ETH zürich

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Computer Science

Course at D-MATH / D-PHYS at ETH Zurich

Autumn 2019

Welcome

to the Course Informatik

at the MATH/PHYS departement of ETH Zürich.

Place and time:

Tuesday 13:15 - 15:00, ML D28, ML E12.
Pause 14:00 - 14:15, slight shift possible.

Course web page

`http://lec.inf.ethz.ch/ifmp`

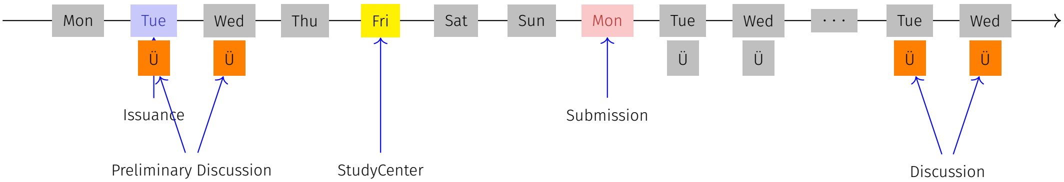
Team

chef assistant	Vytautas Astrauskas	
assistants	Benjamin Rothenberger	Charlotte Franke
	Claire Dick	David Sommer
	Edoardo Mazzoni	Eliza Wszola
	Enis Ulqinaku	Gaspard Zoss
	Janet Greutmann	Jannik Kochert
	Kevin Kaiwen Zhang	Manuel Mekkattu
	Moritz Schneider	Orhan Saeedi
	Raul Rao	Reza Sefidgar
	Sammy Christen	Tanja Kaister
	Tobias Klenze	Viera Klasovita
lecturers	Dr. Malte Schwerhoff / Dr. Felix Friedrich	

Registration for Exercise Sessions

- Registration via web page
- Registration already open

Procedure



- Exercises availabe at lectures
- Preliminary discussion in the following exercise session (on the same/next day)
- StudyCenter (`studycenter.ethz.ch`)
- Solution must be submitted at latest one day before the next lecture (23:59h)
- Discussion of the exercise in the session one week after the submission. Feedback will be provided in the week after the submission.

Exercises

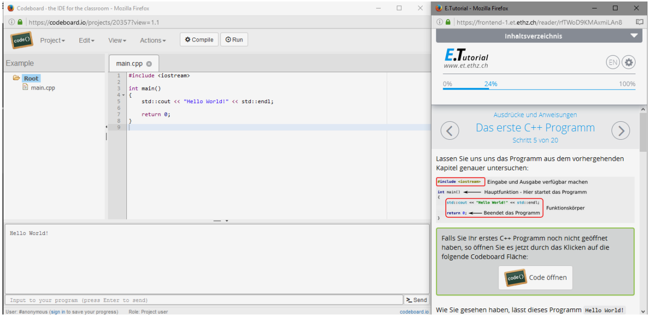
- The solution of the weekly exercises is thus voluntary but **strongly** recommended.

No lacking resources!

For the exercises we use an online development environment that requires only a browser, internet connection and your ETH login.

If you do not have access to a computer: there are a a lot of computers publicly accessible at ETH.

Online Tutorial



For a smooth course entry we provide an **online C++ tutorial**
Goal: leveling of the different programming skills.
Written mini test for your **self assessment** in the second exercise session.

Exams

The exam (in examination period 2018) will cover

- Lectures content (lectures, handouts)
- Exercise content (exercise sessions, exercises).

Written exam.
We will test your practical skills (programming skills) and theoretical knowledge (background knowledge, systematics).

Offer (VVZ)

- During the semester we offer weekly programming exercises that are graded. Points achieved will be taken as a bonus to the exam.
- The bonus is proportional to the score achieved in specially marked bonus tasks, where a full score equals a bonus of 0.25. The admission to specially marked bonus depends on the successful completion of other exercises. The achieved mark bonus expires as soon as the lecture is given anew.

Offer (Concretely)

- 3 bonus exercises in total; 2/3 of the points suffice for the exam bonus of 0.25 marks
- You can, e.g. fully solve 2 bonus exercises, or solve 3 bonus exercises to 66% each, or ...
- Bonus exercises must be unlocked ($\rightarrow$ experience points) by successfully completing the weekly exercises
- It is again not necessary to solve all weekly exercises completely in order to unlock a bonus exercise
- Details: course website, exercise sessions, online exercise system (Code Expert)

Academic integrity

Rule

You submit solutions that you have written yourself and that you have understood.
We check this (partially automatically) and reserve our rights to invite you to interviews.
Should you be invited to an interview: don't panic. Primary we presume your innocence and want to know if you understood what you have submitted.

Credits

- Lecture:
  - Original version by Prof. B. Gärtner and Dr. F. Friedrich
  - With changes from Dr. F. Friedrich, Dr. H. Lehner, Dr. M. Schwerhoff
- Script: Prof. B. Gärtner
- Code Expert: Dr. H. Lehner, David Avanthay and others

Andere Quellen werden hier am Rand in dieser Form angegeben.

1. Introduction

Computer Science: Definition and History, Algorithms, Turing Machine, Higher Level Programming Languages, Tools, The first C++Program and its Syntactic and Semantic Ingredients

What is Computer Science?

- The science of **systematic processing of informations**,...
- ...particularly the automatic processing using digital computers.

(Wikipedia, according to “Duden Informatik”)

Computer Science vs. Computers

Computer science is not about machines, in the same way that astronomy is not about telescopes.

Mike Fellows, US Computer Scientist (1991)

http://larc.unt.edu/ian/research/cseducation/fellows1991.pdf

Computer Science vs. Computers

- Computer science is also concerned with the development of fast computers and networks…
- …but not as an end in itself but for the **systematic processing of informations**.

Computer Science $\neq$ Computer Literacy

Computer literacy: *user knowledge*

- Handling a computer
- Working with computer programs for text processing, email, presentations ...

Computer Science *Fundamental knowledge*

- How does a computer work?
- How do you write a computer program?

Back from the past: This course

- Systematic problem solving with algorithms and the programming language C++.
- Hence: **not only**
  but also programming course.

Algorithm: Fundamental in Computer Science

Algorithm:

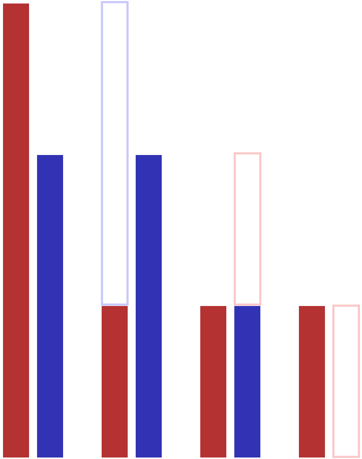
- Instructions to solve a problem step by step
- Execution does not require any intelligence, but precision (even computers can do it)
- according to *Muhammed al-Chwarizmi*, author of an arabic computation textbook (about 825)

“Dixit algorizmi…” (Latin translation)

http://de.wikipedia.org/wiki/Algorithmus

Oldest Nontrivial Algorithm

Euclidean algorithm (from the *elements* from Euklid, 3. century B.C.)



- Input: integers $a > 0$, $b > 0$
- Output: gcd of a und b

While $b \neq 0$

    If $a > b$ then

        $a \leftarrow a - b$

    else:

        $b \leftarrow b - a$

Result: a.

Algorithms: 3 Levels of Abstractions

1. **Core idea** (abstract)**:**
   the essence of any algorithm ("Eureka moment")
2. **Pseudo code** (semi-detailed)**:**
   made for humans (education, correctness and efficiency discussions, proofs
3. **Implementation** (very detailed)**:**
   made for humans & computers (read- & executable, specific programming language, various implementations possible)

Euclid: Core idea and pseudo code shown, implementation yet missing

Euklid in the Box

Speicher

0	1	2	3	4	5	6	7	8	9
[**8**] $\to$ **L**	[**9**] $\to$ **R**	**L** $=0$? *stop*	**R** $>$ **L**? *springe zu* **6**	**L** $-$ **R** $\to$ [**8**]	*springe zu* 0	**R** $-$ **L** $\to$ [**9**]	*springe zu* 0	b	a

Programmcode (0–7) · Daten (8–9)

Daten

Links	**R**echts
b	a

Register

While $b \neq 0$

  If $a > b$ then

    $a \leftarrow a - b$

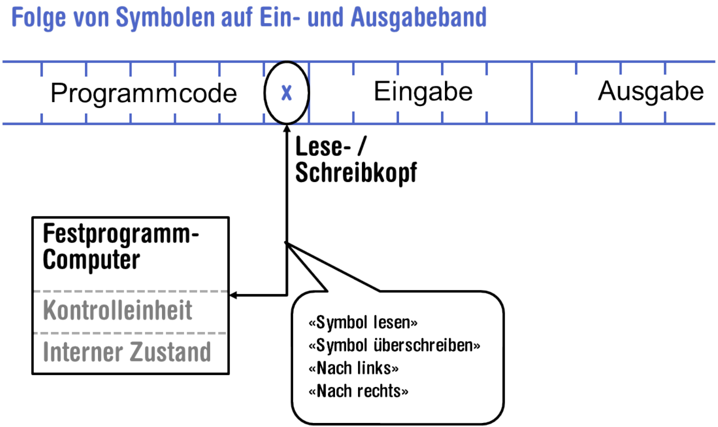
  else:

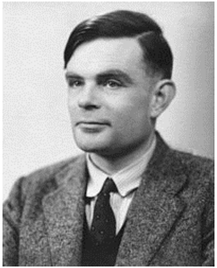
    $b \leftarrow b - a$

Ergebnis: a.

Computers – Concept

A bright idea: universal Turing machine (Alan Turing, 1936)



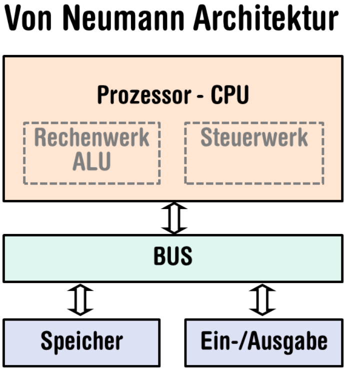


Alan Turing

http://en.wikipedia.org/wiki/Alan_Turing

Computer – Implementation

- Z1 – Konrad Zuse (1938)
- ENIAC – John Von Neumann (1945)



Konrad Zuse

John von Neumann

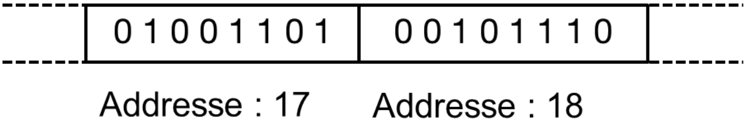
Computer

Ingredients of a *Von Neumann Architecture*

- Memory (RAM) for programs **and** data
- Processor (CPU) to process programs and data
- I/O components to communicate with the world

Memory for data *and* program

- Sequence of bits from $\{0, 1\}$.
- Program state: value of all bits.
- Aggregation of bits to memory cells (often: 8 Bits = 1 Byte)
- Every memory cell has an address.
- Random access: access time to the memory cell is (nearly) independent of its address.



Processor

The processor (CPU)

- executes instructions in machine language
- has an own "fast" memory (registers)
- can read from and write to main memory
- features a set of simplest operations = instructions (e.g. adding to register values)

Programming

- With a **programming language** we issue commands to a computer such that it does exactly what we want.
- The sequence of instructions is the **(computer) program**

The Harvard Computers, human computers, ca.1890

http://en.wikipedia.org/wiki/Harvard_Computers

Computing speed

In the time, on average, that the sound takes to travel from from my mouth to you …

30 m $\widehat{=}$ more than 100.000.000 instructions

a contemporary desktop PC can process more than 100 millions instructions [1]

[1] Uniprocessor computer at 1 GHz.

Why programming?

- Do I study computer science or what ...
- There are programs for everything ...
- I am not interested in programming ...
- because computer science is a mandatory subject here, unfortunately...
- ...

Mathematics used to be the lingua franca of the natural sciences on all universities. Today this is computer science.

Lino Guzzella, president of ETH Zurich 2015-2018, NZZ Online, 1.9.2017

((BTW: Lino Guzzella is not a computer scientist, he is a mechanical engineer and prof. for thermotronics ☺)

This is why programming!

- Any understanding of modern technology requires knowledge about the fundamental operating principles of a computer.
- Programming (with the computer as a tool) is evolving a cultural technique like reading and writing (using the tools paper and pencil)
- Programming is *the* interface between engineering and computer science – the interdisciplinary area is growing constantly.
- Programming is fun (and is useful)!

Programming Languages

- The language that the computer can understand (machine language) is very primitive.
- Simple operations have to be subdivided into (extremely) many single steps
- The machine language varies between computers.

Higher Programming Languages

can be represented as program text that

- can be *understood* by humans
- is *independent* of the computer model

  $\rightarrow$ Abstraction!

Programming langauges – classification

Differentiation into

- Compiled vs. interpreted languages
  - C++, C#, Java, Go, Pascal, Modula, Oberon
    vs.
    Python, Javascript, Matlab
- **Higher** programming languages vs. Assembler
- **Multi-purpose** programming languages vs. single purpose programming languages
- **Procedural, object oriented**, functional and logical languages.

Why $\mathrm{C{+}{+}}$?

Other popular programming languages: Java, C#, Python, Javascript, Swift, Kotlin, Go, ... …

General consensus:

- „The” programming language for systems programming: C
- C has a fundamental weakness: missing (type) safety

Why C++?

Over the years, C++'s greatest strength and its greatest weakness has been its C-Compatibility – B. Stroustrup

B. Stroustrup, Design and Evolution of C++, Kap. 4.5

Why C++?

- C++equips C with the power of the abstraction of a higher programming language
- In this course: C++ introduced as high level language, not as better C
- Approach: traditionally procedural $\rightarrow$ object-oriented.

Syntax and Semantics

- Like our language, programs have to be formed according to certain rules.
  - **Syntax**: Connection rules for elementary symbols (characters)
  - **Semantics**: interpretation rules for connected symbols.
- Corresponding rules for a computer program are simpler but also more strict because computers are relatively stupid.

