separation between declaration and definition (bad for libraries)

Member-Definition: In-Class vs. Out-of-Class

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return n;
    }
    ....
};
```

- No separation between declaration and definition (bad for libraries)

```
class rational {
    int n;
    ...
public:
    int numerator () const;
    ...
};

int rational::numerator () const
{
  return n;
}
```

- This also works.

Initialisation? Constructors!

```
class rational
{
public:
    rational (int num, int den)
        : n (num), d (den)
    {
        assert (den != 0);
    }
...
};
...
rational r (2,3); // r = 2/3
```

Initialisation? Constructors!

```
class rational
{
public:
    rational (int num, int den)
        : n (num), d (den)
    {
        assert (den != 0);
    }
...
};
...
rational r (2,3); // r = 2/3
```

← Initialization of the member variables

← function body.

Initialisation “rational = int”?

```
class rational
{
public:
    rational (int num)
      : n (num), d (1)
    {}
...
};
...
rational r (2); // explicit initialization with 2
rational s = 2; // implicit conversion
```

Initialisation “rational = int”?

```
class rational
{
public:
    rational (int num)
       : n (num), d (1)
    {}   ⟵ empty function body
...
};
...
rational r (2); // explicit initialization with 2
rational s = 2; // implicit conversion
```

The Default Constructor

```
class rational
{
public:
    ...
    rational ()
       : n (0), d (1)
    {}
...
};
...
rational r;   // r = 0
```

empty list of arguments

The Default Constructor

```
class rational
{
public:
    ...
    rational ()
      : n (0), d (1)
    {}
...
};
...
rational r;   // r = 0
```

empty list of arguments

⇒ There are no uninitiatlized variables of type rational any more!

Alterantively: Deleting a Default Constructor

```
class rational
{
public:
    ...
    rational () = delete;
...
};
...
rational r;   // error:  use of deleted function 'rational::rational
```

⇒ There are no uninitiatlized variables of type rational any more!

User Defined Conversions

are defined via constructors with exactly *one* argument

```
rational (int num)
    : n (num), d (1)
  {}

rational r = 2; // implizite Konversion
```

User Defined Conversions

are defined via constructors with exactly *one* argument

```
rational (int num) ←
    : n (num), d (1)
  {}
```

User defined conversion from **int** to **rational**. values of type **int** can now be converted to **rational**.

```
rational r = 2; // implizite Konversion
```

RAT PACK® Reloaded ...

Customer's program now looks like this:

```
// POST: double approximation of r
double to_double (const rational r)
{
  double result = r.numerator();
  return result / r.denominator();
}
```

RAT PACK® Reloaded ...

Customer’s program now looks like this:

```
// POST: double approximation of r
double to_double (const rational r)
{
  double result = r.numerator();
  return result / r.denominator();
}
```

- We can adapt the member functions together with the representation ✓

RAT PACK® Reloaded ...

before

```
class rational {
...
private:
  int n;
  int d;
};
```

RAT PACK® Reloaded ...

before

```
class rational {
...
private:
  int n;
  int d;
};
```

```
int numerator () const
{
  return n;
}
```

RAT PACK® Reloaded ...

before

```
class rational {
...
private:
  int n;
  int d;
};
```

```
int numerator () const
{
  return n;
}
```

after

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

RAT PACK® Reloaded . . .

before

```
class rational {
...
private:
  int n;
  int d;
};
```

```
int numerator () const
{
  return n;
}
```

after

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

RAT PACK® Reloaded ?

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const
{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

RAT PACK® Reloaded ?

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const
{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

- value range of nominator and denominator like before

RAT PACK® Reloaded ?

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const
{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

- value range of nominator and denominator like before
- possible overflow in addition

Encapsulation still Incompleete

Customer’s point of view (`rational.h`):

```
class rational {
public:
   // POST: returns numerator of *this
   int numerator () const;
   ...
private:
  // none of my business
};
```

Encapsulation still Incompleete

Customer’s point of view (`rational.h`):

```
class rational {
public:
   // POST: returns numerator of *this
   int numerator () const;
   ...
private:
  // none of my business
};
```

- We determined denominator and nominator type to be **`int`**

Encapsulation still Incompleete

Customer’s point of view (`rational.h`):

```
class rational {
public:
   // POST: returns numerator of *this
   int numerator () const;
   ...
private:
  // none of my business
};
```

- We determined denominator and nominator type to be **`int`**
- Solution: encapsulate not only data but alsoe types.

Fix: “our” type rational::integer

Customer’s point of view (rational.h):

```
public:
   using integer = long int; // might change
   // POST: returns numerator of *this
   integer numerator () const;
```

Fix: “our” type rational::integer

Customer’s point of view (rational.h):

```
public:
   using integer = long int; // might change
   // POST: returns numerator of *this
   integer numerator () const;
```

- We provide an additional type!

Fix: “our” type rational::integer

Customer’s point of view (rational.h):

```
public:
   using integer = long int; // might change
   // POST: returns numerator of *this
   integer numerator () const;
```

- We provide an additional type!
- Determine only **Functionality**, e.g:
  - implicit conversion int → rational::integer

Fix: “our” type rational::integer

Customer’s point of view (rational.h):

```
public:
    using integer = long int; // might change
    // POST: returns numerator of *this
    integer numerator () const;
```

- We provide an additional type!
- Determine only **Functionality**, e.g:
  - implicit conversion int → rational::integer
  - function double to_double (rational::integer)

RAT PACK® Revolutions

Finally, a customer program that remains stable

```
// POST: double approximation of r
double to_double (const rational r)
{
  rational::integer n = r.numerator();
  rational::integer d = r.denominator();
  return to_double (n) / to_double (d);
}
```

20. Dynamic Data Structures I

Dynamic Memory, Addresses and Pointers, Const-Pointer Arrays, Array-based Vectors

Recap: `vector`<T>

- Can be initialised with arbitrary size **`n`**

Recap: vector<T>

- Can be initialised with arbitrary size **n**
- Supports various operations:

```
e = v[i];            // Get element
v[i] = e;            // Set element
l = v.size();        // Get size
v.push_front(e);     // Prepend element
v.push_back(e);      // Append element
...
```

Recap: vector<T>

- Can be initialised with arbitrary size **n**
- Supports various operations:

```
e = v[i];           // Get element
v[i] = e;           // Set element
l = v.size();       // Get size
v.push_front(e);    // Prepend element
v.push_back(e);     // Append element
...
```

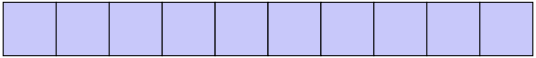
- A vector is a *dynamic data structure*, whose size may change at runtime

Our Own Vector!

- Today, we'll implement our own vector: **`vec`**
- Step 1: **`vec<int>`** (today)
- Step 2: **`vec<`**T**`>`** (later, only superficially)

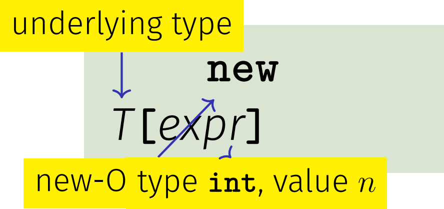
Vectors in Memory

Already known: A vector has a *contiguous* memory layout



Question: How to *allocate* a chunk of memory of *arbitrary* size during runtime, i.e. *dynamically*?

new for Arrays



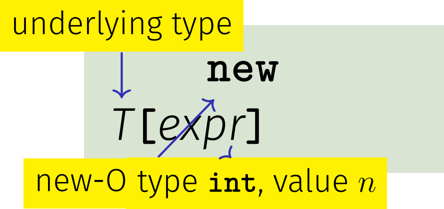
underlying type

new

T[*expr*]

new-O type **int**, value n

`new` for Arrays

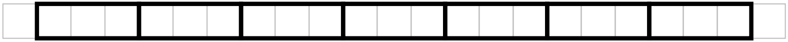


underlying type

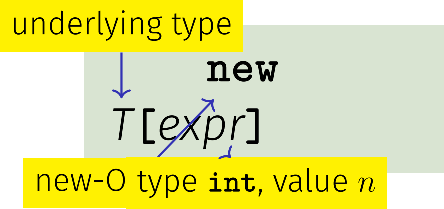
`new` *T*`[`*expr*`]`

new-O type **`int`**, value n

- **Effect**: new contiguous chunk of memory n elements of type *T* is allocated



`new` for Arrays

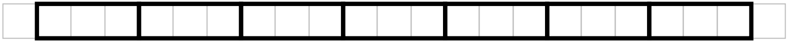


underlying type

`new` *T*[*expr*]

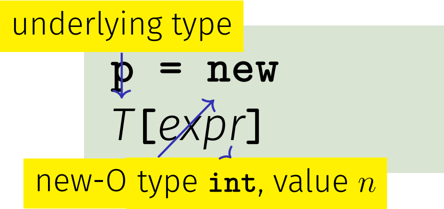
new-O type **`int`**, value n

- **Effect**: new contiguous chunk of memory n elements of type *T* is allocated



- This chunk of memory is called an *array* (of length n)

`new` for Arrays



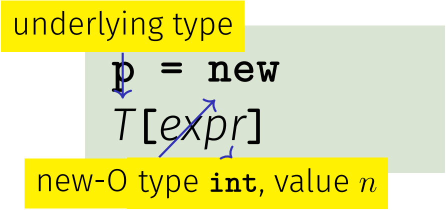
underlying type

`p = new`

T`[`*expr*`]`

new-O type **`int`**, value n

`new` for Arrays

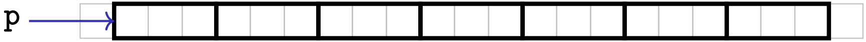


underlying type

```
p = new
T[expr]
```

new-O type **`int`**, value n

- **Value**: the starting address of the memory chunk



- **Type**: A pointer T**`*`** (more soon)

Outlook: `new` and `delete`

`new`

T`[`*expr*`]`

- So far: memory (local variables, function arguments) “lives” only inside a function call

Outlook: `new` and `delete`

`new` *T*`[`*expr*`]`

- So far: memory (local variables, function arguments) “lives” only inside a function call
- But now: memory chunk inside vector must not “die” before the vector itself

Outlook: `new` and `delete`

`new`

T`[`*expr*`]`

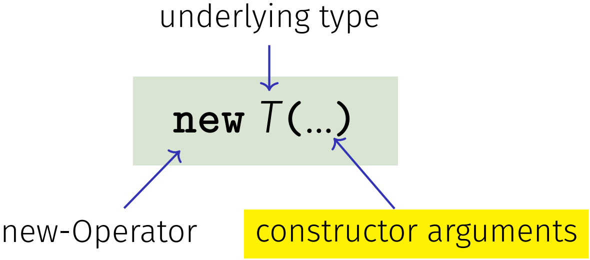
- So far: memory (local variables, function arguments) “lives” only inside a function call
- But now: memory chunk inside vector must not “die” before the vector itself
- Memory allocated with **`new`** is *not* automatically *deallocated* (= released)

Outlook: `new` and `delete`

```
new T[expr]
```

- So far: memory (local variables, function arguments) “lives” only inside a function call
- But now: memory chunk inside vector must not “die” before the vector itself
- Memory allocated with **`new`** is *not* automatically *deallocated* (= released)
- Every **`new`** must have a matching **`delete`** that releases the memory explicitly → **in two weeks**

new (Without Arrays)



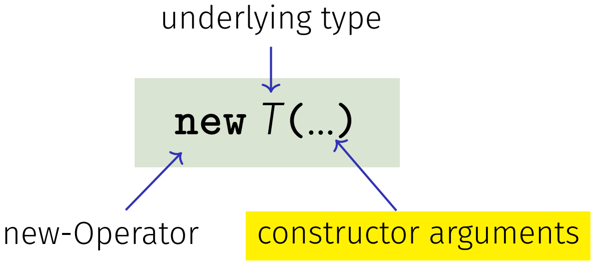
underlying type

new *T*(...)

new-Operator

constructor arguments

`new` (Without Arrays)



underlying type

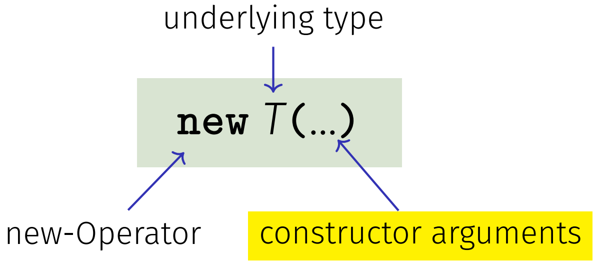
`new` T(...)

new-Operator

constructor arguments

- **Effect**: memory for a new object of type T is allocated ...

`new` (Without Arrays)



underlying type

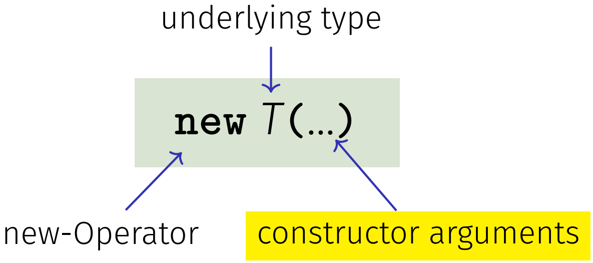
`new` *T*(...)

new-Operator

constructor arguments

- **Effect**: memory for a new object of type *T* is allocated ...
- ...and initialized by means of the matching constructor

`new` (Without Arrays)



underlying type

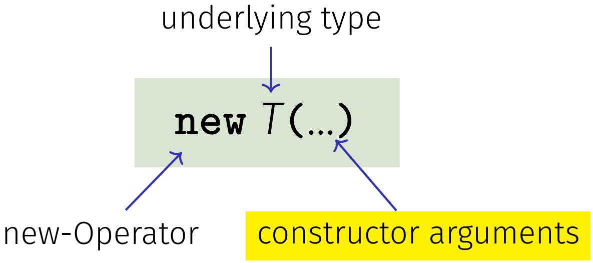
new T(...)

new-Operator

constructor arguments

- **Effect**: memory for a new object of type T is allocated ...
- ...and initialized by means of the matching constructor
- **Value**: address of the new T object, **Type**: Pointer T*****

`new` (Without Arrays)



underlying type

`new` T(...)

new-Operator

constructor arguments

- **Effect**: memory for a new object of type T is allocated …
- …and initialized by means of the matching constructor
- **Value**: address of the new T object, **Type**: Pointer T`*`
- Also true here: object "lives" until deleted explicitly (usefulness will become clearer later)

Pointer Types

`T*` Pointer type for base type **`T`**

An expression of type **`T*`** is called *pointer (to **`T`**)*

Pointer Types

`T*` Pointer type for base type `T`

An expression of type `T*` is called *pointer (to* `T`*)*

```
int* p; // Pointer to an int
std::string* q; // Pointer to a std::string
```

Pointer Types

`T*` Pointer type for base type **`T`**

A $T*$ must actually point to a T

```
int* p = ...;
std::string* q = p; // compiler error!
```

Pointer Types

Value of a pointer to **`T`** is the *address* of an object of type **`T`**

Pointer Types

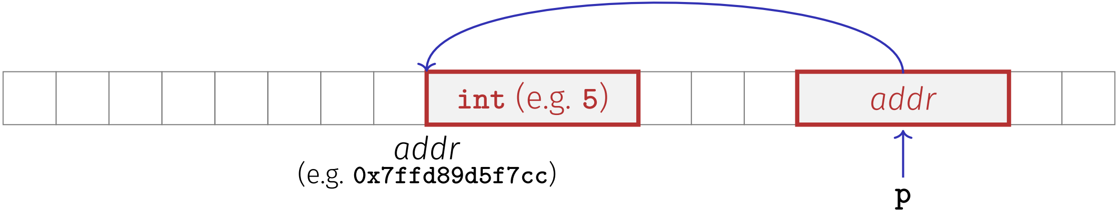
Value of a pointer to **`T`** is the *address* of an object of type **`T`**

```
int* p = ...;
std::cout << p; // e.g. 0x7ffd89d5f7cc
```

Pointer Types

Value of a pointer to `T` is the *address* of an object of type `T`

```
int* p = ...;
std::cout << p; // e.g. 0x7ffd89d5f7cc
```



Address Operator

Question: How to obtain an object's address?

1. Directly, when creating a new object via **`new`**

Address Operator

Question: How to obtain an object’s address?

1. Directly, when creating a new object via **`new`**

2. For existing objects: via the *address operator* **`&`**

&expr ← expr: l-value of type T

Address Operator

Question: How to obtain an object's address?

1. Directly, when creating a new object via **`new`**

2. For existing objects: via the *address operator* **`&`**

&*expr* ← expr: l-value of type T

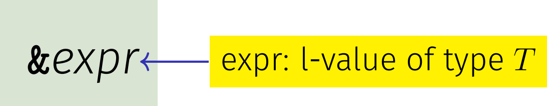
- **Value** of the expression: the *address* of object (l-value) *expr*

Address Operator

Question: How to obtain an object's address?

1. Directly, when creating a new object via **`new`**

2. For existing objects: via the *address operator* **`&`**



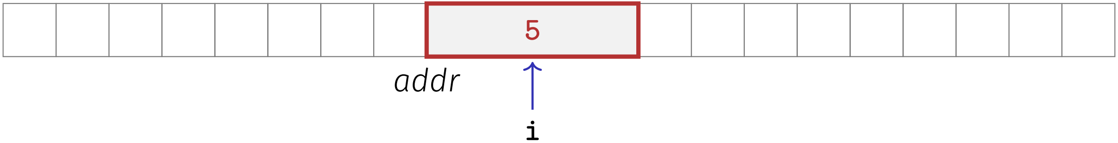
&expr

expr: l-value of type T

- **Value** of the expression: the *address* of object (l-value) *expr*
- **Type** of the expression: A pointer T**`*`** (of type T)

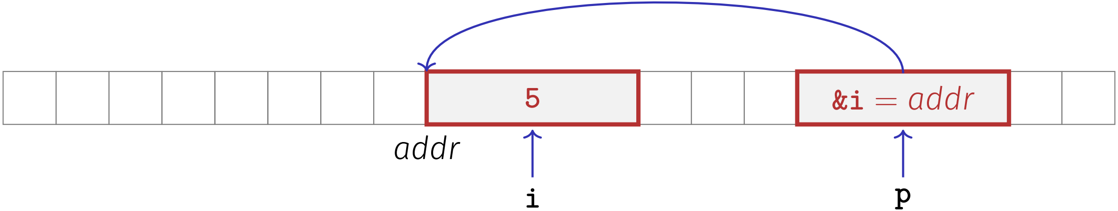
Address Operator

```
int i = 5; // i initialised with 5
```



Address Operator

```
int i = 5; // i initialised with 5
int* p = &i; // p initialised with address of i
```



Address Operator

```
int i = 5; // i initialised with 5
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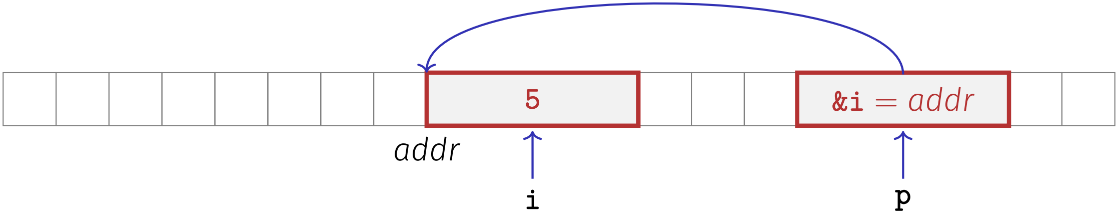
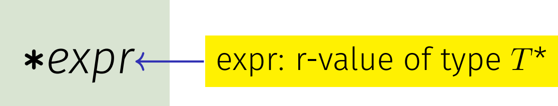


Next question: How to “follow” a pointer?