Deutsch vs. C++

Deutsch

Alleen sind nicht gefährlich, Rasen ist gefährlich!
(Wikipedia: Mehrdeutigkeit)

C++

```
// computation
int b = a * a; // b = a^2
b = b * b;     // b = a^4
```

C++: Kinds of errors illustrated with German sentences

Das Auto fuhr zu schnell.	Syntaktisch und semantisch korrekt.
DasAuto fuh r zu sxhnell.	Syntaxfehler: Wortbildung.
Rot das Auto ist.	Syntaxfehler: Satzstellung.
Man empfiehlt dem Dozenten nicht zu widersprechen	Syntaxfehler: Satzzeichen fehlen .
Sie ist nicht gross und rothaarig.	Syntaktisch korrekt aber mehrdeutig. [kein Analogon]
Die Auto ist rot.	Syntaktisch korrekt, doch semantisch fehlerhaft: Falscher Artikel. [Typfehler]
Das Fahrrad galoppiert schnell.	Syntaktisch und grammatikalisch korrekt! Semantisch fehlerhaft. [Laufzeitfehler]
Manche Tiere riechen gut.	Syntaktisch und semantisch korrekt. Semantisch mehrdeutig. [kein Analogon]

Syntax and Semantics of C++

Syntax:

- When is a text a *C++ program*?
- I.e. is it *grammatically* correct?
- $\rightarrow$ Can be checked by a computer

Semantics:

- What does a program *mean*?
- Which algorithm does a program *implement*?
- $\rightarrow$ Requires human understanding

Syntax and semantics of C++

The ISO/IEC Standard 14822 (1998, 2011, 2014, …)

- is the “law” of C++
- defines the grammar and meaning of C++programs
- since 2011, continuously extended with features for *advanced* programming

Programming Tools

- **Editor:** Program to modify, edit and store C++program texts
- **Compiler:** program to translate a program text into machine language
- **Computer:** machine to execute machine language programs
- **Operating System:** program to organize all procedures such as file handling, editor-, compiler- and program execution.

Language constructs with an example

- Comments/layout
- Include directive
- the main function
- Values effects
- Types and functionality
- literals
- variables
- constants
- identifiers, names
- **expressions**
- L- and R- values
- operators
- statements

The first C++ program

```
// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

Statements: Do something (read in a)!

Expressions: Compute a value (a^2)!

Behavior of a Program

At compile time:

- program accepted by the compiler (syntactically correct)
- Compiler error

During runtime:

- correct result
- incorrect result
- program crashes
- program does not terminate (endless loop)

“Accessories:” Comments

```
// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

comments

Comments and Layout

Comments

- are contained in every good program.
- document *what* and *how* a program does something and how it should be used,
- are ignored by the compiler
- Syntax: “double slash” `// until the line ends.`

The compiler *ignores* additionally

- Empty lines, spaces,
- Indendations that should reflect the program logic

Comments and Layout

The compiler does not care...

```
#include <iostream>
int main(){std::cout << "Compute a^8 for a =? ";
int a; std::cin >> a; int b = a * a; b = b * b;
std::cout << a << "^8 = " << b*b << "\n";return 0;}
```

... but we do!

“Accessories:” Include and Main Function

```
// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

include directive

declaration of the main function

Include Directives

C++ consists of

- the core language
- standard library
    - in-/output (header iostream)
    - mathematical functions (cmath)
    - ...

```
#include <iostream>
```

- makes in- and output available

The main Function

the **`main`**-function

- is provided in any C++ program
- is called by the operating system
- like a mathematical function ...
    - arguments
    - return value
- ... but with an additional **effect**
    - Read a number and output the 8th power.

Statements: Do something!

```
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

expression statements

return statement

Statements

- building blocks of a C++ program
- are *executed* (sequentially)
- end with a semicolon
- Any statement has an **effect** (potentially)

Expression Statements

- have the following form:

  expr;

  where *expr* is an expression
- Effect is the effect of *expr*, the value of *expr* is ignored.

```
b = b*b;
```

Return Statements

- do only occur in functions and are of the form

  return expr;

  where *expr* is an expression
- specify the return value of a function

```
return 0;
```

Statements – Effects

```
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4

    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

effect: output of the string `Compute` .

Effect: input of a number stored in a

Effect: saving the computed value of $a \cdot a$ into b

Effect: saving the computed value of $b \cdot b$ into b

Effect: return the value 0

Effect: output of the value of a and the c

Values and Effects

- determine what a program does,
- are purely semantical concepts:
  - Symbol **`0`** means Value $0 \in \mathbb{Z}$
  - **`std::cin >> a;`** means effect "read in a number"
- depend on the program state (memory content, inputs)

Statements – Variable Definitions

```
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

type names

declaration statement

Declaration Statements

- introduce new names in the program,
- consist of declaration and semicolon Example: **`int a;`**
- can initialize variables Example: **`int b = a * a;`**

Types and Functionality

`int`:

- $\mathrm{C}{+}{+}$ integer type
- corresponds to $(\mathbb{Z}, +, \times)$ in math

In $\mathrm{C}{+}{+}$ each type has a name and

- a domain (e.g. integers)
- functionality (e.g. addition/multiplication)

Fundamental Types

C++ comprises fundamental types for

- integers (**`int`**)
- natural numbers (**`unsigned int`**)
- real numbers (**`float`**, **`double`**)
- boolean values (**`bool`**)
- ...

Variables

- represent (varying) values
- have
  - **name**
  - **type**
  - **value**
  - **address**
- are "visible" in the program context

`int a;` defines a variable with

- name: `a`
- type: `int`
- value: (initially) undefined
- Address: determined by compiler

Identifiers and Names

(Variable-)names are identifiers

- allowed: A,...,Z; a,...,z; 0,...,9;`_`
- First symbol needs to be a character.

There are more names:

- **`std::cin`** (Qualified identifier)

Expressions: compute a value!

Expressions

- represent *Computations*
- are either **primary** (`b`)
- or **composed** (`b*b`)...
- ...from different expressions, using **operators**
- have a type and a value

Analogy: building blocks

Expressions

Building Blocks

composite expression

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;
```

Two times composed expression

```
// output b * b, i.e., a^8
std::cout << a << "^8 = " << b * b << ".\n";

return
```

Four times composed expression

Expressions

- represent *computations*
- are *primary* or *composite* (by other expressions and operations)

  `a * a`

  composed of

  variable name, operator symbol,variable name

  variable name: primary expression
- can be put into parantheses

  `a * a` is equivalent to **`(a * a)`**

Expressions

have **type**, **value** und **effect** (potentially).

`a * a`

- type: `int` (type of the operands)
- Value: product of `a` and `a`
- Effect: none.

`b = b * b`

- type: `int` (Typ der Operanden)
- Value: product of `b` and `b`
- effect: assignment of the product value to `b`

The type of an expression is fixed but the value and effect are only determined by the *evaluation* of the expression

Literals

- represent constant values
- have a fixed **type** and **value**
- are "syntactical values"

- `0` has type `int`, value 0.
- `1.2e5` has type `double`, value $1.2 \cdot 10^5$.

L-Values and R-Values

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;   // b = a^2
b = b * b;       // b = a^4

// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";
return 0;
```

R-Value

L-value (expression + address)

L-value (expression + address)

R-Value

R-Value (expression that is not an L-value)

L-Values and R-Values

L-Wert (“**L**eft of the assignment operator”)

- Expression with **address**
- **Value** is the content at the memory location according to the type of the expression.
- L-Value can change its value (e.g. via assignment)

Example: variable name

L-Values and R-Values

R-Wert ("**R**ight of the assignment operator")

- Expression that is no L-value
- Any L-Value can be used as R-Value (but not the other way round)
- An R-Value *cannot change* its value

Example: literal 0

Operators and Operands   Building Blocks

left operand (output stream)

output operator

right operand (string)

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;
```

right operand (variable name)

input operator

left operand (input stream)

```
// computati
int b = a
b = b * b;      // b = a^4
```

assignment operator

```
// ou            ^8
std::cout << a << "^8 = " << b * b << "\n";
return 0;
```

multiplication operator

Operators

Operators

- combine expressions (*operands*) into new composed expressions
- specify for the operands and the result the types and if the have to be L- or R-values.
- have an arity

Multiplication Operator `*`

- expects two R-values of the same type as operands (arity 2)
- "returns the product as R-value of the same type", that means formally:
  - The composite expression is an R-value; its value is the product of the value of the two operands

Examples: **`a * a`** and **`b * b`**

Assignment Operator =

- **L**eft operand is **L**-value,
- **R**ight operand is **R**-value of the same type.
- Assigns to the left operand the value of the right operand and returns the left operand as L-value

Examples `b = b * b` and `a = b`

Attention, Trap!

The operator **=** corresponds to the assignment operator of mathematics ($:=$), not to the comparison operator ($=$).

Input Operator »

- left operand is L-Value (input stream)
- right operand is L-Value
- assigns to the right operand the next value read from the input stream, *removing it from the input stream* and returns the input stream as L-value Example **`std::cin >> a`** (mostly keyboard input)
- Input stream is being changed and must thus be an L-Value.

Output Operator «

- left operand is L-Value (*output stream*)
- right operand is R-Value
- outputs the value of the right operand, appends it to the output stream and returns the output stream as L-Value

  Example: `std::cout << a` (mostly console output)
- The output stream is being changed and must thus be an L-Value.

Output Operator «

Why returning the output stream?

- allows bundling of output

```
std::cout << a << "^8 = " << b * b << "\n"
```

is parenthesized as follows

```
((((std::cout << a) << "^8 = ") << b * b) << "\n")
```

- `std::cout << a` is the left hand operand of the next `<<` and is thus an L-Value that is no variable name

2. Integers

Evaluation of Arithmetic Expressions, Associativity and Precedence, Arithmetic Operators, Domain of Types **`int`**, **`unsigned int`**

Example: power8.cpp

```
int a; // Input
int r; // Result

std::cout << "Compute a^8 for a = ?";
std::cin >> a;

r = a * a; // r = a^2
r = r * r; // r = a^4

std::cout << "a^8 = " << r*r << '\n';
```

Terminology: L-Values and R-Values

L-Wert ("**L**eft of the assignment operator")

- Expression identifying a **memory location**
- For example a variable
  (we'll see other L-values later in the course)
- **Value** is the content at the memory location according to the type of the expression.
- L-Value can change its value (e.g. via assignment)

Terminology: L-Values and R-Values

R-Wert ("**R**ight of the assignment operator")

- Expression that is no L-value
- Example: integer literal `0`
- Any L-Value can be used as R-Value (but not the other way round) ...
- ... by using the *value* of the L-value
(e.g. the L-value `a` could have the value `2`, which is then used as an R-value)
- An R-Value *cannot change* its value

L-Values and R-Values

R-Value

```
std::cout << "Compute a^8 for a = ? ";
int a;
std::cin >> a;
```

L-value (expression + address)

L-value (expression + address)

```
int r = a * a;  // r = a^2
r = r * r; // r = a^4
```

R-Value

```
std::cout << a<< "^8 = " << r * r << ".\n";

return 0;
```

R-Value (expression that is not an L-value)

Celsius to Fahrenheit

```cpp
// Program: fahrenheit.cpp
// Convert temperatures from Celsius to Fahrenheit.
#include <iostream>

int main() {
  // Input
  std::cout << "Temperature in degrees Celsius =? ";
  int celsius;
  std::cin >> celsius;

  // Computation and output
  std::cout << celsius << " degrees Celsius are "
            << 9 * celsius / 5 + 32  << " degrees Fahrenheit.\n";
  return 0;
}
```

`9 * celsius / 5 + 32`

```
9 * celsius / 5 + 32
```

- Arithmetic expression,
- contains three literals, a variable, three operator symbols

How to put the expression in parentheses?

Precedence

Multiplication/Division before Addition/Subtraction

`9 * celsius / 5 + 32`

bedeutet

`(9 * celsius / 5) + 32`

Rule 1: precedence

Multiplicative operators (`*`, `/`, `%`) have a higher precedence ("bind more strongly") than additive operators (`+`, `-`)

Associativity

From left to right

```
9 * celsius / 5 + 32
```

bedeutet

```
((9 * celsius) / 5) + 32
```

Rule 2: Associativity

Arithmetic operators (`*`, `/`, `%`, `+`, `-`) are left associative: operators of same precedence evaluate from left to right

Arity

Sign

-3 - 4

means

(-3) - 4

Rule 3: Arity

Unary operators **+**, **-** first, then binary operators **+**, **-**.

Parentheses

Any expression can be put in parentheses by means of

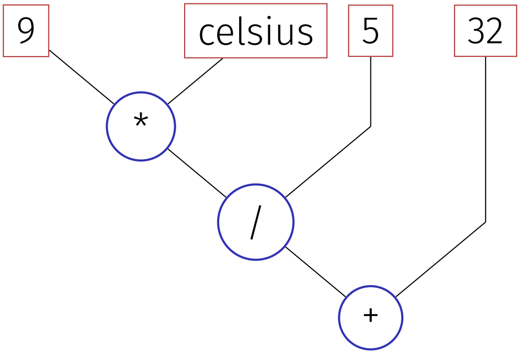
- associativities
- precedences
- arities (number of operands)

of the operands in an unambiguous way (Details in the lecture notes).

Expression Trees

Parentheses yield the expression tree

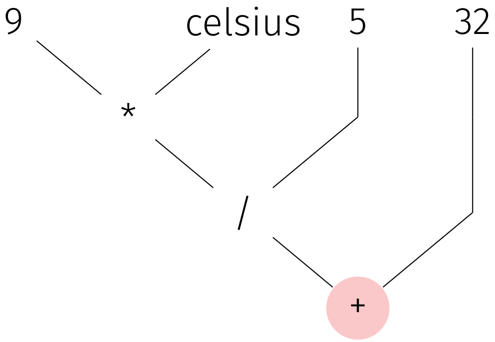
```
(((9 * celsius) / 5) + 32)
```



Evaluation Order

"From top to bottom" in the expression tree

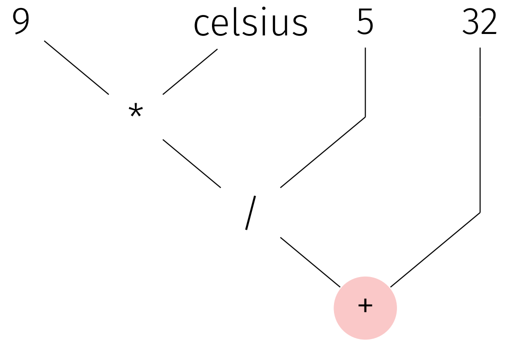
`9 * celsius / 5 + 32`



Evaluation Order

Order is not determined uniquely:

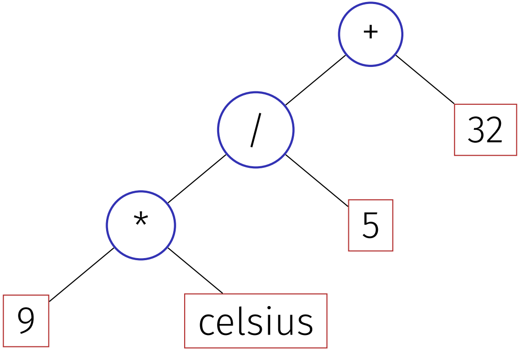
```
9 * celsius / 5 + 32
```



Expression Trees – Notation

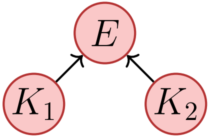
Common notation: root on top

```
9 * celsius / 5 + 32
```



Evaluation Order – more formally

Valid order: any node is evaluated **after** its children



C++: the valid order to be used is not defined.

- "Good expression": any valid evaluation order leads to the same result.
- Example for a “bad expression”: `a*(a=2)`

Evaluation order

Guideline

Avoid modifying variables that are used in the same expression more than once.

Arithmetic operations

	Symbol	Arity	Precedence	Associativity
Unary +	+	1	16	right
Negation	−	1	16	right
Multiplication	*	2	14	left
Division	/	2	14	left
Modulo	%	2	14	links
Addition	+	2	13	left
Subtraction	−	2	13	left

All operators: [R-value $\times$] R-value $\rightarrow$ R-value

Interlude: Assignment expression – in more detail

- Already known: `a = b` means Assignment of `b` (R-value) to `a` (L-value). Returns: L-value.
- What does `a = b = c` mean?
- Answer: assignment is right-associative

`a = b = c` $\Longleftrightarrow$ `a = (b = c)`

Multiple assignment: `a = b = 0` $\Longrightarrow$ `b=0; a=0`

Division

- Operator / implements integer division

`5 / 2` has value 2

- In **`fahrenheit.cpp`**

```
9 * celsius / 5 + 32

15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent...but not in C++!

```
9 / 5 * celsius + 32

15 degrees Celsius are 47 degrees Fahrenheit
```

Loss of Precision

Guideline

- Watch out for potential loss of precision
- Postpone operations with potential loss of precision to avoid “error escalation”

Division and Modulo

- Modulo-operator computes the rest of the integer division

`5 / 2` has value 2, `5 % 2` has value 1.