Dereference Operator

Answer: by using the *dereference operator* *****



**expr*

expr: r-value of type T^*

Dereference Operator

Answer: by using the *dereference operator* *****

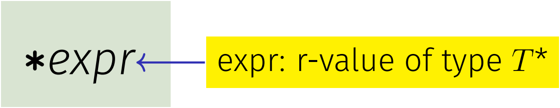
$*expr$

expr: r-value of type T*

- **Value** of the expression: the *value* of the object located at the address denoted by *expr*

Dereference Operator

Answer: by using the *dereference operator* $*$



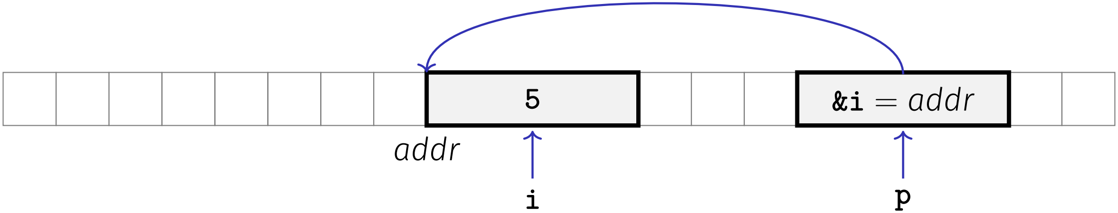
$*$*expr*

expr: r-value of type T*

- **Value** of the expression: the *value* of the object located at the address denoted by *expr*
- **Type** of the expression: T

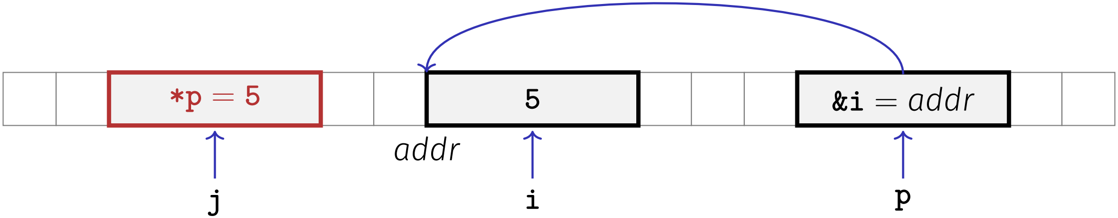
Dereference Operator

```
int i = 5;
int* p = &i; // p = address of i
```

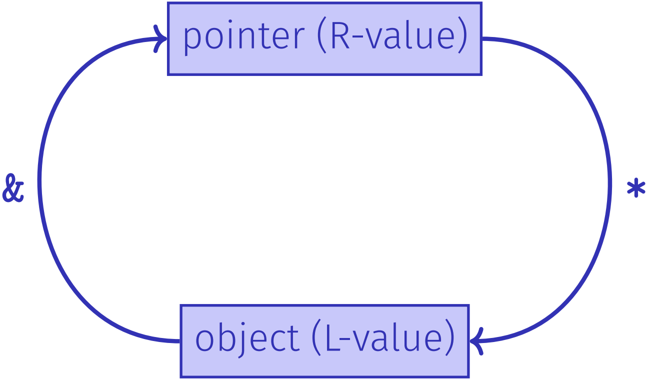


Dereference Operator

```
int i = 5;
int* p = &i; // p = address of i
int j = *p; // j = 5
```



Address and Dereference Operator



Mnenmonic Trick

The declaration

```
T* p;        // p is of the type “pointer to T”
```

Mnenmonic Trick

The declaration

```
T* p;       // p is of the type “pointer to T”
```

can be read as

```
T *p;       // *p is of type T
```

Null-Pointer

- Special pointer value that signals that no object is pointed to
- represented b the literal **`nullptr`** (convertible to **`T*`**)

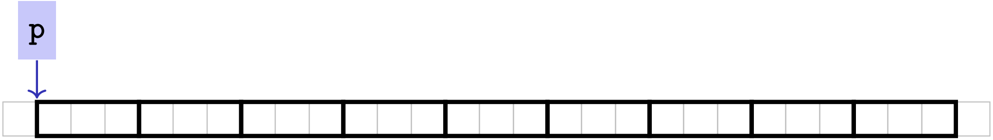
```
int* p = nullptr;
```

- Cannot be dereferenced (runtime error)
- Exists to avoid undefined behaviour

```
int* p; // Accessing p is undefined behaviour
int* q = nullptr; // q explicitly points nowhere
```

Pointer Arithmetic: Pointer plus `int`

```
T* p = new T[n]; // p points to first array element
```



Question: How to point to rear elements?

Pointer Arithmetic: Pointer plus `int`

```
T* p = new T[n]; // p points to first array element
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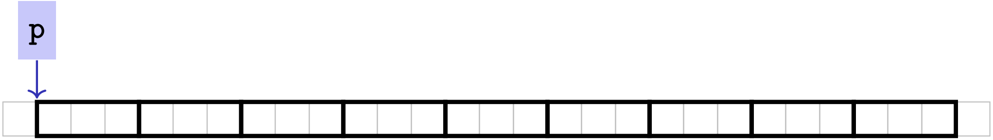

Question: How to point to rear elements? → via *Pointer arithmetic*:

Pointer Arithmetic: Pointer plus `int`

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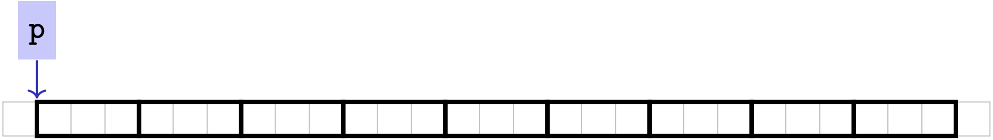

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- `p` yields the *value* of the *first* array element, `*p` its *value*

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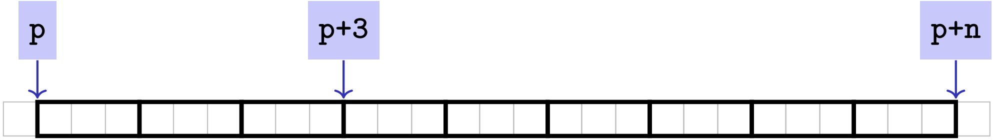

Question: How to point to rear elements? → via *Pointer arithmetic*:

- `p` yields the *value* of the *first* array element, `*p` its *value*
- `*(p + i)` yields the value of the `i`*th* array element, for $0 \leq$ `i` $<$ `n`

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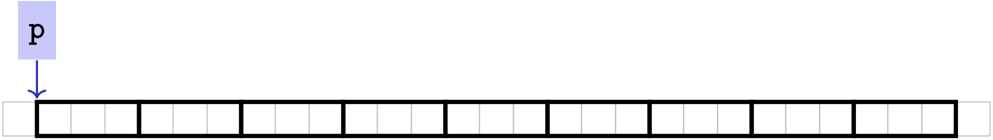

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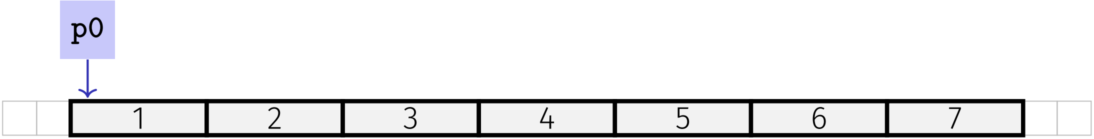
- `p` yields the *value* of the *first* array element, `*p` its *value*
- `*(p + i)` yields the value of the `i`*th* array element, for $0 \leq \texttt{i} < \texttt{n}$
- `*p` is equivalent to `*(p + 0)`

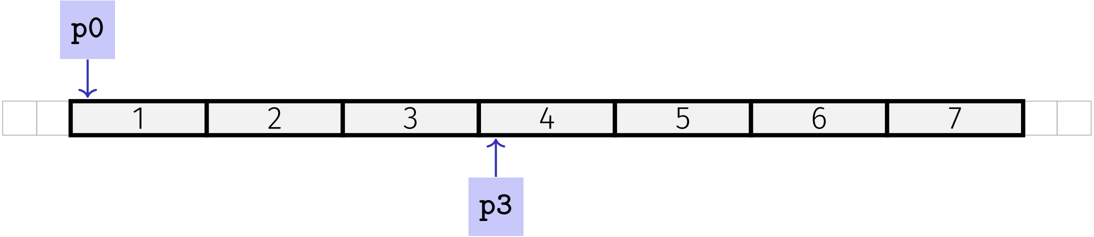
Pointer Arithmetic: Pointer plus `int`

```
int* p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
```



Pointer Arithmetic: Pointer plus `int`

```
int* p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
int* p3 = p0 + 3; // p3 points to 4th element
```



Pointer Arithmetic: Pointer plus int

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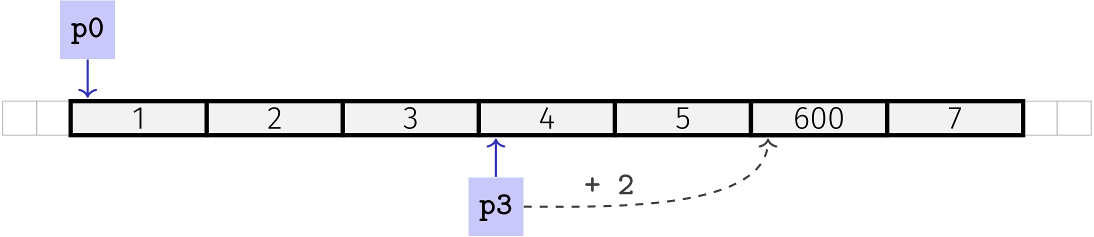
int* p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
int* p3 = p0 + 3; // p3 points to 4th element
*(p3 + 2) = 600; // set value of 6th element to 600
std::cout << *(p0 + 5);
```



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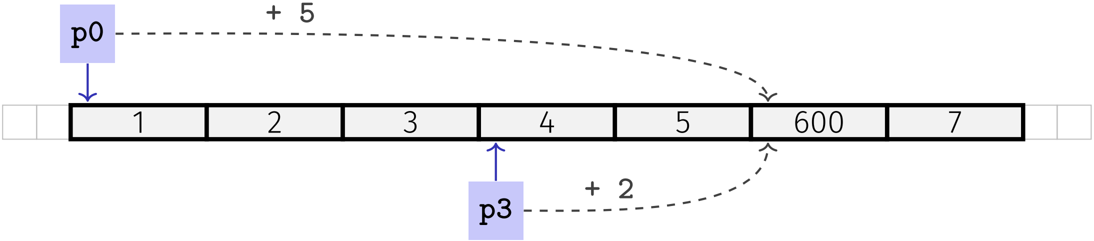
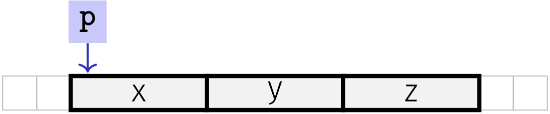
```
int* p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
int* p3 = p0 + 3; // p3 points to 4th element
*(p3 + 2) = 600; // set value of 6th element to 600
std::cout << *(p0 + 5); // output 6th element's value (i.e. 600)
```



Sequential Pointer Iteration

```
char* p = new char[3]{’x’, ’y’, ’z’};
```



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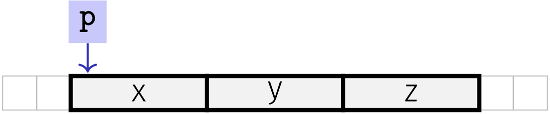

```
  for (char* it = p;
        it != p + 3;
        ++it) {

    std::cout << *it << ' ';
  }
```

Sequential Pointer Iteration

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char* p = new char[3]{'x', 'y', 'z'};
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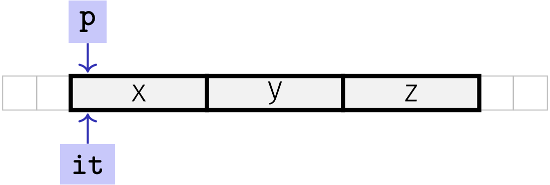

```
  for (char* it = p;
         it != p + 3;
         ++it) {

    std::cout << *it << ' ';
  }
```

it points to first element

Sequential Pointer Iteration

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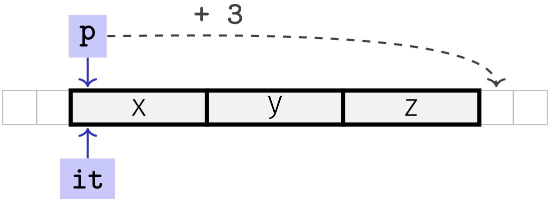

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  for (char* it = p;
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         ++it) {

    std::cout << *it << ' ';
  }
```

Abort if end reached

Sequential Pointer Iteration

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char* p = new char[3]{'x', 'y', 'z'};
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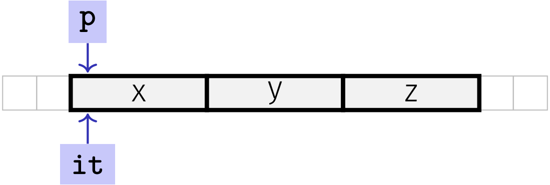

```
  for (char* it = p;
        it != p + 3;
        ++it) {

    std::cout << *it << ' ';
  }
```

Output current element: 'x'

Sequential Pointer Iteration

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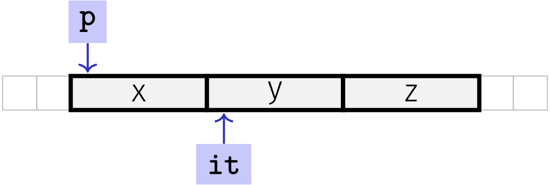

```
  for (char* it = p;
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       ++it) {

    std::cout << *it << ' ';  // x
  }
```

Advance pointer element-wise

Sequential Pointer Iteration

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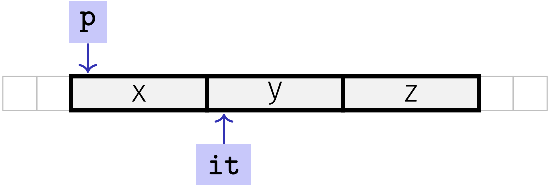

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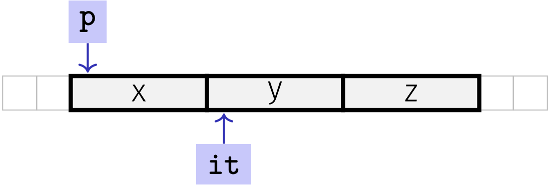

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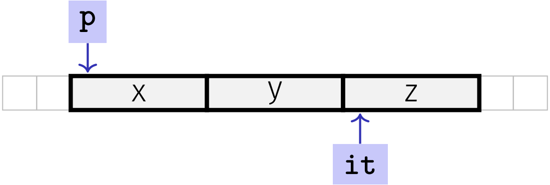

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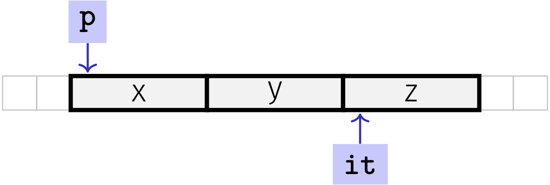

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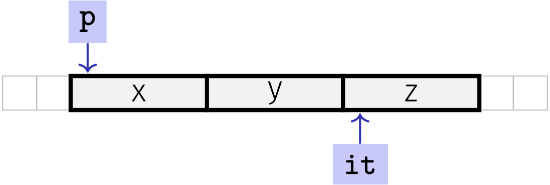

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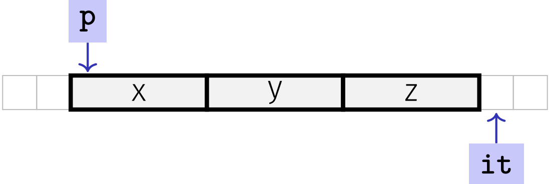

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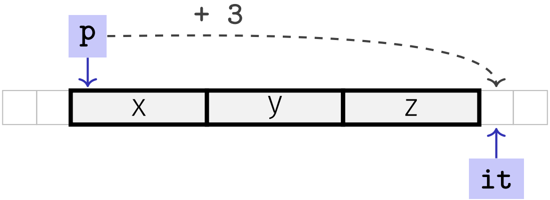

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Random Access to Arrays

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char* p = new char[3]{’x’, ’y’, ’z’};
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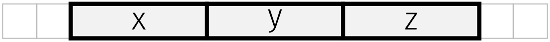

- The expression `*(p + i)`
- can also be written as `p[i]`

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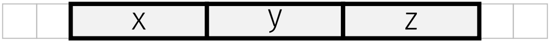

- The expression `*(p + i)`
- can also be written as `p[i]`
- E.g. `p[1] == *(p + 1) == 'y'`

Random Access to Arrays

iteration over an array via indices and *random access*:

```
char* p = new char[3]{’x’, ’y’, ’z’};

for (int i = 0; i < 3; ++i)
  std::cout << p[i] << ’ ’;
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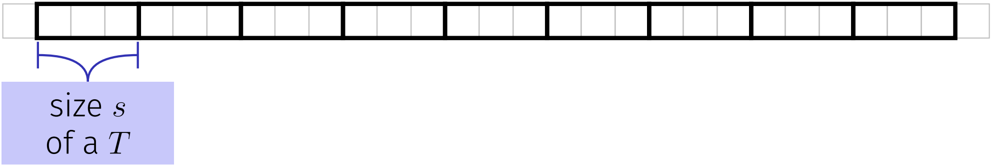
But: this is less *efficient* than the previously shown *sequential* access via pointer iteration

Random Access to Arrays

```
T* p = new T[n];
```



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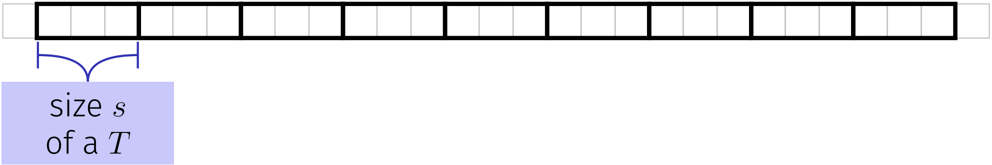
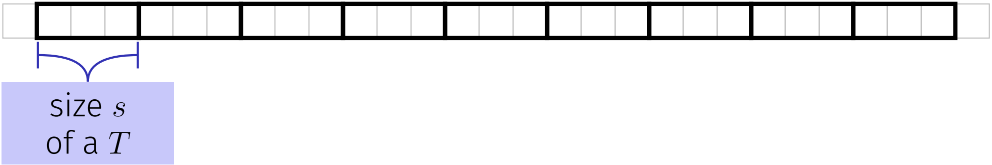


- Access **`p[i]`**, i.e. **`*(p + i)`**, “costs” computation $p + i \cdot s$

Random Access to Arrays

T`* p = new `T`[n];`



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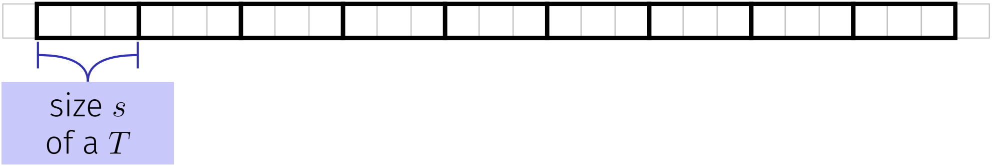

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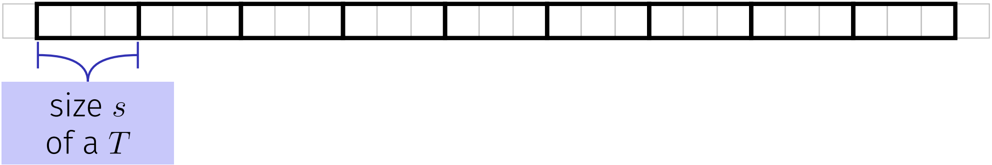

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- Iteration via *sequentiall access* (**++p**, **++p**, …) costs only one addition per access
- Sequential access is thus to be preferred for iterations

Reading a book . . . with random access

Random Access

- open book on page 1
- close book
- open book on pages 2-3
- close book
- open book on pages 4-5
- close book
- ....

Reading a book ...with sequential access

Random Access

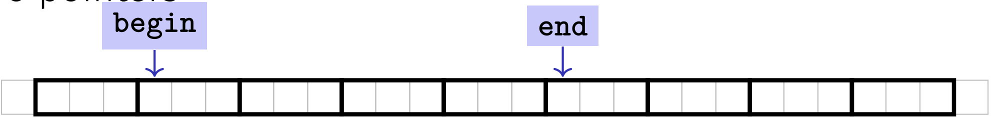
- open book on page 1
- close book
- open book on pages 2-3
- close book
- open book on pages 4-5
- close book
- ....

Sequential Access

- open book on page 1
- turn the page
- turn the page
- turn the page
- turn the page
- turn the page
- ...

Arrays in Functions

C++*covention*: arrays (or a segment of it) are passed using two pointers



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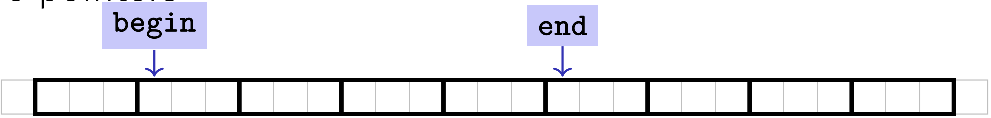

- **`begin`**: Pointer to the first element
- **`end`**: Pointer *past* the last element

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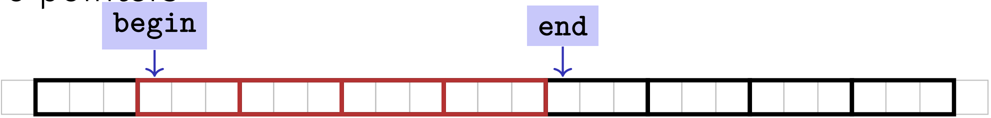

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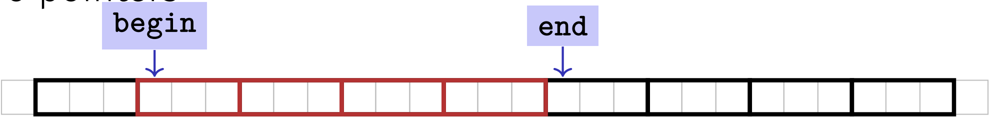

- **`begin`**: Pointer to the first element
- **`end`**: Pointer *past* the last element
- **`[begin, end)`** Designates the elements of the segment of the array
- **`[begin, end)`** is empty if **`begin == end`**
- **`[begin, end)`** must be a *valid range*, i.e. a (pot. empty) array segment

Arrays in (mutating) Functions: `fill`

```cpp
// PRE: [begin, end) is a valid range
// POST: Every element within [begin, end) was set to value
void fill(int* begin, int* end, int value) {
  for (int* p = begin; p != end; ++p)
    *p = value;
}
```

Arrays in (mutating) Functions: fill

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  for (int* p = begin; p != end; ++p)
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}
```

```
int* p = new int[5];
fill(p, p+5, 1); // Array at p becomes {1, 1, 1, 1, 1}
```

Functions with/without Effect

- Pointers can (like references) be used for functions with effect. Example: **`fill`**

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- Pointers can (like references) be used for functions with effect. Example: **`fill`**
- But many functions don’t have an effect, they only read the data
- ⇒ Use of **`const`**
- So far, for example:

```
const int zero = 0;
const int& nil = zero;
```

Positioning of Const

`const` T is equivalent to T **`const`** (and can be written like this):

```
const int zero = ...   ⟺   int const zero = ...
const int& nil = ...   ⟺   int const& nil = ...
```

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const int zero = ...   ⟺   int const zero = ...
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```

Both keyword orders are used in praxis

Const and Pointers

Read the declaration from right to left

`int const p1;`	**`p1`** is a constant integer

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`int* const p3;`	**p3** is a constant pointer to an integer
`int const* const p4;`	**p4** is a constant pointer to a constant integer

Non-mutating Functions: print

```
// PRE: [begin, end) is a valid range
// POST: The values in [begin, end) were printed
void print(
    int const* const begin,
    const int* const end) {

  for (int const* p = begin; p != end; ++p)
    std::cout << *p << ' ';
}
```

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Likewise (but different keyword order)

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Pointer, *not const*, to const int

Arrays, `new`, Pointer: Conclusion

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- Pointer arithmetic enables accessing rear array elements
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- **`new`** T allocates memory for (and initialises) a single T-object, and yields a pointer to it

Arrays, `new`, Pointer: Conclusion

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- Pointers can point to something (not) `const`, and they can be (not) `const` themselves

Arrays, `new`, Pointer: Conclusion

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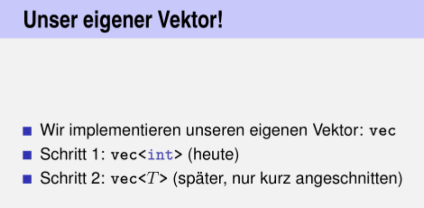
- Memory allocated by **`new`** is *not* automatically released (more on this soon)
- Pointers and references are related, both “link” to objects in memory. See also additional the slides **`pointers.pdf`**)

Array-based Vector

- Vectors . . . that somehow rings a bell 🤔



Unser eigener Vektor!

- Wir implementieren unseren eigenen Vektor: `vec`
- Schritt 1: `vec<int>` (heute)
- Schritt 2: `vec<`T`>` (später, nur kurz angeschnitten)

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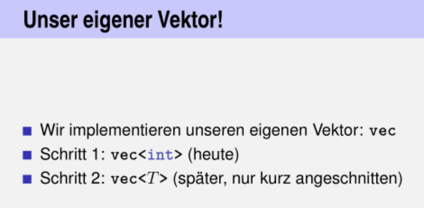

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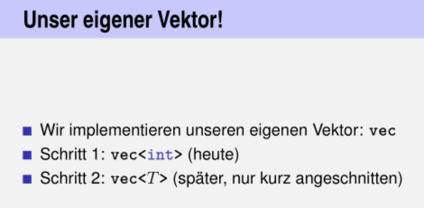

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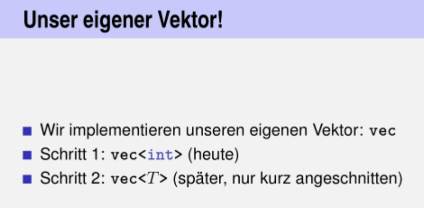
Array-based Vector

- Vectors . . . that somehow rings a bell 🤔
- Now we know how to allocate memory chunks of arbitrary size . . .
- . . . we can implement a vector, based on such a chunk of memory
- **`avec`** – an array-based vector of **`int`** elements



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Array-based Vector avec: Class Signature

```
class avec {
  // Private (internal) state:
  int* elements;
  unsigned int count;




}
```

Pointer to first element

Array-based Vector avec: Class Signature

```
class avec {
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}
```

Number of elements

Array-based Vector avec: Class Signature

```
class avec {
  // Private (internal) state:
  int* elements; // Pointer to first element
  unsigned int count; // Number of elements

public: // Public interface:
  avec(unsigned int size);
  unsigned int size() const;
  int& operator[](int i);
  void print(std::ostream& sink) const;
}
```

Constructor

Array-based Vector avec: Class Signature

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Size of vector

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public: // Public interface:
  avec(unsigned int size); // Constructor
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  int& operator[](int i);
  void print(std::ostream& sink) const;
}
```

Access an element

Array-based Vector avec: Class Signature

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  int* elements; // Pointer to first element
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public: // Public interface:
  avec(unsigned int size); // Constructor
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  int& operator[](int i); // Access an element
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}
```

Output elements

Array-based Vector `avec`: Class Signature

```
class avec {
  // Private (internal) state:
  int* elements; // Pointer to first element
  unsigned int count; // Number of elements

public: // Public interface:
  avec(unsigned int size); // Constructor
  unsigned int size() const; // Size of vector
  int& operator[](int i); // Access an element
  void print(std::ostream& sink) const; // Output elems.
}
```

Constructor `avec::avec()`

```
avec::avec(unsigned int size)
        : count(size) {

  elements = new int[size];
}
```

Save size

Constructor `avec::avec()`

```
avec::avec(unsigned int size)
       : count(size)  {

  elements = new int[size];
}
```

Allocate memory

Constructor `avec::avec()`

```
avec::avec(unsigned int size)
        : count(size)  {

  elements = new int[size];
}
```

Side remark: vector is not initialised with a default value

Excursion: Accessing Member Variables

```
avec::avec(unsigned int size): count(size) {
  elements = new int[size];
}
```

- **`elements`** is a member variable of our **`avec`** instance

Excursion: Accessing Member Variables

```
avec::avec(unsigned int size): count(size) {
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}
```

- **`elements`** is a member variable of our **`avec`** instance
- That instance can be accessed via the *pointer* **`this`**

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```
avec::avec(unsigned int size): count(size) {
  (*this).elements = new int[size];
}
```

- **`elements`** is a member variable of our **`avec`** instance
- That instance can be accessed via the *pointer* **`this`**
- **`elements`** is a shorthand for **`(*this).elements`**

Excursion: Accessing Member Variables

```
avec::avec(unsigned int size): count(size) {
  this->elements = new int[size];
}
```

- `elements` is a member variable of our `avec` instance
- That instance can be accessed via the *pointer* `this`
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- Equivalent, but shorter: `this->elements`

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- That instance can be accessed via the *pointer* **`this`**
- **`elements`** is a shorthand for **`(*this).elements`**
- Equivalent, but shorter: **`this->elements`**
- Mnemonic trick: “Follow the pointer to the member variable”

Function `avec::size()`

```
int avec::size() const {
  return this->count;
}
```

Doesn’t modify the vector

Function `avec::size()`

```
int avec::size() const  {
  return this->count;
}
```

Return size

Usage example:

```
avec v = avec(7);
assert(v.size() == 7); // ok
```

Function `avec::operator[]`

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Return `i`th element

Function avec::operator[]

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Element access with index check:

```
int& avec::at(int i) const {
  assert(0 <= i && i < this->count);

  return this->elements[i];
}
```

Function `avec::operator[]`

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Usage example:

```
avec v = avec(7);
std::cout << v[6]; // Outputs a "random" value
v[6] = 0;
std::cout << v[6]; // Outputs 0
```

Function avec::print()

Output elements using sequential access:

```
void avec::print(std::ostream& sink) const {
  for (int* p = this->elements;
       p != this->elements + this->count;
       ++p)
  {
    sink << *p << ' ';
  }
}
```

Pointer to first element

Function `avec::print()`

Output elements using sequential access:

```
void avec::print(std::ostream& sink) const {
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}
```

Abort iteration if past last element

Function avec::print()

Output elements using sequential access:

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Advance pointer element-wise

Function `avec::print()`

Output elements using sequential access:

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  for (int* p = this->elements;
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  }
}
```

Output current element

Function avec::print()

Finally: overload output operator:

```
____________ operator<<(____________ sink,
                    ____________ vec) {
  vec.print(sink);
  return ____________;
}
```

Function `avec::print()`

Finally: overload output operator:

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std::ostream& operator<<(std::ostream& sink,
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Function **avec::print()**

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Observations:

- Constant reference to **vec**, since unchanged

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  vec.print(sink);
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Observations:

- Constant reference to **`vec`**, since unchanged
- But not to **`sink`**: Outputing elements equals change
- **`sink`** is returned to enable output chaining, e.g. **`std::cout << v << '\n'`**

Further Functions?

```
class avec {
  ...
  void push_front(int e)      // Prepend e to vector
  void push_back(int e)       // Append e to vector
  void remove(unsigned int i) // Cut out ith element
  ...
}
```

Further Functions?

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Commonalities: such operations need to change the vector’s *size*

Resizing arrays

An allocated block of memory (e.g. **`new int[3]`**) cannot be resized later on

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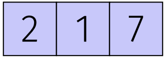

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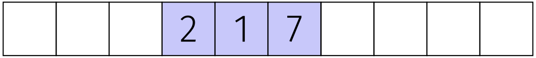

Possibility:

- Allocate more memory than initially necessary

Resizing arrays

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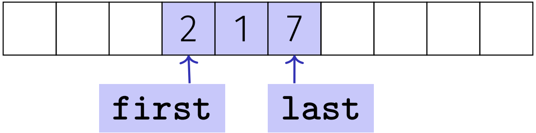
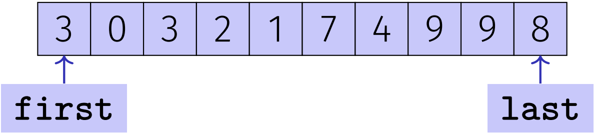


Possibility:

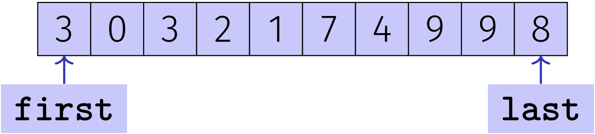
- Allocate more memory than initially necessary
- Fill from inside out, with pointers to first and last element

Resizing arrays



- But eventually, all slots will be in use

Resizing arrays



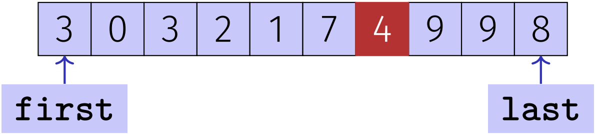
3	0	3	2	1	7	4	9	9	8

`first`

`last`

- But eventually, all slots will be in use
- Then unavoidable: Allocate larger memory block and copy data over

Resizing arrays



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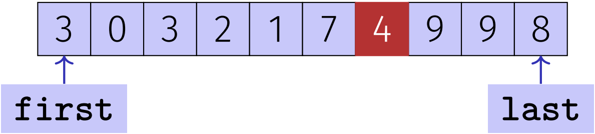
Resizing arrays



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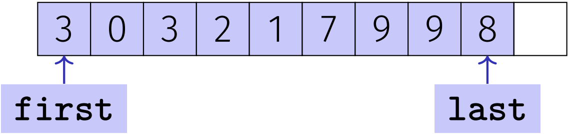

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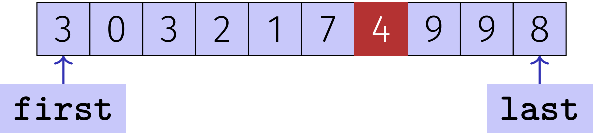

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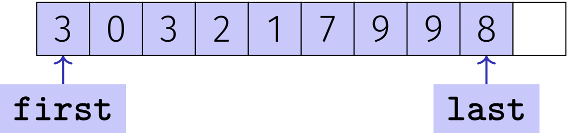

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`first`

`last`

Similar: inserting at arbitrary position

21. Dynamic Data Structures II

Linked Lists, Vectors as Linked Lists

Different Memory Layout: Linked List

- **No** contiguous area of memory and **no** random access

Different Memory Layout: Linked List

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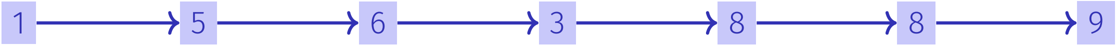
- Each element points to its successor



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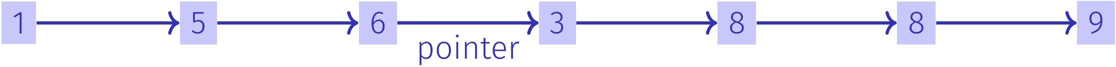

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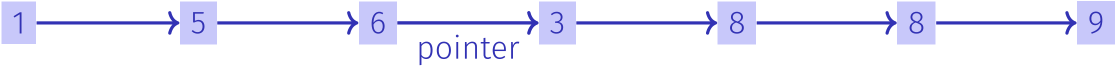

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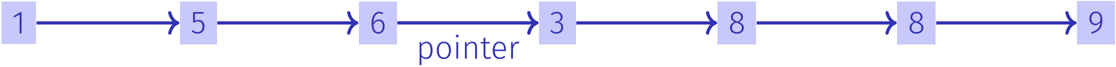
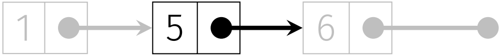


⇒ Our vector can be implemented as a linked list

Linked List: Zoom

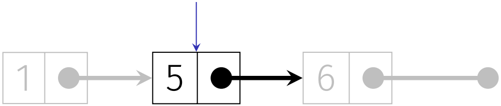


```
struct llnode {
  int value;
  llnode* next;

  llnode(int v, llnode* n): value(v), next(n) {} // Constructor
};
```

Linked List: Zoom

element (type **`struct llnode`**)



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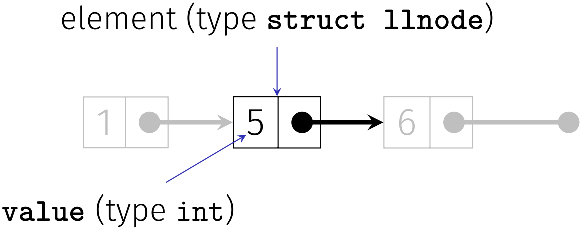

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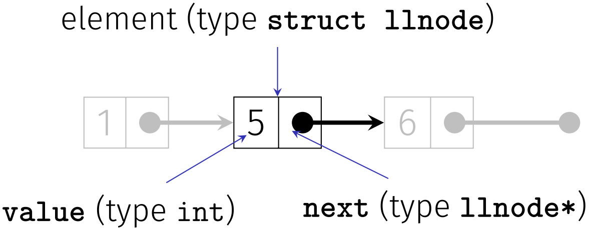

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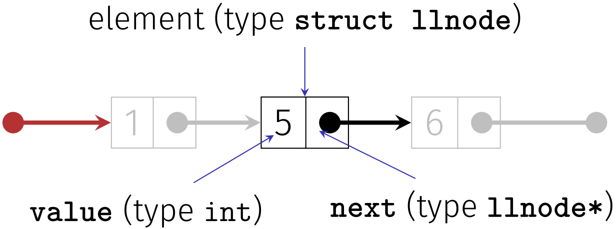
```

Vector = Pointer to the First Element



```
class llvec {
  llnode* head;
public: // Public interface identical to avec's
  llvec(unsigned int size);
  unsigned int size() const;
  ...
};
```

Function `llvec::print()`

```
struct llnode {
  int value;
  llnode* next;
  ...
};
```

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';
  }
}
```

Pointer to first element

Function `llvec::print()`

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struct llnode {
  int value;
  llnode* next;
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}
```

Abort if end reached

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```
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```

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void llvec::print(std::ostream& sink) const {
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       n = n->next)
  {
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  }
}
```

Advance pointer element-wise

Function llvec::print()

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  }
}
```

Output current element

Function `llvec::print()`

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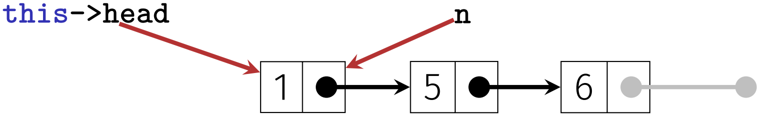

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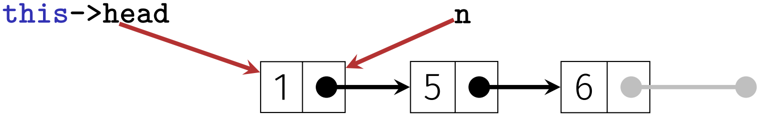

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void llvec::print(std::ostream& sink) const {
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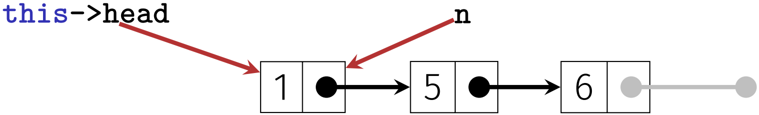
  {
    sink << n->value << ' ';  // 1
  }
}
```



Function `llvec::print()`

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void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';   // 1
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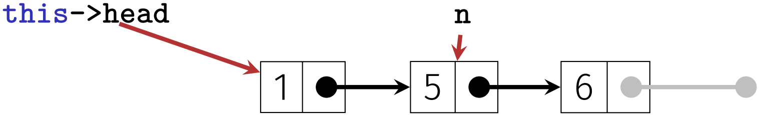

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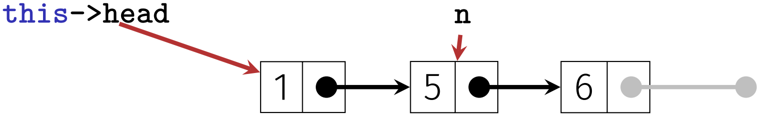

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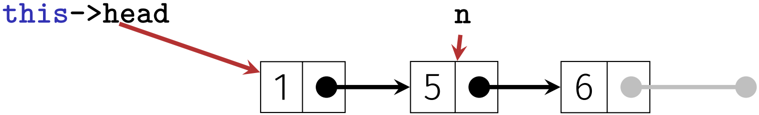

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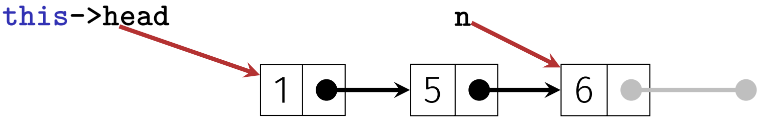

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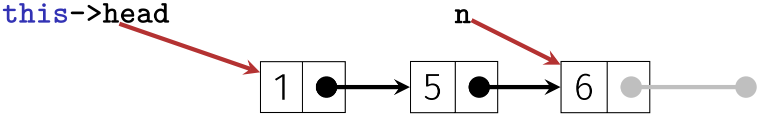

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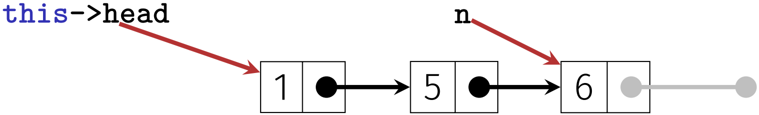

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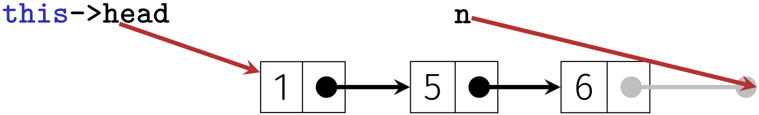


Function llvec::operator[]

Accessing ith Element is implemented similarly to **print()**:

```
int& llvec::operator[](unsigned int i) {
  llnode* n = this->head;

  for (; 0 < i; --i)
    n = n->next;

  return n->value;
}
```

Pointer to first element

Function `llvec::operator[]`

Accessing ith Element is implemented similarly to **`print()`**:

```
int& llvec::operator[](unsigned int i) {
  llnode* n = this->head;

  for (; 0 < i; --i)
    n = n->next;

  return n->value;
}
```

Step to `i`th element

Function `llvec::operator[]`

Accessing ith Element is implemented similarly to **`print()`**:

```
int& llvec::operator[](unsigned int i) {
  llnode* n = this->head;

  for (; 0 < i; --i)
    n = n->next;

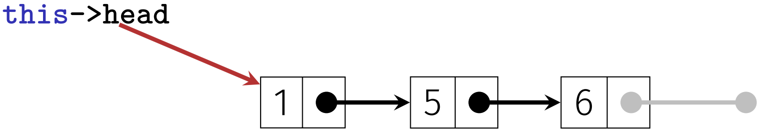
  return n->value;
}
```

Return `i`th element

Function `llvec::push_front()`

Advantage **`llvec`**: Prepending elements is very easy:

```
void llvec::push_front(int e) {
  this->head =
    new llnode{e, this->head};
}
```



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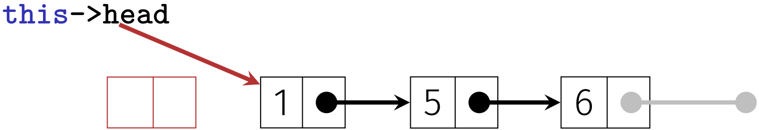

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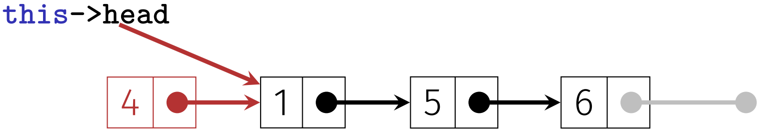

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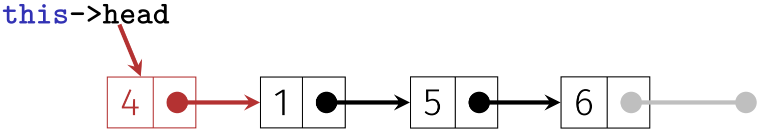

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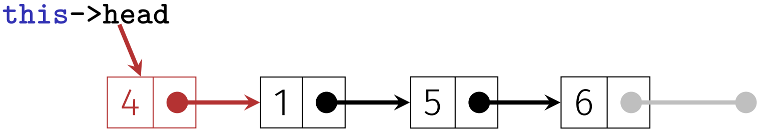

Attention: If the new **`llnode`** weren't allocated *dynamically*, then it would be deleted (= memory deallocated) as soon as **`push_front`** terminates

Function `llvec::llvec()`

Constructor can be implemented using **`push_front()`**:

```
llvec::llvec(unsigned int size) {
  this->head = nullptr;

  for (; 0 < size; --size)
    this->push_front(0);
}
```

`head` initially points to nowhere

Function `llvec::llvec()`

Constructor can be implemented using **`push_front()`**:

```
llvec::llvec(unsigned int size) {
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```

Prepend **`0`** **`size`** times

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llvec::llvec(unsigned int size) {
  this->head = nullptr;

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    this->push_front(0);
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```

Use case:

```
llvec v = llvec(3);
std::cout << v; // 0 0 0
```

Function llvec::push_back()

Simple, but inefficient: traverse linked list to its end and append new element

```
void llvec::push_back(int e) {
  llnode* n = this->head;

  for (; n->next != nullptr; n = n->next);

  n->next =
    new llnode{e, nullptr};
}
```

Start at first element ...