- It holds that

```
(-a)/b == -(a/b)
```

- It also holds:

`(a / b) * b + a % b` has the value of `a`.

- From the above one can conclude the results of division and modulo with negative numbers

Increment and decrement

- Increment / Decrement a number by one is a frequent operation
- works like this for an L-value:

```
expr = expr + 1.
```

Disadvantages

- relatively long
- `expr` is evaluated twice
  - Later: L-valued expressions whose evaluation is “expensive”
  - `expr` could have an effect (but should not, cf. guideline)

In-/Decrement Operators

Post-Increment

```
expr++
```

Value of **`expr`** is increased by one, the **old** value of **`expr`** is returned (as R-value)

Pre-increment

```
++expr
```

Value of **`expr`** is increased by one, the **new** value of **`expr`** is returned (as L-value)

Post-Dekrement

```
expr--
```

Value of **`expr`** is decreased by one, the **old** value of **`expr`** is returned (as R-value)

Prä-Dekrement

```
--expr
```

Value of **`expr`** is increased by one, the **new** value of **`expr`** is returned (as L-value)

In-/decrement Operators

	use	arity	prec	assoz	L-/R-value
Post-increment	`expr++`	1	17	left	L-value $\rightarrow$ R-value
Pre-increment	`++expr`	1	16	right	L-value $\rightarrow$ L-value
Post-decrement	`expr--`	1	17	left	L-value $\rightarrow$ R-value
Pre-decrement	`--expr`	1	16	right	L-value $\rightarrow$ L-value

In-/Decrement Operators

```
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n"; // 8
std::cout << a << "\n"; // 9
```

In-/Decrement Operators

Is the expression

`++expr;` ← we favour this

equivalent to

`expr++;`?

Yes, but

- Pre-increment can be more efficient (old value does not need to be saved)
- Post In-/Decrement are the only left-associative unary operators (not very intuitive)

`C++` vs. `++C`

Strictly speaking our language should be named `++C` because

- it is an advancement of the language `C`
- while `C++` returns the old `C`.

Arithmetic Assignments

```
a += b
```

$$\Longleftrightarrow$$

```
a = a + b
```

analogously for `-`, `*`, `/` and `%`

Arithmetic Assignments

	Gebrauch	Bedeutung
`+=`	`expr1 += expr2`	`expr1 = expr1 + expr2`
`-=`	`expr1 -= expr2`	`expr1 = expr1 - expr2`
`*=`	`expr1 *= expr2`	`expr1 = expr1 * expr2`
`/=`	`expr1 /= expr2`	`expr1 = expr1 / expr2`
`%=`	`expr1 %= expr2`	`expr1 = expr1 % expr2`

Arithmetic expressions evaluate expr1 only once.

Assignments have precedence 4 and are right-associative.

Binary Number Representations

Binary representation (Bits from $\{0, 1\}$)

$$b_n b_{n-1} \dots b_1 b_0$$

corresponds to the number $b_n \cdot 2^n + \cdots + b_1 \cdot 2 + b_0$

`101011` corresponds to **`43`**.

Least Significant Bit (LSB)

Most Significant Bit (MSB)

Computing Tricks

- Estimate the orders of magnitude of powers of two.[2]:

$$
\begin{aligned}
2^{10} &= 1024 = 1\mathrm{Ki} \approx 10^3. \\
2^{20} &= 1\mathrm{Mi} \approx 10^6, \\
2^{30} &= 1\mathrm{Gi} \approx 10^9, \\
2^{32} &= 4 \cdot (1024)^3 = 4\mathrm{Gi}. \\
2^{64} &= 16\mathrm{Ei} \approx 16 \cdot 10^{18}.
\end{aligned}
$$

[2]Decimal vs. binary units: MB - Megabyte vs. MiB - Megabibyte (etc.)
kilo (K, Ki) – mega (M, Mi) – giga (G, Gi) – tera(T, Ti) – peta(P, Pi) – exa (E, Ei)

Hexadecimal Numbers

Numbers with base 16

$$h_n h_{n-1} \dots h_1 h_0$$

corresponds to the number

$$h_n \cdot 16^n + \cdots + h_1 \cdot 16 + h_0.$$

notation in C++: prefix **`0x`**

`0xff` corresponds to **`255`**.

Hex Nibbles

hex	bin	dec
0	0000	0
1	**0001**	**1**
2	**0010**	**2**
3	0011	3
4	**0100**	**4**
5	0101	5
6	0110	6
7	0111	7
8	**1000**	**8**
9	1001	9
a	1010	10
b	1011	11
c	1100	12
d	1101	13
e	1110	14
f	1111	15

Why Hexadecimal Numbers?

- A Hex-Nibble requires exactly 4 bits. Numbers 1, 2, 4 and 8 represent bits 0, 1, 2 and 3.
- “compact representation of binary numbers”

Why Hexadecimal Numbers?

"For programmers and technicians" (user manual of the chess computers *Mephisto II*, 1981)

Beispiele:

8200

a) Anzeige ***8200***
MEPHISTO ist mit genau 2 Bauern-Einheiten im Vorteil.

7F00

b) Anzeige *7F00*
MEPHISTO ist mit genau 1 Bauern-Einheit im Nachteil.

Die Anzeige erfolgt in ***hexadezimaler Schreibweise.*** Im Gegensatz zum gewohnten Dezimalsystem gehen die Ziffern an jeder Stelle von 0 bis F (A = 10, B = 11, ... F = 15).
Für mathematisch Vorgebildete nachstehend die Umrechnungsformel in das dezimale Punktsystem:

$$ABCD = (Ax16^3) + (Bx16^2) + (Cx16^1) + (Dx16^0)$$

Für A gilt: 7 = -1; 8 = 0; 9 = +1 usw.
Eine Bauerneinheit (B) wird ausgedrückt in 16^2 = 256 Punkten.
Dieses auf den ersten Blick vielleicht etwas komplizierte System dient der Service-Freundlichkeit von MEPHISTO, sowie insbesondere der Entwicklungsarbeit an zukünftigen, noch stärkeren Programmen, ist also mehr für unsere Programmierer und Techniker vorgesehen.

Beispiele:

805E

c) Anzeige *805E*
(E=14) Umrechnung nach folgendem Verfahren:
$(14x16^0) + (5x16^1) + (0x16^2) + (0x16^3) = 14+80+0+0 =$
$= $ ***+94 Punkte.***

7F80

d) Anzeige *7F80*
(7=-1; F=15) Umrechnung wie folgt:
$(0x16^0) + (8x16^1) + (15x16^2) - (1x16^3) = 0+128+3840-4096 =$

http://www.zanchetta.net/default.aspx?Categorie=ECHIQUIERS&Page=documentations

Example: Hex-Colors

$$\#\underbrace{00}_{r}\underbrace{FF}_{g}\underbrace{00}_{b}$$

Why Hexadecimal Numbers?

The NZZ could have saved a lot of space ...

Domain of Type `int`

```
// Output the smallest and the largest value of type int.
#include <iostream>
#include <limits>

int main() {
  std::cout << "Minimum int value is "
            << std::numeric_limits<int>::min() << ".\n"
            << "Maximum int value is "
            << std::numeric_limits<int>::max() << ".\n";
  return 0;
}
```

```
Minimum int value is -2147483648.
Maximum int value is 2147483647.
```

Where do these numbers come from?

Domain of the Type `int`

- Representation with B bits. Domain comprises the 2^B integers:

$$\{-2^{B-1}, -2^{B-1}+1, \ldots, -1, 0, 1, \ldots, 2^{B-1}-2, 2^{B-1}-1\}$$

- On most platforms $B = 32$
- For the type `int` $\mathrm{C}{+}{+}$ guarantees $B \geq 16$
- Background: Section 2.2.8 (Binary Representation) in the lecture notes.

Over- and Underflow

- Arithmetic operations (`+,-,*`) can lead to numbers outside the valid domain.
- Results can be incorrect!

  `power8.cpp`: $15^8 = -1732076671$

- There is **no error message!**

The Type `unsigned int`

- Domain
$$\{0, 1, \ldots, 2^B - 1\}$$
- All arithmetic operations exist also for **`unsigned int`**.
- Literals: **`1u, 17u`** …

Mixed Expressions

- Operators can have operands of different type (e.g. **`int`** and **`unsigned int`**).

```
17 + 17u
```

- Such mixed expressions are of the "more general" type **`unsigned int`**.
- **`int`**-operands are **converted** to **`unsigned int`**.

Conversion

`int` Value	Sign	`unsigned int` Value
x	≥ 0	x
x	< 0	$x + 2^B$

Due to a clever representation (two's complement), no addition is internally needed

Conversion “reversed”

The declaration

```
int a = 3u;
```

converts **`3u`** to **`int`**.
The value is preserved because it is in the domain of **`int`**; otherwise the result depends on the implementation.

Signed Numbers

Note: the remaining slides on signed numbers, computing with binary numbers, and the two's complement, are *not* relevant for the exam

Signed Number Representation

- (Hopefully) clear by now: binary number representation without sign, e.g.

$$[b_{31}b_{30}\dots b_0]_u \quad \widehat{=} \quad b_{31}\cdot 2^{31} + b_{30}\cdot 2^{30} + \dots + b_0$$

- Looking for a consistent solution

  The representation with sign should coincide with the unsigned solution as much as possible. Positive numbers should arithmetically be treated equal in both systems.

Computing with Binary Numbers (4 digits)

Simple Addition

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array} \qquad \begin{array}{r} 0010 \\ +0011 \\ \hline 0101_2 \end{array} = 5_{10}$$

Simple Subtraction

$$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array} \qquad \begin{array}{r} 0101 \\ -0011 \\ \hline 0010_2 \end{array} = 2_{10}$$

Computing with Binary Numbers (4 digits)

Addition with Overflow

$$\begin{array}{r} 7 \\ +10 \\ \hline 17 \end{array} \qquad \begin{array}{r} 0111 \\ +1010 \\ \hline (1)0001_2 \end{array} \quad = 1_{10} (= 17 \bmod 16)$$

Subtraction with underflow

$$\begin{array}{r} 5 \\ +(-10) \\ \hline -5 \end{array} \qquad \begin{array}{r} 0101 \\ 1010 \\ \hline (\dots 11)1011_2 \end{array} \quad = 11_{10} (= -5 \bmod 16)$$

Why this works

Modulo arithmetics: Compute on a circle[3]

$11 \equiv 23 \equiv -1 \equiv \dots \mod 12$ $\quad$ $4 \equiv 16 \equiv \dots \mod 12$ $\quad$ $3 \equiv 15 \equiv \dots \mod 12$

[3] The arithmetics also work with decimal numbers (and for multiplication).

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	**111**	-1
2	010	**110**	-2
3	011	**101**	-3
4	**100**	**100**	-4
5	**101**		
6	**110**		
7	**111**		

The most significant bit decides about the sign *and* it contributes to the value.

Two's Complement

- Negation by bitwise negation and addition of 1

$$-2 = -[0010] = [1101] + [0001] = [1110]$$

- Arithmetics of addition and subtraction **identical** to unsigned arithmetics

$$3 - 2 = 3 + (-2) = [0011] + [1110] = [0001]$$

- Intuitive "wrap-around" conversion of negative numbers.

$$-n \to 2^B - n$$

- Domain: $-2^{B-1} \dots 2^{B-1} - 1$

3. Logical Values

Boolean Functions; the Type `bool`; logical and relational operators; shortcut evaluation

Our Goal

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Behavior depends on the value of a ***Boolean expression***

Boolean Values in Mathematics

Boolean expressions can take on one of two values:

0 or **1**

- **0** corresponds to **“false”**
- **1** corresponds to **“true”**

The Type `bool` in C++

- represents **logical values**
- Literals `false` and `true`
- Domain {***false***, ***true***}

```
bool b = true; // Variable with value true
```

Relational Operators

`a < b` (smaller than)

`a >= b` (greater than)

`a == b` (equals)

`a != b` (not equal)

arithmetic type $\times$ arithmetic type $\rightarrow$ **`bool`**

R-value $\times$ R-value $\rightarrow$ R-value

Table of Relational Operators

	Symbol	Arity	Precedence	Associativity
smaller	`<`	2	11	left
greater	`>`	2	11	left
smaller equal	`<=`	2	11	left
greater equal	`>=`	2	11	left
equal	`==`	2	10	left
unequal	`!=`	2	10	left

arithmetic type $\times$ arithmetic type $\rightarrow$ **`bool`**

R-value $\times$ R-value $\rightarrow$ R-value

Boolean Functions in Mathematics

- Boolean function

$$f : \{0, 1\}^2 \to \{0, 1\}$$

- 0 corresponds to “false”.
- 1 corresponds to “true”.