Function llvec::push_back()

Simple, but inefficient: traverse linked list to its end and append new element

```
void llvec::push_back(int e) {
  llnode* n = this->head;

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}
```

… and go to the last element

Function `llvec::push_back()`

Simple, but inefficient: traverse linked list to its end and append new element

```
void llvec::push_back(int e) {
  llnode* n = this->head;

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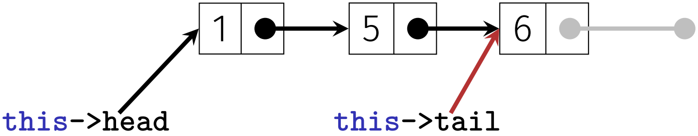
    new llnode{e, nullptr};
}
```

Append new element to currently last

Function `llvec::push_back()`

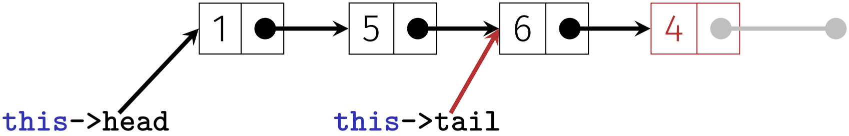
- More efficient, but also slightly more complex:
  1. Second pointer, pointing to the last element: **`this->tail`**



Function `llvec::push_back()`

- More efficient, but also slightly more complex:

  1. Second pointer, pointing to the last element: **`this->tail`**
  2. Using this pointer, it is possible to append to the end directly



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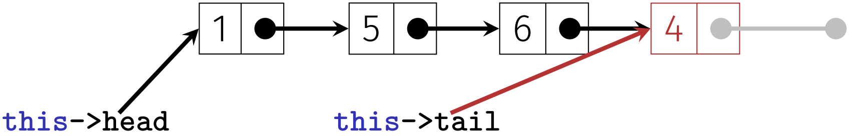

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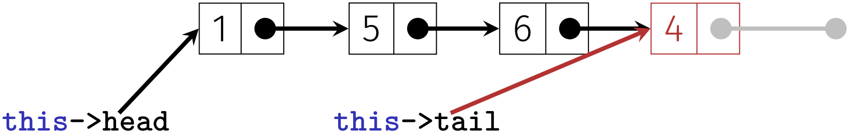

- But: Several corner cases, e.g. vector still empty, must be accounted for

Function `llvec::size()`

Simple, but inefficient: *compute* size by counting

```
unsigned int llvec::size() const {
  unsigned int c = 0;

  for (______________________________;
       ______________________________;
       ______________________________)
  {
    ______________________________;
  }

  return c;
}
```

Function `llvec::size()`

Simple, but inefficient: *compute* size by counting

```
unsigned int llvec::size() const {
  unsigned int c = 0;

  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
    ++c;

  return c;
}
```

Count initially 0

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```

Count linked-list length

Function `llvec::size()`

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Return count

Function `llvec::size()`

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More efficient, but also slightly more complex: *maintain* size as member variable

1. Add member variable **`unsigned int count`** to class **`llvec`**
2. **`this->count`** must now be updated *each* time an operation (such as **`push_front`**) affects the vector's size

Efficiency: Arrays vs. Linked Lists

- Memory: our **`avec`** requires roughly n ints (vector size n), our **`llvec`** roughly $3n$ ints (a pointer typically requires 8 byte)

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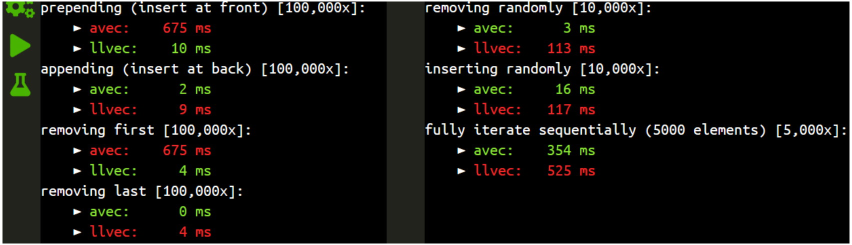
- Memory: our **avec** requires roughly n ints (vector size n), our **llvec** roughly $3n$ ints (a pointer typically requires 8 byte)

- Runtime (with **avec** = **std::vector**, **llvec** = **std::list**):



```
prepending (insert at front) [100,000x]:
    ► avec:     675 ms
    ► llvec:     10 ms
appending (insert at back) [100,000x]:
    ► avec:       2 ms
    ► llvec:      9 ms
removing first [100,000x]:
    ► avec:     675 ms
    ► llvec:      4 ms
removing last [100,000x]:
    ► avec:       0 ms
    ► llvec:      4 ms
```

```
removing randomly [10,000x]:
    ► avec:       3 ms
    ► llvec:    113 ms
inserting randomly [10,000x]:
    ► avec:      16 ms
    ► llvec:    117 ms
fully iterate sequentially (5000 elements) [5,000x]:
    ► avec:     354 ms
    ► llvec:    525 ms
```

22. Containers, Iterators and Algorithms

Containers, Sets, Iterators, const-Iterators, Algorithms, Templates

Vectors are Containers

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    1. A collection of elements
    2. Plus operations on this collection

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- In C++, **`vector<`**T**`>`** and similar data structures are called *container*
- Called *collections* in some other languages, e.g. Java

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- Each container has certain *characteristic properties*
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  - Inserting at/removing from arbitrary index is potentially inefficient
  - Looking for a specific element is potentially inefficient
  - Can contain the same element more than once
  - Elements are in insertion order (ordered but not sorted)

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- Many more are available from 3rd-party libraries, e.g. `https://www.boost.org/doc/libs/1_68_0/doc/html/container.html`, `https://github.com/abseil/abseil-cpp`

Example Container: std::unordered_set<T>

- A *mathematical set* is an unordered, duplicate-free collection of elements:

$$\{1,2,1\} = \{1,2\} = \{2,1\}$$

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  - Efficient “element contained?” check
  - Efficient insertion and removal of elements
- Side remark: implemented as a hash table

Use Case `std::unordered_set<`T`>`

Problem:

- given a sequence of pairs (*name, percentage*) of Code Expert submissions . . .

```
// Input: file submissions.txt
Friedrich 90
Schwerhoff 10
Lehner 20
Schwerhoff 11
```

Use Case std::unordered_set<T>

Problem:

- given a sequence of pairs (*name, percentage*) of Code Expert submissions …

```
// Input: file submissions.txt
Friedrich 90
Schwerhoff 10
Lehner 20
Schwerhoff 11
```

- … determine the submitters that achieved at least 50%

```
// Output
Friedrich
```

Use Case std::unordered_set<T>

```
std::ifstream in("submissions.txt");
std::unordered_set<std::string> names;

std::string name;
unsigned int score;

while (in >> name >> score) {
  if (50 <= score)
    names.insert(name);
}

std::cout << "Unique submitters: "
          << names << '\n';
```

Open submissions.txt

Use Case std::unordered_set<T>

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Set of names, initially empty

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Pair (*name, score*)

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std::cout << "Unique submitters: "
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```

Input next pair

Use Case std::unordered_set<T>

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std::ifstream in("submissions.txt");
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std::string name;
unsigned int score;

while (in >> name >> score) {
  if (50 <= score)
    names.insert(name);
}

std::cout << "Unique submitters: "
          << names << '\n';
```

Record name if score suffices

Use Case std::unordered_set<T>

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std::ifstream in("submissions.txt");
std::unordered_set<std::string> names;

std::string name;
unsigned int score;

while (in >> name >> score) {
  if (50 <= score)
    names.insert(name);
}

std::cout << "Unique submitters: "
          << names << '\n';
```

Output recorded names

Example Container: **`std::set<`**T**`>`**

- Nearly equivalent to **`std::unordered_set<`**T**`>`**, but the elements are *ordered*

$$\{1,2,1\} = \{1,2\} \neq \{2,1\}$$

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- Side remark: implemented as a red-black tree

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```

set instead of unsorted_set . . .

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}

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```

. . . and the output is in alphabetical order

Printing Containers

- Recall: **`avec::print()`** and **`llvec::print()`**

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- Recall: **`avec::print()`** and **`llvec::print()`**
- What about printing **`set`**, **`unordered_set`**, . . .?
- Commonality: iterate over container elements and print them

Similar Functions

- Lots of other useful operations can be implemented by iterating over a container:
- **`contains(c, e)`**: true iff container **`c`** contains element **`e`**

Similar Functions

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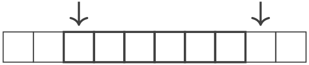
- . . .

Recall: Iterating With Pointers

- Iteration over an *array*:



Recall: Iterating With Pointers

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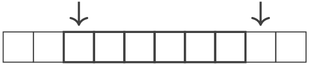

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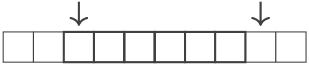

Recall: Iterating With Pointers

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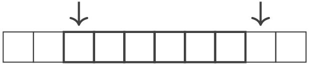

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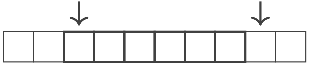

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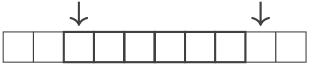

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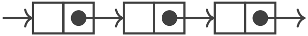

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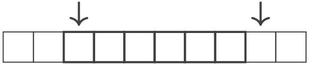

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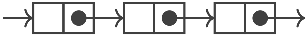

Recall: Iterating With Pointers

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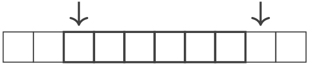

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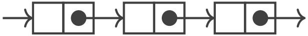

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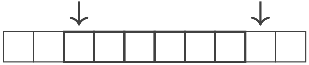

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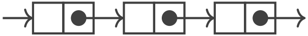

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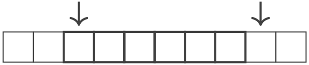

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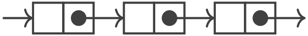

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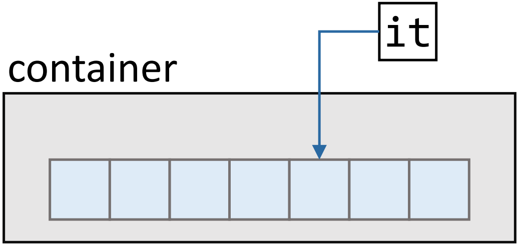
  - **`*it`**: Access current element
  - **`++it`**: Advance iterator by one element
- Iterators are essentially pimped pointers

Iterators

- Iterators allow accessing different containers in a *uniform* way: **`*it`**, **`++it`**, etc.



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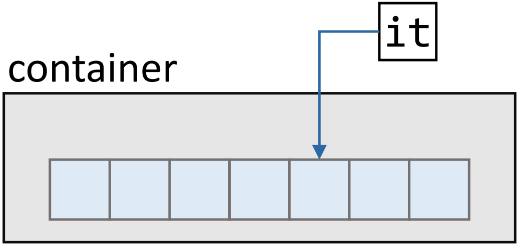

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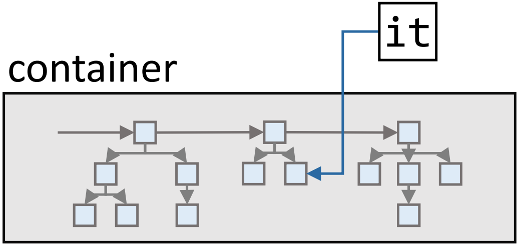

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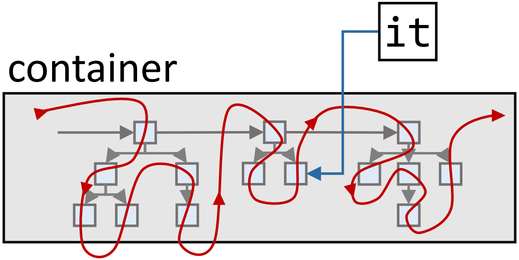

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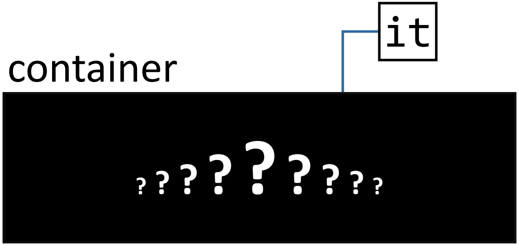

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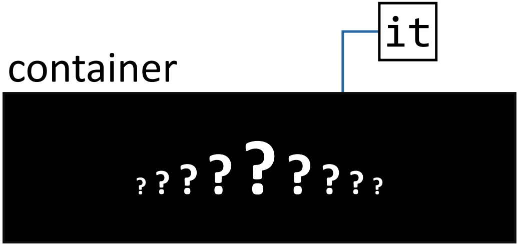

Example: Iterate over std::vector

it is an iterator specific to std::vector<int>

```
std::vector<int> v = {1, 2, 3};

for (std::vector<int>::iterator it = v.begin();
     it != v.end();
     ++it)  {

  *it = -*it;
}

std::cout << v; // -1 -2 -3
```

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`it` initially points to the first element

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Abort if `it` reached the end

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Advance it element-wise

Example: Iterate over std::vector

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std::cout << v; // -1 -2 -3
```

Negate current element ($e \to -e$)

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  *it = -*it;
}

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```

Example: Iterate over `std::vector`

Recall: type aliases can be used to shorten often-used type names

```
using ivit = std::vector<int>::iterator; // int-vector iterator

for (ivit it = v.begin();
     ...
```

Negate as a Function

As before: passing a *range (interval)* to work on

```
void neg(std::vector<int>::iterator begin;
         std::vector<int>::iterator end) {

  for (std::vector<int>::iterator it = begin;
       it != end;
       ++it) {

    *it = -*it;
  }
}
```

Negate elements in interval [**begin**, **end**)

Negate as a Function

As before: passing a *range (interval)* to work on

```
void neg(std::vector<int>::iterator begin;
         std::vector<int>::iterator end);
```

```
// in main():
std::vector<int> v = {1, 2, 3};
neg(v.begin(), v.begin() + (v.size() / 2));
```

← Negate first half

Algorithms Library in C++

- The C++standard library includes lots of useful algorithms (functions) that work on iterator-defined intervals [*begin*, *end*)
- For example **`find`**, **`fill`** and **`sort`**; see also `https://en.cppreference.com/w/cpp/algorithm`

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- Thanks to iterators, these $\geq$ 100 (!) algorithms can be applied to any* container: the 17 (!) C++standard container, our **`avec`** and **`llvec`** (discussed next), etc.
- Without this uniform access to container elements, we would have to duplicate *lots* of code

Not every algorithm can be applied to every container. It is, e.g. Not possible to sort a **`std::set`**.

An iterator for `llvec`

We need:

1. An **`llvec`**-specific iterator with at least the following functionality:
   - Access current element: **`operator*`**
   - Advance iterator: **`operator++`**
   - End-reached check: **`operator!=`** (or **`operator==`**)

An iterator for **llvec**

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   - Access current element: **operator***
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2. Member functions **begin()** and **end()** for **llvec** to get an iterator to the beginning and past the end, respectively

Iterator `llvec::iterator` (Step 1/2)

```
class llvec {
  ...
public:
  class iterator {
    ...
  };


  ...
}
```

- The iterator belongs to our vector, that’s why **`iterator`** is a public *inner class* of **`llvec`**

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  ...
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```

- The iterator belongs to our vector, that’s why **`iterator`** is a public *inner class* of **`llvec`**
- Instances of our iterator are of type **`llvec::iterator`**

Iterator `llvec::iterator` (Step 1/2)

```
class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
  bool operator!=(const iterator& other) const;
};
```

Pointer to current vector element

Iterator `llvec::iterator` (Step 1/2)

```
class iterator {
  llnode* node;

public:
  iterator(llnode* n);
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  int& operator*() const;
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};
```

Create iterator to specific element

Iterator `llvec::iterator` (Step 1/2)

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public:
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};
```

Advance iterator by one element

Iterator `llvec::iterator` (Step 1/2)

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class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
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```

Access current element

Iterator `llvec::iterator` (Step 1/2)

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class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
  bool operator!=(const iterator& other) const;
};
```

Compare with other iterator

Iterator `llvec::iterator` (Step 1/2)

```
// Constructor
llvec::iterator::iterator(llnode* n): node(n) {}



// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
  assert(this->node != nullptr);

  this->node = this->node->next;

  return *this;
}
```

Iterator `llvec::iterator` (Step 1/2)

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```

Let iterator point to **`n`** initially

Iterator `llvec::iterator` (Step 1/2)

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// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
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Advance iterator by one element

Iterator llvec::iterator (Step 1/2)

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// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
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  this->node = this->node->next;

  return *this;
}
```

Return reference to advanced iterator

Iterator llvec::iterator (Step 1/2)

```
// Element access
int& llvec::iterator::operator*() const {
  return this->node->value;
}


// Comparison: when are two iterators not equal?
bool llvec::iterator::operator!=(
    const llvec::iterator& other) const
{
  return this->node != other.node;
}
```

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Access current element

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  return this->node != other.node;
}
```

this iterator different from **other** if they point to different element

An iterator for `llvec` (Repetition)

We need:

1. An `llvec`-specific iterator with at least the following functionality:
   - Access current element: `operator*`
   - Advance iterator: `operator++`
   - End-reached check: `operator!=` (or `operator==`)

   ✓

2. Member functions `begin()` and `end()` for `llvec` to get an iterator to the beginning and past the end, respectively

Iterator `llvec::iterator` (Step 2/2)

```
class llvec {
  ...
public:
  class iterator {...};

  iterator begin();
  iterator end();


  ...
}
```

`llvec` needs member functions to issue iterators pointing *to the beginning* and *past the end*, respectively, of the vector

Iterator `llvec::iterator` (Step 2/2)

```
llvec::iterator llvec::begin() {
  return llvec::iterator(this->head);
}

llvec::iterator llvec::end() {
  return llvec::iterator(nullptr);
}
```

Iterator to first vector element

Iterator `llvec::iterator` (Step 2/2)

```
llvec::iterator llvec::begin() {
  return llvec::iterator(this->head);
}

llvec::iterator llvec::end() {
  return llvec::iterator(nullptr);
}
```

Iterator past last vector element

Type-generic Container

Type-specific containers

E-BIKES
CHAIRS
PEARS
LOOT

Type-generic container

`https://upload.wikimedia.org/wikipedia/commons/d/df/Container_01_KMJ.jpg` (CC BY-SA 3.0)

(Type-generic containers (templates in general) aren't relevant for the exam)

Type-generic Container

Class **cell**: a simple, single-element container for **int**

```
class cell {
  int element;

public:
  cell(int e);
  int& value();
};
```

container's element

```
cell::cell(int e)
    : element(e) {}

int& cell::value() {
  return this->element;
}
```

Type-generic Container

Class **`cell`**: a simple, single-element container for **`int`**

```
class cell {
  int element;

public:
  cell(int e);
  int& value();
};
```

Constructor stores **`e`** in container

```
cell::cell(int e)
  : element(e) {}

int& cell::value() {
  return this->element;
}
```

Type-generic Container

Class **`cell`**: a simple, single-element container for **`int`**

```
class cell {
  int element;

public:
  cell(int e);
  int& value();
};
```

```
cell::cell(int e)
    : element(e) {}

int& cell::value() {
  return this->element;
}
```

Access the element

Type-generic Container

Class **cell**: a simple, single-element container for **int**

```
class cell {
  int element;

public:
  cell(int e);
  int& value();
};
```

```
cell::cell(int e)
    : element(e) {}

int& cell::value() {
  return this->element;
}
```

Better: generic **cell<E>** for every element type **E** (analogous to **std::vector<E>**)

Type-generic Container with Templates

Templates enable *type-generic* functions and classes:

```
template<typename E>
class cell {
  E element;

public:
  cell(E e);
  E& value();
};
```

Let **E** an arbitrary type ...

- Types can be used as *parameters*

Type-generic Container with Templates

Templates enable *type-generic* functions and classes:

```
template<typename E>
class cell {
  E element;

public:
  cell(E e);
  E& value();
};
```

Let **E** an arbitrary type . . .

. . . then **cell** manages an element of type **E**

- Types can be used as *parameters*
- Type parameters are valid in the “templated” scope

Type-generic Container with Templates

- Signatures and implementations must be “templated”
- For separately provided implementations, the class prefix must be written in generic form

```
template<typename E>
class cell {
  E element;

public:
  cell(E e);
  E& value();
};
```

```
template<typename E>
cell<E>::cell(E e)
    : element(e) {}

template<typename E>
E& cell<E>::value() {
  return this->element;
}
```

Type-generic Container with Templates