$\mathrm{AND}(x, y)$ $\qquad x \wedge y$

- "logical And"

$$f : \{0,1\}^2 \to \{0,1\}$$

- 0 corresponds to "false".
- 1 corresponds to "true".

x	y	$\mathrm{AND}(x, y)$
0	0	0
0	1	0
1	0	0
1	1	1

Logical Operator `&&`

`a && b` (logical and)

$$\texttt{bool} \times \texttt{bool} \rightarrow \texttt{bool}$$

$$\text{R-value} \times \text{R-value} \rightarrow \text{R-value}$$

```
int n = -1;
int p = 3;
bool b = (n < 0) && (0 < p); // b = true
```

$\mathrm{OR}(x, y)$ $\qquad x \vee y$

- "logical Or"

$$f : \{0,1\}^2 \to \{0,1\}$$

- 0 corresponds to "false".
- 1 corresponds to "true".

x	y	$\mathrm{OR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	1

Logical Operator ||

`a || b` (logical or)

$\texttt{bool} \times \texttt{bool} \to \texttt{bool}$

R-value × R-value → R-value

```
int n = 1;
int p = 0;
bool b = (n < 0) || (0 < p); // b = false
```

$\mathrm{NOT}(x)$ $\neg x$

- "logical Not"

$$f : \{0,1\} \to \{0,1\}$$

- 0 corresponds to "false".
- 1corresponds to "true".

x	$\mathrm{NOT}(x)$
0	1
1	0

Logical Operator `!`

`!b` (logical not)

`bool` → `bool`

R-value → R-value

```
int n = 1;
bool b = !(n < 0); // b = true
```

Precedences

```
!b && a
```

$$\Updownarrow$$

```
(!b) && a
```

```
a && b || c && d
```

$$\Updownarrow$$

```
(a && b) || (c && d)
```

```
a || b   &&   c || d
```

$$\Updownarrow$$

```
a || (b && c) || d
```

Table of Logical Operators

	Symbol	Arity	Precedence	Associativity
Logical and (AND)	`&&`	2	6	left
Logical or (OR)	`||`	2	5	left
Logical not (NOT)	`!`	1	16	right

Precedences

The unary logical operator `!`
binds more strongly than
binary arithmetic operators. These
bind more strongly than
relational operators,
and these bind more strongly than
binary logical operators.

```
 7 + x < y && y != 3 * z || ! b
7 + x < y && y != 3 * z || (!b)
```

Completeness

- AND, OR and NOT are the boolean functions available in C++.
- Any other *binary* boolean function can be generated from them.

x	y	$\mathrm{XOR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	0

Completeness: $\mathrm{XOR}(x, y)$ $x \oplus y$

$$\mathrm{XOR}(x, y) = \mathrm{AND}(\mathrm{OR}(x, y), \mathrm{NOT}(\mathrm{AND}(x, y))).$$

$$x \oplus y = (x \vee y) \wedge \neg(x \wedge y).$$

```
(x || y) && !(x && y)
```

Completeness Proof

- Identify binary boolean functions with their characteristic vector.

x	y	$\mathrm{XOR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	0

characteristic vector: 0110

$$\mathrm{XOR} = f_{0110}$$

Completeness Proof

- Step 1: generate the *fundamental* functions f_{0001}, f_{0010}, f_{0100}, f_{1000}

$$
\begin{aligned}
f_{0001} &= \mathrm{AND}(x, y) \\
f_{0010} &= \mathrm{AND}(x, \mathrm{NOT}(y)) \\
f_{0100} &= \mathrm{AND}(y, \mathrm{NOT}(x)) \\
f_{1000} &= \mathrm{NOT}(\mathrm{OR}(x, y))
\end{aligned}
$$

Completeness Proof

- Step 2: generate all functions by applying logical or

$$f_{1101} = \mathrm{OR}(f_{1000}, \mathrm{OR}(f_{0100}, f_{0001}))$$

- Step 3: generate f_{0000}

$$f_{0000} = 0.$$

`bool` vs `int`: Conversion

- `bool` can be used whenever `int` is expected – and vice versa.
- Many existing programs use `int` instead of `bool`

  This is bad style originating from the language C **.**

`bool`	$\rightarrow$	`int`
true	$\rightarrow$	1
false	$\rightarrow$	0

`int`	$\rightarrow$	`bool`
$\neq$0	$\rightarrow$	*true*
0	$\rightarrow$	*false*

```
bool b = 3; // b=tru
```

DeMorgan Rules

- `!(a && b) == (!a || !b)`
- `!(a || b) == (!a && !b)`

`!` (rich *and* beautiful) == (poor *or* ugly)

Application: either ... or (XOR)

`(x || y) && !(x && y)`	x or y, and not both
`(x || y) && (!x || !y)`	x or y, and one of them not
`!(!x && !y) && !(x && y)`	not none and not both
`!(!x && !y || x && y)`	not: both or none

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

```
x != 0 && z / x > y
```

$\Rightarrow$ No division by 0

4. Defensive Programming

Constants and Assertions

Sources of Errors

- Errors that the compiler can find:
  syntactical and some semantical errors
- Errors that the compiler cannot find:
  runtime errors (always semantical)

The Compiler as Your Friend: Constants

Constants

- are variables with immutable value

```
const int speed_of_light = 299792458;
```

- Usage: `const` before the definition

The Compiler as Your Friend: Constants

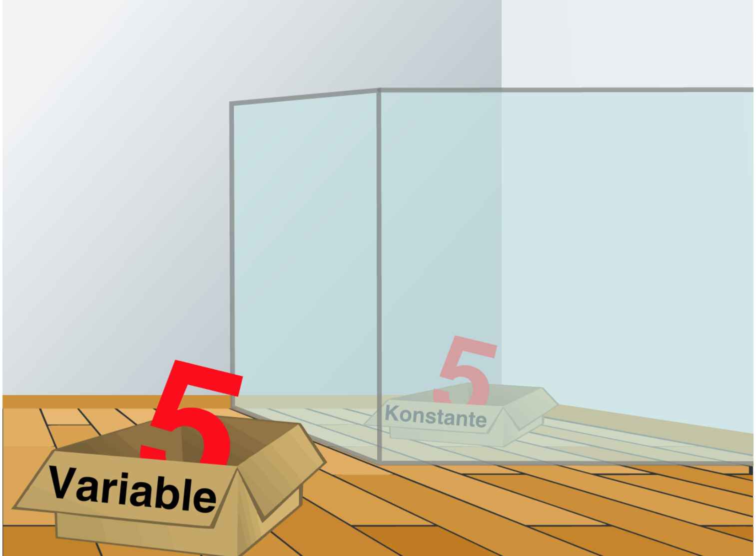
- Compiler checks that the **`const`**-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise :*“value does not change”*

Constants: Variables behind Glass



The `const`-guideline

> **const-guideline**
>
> For *each variable*, think about whether it will change its value in the lifetime of a program. If not, use the keyword `const` in order to make the variable a constant.

A program that adheres to this guideline is called `const`-correct.

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior
2. Check at many places in the code if the program is still on track
3. Question the (seemingly) obvious, there could be a typo in the code

Against Runtime Errors: *Assertions*

`assert(expr)`

- halts the program if the boolean expression **`expr`** is false
- requires **`#include <cassert>`**
- can be switched off (potential performance gain)

Assertions for the $gcd(x, y)$

Check if the program is on track ...

```
// Input x and y
std::cout << "x =? ";
std::cin >> x;
std::cout << "y =? ";
std::cin >> y;
```

Input arguments for calculation

```
// Check validity of inputs
assert(x > 0 && y > 0);
```

Precondition for the ongoing computation

```
... // Compute gcd(x,y), store result in variable a
```

Assertions for the $gcd(x, y)$

... and question the obvious! ...

```
...
assert(x > 0 && y > 0);
```

Precondition for the ongoing computation

```
... // Compute gcd(x,y), store result in variable a

assert (a >= 1);
assert (x % a == 0 && y % a == 0);
for (int i = a+1; i <= x && i <= y; ++i)
  assert(!(x % i == 0 && y % i == 0));
```

Properties of the gcd

Switch off Assertions

```
#define NDEBUG // To ignore assertions
#include<cassert>

...
assert(x > 0 && y > 0); // Ignored

... // Compute gcd(x,y), store result in variable a

assert(a >= 1); // Ignored
...
```

Fail-Fast with Assertions

- Real software: many C++ files, complex control flow
- Errors surface late(r) → impedes error localisation
- Assertions: Detect errors early

5. Control Structures I

Selection Statements, Iteration Statements, Termination, Blocks

Control Flow

- Up to now: *linear* (from top to bottom)
- Interesting programs require “branches” and “jumps”

```
// Project Hangman
...
while (game_not_over) {
  ...
  if (word.contains(guess)) {
    ...
  } else {
    ...
  }
}
...
```

Selection Statements

implement branches

- **`if`** statement
- **`if-else`** statement

`if`-Statement

`if` (*condition*)
statement

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

If *condition* is true then *statement* is executed

- *statement*: arbitrary statement (*body* of the **`if`**-Statement)
- *condition*: convertible to **`bool`**

`if-else`-statement

`if` (*condition*)
    statement1
else
    statement2

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

If *condition* is true then *statement1* is executed, otherwise *statement2* is executed.

- *condition*: convertible to `bool`.
- *statement1*: *body* of the `if`-branch
- *statement2*: *body* of the `else`-branch

Layout!

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Indentation

Indentation

Iteration Statements

implement loops

- `for`-statement
- `while`-statement
- `do`-statement

Compute $1 + 2 + ... + n$

```
// Program: sum_n.cpp
// Compute the sum of the first n natural numbers.

#include <iostream>

int main()
{
  // input
  std::cout << "Compute the sum 1+...+n for n =? ";
  unsigned int n;
  std::cin >> n;

  // computation of sum_{i=1}^n i
  unsigned int s = 0;
  for (unsigned int i = 1; i <= n; ++i) s += i;

  // output
```

`for`-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

`i`		`s`
`i==1`	wahr	`s == 1`
`i==2`	wahr	`s == 3`
`i==3`	falsch	

`s == 3`

Gauß as a Child (1777 - 1855)

- As you probably know, there exists a more efficient way to compute the sum of the first n natural numbers. Here's a corresponding anecdote:
- Math-teacher wanted to keep the pupils busy with the following task:

> *Compute the sum of numbers from 1 to 100!*

- Gauß finished after one minute.

The Solution of Gauß

- The requested number is

$$1 + 2 + 3 + \cdots + 98 + 99 + 100.$$

- This is half of

$$\begin{array}{rrrrrrrrr} & 1 & + & 2 & + & \cdots & + & 99 & + & 100 \\ + & 100 & + & 99 & + & \cdots & + & 2 & + & 1 \\ \hline = & 101 & + & 101 & + & \cdots & + & 101 & + & 101 \end{array}$$

- Answer: $100 \cdot 101/2 = 5050$

`for`-Statement: Syntax

`for` (*init statement* **`;`** *condition* **`;`** *expression*)
    body statement

- *init statement*: expression statement, declaration statement, null statement
- *condition*: convertible to **`bool`**
- *expression*: any expression
- *body statement*: any statement (*body* of the for-statement)

`for`-Statement: semantics

`for` (*init statement* *condition* `;` *expression*)
    statement

- *init-statement* is executed
- *condition* is evaluated
    - `true`: Iteration starts
      statement is executed
      expression is executed
    - false: `for`-statement is ended.

`for`-Statement: Termination

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

Here and in most cases:

- *expression* changes its value that appears in *condition* .
- After a finite number of iterations *condition* becomes false: **Termination**

Infinite Loops

- Infinite loops are easy to generate:

```
for ( ; ; ) ;
```

  - Die *empty condition* is true.
  - Die *empty expression* has no effect.
  - Die *null statement* has no effect.

- ... but can in general not be automatically detected.

```
for (init; cond; expr) stmt;
```

Halting Problem

Undecidability of the Halting Problem

There is no $\mathrm{C{+}{+}}$ program that can determine for each $\mathrm{C{+}{+}}$-Program P and each input I if the program P terminates with the input I.

This means that the correctness of programs can in general *not* be automatically checked.[4]

[4]Alan Turing, 1936. Theoretical questions of this kind were the main motivation for Alan Turing to construct a computing machine.

Example: Prime Number Test

Def.: a natural number $n \geq 2$ is a prime number, if no $d \in \{2, \ldots, n-1\}$ divides n .

A loop that can test this:

```
unsigned int d;
for (d=2; n%d != 0; ++d);
```

Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value `d=2`, then plus 1 in every iteration (`++d`)
- Exit: `n%d != 0` evaluates to `false` as soon as a divisor is found — at the latest, once `d == n`
- Progress guarantees that the exit condition will be reached

Example: Correctness

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

Every potential divisor **`2 <= d <= n`** will be tested. If the loop terminates with **`d == n`** then and only then is **`n`** prime.

Blocks

- Blocks group a number of statements to a new statement

```
{statement1 statement2 ... statementN}
```

- Example: body of the main function

```
int main() {
    ...
}
```

- Example: loop body

```
for (unsigned int i = 1; i <= n; ++i) {
    s += i;
    std::cout << "partial sum is " << s << "\n";
```

6. Control Statements II

Visibility, Local Variables, While Statement, Do Statement, Jump Statements

Visibility

Declaration in a block is not *visible* outside of the block.

```
int main()
{
    {
        int i = 2;
    }
    std::cout << i; // Error: undeclared name
    return 0;
}
```

main block

block

„Blickrichtung“

Control Statement defines Block

In this respect, statements behave like blocks.

```
int main()
{
    for (unsigned int i = 0; i < 10; ++i)
        s += i;
    std::cout << i; // Error: undeclared name
    return 0;
}
```

block

Scope of a Declaration

Potential scope: from declaration until end of the part that contains the declaration.

in the block

```
{
    ...
    int i = 2;
    ...
}
```

scope

in function body

```
int main() {
    ...
    int i = 2;
    ...
    return 0;
}
```

scope

in control statement

```
for (int i = 0; i < 10; ++i) {s += i; ... }
```

scope

Scope of a Declaration

Real scope = potential scope minus potential scopes of declarations of symbols with the same name

```
int main()
{
  int i = 2;
  for (int i = 0; i < 5; ++i)
     // outputs 0,1,2,3,4
     std::cout << i;
   // outputs 2
   std::cout << i;
  return 0;
}
```

scope of i in main

i_2 in for

Automatic Storage Duration

Local Variables (declaration in block)

- are (re-)created each time their declaration is reached
  - memory address is assigned (allocation)
  - potential initialization is executed
- are deallocated at the end of their declarative region (memory is released, address becomes invalid)

Local Variables

```
int main()
{
    int i = 5;
    for (int j = 0; j < 5; ++j) {
        std::cout << ++i; // outputs 6, 7, 8, 9, 10
        int k = 2;
        std::cout << --k; // outputs 1, 1, 1, 1, 1
    }
}
```

Local variables (declaration in a block) have *automatic storage duration.*

`while` Statement

```
while (condition)
  statement
```

- *statement*: arbitrary statement, body of the **`while`** statement.
- *condition*: convertible to **`bool`**.

`while` Statement

```
while (condition)
  statement
```

is equivalent to

```
for (; condition; )
  statement
```

`while`-Statement: Semantics

```
while (expression)
  statement
```

- *condition* is evaluated
  - `true`: iteration starts
    statement is executed
  - `false`: `while`-statement ends.

`while`-statement: why?

- In a `for`-statement, the expression often provides the progress (“counting loop”)

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

- If the progress is not as simple, `while` can be more readable.