```
cell<int> c1(313);
cell<std::string> c2("terrific!")
```

- For *declarations*, e.g. **`cell<int>`**, type parameters must be provided explicitly . . .
- . . . but they are *inferred* by the compiler everywhere else, e.g. for **`c1(313)`**, i.e. when invoking the generic constructor **`cell(E e)`** (where type parameter **`E`** is instantiated by the compiler with **`int`**)

Templates: Conclusion

- Templates realise *static code generation*/*static metaprogramming* in C++
- Template code is *copied* per type instantiation. When using **`cell<int>`** and **`cell<std::string>`**, the compiler creates two *instantiated copies* of **`cell`**'s code: conceptually, the two (no longer generic) classes **`cell_int`** and **`cell_stdstring`**.
- Templates reduce code duplication and facilitate code reuse
- Compiler errors that refer to templates are unfortunately often even more complex than C++ errors usually already are

23. Dynamic Datatypes and Memory Management

Problem

Last week: dynamic data type

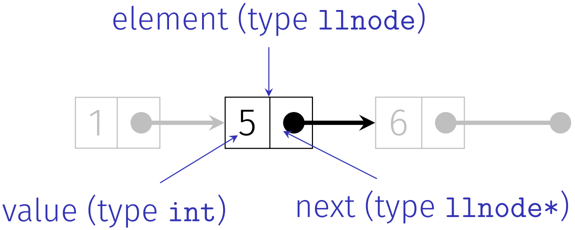
Have allocated dynamic memory, but not released it again. In particular: no functions to remove elements from **`llvec`**.

Today: correct memory management!

Goal: class stack with memory management

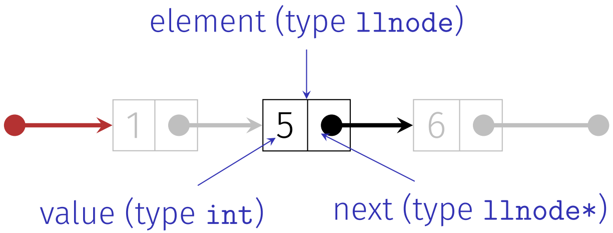
```
class stack{
public:
  // post: Push an element onto the stack
  void push(int value);
  // pre: non-empty stack
  // post: Delete top most element from the stack
  void pop();
  // pre: non-empty stack
  // post: return value of top most element
  int top() const;
  // post: return if stack is empty
  bool empty() const;
  // post: print out the stack
  void print(std::ostream& out) const;
...
```

Recall the Linked List



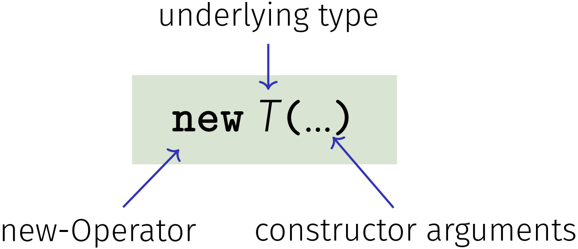
```
struct llnode {
  int value;
  llnode* next;
  // constructor
  llnode (int v, llnode* n) : value (v), next (n) {}
};
```

Stack = Pointer to the Top Element



```
class stack {
public:
  void push (int value);
  ...
private:
  llnode* topn;
};
```

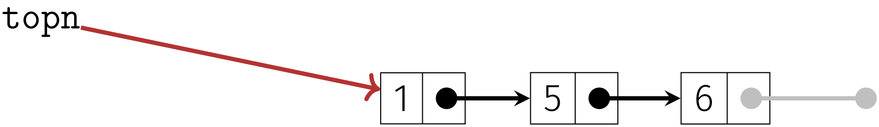
Recall the `new` Expression



- **Effect**: memory for a new object of type T is allocated . . .
- . . . and initialized by means of the matching constructor
- **Value**: address of the new T object, **Type**: Pointer **T***!

The new Expression push(4)

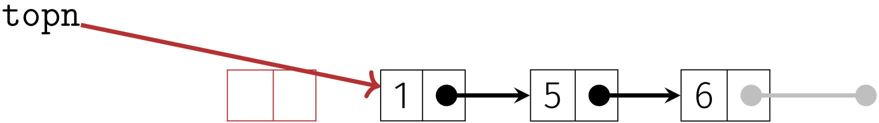
```
void stack::push(int value) {
  topn = new llnode(value, topn);
}
```



The `new` Expression push(4)

- **Effect:** new object of type *T* is allocated in memory …

```
void stack::push(int value) {
  topn = new llnode(value, topn);
}
```



The `new` Expression push(4)

- **Effect:** new object of type *T* is allocated in memory …
- …and intialized by means of the matching constructor

```
void stack::push(int value) {
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}
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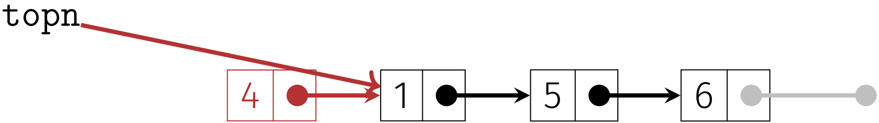

The `new` Expression push(4)

- **Effect:** new object of type *T* is allocated in memory . . .
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```
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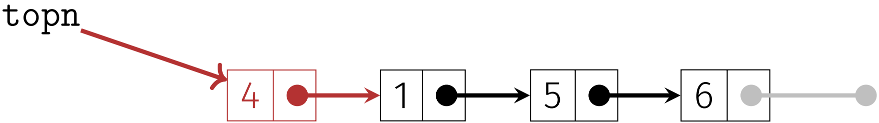
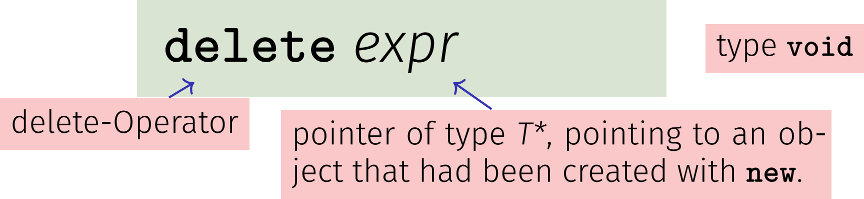


The `delete` Expression

Objects generated with `new` have **dynamic storage duration:** they “live” until they are explicitly *deleted*

The `delete` Expression

Objects generated with **`new`** have **dynamic storage duration:** they “live” until they are explicitly *deleted*



`delete` *expr*

type **`void`**

delete-Operator

pointer of type *T**, pointing to an object that had been created with **`new`**.

The `delete` Expression

Objects generated with **`new`** have **dynamic storage duration:** they “live” until they are explicitly *deleted*

`delete` *expr*

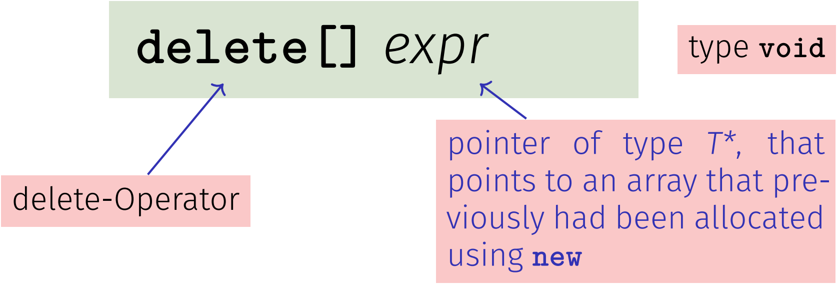
type **`void`**

delete-Operator

pointer of type *T**, pointing to an object that had been created with **`new`**.

- **Effect:** object is **deconstructed** (explanation below) … and **memory is released**.

`delete` for Arrays



`delete[]` *expr*

type `void`

delete-Operator

pointer of type *T**, that points to an array that previously had been allocated using **`new`**

- **Effect:** array is deleted and memory is released

Who is born must die. . .

Guideline “Dynamic Memory”

For each **`new`** there is a matching **`delete`**!

Who is born must die. . .

Guideline “Dynamic Memory”

For each **`new`** there is a matching **`delete`**!

Non-compliance leads to memory leaks

Who is born must die...

Guideline "Dynamic Memory"

For each **`new`** there is a matching **`delete`**!

Non-compliance leads to memory leaks

- old objects that occupy memory...

Who is born must die. . .

Guideline “Dynamic Memory”

For each **`new`** there is a matching **`delete`**!

Non-compliance leads to memory leaks

- old objects that occupy memory. . .
- . . . until it is full (heap overflow)

Careful with `new` and `delete`!

```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
```

Careful with `new` and `delete`!

```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
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memory for t is allocated

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rational* t = new rational;
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int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

Careful with `new` and `delete`!

```
rational* t = new rational;
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memory for t is allocated

other pointers may point to the same object

... and used for releaseing the object

Careful with `new` and `delete`!

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rational* t = new rational;
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delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

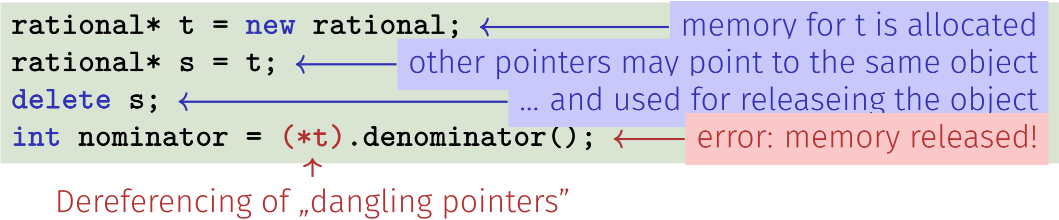
... and used for releaseing the object

error: memory released!

Dereferencing of „dangling pointers”

- Pointer to released objects: **dangling pointers**

Careful with `new` and `delete`!



```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

... and used for releaseing the object

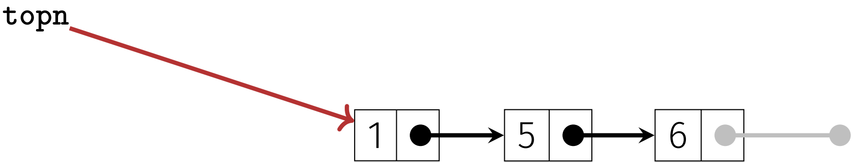
error: memory released!

Dereferencing of „dangling pointers”

- Pointer to released objects: **dangling pointers**
- Releasing an object more than once using **`delete`** is a similar severe error

Stack Continued: pop()

```
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```



Stack Continued: pop()

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void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
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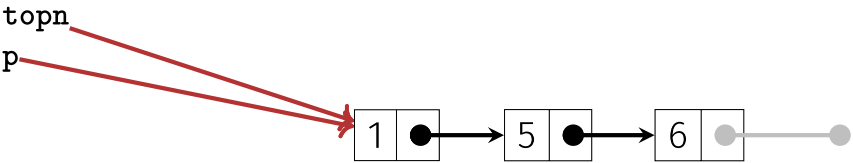
}
```



Stack Continued: pop()

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void stack::pop(){
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    topn = topn->next;
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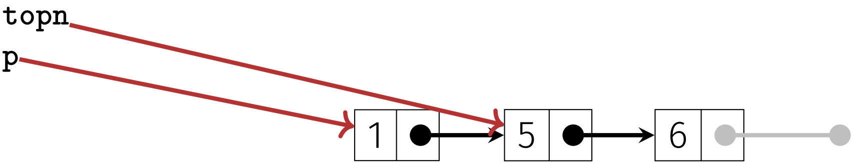
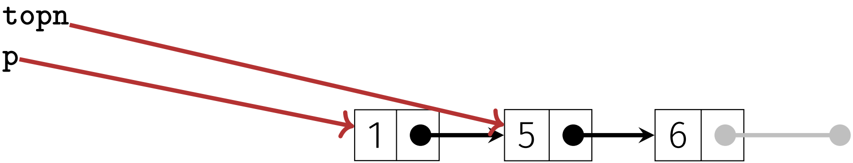
}
```



Stack Continued: pop()

```
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```

reminder: shortcut for `(*topn).next`



Stack Continued: pop()

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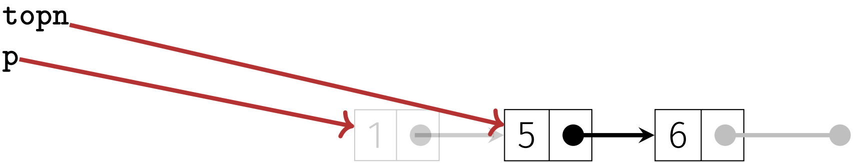
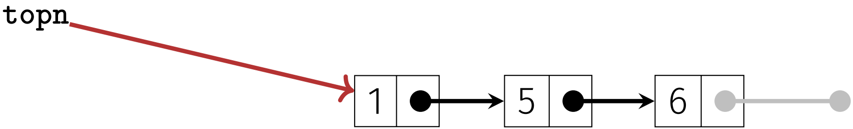


Print the Stack `print()`

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " ";
}
```



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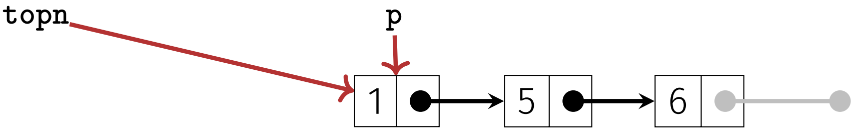
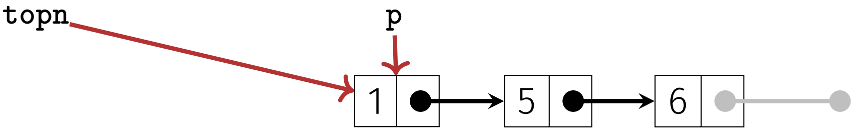


Print the Stack print()

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1
}
```



Print the Stack

print()

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
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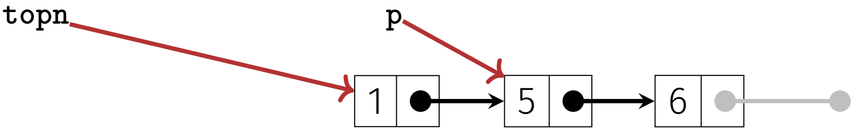
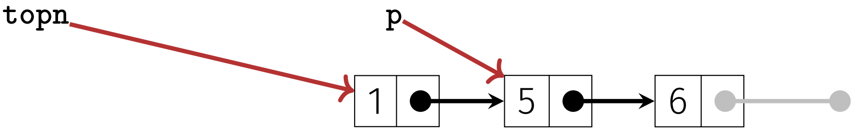
}
```



Print the Stack `print()`

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1 5
}
```



Print the Stack

print()

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}
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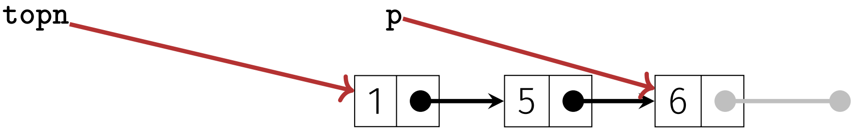
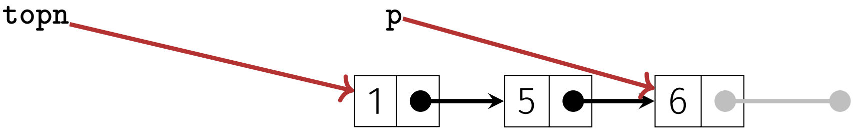


Print the Stack print()

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1 5 6
}
```



Print the Stack print()

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}
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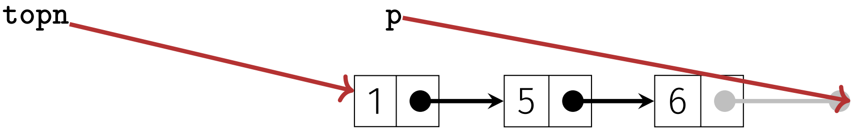

Output Stack: operator«

```
class stack {
public:
   void push (int value);
   void pop();
   void print (std::ostream& o) const;
   ...
private:
   llnode* topn;
};

// POST: s is written to o
std::ostream& operator<< (std::ostream& o, const stack& s){
   s.print (o);
   return o;
}
```

empty(), top()

```
bool stack::empty() const {
  return top == nullptr;
}

int stack::top() const {
  assert(!empty());
  return topn->value;
}
```

Empty Stack

```
class stack{
public:
  stack() : topn (nullptr) {} // default constructor

  void push(int value);
  void pop();
  void print(std::ostream& out) const;
  int top() const;
  bool empty() const;
private:
  llnode* topn;
}
```

Zombie Elements

```
{
  stack s1; // local variable
  s1.push (1);
  s1.push (3);
  s1.push (2);
  std::cout << s1 << "\n"; // 2 3 1
}
// s1 has died (become invalid)...
```

Zombie Elements

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  stack s1; // local variable
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- ...but the three elements of the stack `s1` continue to live (memory leak)!

Zombie Elements

```
{
  stack s1; // local variable
  s1.push (1);
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  s1.push (2);
  std::cout << s1 << "\n"; // 2 3 1
}
// s1 has died (become invalid)...
```

- ...but the three elements of the stack `s1` continue to live (memory leak)!
- They should be released together with `s1`.

The Destructor

- The Destructor of class *T* is the unique member function with declaration

  $$\sim T\ (\ );$$

- is automatically called when the memory duration of a class object ends – i.e. when **`delete`** is called on an object of type **`T*`** or when the enclosing scope of an object of type **`T`** ends.
- If no destructor is declared, it is automatically generated and calls the destructors for the member variables (pointers **`topn`**, no effect – reason for zombie elements

Using a Destructor, it Works

```
// POST: the dynamic memory of *this is deleted
stack::~stack(){
  while (topn != nullptr){
    llnode* t = topn;
    topn = t->next;
    delete t;
  }
}
```

Using a Destructor, it Works

```
// POST: the dynamic memory of *this is deleted
stack::~stack(){
  while (topn != nullptr){
    llnode* t = topn;
    topn = t->next;
    delete t;
  }
}
```

- automatically deletes all stack elements when the stack is being released

Using a Destructor, it Works

```
// POST: the dynamic memory of *this is deleted
stack::~stack(){
  while (topn != nullptr){
    llnode* t = topn;
    topn = t->next;
    delete t;
  }
}
```

- automatically deletes all stack elements when the stack is being released
- Now our stack class seems to follow the guideline “dynamic memory” (?)

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n";

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

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stack s1;
s1.push (1);
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s1.pop ();
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std::cout << s1 << "\n";

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stack s1;
s1.push (1);
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std::cout << s1 << "\n";

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

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stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop (); // Oops, crash!
```

Stack Done?

Obviously not...

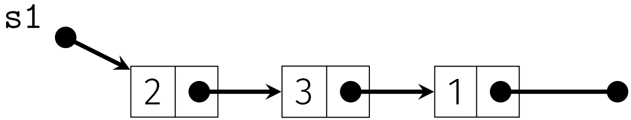
```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop (); // Oops, crash!
```

What has gone wrong?

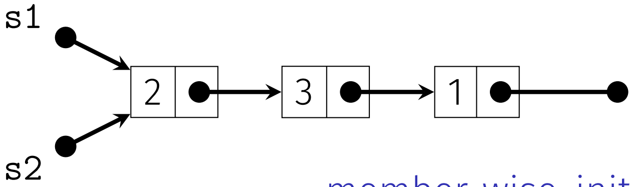


```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?



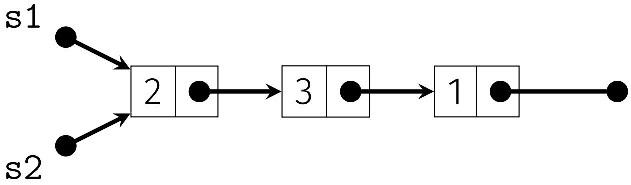
member-wise initialization: copies the topn pointer only.

```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?

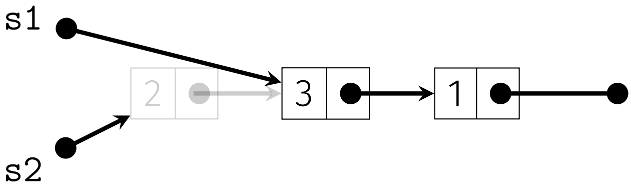


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?

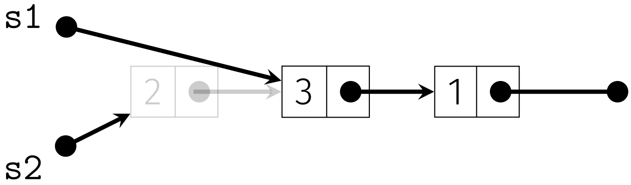


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?



```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

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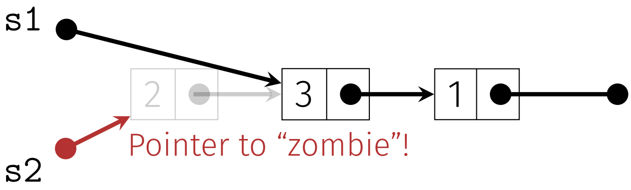
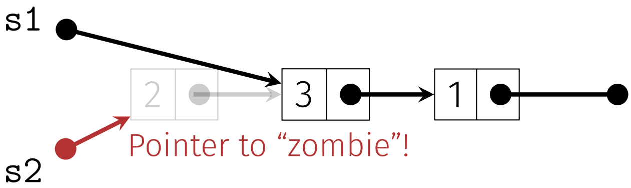


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

What has gone wrong?



Pointer to “zombie”!

```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();  // Oops, crash!
```

The actual problem

Already this goes wrong:

```
{
  stack s1;
  s1.push(1);
  stack s2 = s1;
}
```

When leaving the scope, both stacks are deconstructed. But both stacks try to delete the same data, because both stacks have **access to the same pointer**.

Possible solutions

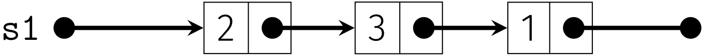
Smart-Pointers (we will not go into details here):

- Count the number of pointers referring to the same objects and delete only when that number goes down to 0 **`std::shared_pointer`**
- Make sure that not more than one pointer can point to an object: **`std::unique_pointer`**.

or:

- We make a real copy of all data – as discussed below.