Example: The Collatz-Sequence $(n \in \mathbb{N})$

- $n_0 = n$
- $n_i = \begin{cases} \dfrac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1} + 1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

n=5: 5, 16, 8, 4, 2, 1, 4, 2, 1, ... (repetition at 1)

The Collatz Sequence in C++

```
// Program collatz.cpp. Computes the Collatz sequence of a number n.

#include <iostream>

int main() {
  // Input
  std::cout << "Compute the Collatz sequence for n =? ";
  unsigned int n;
  std::cin >> n;

  // Iteration
  while (n > 1) {
    if (n % 2 == 0) n = n / 2;
    else n = 3 * n + 1;
    std::cout << n << " ";
  }
  std::cout << "\n";

  return 0;
}
```

The Collatz Sequence in C++

```
n = 27:
82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484,
242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466,
233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890,
445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283,
850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238,
1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051,
6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300,
650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106,
53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1
```

The Collatz-Sequence

Does 1 occur for each n?

- It is conjectured, but nobody can prove it!
- If not, then the **`while`**-statement for computing the Collatz-sequence can theoretically be an endless loop for some n.

`do` Statement

```
do
  statement
while (condition);
```

- *statement*: arbitrary statement, body of the **`do`** statement.
- *condition*: convertible to **`bool`**.

`do` Statement

```
do
  statement
while (condition);
```

is equivalent to

```
statement
while (condition)
  statement
```

`do`-Statement: Semantics

```
do
  statement
while (condition);
```

- Iteration starts
  - *statement* is executed.
- *condition* is evaluated
  - **`true`**: iteration begins
  - **`false`**: **`do`**-statement ends.

do-Statement: Example Calculator

Sum up integers (if 0 then stop):

```
int a;     // next input value
int s = 0; // sum of values so far
do {
    std::cout << "next number =? ";
    std::cin >> a;
    s += a;
    std::cout << "sum = " << s << "\n";
} while (a != 0);
```

Conclusion

- Selection (conditional *branches*)
  - **`if`** and **`if-else`**-statement
- Iteration (conditional *jumps*)
  - **`for`**-statement
  - **`while`**-statement
  - **`do`**-statement
- Blocks and scope of declarations

Jump Statements

- `break;`
- `continue;`

`break`-Statement

```
break;
```

- Immediately leave the enclosing iteration statement
- useful in order to be able to break a loop “in the middle” [5]

[5] and indispensible for switch-statements

Calculator with **break**

Sum up integers (if 0 then stop)

```
int a;
int s = 0;
do {
    std::cout << "next number =? ";
    std::cin >> a;
    s += a; /* irrelevant in last iteration */
    std::cout << "sum = " << s << "\n";
} while (a != 0);
```

Calculator with `break`

Suppress irrelevant addition of 0:

```
int a;
int s = 0;
do {
    std::cout << "next number =? ";
    std::cin >> a;
    if (a == 0) break; // exit loop in the middle
    s += a;
    std::cout << "sum = " << s << "\n";
} while (a != 0)
```

Calculator with **break**

Equivalent and yet more simple:

```
int a;
int s = 0;
for (;;) {
    std::cout << "next number =? ";
    std::cin >> a;
    if (a == 0) break; // exit loop in the middle
    s += a;
    std::cout << "sum = " << s << "\n";
}
```

Calculator *without* `break`

Version without `break` evaluates `a != 0` twice (and requires an additional block).

```
int a = 1;
int s = 0;
for (; a != 0; ) {
    std::cout << "next number =? ";
    std::cin >> a;
    if (a != 0) {
        s += a;
        std::cout << "sum = " << s << "\n";
    }
}
```

`continue`-Statement

```
continue;
```

- Jump over the rest of the body of the enclosing iteration statement
- Iteration statement is *not* left.

`break` and `continue` in practice

- Advantage: Can avoid nested **`if-else`**blocks (or complex disjunctions)
- But they result in additional jumps and thus potentially complicate the control flow
- Their use is thus controversial, and should be carefully considered

Calculator with `continue`

Ignore negative input:

```
for (;;) {
    std::cout << "next number =? ";
    std::cin >> a;
    if (a < 0) continue; // jump to }
    if (a == 0) break;
    s += a;
    std::cout << "sum = " << s << "\n";
}
```

Equivalence of Iteration Statements

We have seen:

- **`while`** and **`do`** can be simulated with **`for`**

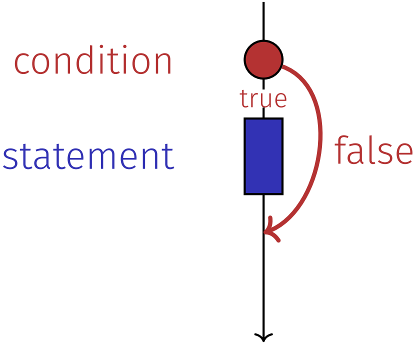
It even holds:

- The three iteration statements provide the same "expressiveness" (lecture notes)
- Not so simple if a `continue` is used

Control Flow

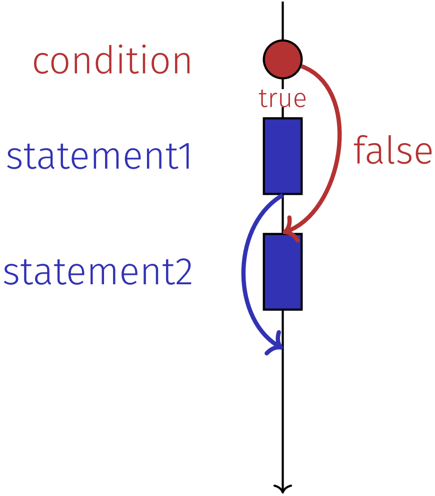
Order of the (repeated) execution of statements

- generally from top to bottom…
- …except in selection and iteration statements



`if` (*condition*)
    statement

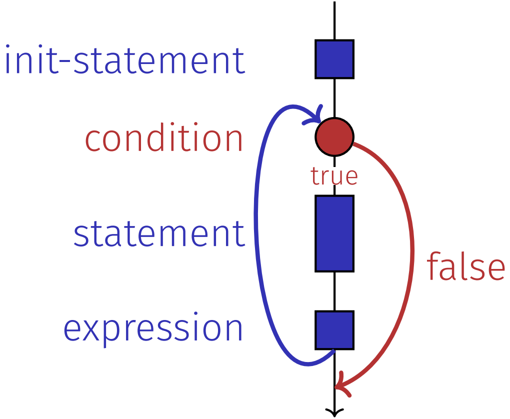
Control Flow `if else`



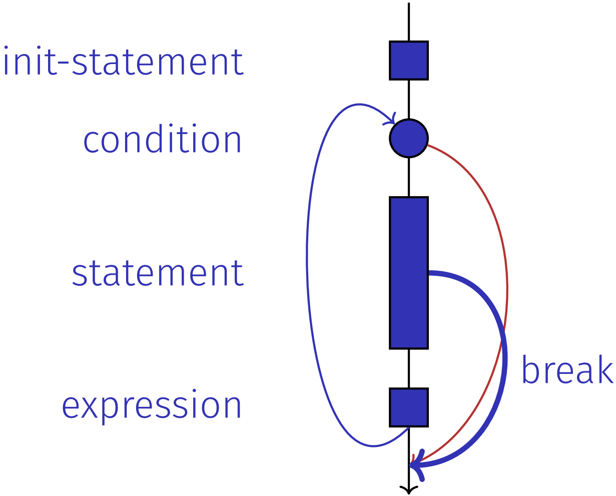
`if` (*condition*)
    statement1
else
    statement2

Control Flow `for`

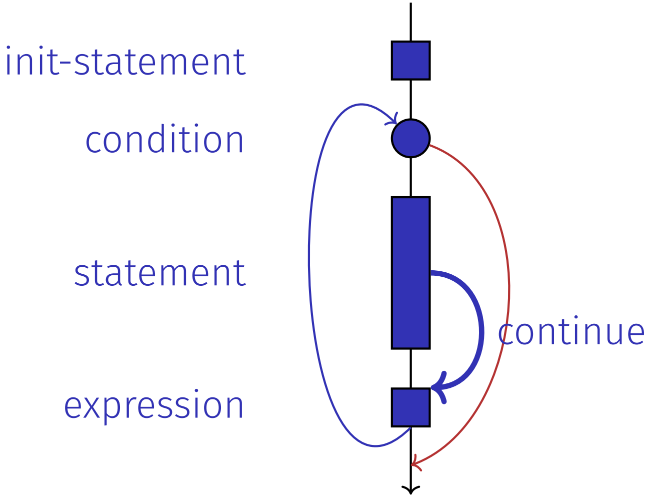
`for` (*init statement*   *condition* **;** *expression*)
      statement



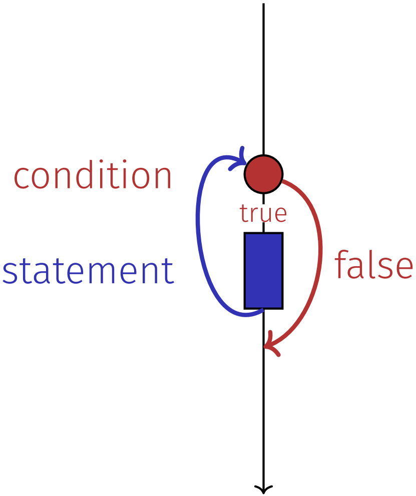
Control Flow `break` in for



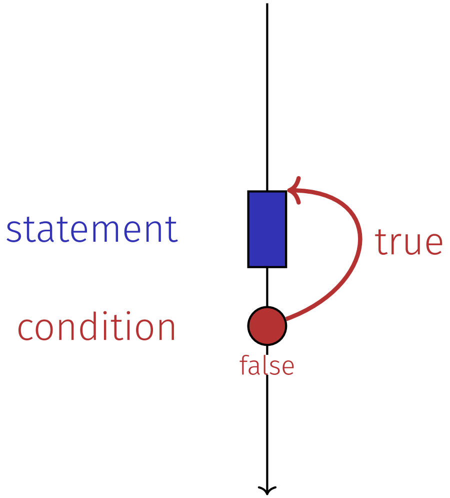
Control Flow `continue` in for



Control Flow `while`



Control Flow `do while`



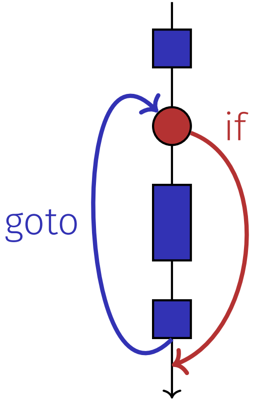
Control Flow: the Good old Times?

Observation

Actually, we only need **`if`** and jumps to arbitrary places in the program (**`goto`**).

Languages based on them:

- Machine Language
- Assembler ("higher" machine language)
- BASIC, the first programming language for the general public (1964)



BASIC and home computers…

…allowed a whole generation of young adults to program.

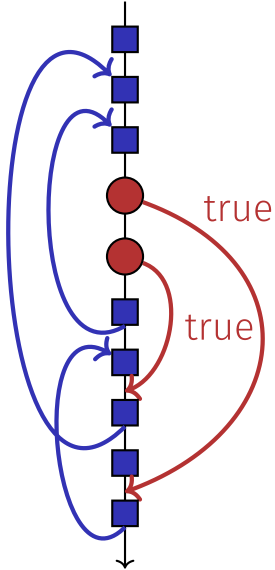
Home-Computer Commodore C64 (1982)

http://de.wikipedia.org/wiki/Commodore_64

Spaghetti-Code with `goto`

Output of of ???????????all prime numbers
using the programming language BASIC:

```
10  N=2
20  D=1
30  D=D+1
40  IF N=D GOTO 100
50  IF N/D = INT(N/D) GOTO 70
60  GOTO 30
70  N=N+1
80  GOTO 20
100 PRINT N
110 GOTO 70
```



The “right” Iteration Statement

Goals: readability, conciseness, in particular

- few statements
- few lines of code
- simple control flow
- simple expressions

Often not all goals can be achieved simultaneously.

Odd Numbers in $\{0, \ldots, 100\}$

First (correct) attempt:

```
for (unsigned int i = 0; i < 100; ++i) {
    if (i % 2 == 0)
        continue;
    std::cout << i << "\n";
}
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, **less** lines:

```
for (unsigned int i = 0; i < 100; ++i) {
    if (i % 2 != 0)
        std::cout << i << "\n";
}
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, **simpler** control flow:

```
for (unsigned int i = 1; i < 100; i += 2)
    std::cout << i << "\n";
```

This is the “right” iteration statement

Jump Statements

- implement unconditional jumps.
- are useful, such as **`while`** and **`do`** but not indispensible
- should be used with care: only where the control flow is *simplified* instead of making it *more complicated*

Outputting Grades

1. Functional requirement:

$$
\begin{aligned}
6 &\rightarrow \texttt{"Excellent ... You passed!"} \\
5, 4 &\rightarrow \texttt{"You passed!"} \\
3 &\rightarrow \texttt{"Close, but ... You failed!"} \\
2, 1 &\rightarrow \texttt{"You failed!"} \\
\textit{otherwise} &\rightarrow \texttt{"Error!"}
\end{aligned}
$$

2. Moreover: Avoid duplication of text and code

Outputting Grades with `if` Statements

```
int grade;
...
if (grade == 6) std::cout << "Excellent ... ";
if (4 <= grade && grade <= 6) {
    std::cout << "You passed!";
} else if (1 <= grade && grade < 4) {
    if (grade == 3) std::cout << "Close, but ... ";
    std::cout << "You failed!";
} else std::cout << "Error!";
```

Disadvantage: Control flow – and thus program behaviour – not quite obvious

Outputting Grades with `switch` Statement

```
switch (grade) {
  case 6: std::cout << "Excellent ... ";
  case 5:
  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

Jump to matching **case**

Fall-through

Exit **switch**

Fall-through

Exit **switch**

In all other cases

Advantage: Control flow clearly recognisable

The `switch`-Statement

```
switch (expression)
  statement
```

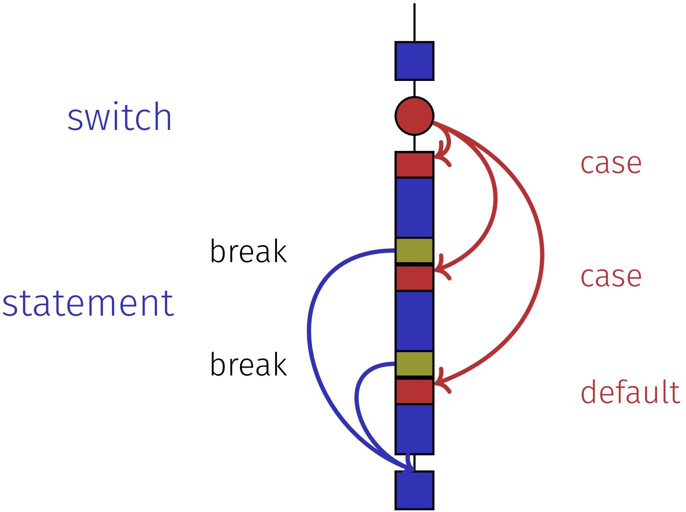
- *expression*: Expression, convertible to integral type
- *statement* : arbitrary statemet, in which **`case`** and **`default`**-lables are permitted, **`break`** has a special meaning.
- Use of fall-through property is controversial and should be carefully considered (corresponding compiler warning can be enabled)

Semantics of the `switch`-statement

> **`switch`** (*expression*)
>   *statement*

- **`expression`** is evaluated.
- If **`statement`** contains a **`case`**-label with (constant) value of **`condition`**, then jump there
- otherwise jump to the **`default`**-lable, if available. If not, jump over **`statement`**.
- The **`break`** statement ends the **`switch`**-statement.

Control Flow `switch`



7. Floating-point Numbers I

Types **`float`** and **`double`**; Mixed Expressions and Conversion; Holes in the Value Range

“Proper” Calculation

```
// Program: fahrenheit_float.cpp
// Convert temperatures from Celsius to Fahrenheit.

#include <iostream>

int main() {
  // Input
  std::cout << "Temperature in degrees Celsius =? ";
  float celsius;
  std::cin >> celsius;

  // Computation and output
  std::cout << celsius << " degrees Celsius are "
            << 9 * celsius / 5 + 32 << " degrees Fahrenheit.\n";
  return 0;
}
```

Fixed-point numbers

- fixed number of integer places (e.g. 7)
- fixed number of decimal places (e.g. 3)

```
0.0824 = 0000000.082
```

third place truncated

Disadvantages

- Value range is getting *even* smaller than for integers.
- Representability depends on the position of the decimal point.

Floating-point numbers

- Observation: same number, different representations with varying "efficiency", e.g.

$$\begin{aligned} 0.0824 &= 0.00824 \cdot 10^{1} &&= 0.824 \cdot 10^{-1} \\ &= 8.24 \cdot 10^{-2} &&= 824 \cdot 10^{-4} \end{aligned}$$

  Number of *significant digits* remains constant

- Floating-point number representation thus:
  - Fixed number of significant places (e.g. 10),
  - Plus position of the decimal point via exponent
  - Number is $\mathit{Mantissa} \times 10^{\mathit{Exponent}}$

Types `float` and `double`

- are the fundamental C++ types for floating point numbers
- approximate the field of real numbers $(\mathbb{R}, +, \times)$ from mathematics
- have a big value range, sufficient for many applications:
  - **`float`**: approx. 7 digits, exponent up to ± 38
  - **`double`**: approx. 15 digits, exponent up to ± 308
- are fast on most computers (hardware support)

Arithmetic Operators

Analogous to **`int`**, but ...

- Division operator **`/`** models a “proper” division (real-valued, not integer)
- No modulo operator, i.e. no **`%`**

Literals

are different from integers by providing

- decimal point

  `1.0` : type **`double`**, value 1

  `1.27f` : type **`float`**, value 1.27

- and / or exponent.

  `1e3` : type **`double`**, value 1000

  `1.23e-7` : type **`double`**, value $1.23 \cdot 10^{-7}$

  `1.23e-7f` : type **`float`**, value $1.23 \cdot 10^{-7}$

1.23e-7f

integer part

exponent

fractional part

Computing with `float`: Example

Approximating the Euler-Number

$$e = \sum_{i=0}^{\infty} \frac{1}{i!} \approx 2.71828\ldots$$

using the first 10 terms.

Computing with `float`: Euler Number

```
std::cout << "Approximating the Euler number... \n";

// values for i-th iteration, initialized for i = 0
float t = 1.0f; // term 1/i!
float e = 1.0f; // i-th approximation of e

// iteration 1, ..., n
for (unsigned int i = 1; i < 10; ++i) {
    t /= i;    // 1/(i-1)! -> 1/i!
    e += t;
    std::cout << "Value after term " << i << ": "
              << e << "\n";
}
```

Computing with `float`: Euler Number

```
Value after term 1: 2
Value after term 2: 2.5
Value after term 3: 2.66667
Value after term 4: 2.70833
Value after term 5: 2.71667
Value after term 6: 2.71806
Value after term 7: 2.71825
Value after term 8: 2.71828
Value after term 9: 2.71828
```

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.
- In mixed expressions integers are converted to floating point numbers.

```
9 * celsius / 5  + 32
```

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;

float n2;
std::cout << "Second number =? ";
std::cin >> n2;

float d;
std::cout << "Their difference =? ";
std::cin >> d;

std::cout << "Computed difference - input difference = "
          << n1 - n2 - d << "\n";
```

input 1.1

input 1.0

input 0.1

output 2.23517e-8

What is going on here?

Value range

Integer Types:

- Over- and Underflow relatively frequent, but ...
- the value range is contiguous (no holes): $\mathbb{Z}$ is “discrete”.

Floating point types:

- Overflow and Underflow seldom, but ...
- there are holes: $\mathbb{R}$ is “continuous”.

8. Floating-point Numbers II

Floating-point Number Systems; IEEE Standard; Limits of Floating-point Arithmetics; Floating-point Guidelines; Harmonic Numbers

Floating-point Number Systems

A Floating-point number system is defined by the four natural numbers:

- $\beta \geq 2$, the base,
- $p \geq 1$, the precision (number of places),
- $e_{\min}$, the smallest possible exponent,
- $e_{\max}$, the largest possible exponent.

Notation:

$$F(\beta, p, e_{\min}, e_{\max})$$

Floating-point number Systems

$F(\beta, p, e_{\min}, e_{\max})$ contains the numbers

$$\pm \sum_{i=0}^{p-1} d_i \beta^{-i} \cdot \beta^e,$$

$$d_i \in \{0, \ldots, \beta - 1\}, \quad e \in \{e_{\min}, \ldots, e_{\max}\}.$$

represented in base β:

$$\pm\, d_0 \bullet d_1 \ldots d_{p-1} \times \beta^e,$$

Floating-point Number Systems

Representations of the decimal number 0.1 (with $\beta = 10$):

$$1.0 \cdot 10^{-1}, \quad 0.1 \cdot 10^{0}, \quad 0.01 \cdot 10^{1}, \quad \ldots$$

Different representations due to choice of exponent

Normalized representation

Normalized number:

$$\pm\, d_0{\bullet} d_1 \dots d_{p-1} \times \beta^e, \qquad d_0 \neq 0$$

Remark 1

The normalized representation is unique and therefore preferred.

Remark 2

The number 0, as well as all numbers smaller than $\beta^{e_{\min}}$, have no normalized representation (we will come back to

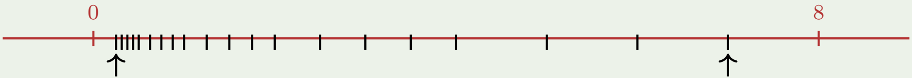
Set of Normalized Numbers

$$F^*(\beta, p, e_{\min}, e_{\max})$$

Normalized Representation

Example $F^*(2, 3, -2, 2)$ (only positive numbers)

$d_0{\bullet}d_1d_2$	$e=-2$	$e=-1$	$e=0$	$e=1$	$e=2$
1.00_2	0.25	0.5	1	2	4
1.01_2	0.3125	0.625	1.25	2.5	5
1.10_2	0.375	0.75	1.5	3	6
1.11_2	0.4375	0.875	1.75	3.5	7



$1.00 \cdot 2^{-2} = \frac{1}{4}$

$1.11 \cdot 2^{2} = 7$

Binary and Decimal Systems

- Internally the computer computes with $\beta = 2$ (binary system)
- Literals and inputs have $\beta = 10$ (decimal system)
- Inputs have to be converted!

Conversion Decimal $\to$ Binary

Assume, $0 < x < 2$.
Binary representation:

$$
\begin{aligned}
x &= \sum_{i=-\infty}^{0} b_i 2^i = b_0{\bullet}b_{-1}b_{-2}b_{-3}\dots \\
&= b_0 + \sum_{i=-\infty}^{-1} b_i 2^i = b_0 + \sum_{i=-\infty}^{0} b_{i-1} 2^{i-1} \\
&= b_0 + \underbrace{\left(\sum_{i=-\infty}^{0} b_{i-1} 2^i \right)}_{x'=b_{-1}{\bullet}b_{-2}b_{-3}b_{-4}} /2
\end{aligned}
$$

Conversion Decimal $\to$ Binary

Assume $0 < x < 2$.

- Hence: $x' = b_{-1} \bullet b_{-2} b_{-3} b_{-4} \ldots = 2 \cdot (x - b_0)$
- Step 1 (for x): Compute b_0:

$$b_0 = \begin{cases} 1, & \text{if } x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

- Step 2 (for x): Compute $b_{-1}, b_{-2}, \ldots$:

  Go to step 1 (for $x' = 2 \cdot (x - b_0)$)

Binary representation of 1.1_{10}

x	b_i	$x - b_i$	$2(x - b_i)$
1.1	$b_0 = 1$	0.1	0.2
0.2	$b_1 = 0$	0.2	0.4
0.4	$b_2 = 0$	0.4	0.8
0.8	$b_3 = 0$	0.8	1.6
1.6	$b_4 = 1$	0.6	1.2
1.2	$b_5 = 1$	0.2	0.4

$\Rightarrow$ $1.0\overline{0011}$, periodic, *not* finite

Binary Number Representations of 1.1 and 0.1

- are not finite, hence there are errors when converting into a (finite) binary floating-point system.
- `1.1f` and `0.1f` do not equal 1.1 and 0.1, but are slightly inaccurate approximation of these numbers.
- In `diff.cpp`: $1.1 - 1.0 \neq 0.1$

Binary Number Representations of 1.1 and 0.1

on my computer:

$$\begin{aligned} \texttt{1.1} &= \underline{1.10000000000000000}888178\ldots \\ \texttt{1.1f} &= \underline{1.1000000}238418\ldots \end{aligned}$$

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$
\begin{array}{rl}
 & 1.111 \cdot 2^{-2} \\
+ & 1.011 \cdot 2^{-1} \\
\hline
= & 1.001 \cdot 2^{0}
\end{array}
$$

1. adjust exponents by denormalizing one number 2. binary addition of the significands 3. renormalize 4. round to p significant places, if necessary

The IEEE Standard 754

defines floating-point number systems and their rounding behavior and is used nearly everywhere

- Single precision (**`float`**) numbers:

  $F^*(2, 24, -126, 127)$ (32 bit) plus $0, \infty, \ldots$

- Double precision (**`double`**) numbers:

  $F^*(2, 53, -1022, 1023)$ (64 bit) plus $0, \infty, \ldots$

- All arithmetic operations round the *exact* result to the next representable number

The IEEE Standard 754

Why

$$F^*(2, 24, -126, 127)?$$

- 1 sign bit
- 23 bit for the significand (leading bit is 1 and is not stored)
- 8 bit for the exponent (256 possible values)(254 possible exponents, 2 special values: 0, ∞,...)

$\Rightarrow$ 32 bit in total.

The IEEE Standard 754

Why

$$F^*(2, 53, -1022, 1023)?$$

- 1 sign bit
- 52 bit for the significand (leading bit is 1 and is not stored)
- 11 bit for the exponent (2046 possible exponents, 2 special values: 0, ∞,…)

$\Rightarrow$ 64 bit in total.

Example: 32-bit Representation of a Floating Point Number

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

$\pm$ Exponent Mantisse

$$\pm \begin{array}{c} 2^{-126}, \ldots, 2^{127} \\ 0, \infty, \ldots \end{array} \qquad \begin{array}{c} 1.00000000000000000000000 \\ \cdots \\ 1.11111111111111111111111 \end{array}$$

Floating-point Rules

Rule 1

Do not test rounded floating-point numbers for equality.

```
for (float i = 0.1; i != 1.0; i += 0.1)
    std::cout << i << "\n";
```

endless loop because i never becomes exactly 1

Floating-point Rules

Rule 2

Do not add two numbers of very different orders of magnitude!

$$\begin{aligned} & 1.000 \cdot 2^5 \\ & +1.000 \cdot 2^0 \\ = \; & 1.00001 \cdot 2^5 \\ \text{“}{=}\text{”} \; & 1.000 \cdot 2^5 \quad \text{(Rounding on 4 places)} \end{aligned}$$

Addition of 1 does not have any effect!

Harmonic Numbers Rule 2

- The n-the harmonic number is

$$H_n = \sum_{i=1}^{n} \frac{1}{i} \approx \ln n.$$

- This sum can be computed in forward or backward direction, which is mathematically clearly equivalent

Harmonic Numbers Rule 2

```
// Program: harmonic.cpp
// Compute the n-th harmonic number in two ways.

#include <iostream>

int main()
{
  // Input
  std::cout << "Compute H_n for n =? ";
  unsigned int n;
  std::cin >> n;

  // Forward sum
  float fs = 0;
  for (unsigned int i = 1; i <= n; ++i)
    fs += 1.0f / i;

  // Backward sum
  float bs = 0;
  for (unsigned int i = n; i >= 1; --i)
    bs += 1.0f / i;

  // Output
  std::cout << "Forward sum  = " << fs << "\n"
            << "Backward sum = " << bs << "\n";
  return 0;
}
```

Harmonic Numbers Rule 2

Results:

- ```
  Compute H_n for n =?  10000000
  Forward sum = 15.4037
  Backward sum = 16.686
  ```
- ```
  Compute H_n for n =?  100000000
  Forward sum = 15.4037
  Backward sum = 18.8079
  ```

Harmonic Numbers Rule 2

Observation:

- The forward sum stops growing at some point and is “really” wrong.
- The backward sum approximates H_n well.

Explanation:

- For $1 + 1/2 + 1/3 + \cdots$, later terms are too small to actually contribute
- Problem similar to $2^5 + 1$ “$=$” 2^5

Floating-point Guidelines Rule 3

Rule 4

Do not subtract two numbers with a very similar value.

Cancellation problems, cf. lecture notes.

Literature

David Goldberg: What Every Computer Scientist Should Know About Floating-Point Arithmetic (1991)

Randy Glasbergen, 1996

9. Functions I

Defining and Calling Functions, Evaluation of Function Calls, the Type `void`

Functions

- encapsulate functionality that is frequently used (e.g. computing powers) and make it easily accessible
- structure a program: partitioning into small sub-tasks, each of which is implemented as a function

$\Rightarrow$ Procedural programming; procedure: a different word for function.