We make a real copy

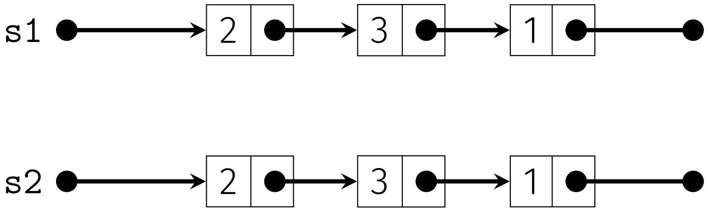


```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

We make a real copy



```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

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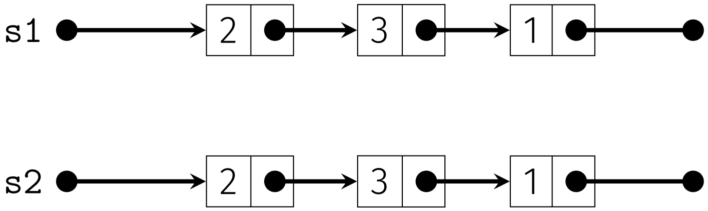

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...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
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s2.pop ();
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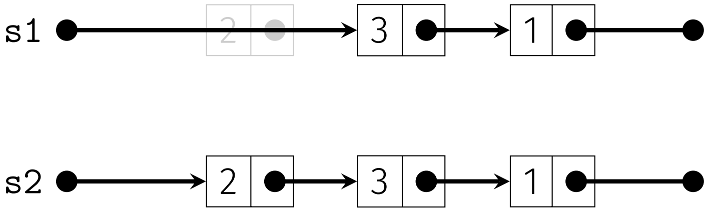

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s1.pop ();
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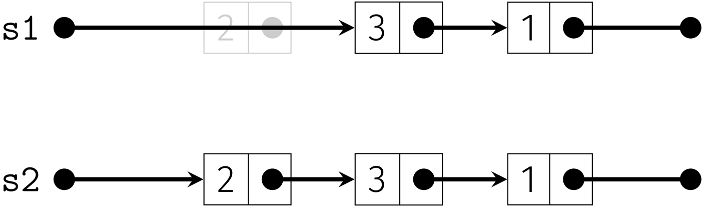

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std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
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s2.pop ();
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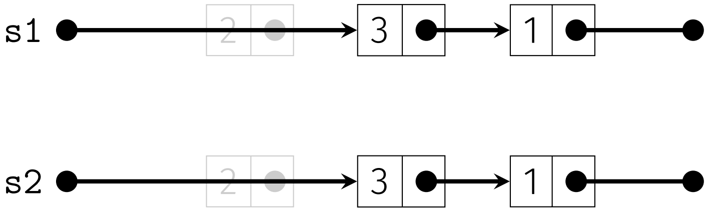

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std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
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s2.pop ();
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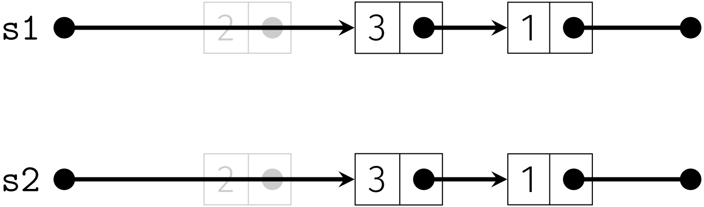


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();  // ok
```

The Copy Constructor

- The copy constructor of a class *T* is the unique constructor with declaration

  T (**`const`** *T***&** x);

- is automatically called when values of type *T* are initialized with values of type **`T`**

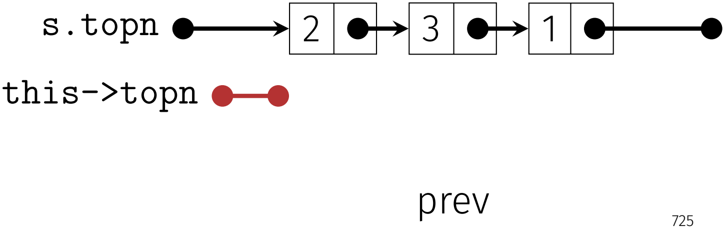
```
T x = t;
T x (t);
```

  (t of type T)

- If there is no copy-constructor declared then it is generated automatically (and initializes member-wise – reason for the problem above

It works with a Copy Constructor

```
// POST: *this is initialized with a copy of s
stack::stack (const stack& s) : topn (nullptr) {
  if (s.topn == nullptr) return;
  topn = new llnode(s.topn->value, nullptr);
  llnode* prev = topn;
  for(llnode* n = s.topn->next; n != nullptr; n = n->next){
    llnode* copy = new llnode(n->value, nullptr);
    prev->next = copy;
    prev = copy;
  }
}
```



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    prev = copy;
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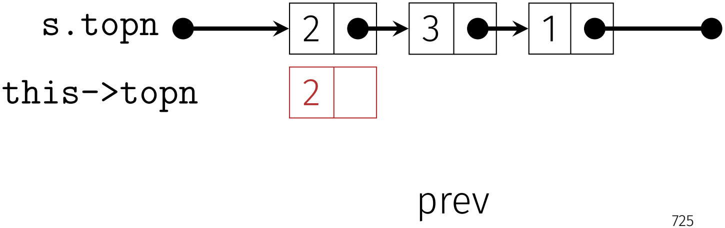
}
```



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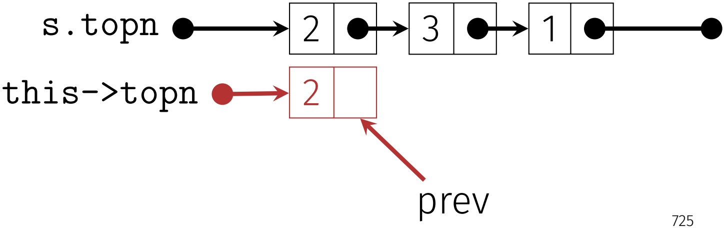
}
```



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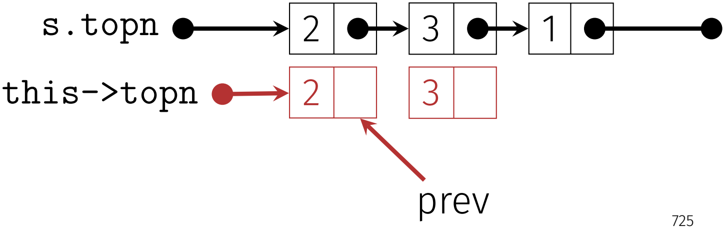
}
```



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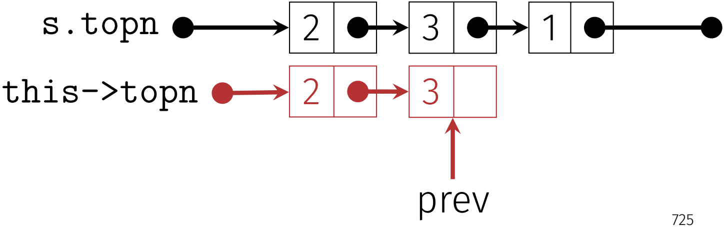

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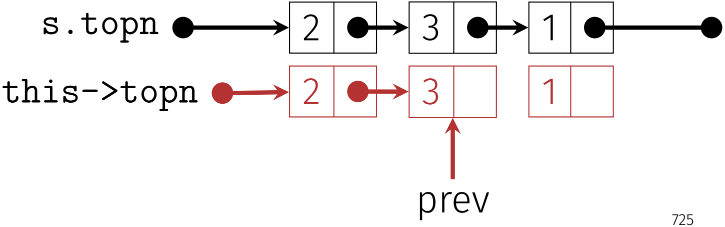
}
```



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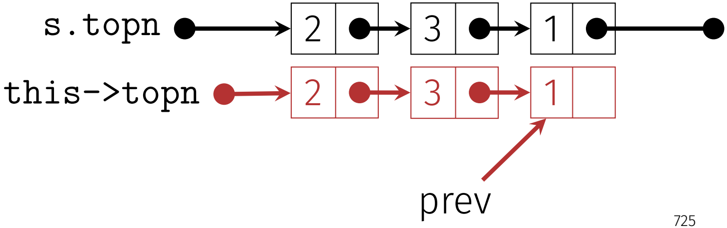

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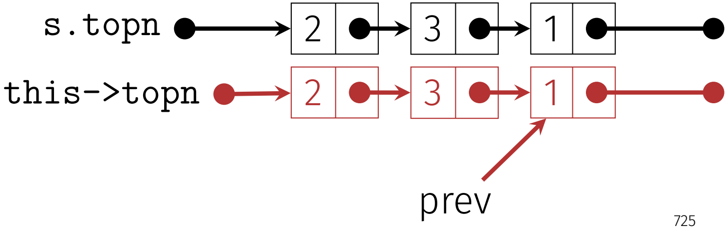

Initialization $\neq$ Assignment!

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1; // Initialisierung


s1.pop ();
std::cout << s1 << "\n"; // 3 1
s2.pop (); // ok:  Copy Constructor!
```

Initialization $\neq$ Assignment!

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2;
s2 = s1; // Zuweisung

s1.pop ();
std::cout << s1 << "\n"; // 3 1
s2.pop (); // Oops, Crash!
```

The Assignment Operator

- Overloading **`operator=`** as a member function
- Like the copy-constructor without initializer, but additionally
  - Releasing memory for the “old” value
  - Check for self-assignment (`s1=s1`) that should not have an effect

- If there is no assignment operator declared it is automatically generated (and assigns member-wise – reason for the problem above

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//           copy of s (right operand)
stack& stack::operator= (const stack& s)
```

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//            copy of s (right operand)
stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
```

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//             copy of s (right operand)
stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
    stack copy = s; // Copy Construction
```

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//            copy of s (right operand)
stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
    stack copy = s; // Copy Construction
    std::swap(topn, copy.topn); // now copy has the garbage!
```

It works with an Assignment Operator!

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stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
    stack copy = s; // Copy Construction
    std::swap(topn, copy.topn); // now copy has the garbage!
  } // copy is cleaned up -> deconstruction
  return *this; // return as L-Value (convention)
}
```

It works with an Assignment Operator!

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    std::swap(topn, copy.topn); // now copy has the garbage!
  } // copy is cleaned up -> deconstruction
  return *this; // return as L-Value (convention)
}
```

Cooool trick! 🙂

Done

```
class stack{
public:
  stack(); // constructor
  ~stack(); // destructor
  stack(const stack& s); // copy constructor
  stack& operator=(const stack& s); // assignment operator

  void push(int value);
  void pop();
  int top() const;
  bool empty() const;
  void print(std::ostream& out) const;
private:
  llnode* topn;
}
```

Dynamic Datatype

- Type that manages dynamic memory (e.g. our class for a stack)
- Minimal Functionality:
  - Constructors
  - Destructor
  - Copy Constructor
  - Assignment Operator

Dynamic Datatype

- Type that manages dynamic memory (e.g. our class for a stack)
- Minimal Functionality:
  - Constructors
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  - Copy Constructor
  - Assignment Operator

Rule of Three: if a class defines at least one of them, it must define all three

Trees

Trees are

- Generalized lists: nodes can have more than one successor
- Special graphs: graphs consist of nodes and edges. A tree is a fully connected, directed, acyclic graph.

Trees

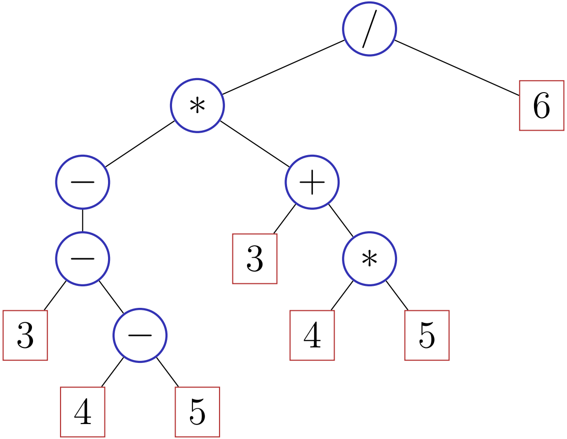
Use

- Decision trees: hierarchic representation of decision rules
- Code tress: representation of a code, e.g. morse alphabet, huffman code
- Search trees: allow efficient searching for an element by value
- syntax trees: parsing and traversing of expressions, e.g. in a compiler

Trees are treated in more detail in other courses (Datastructures and Algorithms (CSE), Algorithms and Complexity (Math Bachelor))

(Expression) Trees

`-(3-(4-5))*(3+4*5)/6`



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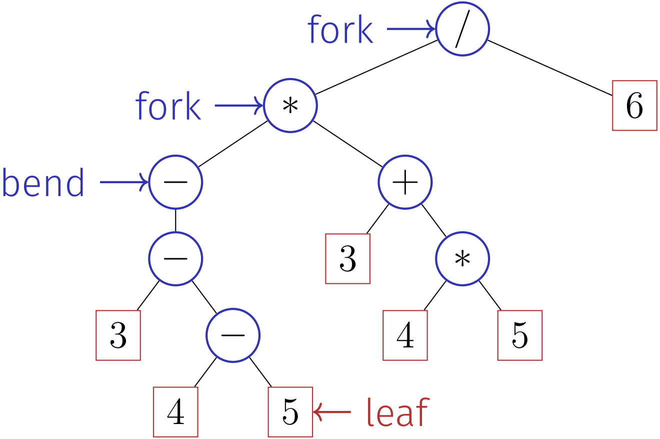

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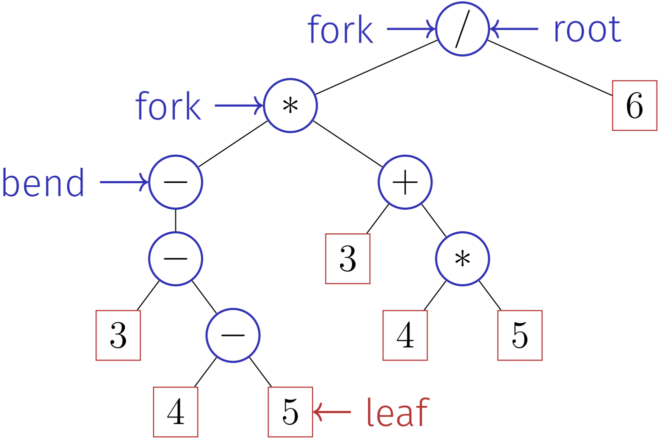

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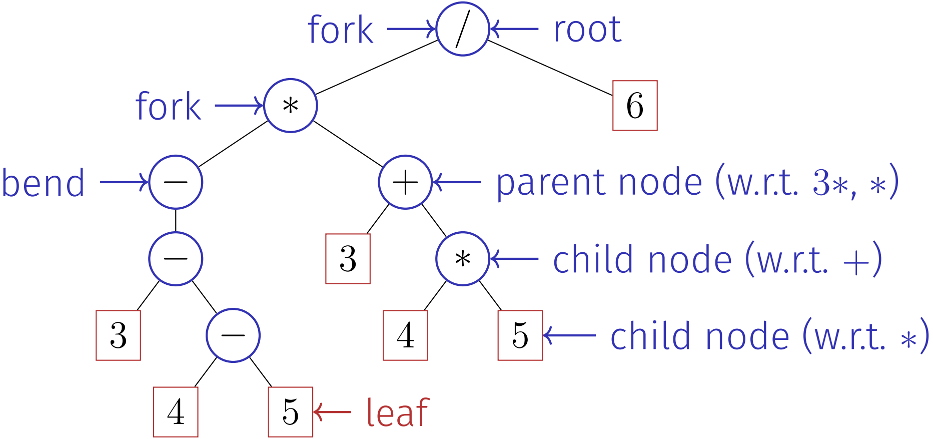
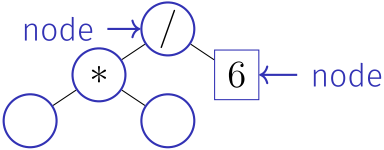


Nodes: Forks, Bends or Leaves



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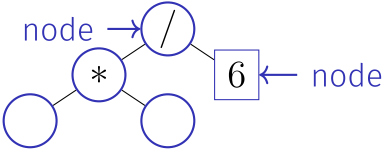

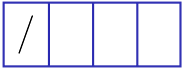

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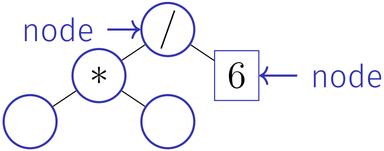

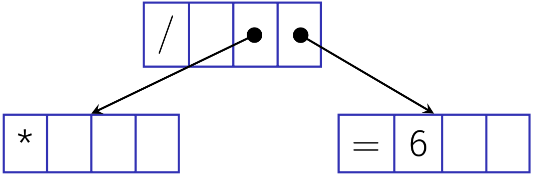

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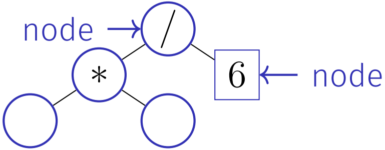

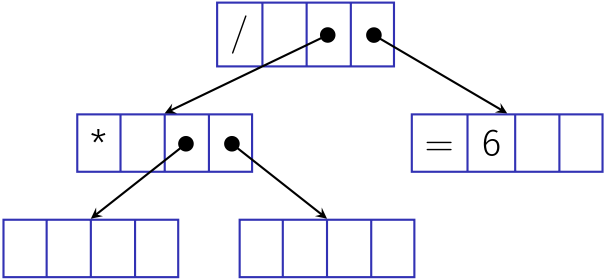

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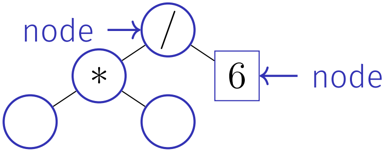

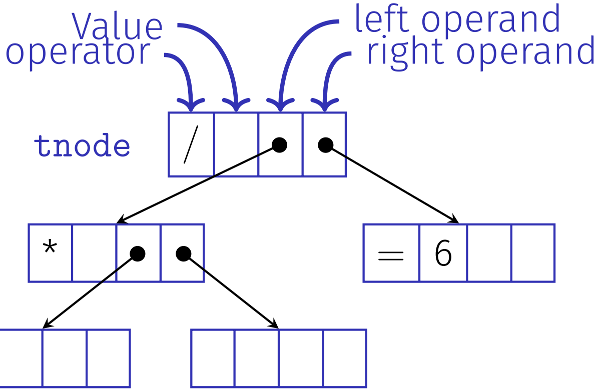

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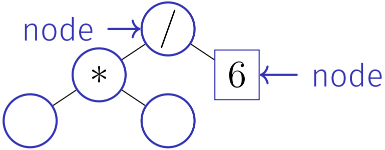
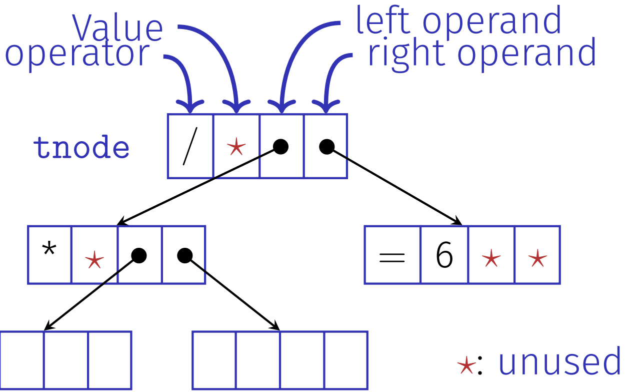
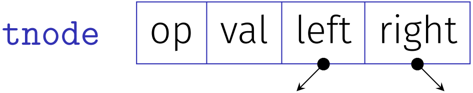




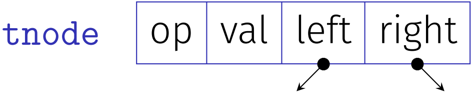
Nodes (struct tnode)



```
struct tnode {
  char op; // leaf node: op is ’=’
           // internal node: op is ’+’, ’-’, ’*’ or ’/’
  double val;
  tnode* left;
  tnode* right;

  tnode(char o, double v, tnode* l, tnode* r)
    : op(o), val(v), left(l), right(r) {}
};
```

Nodes (struct tnode)



```
struct tnode {
  char op; // leaf node: op is '='
           // internal node: op is '+', '-', '*' or '/'
  double val;
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  tnode* right;

  tnode(char o, double v, tnode* l, tnode* r)
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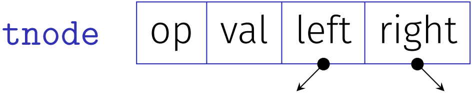
};
```

Nodes (struct tnode)



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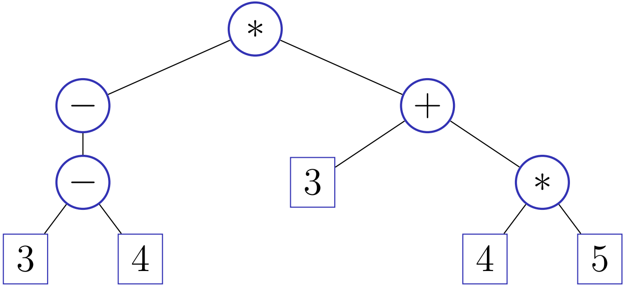
};
```

Size = Count Nodes in Subtrees



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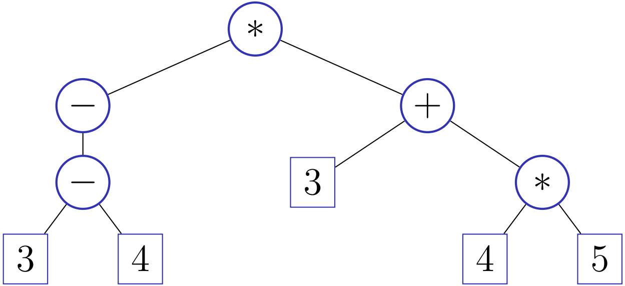
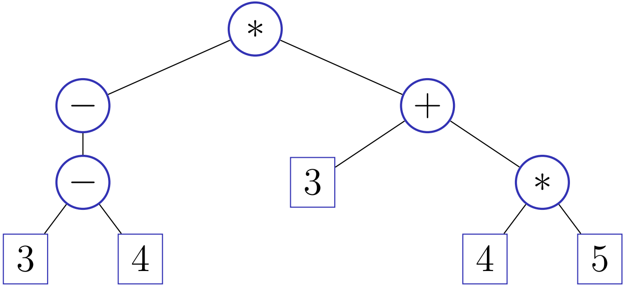