Example: Computing Powers

```
double a;
int n;
std::cin >> a; // Eingabe a
std::cin >> n; // Eingabe n
```

```
double result = 1.0;
if (n < 0) { // a^n = (1/a)^(-n)
  a = 1.0/a;
  n = -n;
}
for (int i = 0; i < n; ++i)
  result *= a;
```

"Funktion pow"

```
std::cout << a << "^" << n << " = " << result << ".\n";
```

Function to Compute Powers

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Function to Compute Powers

```
// Prog: callpow.cpp
// Define and call a function for computing powers.
#include <iostream>

double pow(double b, int e){...}

int main()
{
  std::cout << pow( 2.0, -2) << "\n"; // outputs 0.25
  std::cout << pow( 1.5, 2) << "\n"; // outputs 2.25
  std::cout << pow(-2.0, 9) << "\n"; // outputs -512

  return 0;
}
```

Function Definitions

return type *argument types*

T fname (T_1 pname$_1$, T_2 pname$_2$, $\ldots$, T_N pname$_N$)
block

body

function name *formal arguments*

Defining Functions

- may not occur *locally*, i.e. not in blocks, not in other functions and not within control statements
- can be written consecutively without separator in a program

```
double pow (double b, int e)
{
    ...
}

int main ()
{
    ...
}
```

Example: Xor

```
// post: returns l XOR r
bool Xor(bool l, bool r)
{
    return l && !r || !l && r;
}
```

Example: Harmonic

```
// PRE: n >= 0
// POST: returns nth harmonic number
//       computed with backward sum
float Harmonic(int n)
{
    float res = 0;
    for (unsigned int i = n; i >= 1; --i)
        res += 1.0f / i;
    return res;
}
```

Example: min

```
// POST: returns the minimum of a and b
int min(int a, int b)
{
    if (a<b)
        return a;
    else
        return b;
}
```

Function Calls

fname (*expression*$_1$, *expression*$_2$, $\ldots$, *expression*$_N$)

- All call arguments must be convertible to the respective formal argument types.
- The function call is an expression of the return type of the function. Value and effect as given in the postcondition of the function *fname*.

Example: **`pow(a,n)`**: Expression of type **`double`**

Function Calls

For the types we know up to this point it holds that:

- Call arguments are R-values
  $\hookrightarrow$ *call-by-value* (also *pass-by-value*), more on this soon
- The function call is an R-value.

$$\textit{fname}\text{:}\ \text{R-value} \times \text{R-value} \times \cdots \times \text{R-value} \longrightarrow \text{R-value}$$

Evaluation of a Function Call

- Evaluation of the call arguments
- Initialization of the formal arguments with the resulting values
- Execution of the function body: formal arguments behave laike local variables
- Execution ends with
  `return` *expression*;

  Return value yiels the value of the function call.

Example: Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

Call of pow

Return

sometimes em formal arguments

- Declarative region: function definition
- are *invisible* outside the function definition
- are allocated for each call of the function (automatic storage duration)
- modifications of their value do not have an effect to the values of the call arguments (call arguments are R-values)

Scope of Formal Arguments

```
double pow(double b, int e){
    double r = 1.0;
    if (e<0) {
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        r * = b;
    return r;
}
```

```
int main(){
    double b = 2.0;
    int e = -2;
    double z = pow(b, e);

    std::cout << z; // 0.25
    std::cout << b; // 2
    std::cout << e; // -2
    return 0;
}
```

Not the formal arguments `b` and `e` of pow but the variables defined here locally in the body of `main`

The type `void`

```
// POST: "(i, j)" has been written to standard output
void print_pair(int i, int j) {
    std::cout << "(" << i << ", " << j << ")\n";
}

int main() {
    print_pair(3,4); // outputs (3, 4)
    return 0;
}
```

The type `void`

- Fundamental type with empty value range
- Usage as a return type for functions that do *only* provide an effect

`void`-Functions

- do not require **`return`**.
- execution ends when the end of the function body is reached or if
- **`return;`** is reached
  or
- **`return`** *expression* **`;`** is reached.

Expression with type **`void`** (e.g. a call of a function with return type **`void`**

Functions and `return`

The behavior of a function with non-**`void`** return type is **undefined** if the end of the function body is reached without a **`return`** statement.

Wrong:

```
bool compare(float x, float y) {
  float delta = x - y;
  if (delta*delta < 0.001f) return true;
}
```

Here the value of **`compare(10,20)`** is undefined.

Functions and `return`

The behavior of a function with non-**`void`** return type is **undefined** if the end of the function body is reached without a **`return`** statement.

Better:

```
bool compare(float x, float y) {
   float delta = x - y;
   if (delta*delta < 0.001f)
    return true;
   else
    return false;
}
```

All execution paths reach a **`return`**

Functions and `return`

The behavior of a function with non-**`void`** return type is **undefined** if the end of the function body is reached without a **`return`** statement.

Even better and simpler

```
bool compare(float x, float y) {
   float delta = x - y;
   return delta*delta < 0.001f;
 }
```

10. Functions II

Pre- and Postconditions Stepwise Refinement, Scope, Libraries and Standard Functions

Pre- and Postconditions

- characterize (as complete as possible) what a function does
- document the function for users and programmers (we or other people)
- make programs more readable: we do not have to understand *how* the function works
- are ignored by the compiler
- Pre and postconditions render statements about the correctness of a program possible – provided they are correct.

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

0^e is undefined for $e < 0$

```
// PRE: e >= 0 || b != 0.0
```

Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Here only value, no effect.

```
// POST: return value is b^e
```

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Funktion **`pow`**: works for all numbers $b \neq 0$

Pre- and Postconditions

- We do not make a statement about what happens if the precondition does not hold.
- C++-standard-slang: "Undefined behavior".

Function **`pow`**: division by 0

Pre- and Postconditions

- pre-condition should be as **weak** as possible (largest possible domain)
- post-condition should be as **strong** as possible (most detailed information)

White Lies. . .

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect:

- Overflow if e or b are too large
- b^e potentially not representable as a double (holes in the value range!)

White Lies are Allowed

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

The exact pre- and postconditions are platform-dependent and often complicated. We abstract away and provide the mathematical conditions.
$\Rightarrow$ compromise between formal correctness and lax practice.

Checking Preconditions...

- Preconditions are only comments.
- How can we ensure that they hold when the function is called?

...with assertions

```
#include <cassert>
...
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e) {
    assert (e >= 0 || b != 0);
    double result = 1.0;
    ...
}
```

Postconditions with Asserts

- The result of “complex” computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

```
// PRE: the discriminant p*p/4 - q is nonnegative
// POST: returns larger root of the polynomial x^2 + p x + q
double root(double p, double q)
{
    assert(p*p/4 >= q); // precondition
    double x1 = - p/2 + sqrt(p*p/4 - q);
    assert(equals(x1*x1+p*x1+q,0)); // postcondition
    return x1;
}
```

Exceptions

- Assertions are a rough tool; if an assertions fails, the program is halted in a unrecoverable way.
- C++provides more elegant means (exceptions) in order to deal with such failures depending on the situation and potentially without halting the program
- Failsafe programs should only halt in emergency situations and therefore should work with exceptions. For this course, however, this goes too far.

Stepwise Refinement

A simple *technique* to solve complex problems

Niklaus Wirth. Program development by stepwise refinement. Commun. ACM 14, 4, 1971

Education P. Wegner Editor

Program Development by Stepwise Refinement

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The creative activity of programming—to be distinguished from coding—is usually taught by examples serving to exhibit certain techniques. It is here considered as a sequence of design decisions concerning the decomposition of tasks into subtasks and of data into data structures. The process of successive refinement of specifications is illustrated by a short but nontrivial example, from which a number of conclusions are drawn regarding the art and the instruction of programming.

Key Words and Phrases: education in programming, programming techniques, stepwise program construction
CR Categories: 1.50, 4.0

1. Introduction

Programming is usually taught by examples. Experience shows that the success of a programming course critically depends on the choice of these examples. Unfortunately, they are too often selected with the prime intent to demonstrate what a computer can do. Instead, a main criterion for selection should be their suitability to exhibit certain widely applicable *techniques*. Furthermore, examples of programs are commonly presented as finished "products" followed by explanations of their purpose and their linguistic details. But active programming consists of the design of *new* programs, rather than contemplation of old programs. As a consequence of these teaching methods, the student obtains the impression that programming consists mainly of mastering a language (with all the peculiarities and intricacies so abundant in modern PL's) and relying on one's intuition to somehow transform ideas into finished programs. Clearly, programming courses should teach methods of design and construction, and the selected examples should be such that a gradual *development* can be nicely demonstrated.

This paper deals with a single example chosen with these two purposes in mind. Some well-known techniques are briefly demonstrated and motivated (strategy of preselection, stepwise construction of trial solutions, introduction of auxiliary data, recursion), and the program is gradually developed in a sequence of *refinement steps*.

In each step, one or several instructions of the given program are decomposed into more detailed instructions. This successive decomposition or refinement of specifications terminates when all instructions are expressed in terms of an underlying computer or programming language, and must therefore be guided by the facilities available on that computer or language. The result of the execution of a program is expressed in terms of data, and it may be necessary to introduce further data for communication between the obtained subtasks or instructions. As tasks are refined, so the data may have to be refined, decomposed, or structured, and it is natural to *refine program and data specifications in parallel*.

Every refinement step implies some design decisions. It is important that these decision be made explicit, and that the programmer be aware of the underlying criteria and of the existence of alternative solutions. The possible solutions to a given problem emerge as the leaves of a tree, each node representing a point of deliberation and decision. Subtrees may be considered as *families of solutions* with certain common characteristics and structures. The notion of such a tree may be particularly helpful in the situation of changing purpose and environment to which a program may sometime have to be adapted.

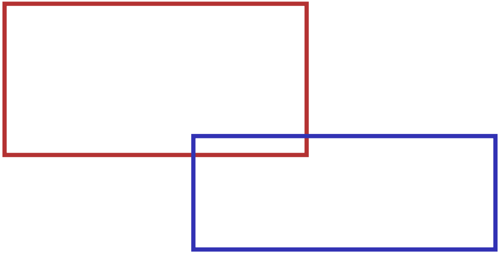
A guideline in the process of stepwise refinement should be the principle to decompose decisions as much as possible, to untangle aspects which are only seemingly interdependent, and to defer those decisions which concern details of representation as long as possible. This

Stepwise Refinement

- Solve the problem step by step. Start with a coarse solution on a high level of abstraction (only comments and abstract function calls)
- At each step, comments are replaced by program text, and functions are implemented (using the same principle again)
- The refinement also refers to the development of data representation (more about this later).
- If the refinement is realized as far as possible by functions, then partial solutions emerge that might be used for other problems.
- Stepwise refinement supports (but does not replace) the structural understanding of a problem.

Example Problem

Find out if two rectangles intersect!

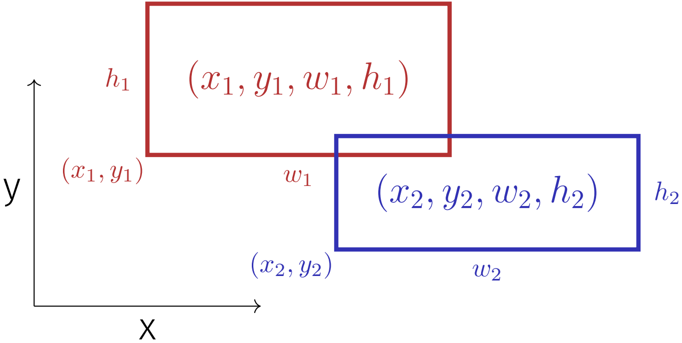


Coarse Solution

(include directives omitted)

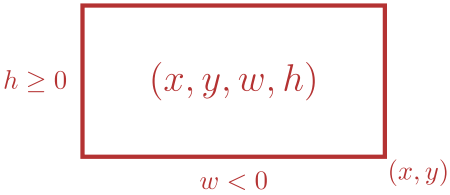
```
int main()
{
    // input rectangles

    // intersection?

    // output solution

    return 0;
}
```

Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

Width w and height h may be negative.



Refinement 1: Input Rectangles

```
int main()
{
    std::cout << "Enter two rectangles [x y w h each] \n";
    int x1, y1, w1, h1;
    std::cin >> x1 >> y1 >> w1 >> h1;
    int x2, y2, w2, h2;
    std::cin >> x2 >> y2 >> w2 >> h2;

    // intersection?

    // output solution

    return 0;
}
```

Refinement 2: Intersection? and Output

```
int main()
{
    input rectangles ✓

    bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);

    if (clash)
        std::cout << "intersection!\n";
    else
        std::cout << "no intersection!\n";

    return 0;
}
```

Refinement 3: Intersection Function...

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}

int main() {
    input rectangles ✓
    intersection? ✓
    output solution ✓
    return 0;
}
```

Refinement 3: Intersection Function...

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

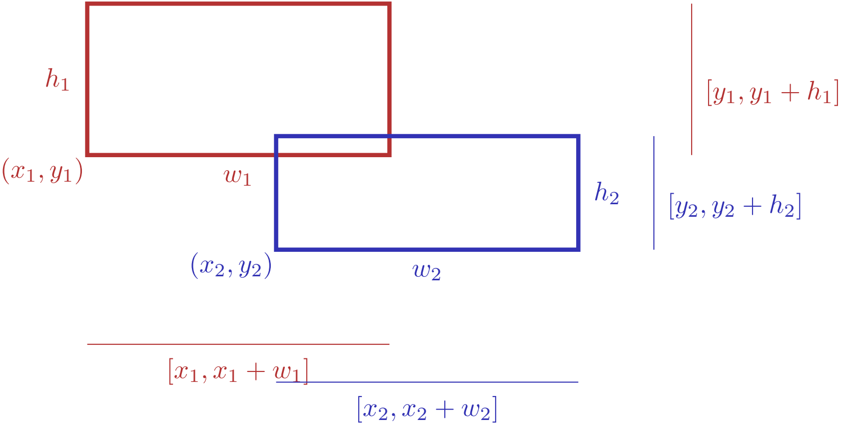
Function main ✓

Refinement 3: ...with PRE and POST

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles,
//      where w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1) and
//      (x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
   return false; // todo
}
```

Refinement 4: Interval Intersection

Two rectangles intersect if and only if their x and y-intervals intersect.



Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2); ✓
}
```

Refinement 4: Interval Intersections

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return false; // todo
}
```

Function rectangles_intersect ✓

Function main ✓

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2); ✓
}
```

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

already exists in the standard library

Function intervals_intersect ✓

Function rectangles_intersect ✓

Function main ✓

Back to Intervals

```
// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return std::max(a1, b1) >= std::min(a2, b2)
        && std::min(a1, b1) <= std::max(a2, b2); ✓
}
```

Look what we have achieved step by step!

```cpp
#include <iostream>
#include <algorithm>

// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
  return std::max(a1, b1) >= std::min(a2, b2)
      && std::min(a1, b1) <= std::max(a2, b2);
}

// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//       w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

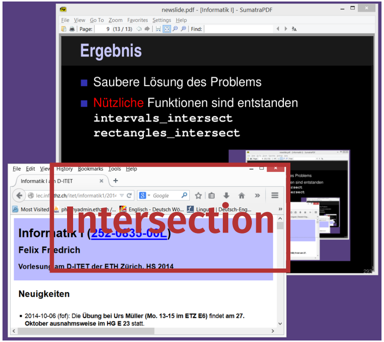
```cpp
int main ()
{
  std::cout << "Enter two rectangles [x y w h each]\n";
  int x1, y1, w1, h1;
  std::cin >> x1 >> y1 >> w1 >> h1;
  int x2, y2, w2, h2;
  std::cin >> x2 >> y2 >> w2 >> h2;
  bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);
  if (clash)
    std::cout << "intersection!\n";
  else
    std::cout << "no intersection!\n";
  return 0;
}
```

Result

- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

  `rectangles_intersect`



Where can a Function be Used?

```
#include <iostream>

int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

Scope of a Function

- is the part of the program where a function can be called
- is defined as the union of all scopes of its declarations (there can be more than one)

declaration of a function: like the definition but without `{...}`.