- Size of a leave: 1
- Size of other nodes: 1 + sum of child nodes’ size

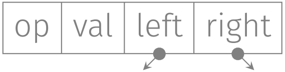
Size = Count Nodes in Subtrees



- Size of a leave: 1
- Size of other nodes: 1 + sum of child nodes’ size
- E.g. size of the "+"-node is 5

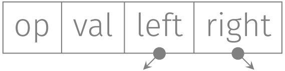
Count Nodes in Subtrees

```
// POST: returns the size (number of nodes) of
//       the subtree with root n
int size (const tnode* n) {
  if (n){ // shortcut for n != nullptr
    return size(n->left) + size(n->right) + 1;
  }
  return 0;
}
```



op	val	left	right

Evaluate Subtrees

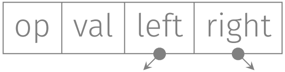


op | val | left | right

```
// POST: evaluates the subtree with root n
double eval(const tnode* n){
  assert(n);
  if (n->op == '=') return n->val; ← leaf...
  double l = 0;                      ...or fork:
  if (n->left) l = eval(n->left); ⟵ op unary, or left branch
  double r = eval(n->right);⟵——— right branch
  switch(n->op){
    case '+': return l+r;
    case '-': return l-r;
    case '*': return l*r;
    case '/': return l/r;
    default: return 0;
  }
}
```

Cloning Subtrees

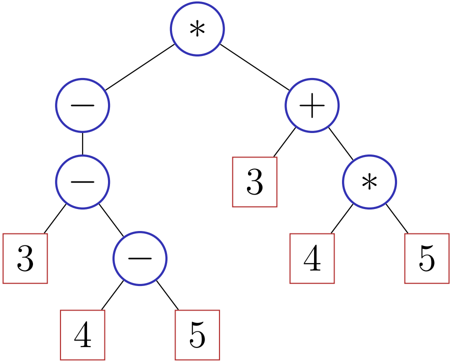
```
// POST: a copy of the subtree with root n is made
//       and a pointer to its root node is returned
tnode* copy (const tnode* n) {
  if (n == nullptr)
    return nullptr;
  return new tnode (n->op, n->val, copy(n->left), copy(n->right));
}
```



op val left right

Felling Subtrees

```
// POST: all nodes in the subtree with root n are deleted
void clear(tnode* n) {
  if(n){
    clear(n->left);
    clear(n->right);
    delete n;
  }
}
```



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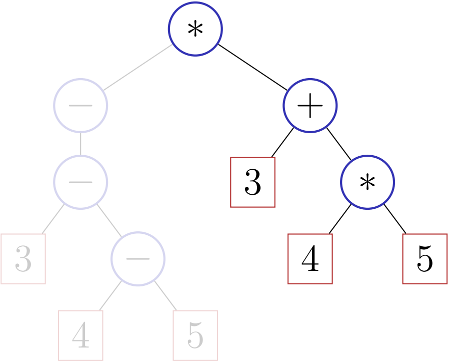

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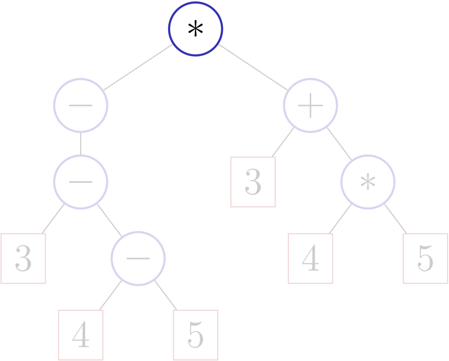

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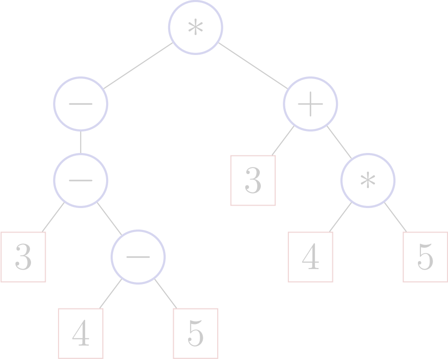

Using Expression Subtrees

```
// Construct a tree for 1 - (-(3 + 7))
tnode* n1 = new tnode(’=’, 3, nullptr, nullptr);
tnode* n2 = new tnode(’=’, 7, nullptr, nullptr);
tnode* n3 = new tnode(’+’, 0, n1, n2);
tnode* n4 = new tnode(’-’, 0, nullptr, n3);
tnode* n5 = new tnode(’=’, 1, nullptr, nullptr);
tnode* root = new tnode(’-’, 0, n5, n4);

// Evaluate the overall tree
std::cout << "1 - (-(3 + 7)) = " << eval(root) << ’\n’;

// Evaluate a subtree
std::cout << "3 + 7 = " << eval(n3) << ’\n’;

clear(root); // free memory
```

Planting Trees

```
class texpression {
public:
    texpression (double d)
        : root (new tnode ('=', d, 0, 0)) {}
    ...
private:
    tnode* root;
};
```

creates a tree with one leaf

Letting Trees Grow

```
texpression& texpression::operator-= (const texpression& e)
{
  assert (e.root);
  root = new tnode ('-', 0, root, copy(e.root));
  return *this;
}
```

root
*this
e.root
e

Letting Trees Grow

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texpression& texpression::operator-= (const texpression& e)
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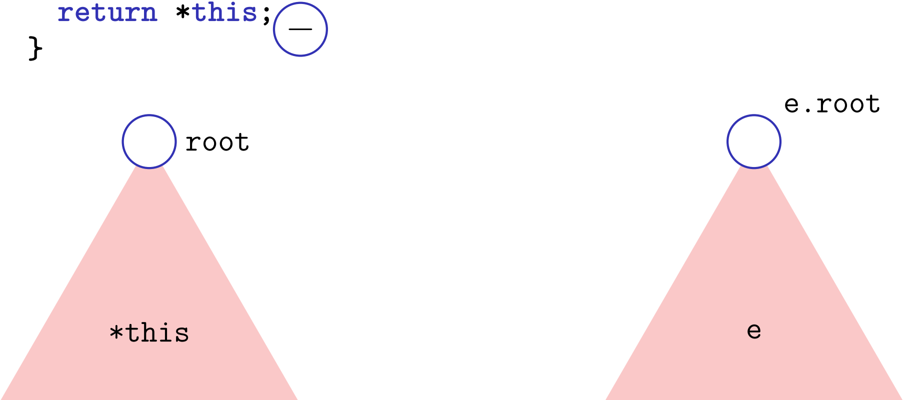

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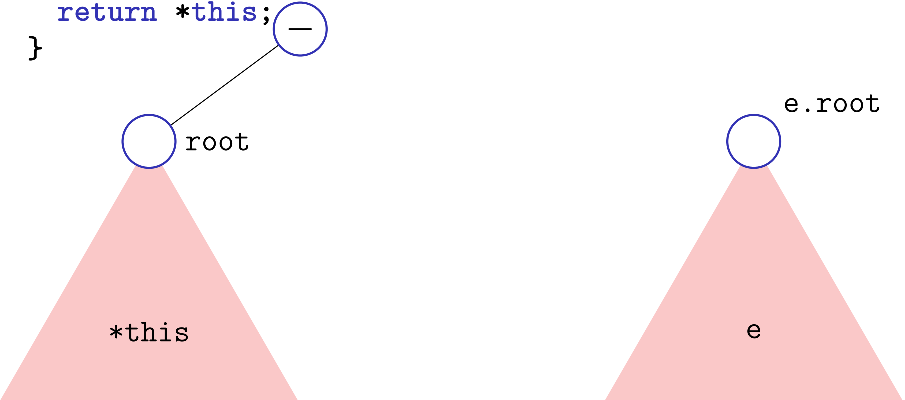

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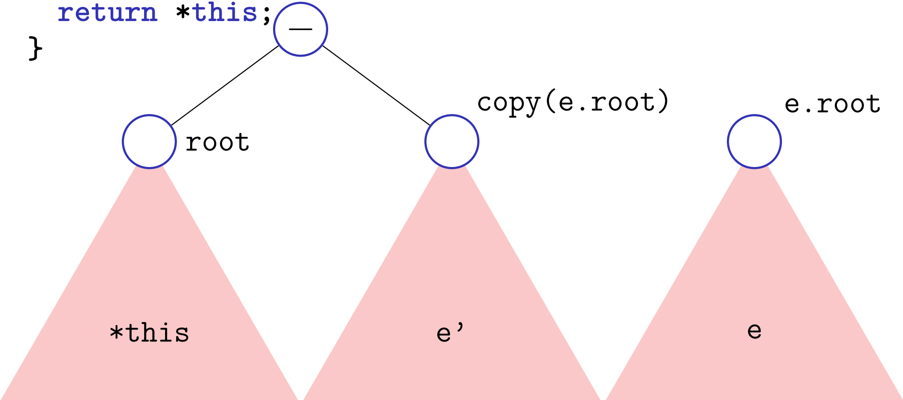

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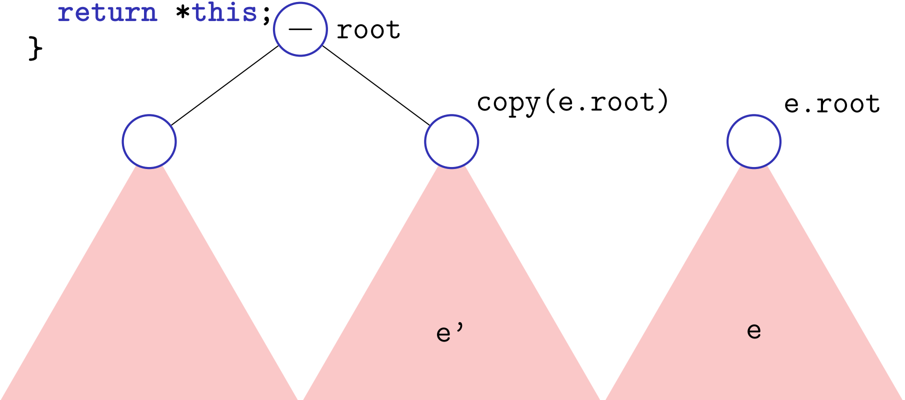

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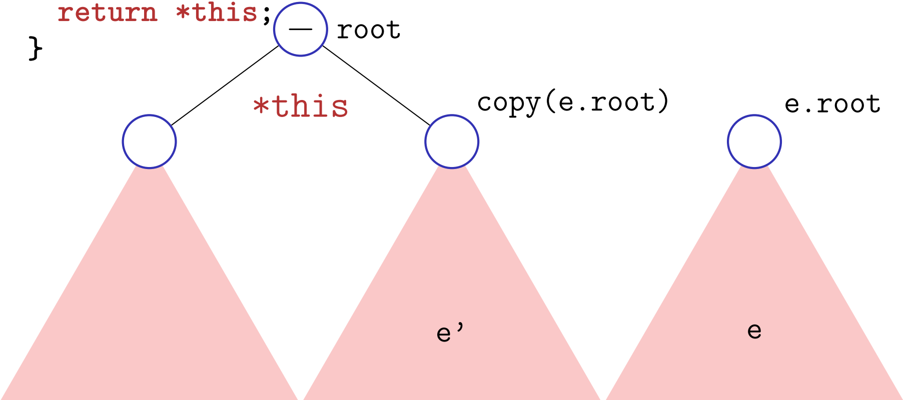

Raising Trees

```
texpression operator- (const texpression& l,
                       const texpression& r){
    texpression result = l;
    return result -= r;
}

texpression a = 3;
texpression b = 4;
texpression c = 5;
texpression d = a-b-c;
```

Raising Trees

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3

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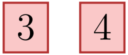

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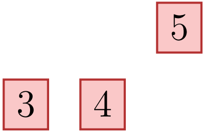

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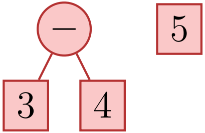

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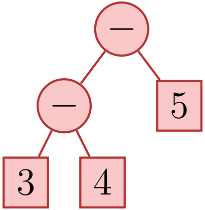

Rule of three: Clone, reproduce and cut trees

```
texpression::~texpression(){
  clear(root);
}

texpresssion::texpression (const texpression& e)
     : root(copy(e.root)) { }

texpression& texpression::operator=(const texpression& e){
  if (root != e.root){
    texpression cp = e;
    std::swap(cp.root, root);
  }
  return *this;
}
```

Concluded

```
class texpression{
public:
  texpression (double d); // constructor
  ~texpression(); // destructor
  texpression (const texpression& e); // copy constructor
  texpression& operator=(const texpression& e); // assignment op
  texpression operator-();
  texpression& operator-=(const texpression& e);
  texpression& operator+=(const texpression& e);
  texpression& operator*=(const texpression& e);
  texpression& operator/=(const texpression& e);
  double evaluate();
private:
 tnode* root;
};
```

From values to trees!

```
// term = factor { "*" factor | "/" factor }
double term (std::istream& is){
  double value = factor (is);
  while (true) {
    if (consume (is, ’*’))
      value *= factor (is);
    else if (consume (is, ’/’))
      value /= factor (is);
  else
      return value;
  }
}
```

calculator.cpp
(expression value)

From values to trees!

```
using number_type = double;
```

```
// term = factor { "*" factor | "/" factor }
number_type term (std::istream& is){
  number_type value = factor (is);
  while (true) {
    if (consume (is, '*'))
      value *= factor (is);
    else if (consume (is, '/'))
      value /= factor (is);
  else
      return value;
  }
}
```

`double_calculator.cpp`
(expression value)

From values to trees!

```
using number_type = texpression ;

// term = factor { "*" factor | "/" factor }
number_type term (std::istream& is){
  number_type value = factor (is);
  while (true) {
    if (consume (is, '*'))
      value *= factor (is);
    else if (consume (is, '/'))
      value /= factor (is);
  else
      return value;
  }
}
```

double_calculator.cpp (expression value)
→
texpression_calculator.cpp (expression tree)

Concluding Remark

- In this lecture, we have intentionally refrained from implementing member functions in the node classes of the list or tree.[7]
- When there is inheritace and polymorphism used, the implementation of the functionality such as evaluate, print, clear (etc:.) is better implemented in member functions.
- In any case it is not a good idea to implement the memory management of the composite data strcuture list or tree within the nodes.

[7] Parts of the implementations are even simpler (because the case `n==nullptr` can be caught more easily

24. Subtyping, Inheritance and Polymorphism

Expression Trees, Separation of Concerns and Modularisation, Type Hierarchies, Virtual Functions, Dynamic Binding, Code Reuse, Concepts of Object-Oriented Programming

Last Week: Expression Trees

- Goal: Represent arithmetic expressions, e.g.

```
2 + 3 * 2
```

Last Week: Expression Trees

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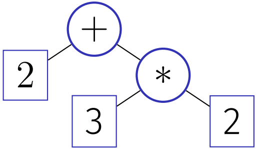
- Arithmetic expressions form a *tree structure*



Last Week: Expression Trees

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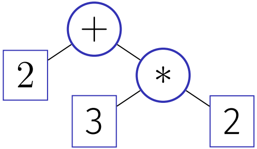

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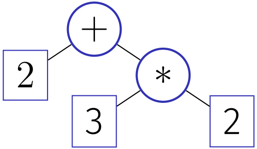
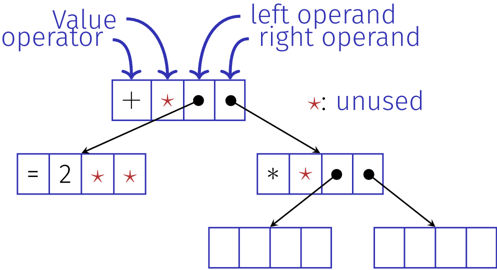


- Expression trees comprise *different* nodes: literals (e.g. 2), binary operators (e.g. $+$), unary operators (e.g. $\sqrt{\ }$), function applications (e.g. $\cos$), etc.

Disadvantages

Implemented via *a single* node type:

```
struct tnode {
  char op; // Operator ('=' for literals)
  double val; // Literal's value
  tnode* left; // Left child (or nullptr)
  tnode* right; // ...
  ...
};
```



Observation: **`tnode`** is the "sum" of all required nodes (constants, addition, ...) ⇒ memory wastage, inelegant

Disadvantages

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Observation: `tnode` is the “sum” of all required nodes – and every function must “dissect” this “sum”, e.g.:

```
double eval(const tnode* n) {
  if (n->op == ’=’) return n->val; // n is a constant
  double l = 0;
  if (n->left) l = eval(n->left); // n is not a unary operator
  double r = eval(n->right);
  switch(n->op) {
    case ’+’: return l+r; // n is an addition node
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    ...
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    ...
```

⇒ Complex, and therefore error-prone

Disadvantages

```
struct tnode {
  char op;
  double val;
  tnode* left;
  tnode* right;
  ...
};
```

```
double eval(const tnode* n) {
  if (n->op == ’=’) return n->val;
  double l = 0;
  if (n->left) l = eval(n->left);
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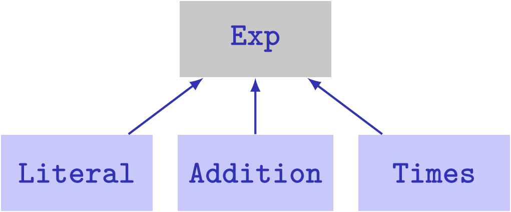
  switch(n->op) {
    case ’+’: return l+r;
    case ’*’: return l*r;
    ...
```

This code isn’t *modular* – we’ll change that today!

New Concepts Today

1. Subtyping

- Type hierarchy: **`Exp`** represents general expressions, **`Literal`** etc. are concrete expression



New Concepts Today

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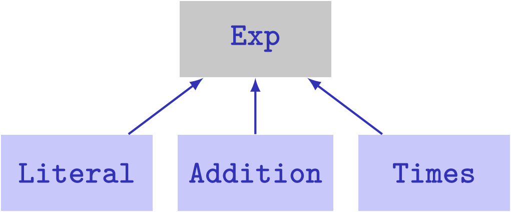
- Every **`Literal`** etc. also is an **`Exp`** (subtype relation)



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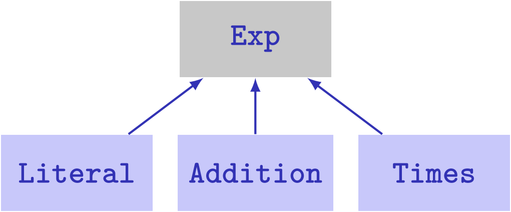


- That's why a **`Literal`** etc. can be used everywhere, where an **`Exp`** is expected:

```
Exp* e = new Literal(132);
```

New Concepts Today

2. Polymorphism and Dynamic Dispatch

- A variable of *static* type **Exp** can “host” expressions of different *dynamic* types:

```
Exp* e = new Literal(2); // e is the literal 2
e = new Addition(e, e); // e is the addition 2 + 2
```

New Concepts Today

2. Polymorphism and Dynamic Dispatch

- A variable of *static* type **Exp** can “host” expressions of different *dynamic* types:

```
Exp* e = new Literal(2); // e is the literal 2
e = new Addition(e, e); // e is the addition 2 + 2
```

- Executed are the member functions of the *dynamic* type:

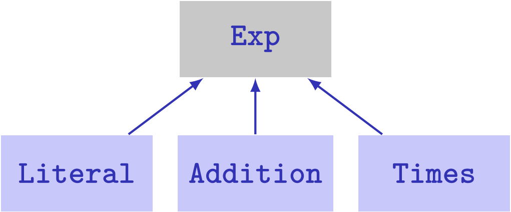
```
Exp* e = new Literal(2);
std::cout << e->eval(); // 2

e = new Addition(e, e);
std::cout << e->eval(); // 4
```

New Concepts Today

3. Inheritance

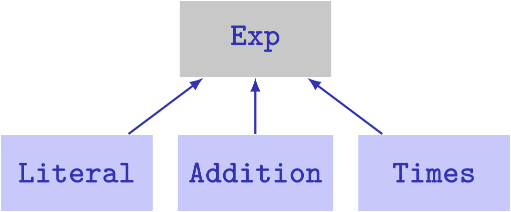
- Certain functionality is shared among type hierarchy members



New Concepts Today

3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (**`Addition`**, **`Times`**):



$$1 + \mathit{size}(\mathit{left\ operand}) + \mathit{size}(\mathit{right\ operand})$$

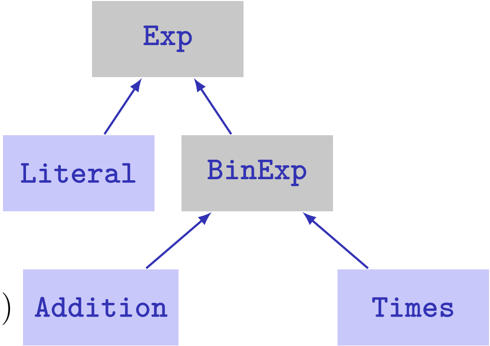
New Concepts Today

3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (**`Addition`**, **`Times`**):

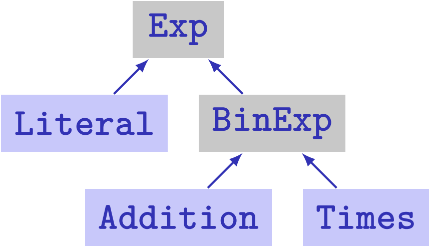
$$1 + \mathit{size}(\mathit{left\ operand}) + \mathit{size}(\mathit{right\ operand})$$

⇒ Implement functionality once, and let subtypes *inherit* it



Advantages

- Subtyping, inheritance and dynamic binding enable *modularisation through spezialisation*

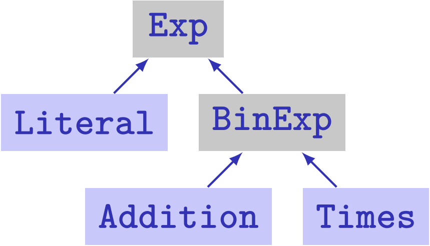


```
Exp* e = new Literal(2);
std::cout << e->eval();

e = new Addition(e, e);
std::cout << e->eval();
```

Advantages

- Subtyping, inheritance and dynamic binding enable *modularisation through spezialisation*
- Inheritance enables sharing common code across modules $\Rightarrow$ *avoid code duplication*



```
Exp* e = new Literal(2);
std::cout << e->eval();

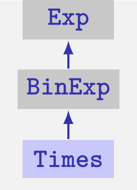
e = new Addition(e, e);
std::cout << e->eval();
```

Syntax and Terminology

```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```

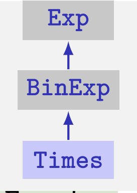


Syntax and Terminology

```
struct Exp {
  ...
}

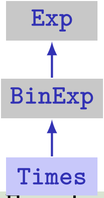
struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```



Note: Today, we focus on the new concepts (subtyping, ...) and ignore the orthogonal aspect of encapsulation (**`class`**, **`private`** vs. **`public`** member variables)

Syntax and Terminology



```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```

- **`BinExp`** is a *subclass*[1] of **`Exp`**

[1] derived class, child class

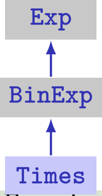
[2] base class, parent class

Syntax and Terminology

```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```



- **`BinExp`** is a *subclass*[1] of **`Exp`**
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[1] derived class, child class

[2] base class, parent class

Syntax and Terminology

```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
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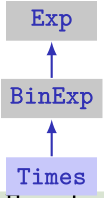
struct Times : public BinExp {
  ...
}
```



- **`BinExp`** is a *subclass*[1] of **`Exp`**
- **`Exp`** is the *superclass*[2] of **`BinExp`**
- **`BinExp`** *inherits* from **`Exp`**

[1] derived class, child class

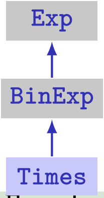
[2] base class, parent class

Syntax and Terminology

```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```



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- **BinExp** *publicly* inherits from **Exp** (**public**), that's why **BinExp** is a *subtype* of **Exp**

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Syntax and Terminology

```
struct Exp {
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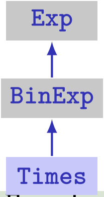
struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```



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- Analogously: **Times** and **BinExp**

[1] derived class, child class

[2] base class, parent class

Syntax and Terminology

```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
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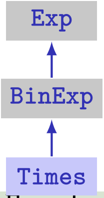
struct Times : public BinExp {
  ...
}
```



- **`BinExp`** is a *subclass*[1] of **`Exp`**
- **`Exp`** is the *superclass*[2] of **`BinExp`**
- **`BinExp`** *inherits* from **`Exp`**
- **`BinExp`** *publicly* inherits from **`Exp`** (**`public`**), that's why **`BinExp`** is a *subtype* of **`Exp`**
- Analogously: **`Times`** and **`BinExp`**
- Subtype relation is transitive: **`Times`** is also a subtype of **`Exp`**

[1] derived class, child class

[2] base class, parent class

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

Activates dynamic dispatch

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

Enforces implementation by derived classes ...

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

.that makes **Exp** an *abstract* class

Enforces implementation by derived classes . . .

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {
  double val;

  Literal(double v);
  int size() const;
  double eval() const;
};
```

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {
  double val;

  Literal(double v);
  int size() const;
  double eval() const;
};
```

Literal inherits from **Exp** ...

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {
  double val;

  Literal(double v);
  int size() const;
  double eval() const;
};
```

Literal inherits from **Exp** . . .

. . . but is otherwise just a regular class

`Literal`: Implementation

```cpp
Literal::Literal(double v): val(v) {}
```

Literal: Implementation

```
Literal::Literal(double v): val(v) {}
```

```
int Literal::size() const {
  return 1;
}
```

Literal: Implementation

```
Literal::Literal(double v): val(v) {}
```

```
int Literal::size() const {
  return 1;
}
```

```
double Literal::eval() const {
  return this->val;
}
```

Subtyping: A Literal is an Expression

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);
```

Subtyping: A Literal is an Expression

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);
Exp* e = lit; // OK: Literal is a subtype of Exp
```

Subtyping: A Literal is an Expression

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);
Exp* e = lit; // OK: Literal is a subtype of Exp
```

But not vice versa:

```
Exp* e = ...
Literal* lit = e; // ERROR: Exp is not a subtype of Literal
```

Polymorphie: a Literal Behaves Like a Literal

```
struct Exp {
  ...
  virtual double eval();
};

double Literal::eval() {
  return this->val;
}
```

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

Polymorphie: a Literal Behaves Like a Literal

```
struct Exp {
  ...
  virtual double eval();
};

double Literal::eval() {
  return this->val;
}
```

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Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

- *virtual* member function: the *dynamic* (here: **`Literal`**) type determines the member function to be executed
  ⇒ *dynamic binding*

Polymorphie: a Literal Behaves Like a Literal

```
struct Exp {
  ...
  virtual double eval();
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double Literal::eval() {
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```

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Exp* e = new Literal(3);
std::cout << e->eval(); // 3
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- *virtual* member function: the *dynamic* (here: **`Literal`**) type determines the member function to be executed ⇒ *dynamic binding*
- Without **`Virtual`** the *static type* (hier: **`Exp`**) determines which function is executed

Polymorphie: a Literal Behaves Like a Literal

```
struct Exp {
  ...
  virtual double eval();
};

double Literal::eval() {
  return this->val;
}
```

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

- *virtual* member function: the *dynamic* (here: **`Literal`**) type determines the member function to be executed ⇒ *dynamic binding*
- Without **`Virtual`** the *static type* (hier: **`Exp`**) determines which function is executed
- We won’t go into further details

Further Expressions: Addition and Times

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

Further Expressions: Addition and Times

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

```
struct Times : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

Further Expressions: Addition and Times

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

```
int Addition::size() const {
  return 1 + left->size()
           + right->size();
}
```

```
struct Times : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

Further Expressions: Addition and Times

```
struct Addition : public Exp {
  Exp* left; // left operand
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int Addition::size() const {
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```
struct Times : public Exp {
  Exp* left; // left operand
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  ...
};
```

```
int Times::size() const {
  return 1 + left->size()
           + right->size();
}
```

Further Expressions: Addition and Times

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struct Addition : public Exp {
  Exp* left; // left operand
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int Addition::size() const {
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```
struct Times : public Exp {
  Exp* left; // left operand
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  ...
};
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```
int Times::size() const {
  return 1 + left->size()
           + right->size();
}
```

😃 Separation of concerns

Further Expressions: Addition and Times

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
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int Addition::size() const {
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struct Times : public Exp {
  Exp* left; // left operand
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  ...
};
```

```
int Times::size() const {
  return 1 + left->size()
           + right->size();
}
```

😃 Separation of concerns

🤬 Code duplication

Extracting Commonalities ...: BinExp

```
struct BinExp : public Exp {
  Exp* left;
  Exp* right;

  BinExp(Exp* l, Exp* r);
  int size() const;
};
```

```
BinExp::BinExp(Exp* l, Exp* r): left(l), right(r) {}
```

Extracting Commonalities . . . : BinExp

```
struct BinExp : public Exp {
  Exp* left;
  Exp* right;

  BinExp(Exp* l, Exp* r);
  int size() const;
};
```

```
BinExp::BinExp(Exp* l, Exp* r): left(l), right(r) {}
```

```
int BinExp::size() const {
  return 1 + this->left->size() + this->right->size();
}
```

Note: **BinExp** does not implement **eval** and is therefore also an abstract class, just like **Exp**

…Inheriting Commonalities: Addition

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

…Inheriting Commonalities: Addition

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

Addition inherits member variables (**left**, **right**) and functions (**size**) from **BinExp**

...Inheriting Commonalities: `Addition`

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

```
Addition::Addition(Exp* l, Exp* r): BinExp(l, r) {}
```

Calling the *super constructor* (constructor of **`BinExp`**) initialises the member variables **`left`** and **`right`**

...Inheriting Commonalities: Addition

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

```
Addition::Addition(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Addition::eval() const {
  return
    this->left->eval() +
    this->right->eval();
}
```

...Inheriting Commonalities: `Times`

```
struct Times : public BinExp {
  Times(Exp* l, Exp* r);
  double eval() const;
};
```

```
Times::Times(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Times::eval() const {
  return
    this->left->eval() *
    this->right->eval();
}
```

Observation: **`Additon::eval()`** and **`Times::eval()`** are very similar and could also be unified. However, this would require the concept of *functional programming*, which is outside the scope of this course.

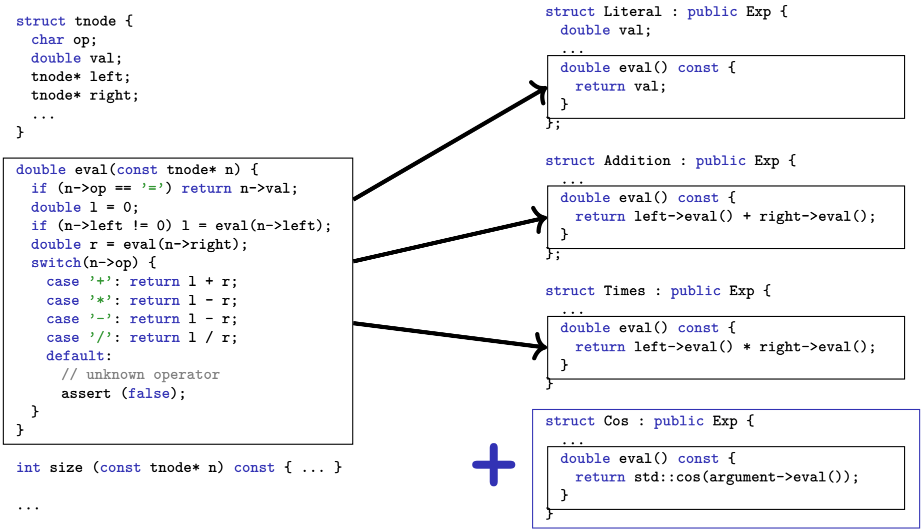
Further Expressions and Operations

- Further expressions, as classes derived from **`Exp`**, are possible, e.g. $-$, $/$, $\sqrt{\ }$, $\cos$, $\log$

Further Expressions and Operations

- Further expressions, as classes derived from **`Exp`**, are possible, e.g. $-$, $/$, $\sqrt{\ }$, $\cos$, $\log$
- A former bonus exercise (included in today's lecture examples on Code Expert) illustrates possibilities: variables, trigonometric functions, parsing, pretty-printing, numeric simplifications, symbolic derivations, . . .

Mission: Monolithic → Modular ✓



```
struct tnode {
  char op;
  double val;
  tnode* left;
  tnode* right;
  ...
}
```

```
double eval(const tnode* n) {
  if (n->op == '=') return n->val;
  double l = 0;
  if (n->left != 0) l = eval(n->left);
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l + r;
    case '*': return l - r;
    case '-': return l - r;
    case '/': return l / r;
    default:
      // unknown operator
      assert (false);
  }
}
```

```
int size (const tnode* n) const { ... }

...
```

```
struct Literal : public Exp {
  double val;
  ...
  double eval() const {
    return val;
  }
};
```

```
struct Addition : public Exp {
  ...
  double eval() const {
    return left->eval() + right->eval();
  }
};
```

```
struct Times : public Exp {
  ...
  double eval() const {
    return left->eval() * right->eval();
  }
}
```

+

```
struct Cos : public Exp {
  ...
  double eval() const {
    return std::cos(argument->eval());
  }
}
```

And there is so much more ...

Not shown/discussed:

- Private inheritance (`class B : ~~public~~ A`)
- Subtyping and polymorphism without pointers
- Non-virtuell member functions and static dispatch (`~~virtual~~ double eval()`)
- Overriding inherited member functions and invoking overridden implementations
- Multiple inheritance
- ...

Object-Oriented Programming

In the last 3rd of the course, several concepts of *object-oriented programming* were introduced, that are briefly summarised on the upcoming slides.

Encapsulation (weeks 10-13):

- Hide the implementation details of types (private section) from users
- Definition of an interface (public area) for accessing values and functionality in a controlled way
- Enables ensuring invariants, and the modification of implementations without affecting user code

Object-Oriented Programming

Subtyping (week 14):

- Type hierarchies, with super- and subtypes, can be created to model relationships between more abstract and more specialised entities
- A subtype supports at least the functionality that its supertype supports – typically more, though, i.e. a subtype extends the interface (public section) of its supertype
- That's why supertypes can be used anywhere, where subtypes are required . . .
- . . . and functions that can operate on more abstract type (supertypes) can also operate on more specialised types (subtypes)
- The streams introduced in week 7 form such a type hierarchy: **`ostream`** is the abstract supertyp, **`ofstream`** etc. are specialised subtypes

Object-Oriented Programming

Polymorphism and *dynamic binding* (week 14):

- A pointer of static typ T_1 can, at runtime, point to objects of (dynamic) type T_2, if T_2 is a subtype of T_1
- When a virtual member function is invoked from such a pointer, the dynamic type determines which function is invoked
- I.e.: despite having the same static type, a different behaviour can be observed when accessing the common interface (member functions) of such pointers
- In combination with subtyping, this enables adding further concrete types (streams, expressions, ...) to an existing system, without having to modify the latter

Object-Oriented Programming

Inheritance (week 14):

- Derived classes inherit the functionality, i.e. the implementation of member functions, of their parent classes
- This enables sharing common code and thereby avoids code duplication
- An inherited implementation can be overridden, which allows derived classes to behave differently than their parent classes (not shown in this course)

25. Conclusion

Purpose and Format

Name the most important key words to each chapter.
Checklist: “does every notion make some sense for me?”

- Ⓜ motivating example for each chapter
- Ⓒ concepts that do not depend from the implementation (language)
- Ⓛ language (C++): all that depends on the chosen language
- Ⓔ examples from the lectures

Kapitelüberblick

- 1. Introduction
- 2. Integers
- 3. Booleans
- 4. Defensive Programming
- 5./6. Control Statements
- 7./8. Floating Point Numbers
- 9./10. Functions
- 11. Reference Types
- 12./13. Vectors and Strings
- 14./15. Recursion
- 16. Structs and Overloading
- 17. Classes
- 18./19. Dynamic Datastructures
- 20. Containers, Iterators and Algorithms
- 21. Dynamic Datatypes and Memory Management
- 22. Subtyping, Polymorphism and Inheritance

1. Introduction

M
- Euclidean algorithm

C
- algorithm, Turing machine, programming languages, compilation, syntax and semantics
- values and effects, fundamental types, literals, variables

L
- include directive `#include <iostream>`
- main function `int main(){...}`
- comments, layout `// Kommentar`
- types, variables, L-value `a` , R-value `a+b`
- expression statement `b=b*b;` , declaration statement `int a;`, return statement `return 0;`

2. Integers

Ⓜ

- Celsius to Fahrenheit

Ⓒ

- associativity and precedence, arity
- expression trees, evaluation order
- arithmetic operators
- binary representation, hexadecimal numbers
- signed numbers, twos complement

Ⓛ

- arithmetic operators `9 * celsius / 5 + 32`
- increment / decrement `expr++`
- arithmetic assignment `expr1 += expr2`
- conversion `int` ↔ `unsigned int`

Ⓔ

- Celsius to Fahrenheit, equivalent resistance

3. Booleans

Ⓒ

- Boolean functions, completeness
- DeMorgan rules

Ⓛ

- the type `bool`
- logical operators `a && !b`
- relational operators `x < y`
- precedences `7 + x < y && y != 3 * z`
- short circuit evaluation `x != 0 && z / x > y`
- the **`assert`**-statement, **`#include <cassert>`**

Ⓔ

- Div-Mod identity.

4. Definsive Programming

Ⓒ
- Assertions and Constants

Ⓛ
- The **`assert`**-statement, **`#include <cassert>`**
- **`const int speed_of_light=2999792458`**

Ⓔ
- Assertions for the GCD

5./6. Control Statements

M

- linear control flow vs. interesting programs

C

- selection statements, iteration statements
- (avoiding) endless loops, halting problem
- Visibility and scopes, automatic memory
- equivalence of iteration statement

L

- if statements `if (a % 2 == 0) {..}`
- for statements `for (unsigned int i = 1; i <= n; ++i) ...`
- while and do-statements `while (n > 1) {...}`
- blocks and branches `if (a < 0) continue;`
- Switch statement `switch(grade) {case 6: }`

E

- sum computation (Gauss), prime number tests, Collatz sequence, Fibonacci numbers, calculator, output grades

7./8. Floating Point Numbers

Ⓜ

- correct computation: Celsius / Fahrenheit

Ⓒ

- fixpoint vs. floating point
- holes in the value range
- compute using floating point numbers
- floating point number systems, normalisation, IEEE standard 754
- *guidelines for computing with floating point numbers*

Ⓛ

- types `float`, `double`
- floating point literals `1.23e-7f`

Ⓔ

- Celsius/Fahrenheit, Euler, Harmonic Numbers

9./10. Functions

Ⓜ

- Computation of Powers

Ⓒ

- Encapsulation of Functionality
- functions, formal arguments, arguments
- scope, forward declarations
- procedural programming, modularization, separate compilation
- *Stepwise Refinement*

Ⓛ

- declaration and definition of functions
  `double pow(double b, int e){ ... }`
- function call `pow (2.0, -2)`
- the type `void`

Ⓔ

- powers, perfect numbers, minimum, calendar

11. Reference Types

Ⓜ

- Swap

Ⓒ

- value- / reference- semantics, pass by value, pass by reference, return by reference
- lifetime of objects / temporary objects
- constants

Ⓛ

- reference type `int& a`
- call by reference, return by reference `int& increment (int& i)`
- const guideline, const references, reference guideline

Ⓔ

- swap, increment

12./13. Vectors and Strings

Ⓜ

- Iterate over data: sieve of erathosthenes

Ⓒ

- vectors, memory layout, random access
- (missing) bound checks
- vectors
- characters: ASCII, UTF8, texts, strings

Ⓛ

- vector types `std::vector<int> a {4,3,5,2,1};`
- characters and texts, the type char `char c = ’a’;`, Konversion nach **`int`**
- vectors of vectors
- Streams `std::istream, std::ostream`

Ⓔ

- sieve of Erathosthenes, Caesar-code, shortest paths

14./15. Recursion

Ⓜ

- recursive math. functions, the n-Queen problem, Lindenmayer systems, a command line calculator

Ⓒ

- recursion
- call stack, memory of recursion
- correctness, termination,
- recursion vs. iteration
- Backtracking, EBNF, formal grammars, parsing

Ⓔ

- factorial, GCD, sudoku-solver, command line calcoulator

16. Structs and Overloading

Ⓜ

- build your own rational number

Ⓒ

- heterogeneous data types
- function and operator overloading
- encapsulation of data

Ⓛ

- struct definition `struct rational {int n; int d;};`
- member access `result.n = a.n * b.d + a.d * b.n;`
- initialization and assignment,
- function overloading `pow(2)` vs. `pow(3,3);`, operator overloading

Ⓔ

- rational numbers, complex numbers

17. Classes

Ⓜ
- rational numbers with encapsulation

Ⓒ
- Encapsulation, Construction, Member Functions

Ⓛ
- classes `class rational { ... };`
- access control `public:` / `private:`
- member functions `int rational::denominator () const`
- The implicit argument of the member functions

Ⓔ
- finite rings, complex numbers

18./19. Dynamic Datastructures

Ⓜ

- Our own vector

Ⓒ

- linked list, allocation, deallocation, dynamic data type

Ⓛ

- The **`new`** statement
- pointer `int* x;`, Null-pointer `nullptr`.
- address and derference operator `int *ip = &i; int j = *ip;`
- pointer and const `const int *a;`

Ⓔ

- linked list, stack

20. Containers, Iterators and Algorithms

Ⓜ
- vectors are containers

Ⓒ
- iteration with pointers
- containers and iterators
- algorithms

Ⓛ
- Iterators **`std::vector<int>::iterator`**
- Algorithms of the standard library `std::fill (a, a+5, 1);`
- implement an iterator
- iterators and const

Ⓔ
- output a vector, a set

21. Dynamic Datatypes and Memory Management

Ⓜ

- Stack
- Expression Tree

Ⓒ

- Guideline ”dynamic memory“
- Pointer sharing
- Dynamic Datatype
- Tree-Structure

Ⓛ

- **`new`** and **`delete`**
- Destructor **`stack::~stack()`**
- Copy-Constructor **`stack::stack(const stack& s)`**
- Assignment operator
  `stack& stack::operator=(const stack& s)`
- Rule of Three

Ⓔ

- Binary Search Tree

22. Subtyping, Polymorphism and Inheritance

Ⓜ

- extend and generalize expression trees

Ⓒ

- Subtyping
- polymorphism and dynamic binding
- Inheritance

Ⓛ

- base class **`struct Exp{}`**
- derived class **`struct BinExp: public Exp{}`**
- abstract class **`struct Exp{virtual int size() const = 0...}`**
- polymorphie **`virtual double eval()`**

Ⓔ

- expression node and extensions

The End

End of the Course