```
double pow(double b, int e);
```

This does not work...

```
#include <iostream>


int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

…but this works!

```
#include <iostream>
int f(int i); // Gueltigkeitsbereich von f ab hier

int main()
{
    std::cout << f(1);
    return 0;
}

int f(int i)
{
    return i;
}
```

Forward Declarations, why?

Functions that mutually call each other:

Gültigkeit g

Gültigkeit f

```
int g(...); // forward declaration

int f(...) // f valid from here
{
    g(...) // ok
}

int g(...)
{
    f(...) // ok
}
```

Reusability

- Functions such as **`rectangles_intersect`** and **`pow`** are useful in many programs.
- “Solution”: copy-and-paste the source code
- Main disadvantage: when the function definition needs to be adapted, we have to change **all** programs that make use of the function

Level 1: Outsource the Function

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Level 1: Include the Function

```
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"
```

file in working directory

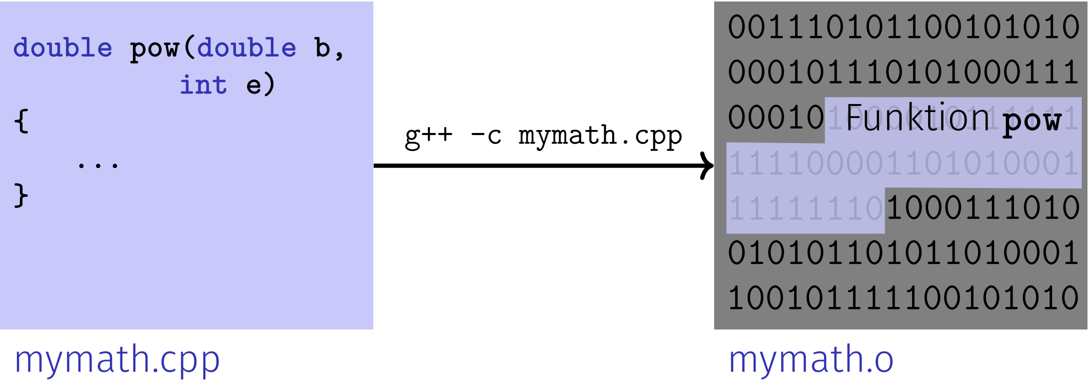
```
int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

Disadvantage of Including

- **`#include`** copies the file (**`mymath.cpp`**) into the main program (**`callpow2.cpp`**).
- The compiler has to (re)compile the function definition for each program
- This can take long for many and large functions.

Level 2: Separate Compilation

of **`mymath.cpp`** independent of the main program:



Level 2: Separate Compilation

Declaration of all used symbols in so-called **header** file.

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e);
```

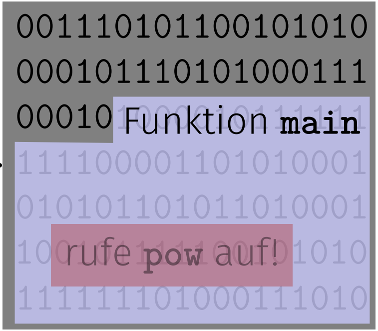
mymath.h

Level 2: Separate Compilation

of the main program, independent of **`mymath.cpp`**, if a *declaration* from **`mymath`** is included.

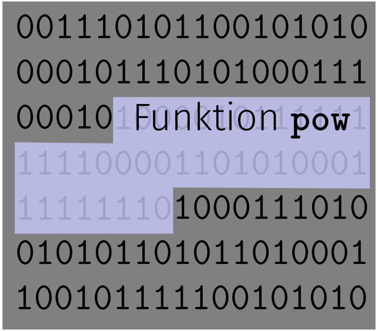
```
#include <iostream>
#include "mymath.h"
int main()
{
  std::cout << pow(2,-2) << "\n";
  return 0;
}
```

callpow3.cpp



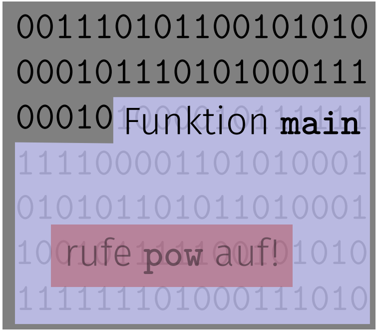
callpow3.o

The linker unites...



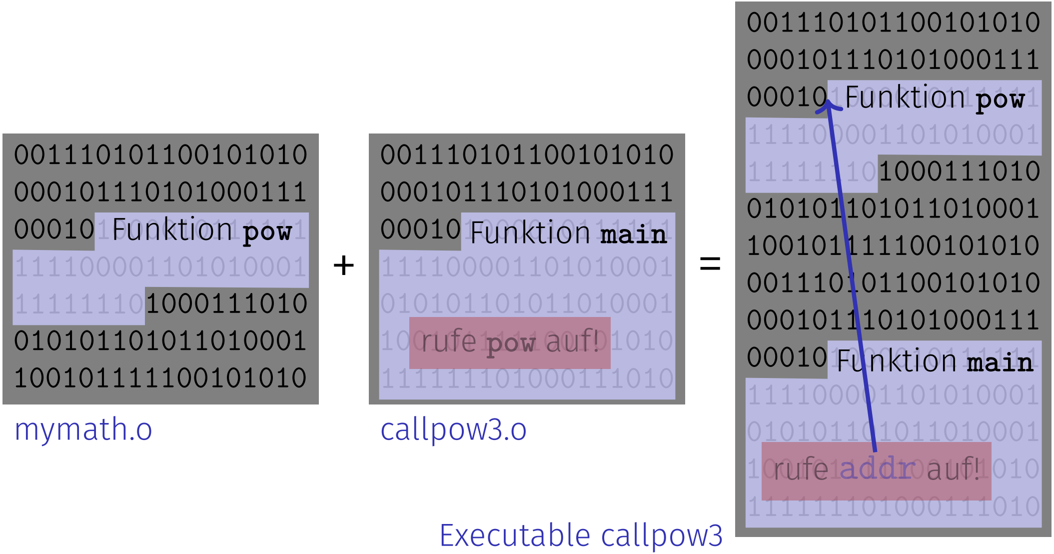
mymath.o

+



callpow3.o

… what belongs together



Availability of Source Code?

Observation

`mymath.cpp` (source code) is not required any more when the **`mymath.o`** (object code) is available.

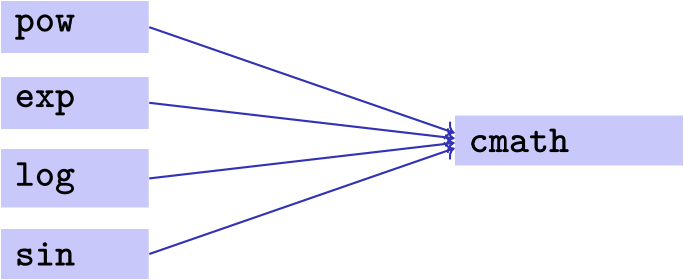
Many vendors of libraries do not provide source code.
Header files then provide the *only* readable informations.

Open-Source Software

- Source code is generally available.
- Only this allows the continued development of code by users and dedicated “hackers”.
- Even in commercial domains, open-source software gains ground.
- Certain licenses force naming sources and open development. Example GPL (GNU Genereal Public License)
- Known open-source software: Linux (operating system), Firefox (browser), Thunderbird (email program)...

Libraries

- Logical grouping of similar functions



Name Spaces...

```
// cmath
namespace std {

  double pow(double b, int e);

  ....
  double exp(double x);
  ...
}
```

…Avoid Name Conflicts

```cpp
#include <cmath>
#include "mymath.h"

int main()
{
    double x = std::pow(2.0, -2); // <cmath>
    double y = pow(2.0, -2); // mymath.h
}
```

Name Spaces / Compilation Units

In C++ the concept of separate compilation is *independent* of the concept of name spaces
In some other languages,e.g. Modula / Oberon (partially also for Java) the compilation unit can define a name space.

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with **`std::pow`**);
- lead to interesting and efficient programs in a simple way;
- guarantee a quality standard that cannot easily be achieved with code written from scratch.

Example: Prime Number Test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, n-1\}$ dividing n .

```
unsigned int d;
for (d=2; n % d != 0; ++d);
```

Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

- This works because `std::sqrt` rounds to the next representable `double` number (IEEE Standard 754).

Prime Number test with `sqrt`

```
// Test if a given natural number is prime.
#include <iostream>
#include <cassert>
#include <cmath>

int main ()
{
  // Input
  unsigned int n;
  std::cout << "Test if n>1 is prime for n =? ";
  std::cin >> n;
  assert (n > 1);

  // Computation: test possible divisors d up to sqrt(n)
  unsigned int bound = std::sqrt(n);
```

Functions Should be More Capable! Swap ?

```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // fail!
}
```

Functions Should be More Capable! Swap ?

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // ok!
}
```

Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.
- Not a new concept specific to functions, but rather a new class of types

Reference types (e.g. `int&`)

11. Reference Types

Reference Types: Definition and Initialization, Pass By Value, Pass by Reference, Temporary Objects, Const-References

Swap!

```
// POST: values of x and y have been exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}

int main() {
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a == 1 && b == 2); // ok!
}
```

Reference Types

- We can make functions change the values of the call arguments
- not a function-specific concept, but a new class of types: *reference types*

Reference Types: Definition

$T\&$ read as "T-reference"

underlying type

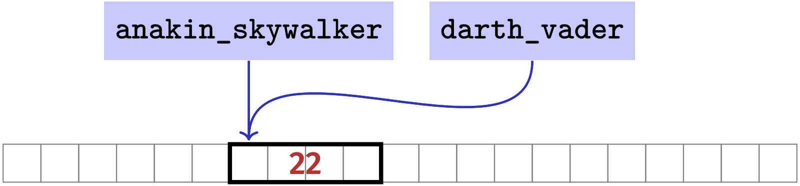
- $T\&$ has the same range of values and functionality as T ...
- ... but initialization and assignment work differently

Anakin Skywalker alias Darth Vader

Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; // Alias
darth_vader = 22;

std::cout << anakin_skywalker; // 22
```

assignment to the L-value behind the alias



Reference Types: Intialization and Assignment

```
int& darth_vader = anakin_skywalker;
darth_vader = 22; // effect: anakin_skywalker = 22
```

- A variable of reference type (a *reference*) must be initialized with an L-Value
- The variable becomes an *alias* of the L-value (a different name for the referenced object)
- Assignment to the reference updates the object *behind* the alias

Reference Types: Implementation

Internally, a value of type *T&* is represented by the address of an object of type *T*.

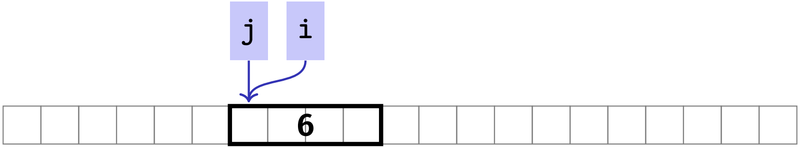
```
int& j; // Error: j must be an alias of something

int& k = 5; // Error: literal 5 has no address
```

Pass by Reference

Reference types make it possible that functions modify the value of their call arguments

```
void increment (int& i){
  ++i;
}
...
int j = 5;
increment (j);
std::cout << j; // 6
```

initialization of the formal arguments: `i` becomes an alias of call argument `j`



Pass by Reference

Formal argument *is of* reference type:

$\Rightarrow$ *Pass by Reference*

Formal argument is (internally) initialized with the **address** of the call argument (L-value) and thus becomes an **alias**.

Pass by Value

Formal argument *is not of* reference type:

$\Rightarrow$ *Pass by Value*

Formal argument is initialized with the *value* of the actual parameter (R-Value) and thus becomes a *copy*.

References in the Context of intervals_intersect

```
// PRE:  [a1, b1], [a2, b2] are (generalized) intervals,
// POST: returns true if [a1, b1], [a2, b2] intersect, in which case
//       [l, h] contains the intersection of [a1, b1], [a2, b2]
bool intervals_intersect(int& l, int& h,
                         int a1, int b1, int a2, int b2) {
  sort(a1, b1);
  sort(a2, b2);
  l = std::max(a1, a2); // Assignments
  h = std::min(b1, b2); // via references
  return l <= h;
}
...
int lo = 0; int hi = 0;
if (intervals_intersect(lo, hi, 0, 2, 1, 3)) // Initialization
   std::cout << "[" << lo << "," << hi << "]" << "\n"; // [1,2]
```

References in the Context of intervals_intersect

```
// POST: a <= b
void sort(int& a, int& b) {
   if (a > b)
     std::swap(a, b); // Initialization ("passing through" a, b
}

bool intervals_intersect(int& l, int& h,
                         int a1, int b1, int a2, int b2) {
  sort(a1, b1); // Initialization
  sort(a2, b2); // Initialization
  l = std::max(a1, a2);
  h = std::min(b1, b2);
  return l <= h;
}
```

Return by Reference

- Even the return type of a function can be a reference type: *Return by Reference*

```
int& inc(int& i) {
  return ++i;
}
```

- call **inc(x)**, for some **int** variable **x**, has exactly the semantics of the pre-increment **++x**
- Function call *itself* now is an L-value
- Thus possible: **inc(inc(x))** or **++(inc(x))**

Temporary Objects

What is wrong here?

```
int& foo(int i) {
  return i;
}
```

Return value of type `int&` becomes an alias of the formal argument (local variable `i`), whose memory lifetime ends after the call

```
int k = 3;
int& j = foo(k); // j is an alias of a zombie
std::cout << j; // undefined behavior
```

The Reference Guidline

Reference Guideline

When a reference is created, the object referred to must "stay alive" at least as long as the reference.

Const-References

- have type **`const`** *T* **`&`**
- type can be interpreted as "(**`const`** *T*) **`&`**"
- can be initialized with R-Values (compiler generates a temporary object with sufficient lifetime)

`const` *T***`&`** **`r`** = *lvalue*;

`r` is initialized with the address of *lvalue* (efficient)

`const` *T***`&`** **`r`** = *rvalue*;

`r` is initialized with the address of a temporary object with the value of the *rvalue* (pragmatic)

What exactly does Constant Mean?

Consider L-value of type **`const`** T. **Case: 1** T *is no* reference type.

$\Rightarrow$ Then the *L-value is a constant*

```
const int n = 5;
int& a = n; // Compiler error: const-qualification discarded
a = 6;
```

The compiler detects our *cheating attempt*

What exactly does Constant Mean?

Consider L-value of type **`const`** *T*. **Case 2:** *T is* reference type.

⇒ Then the *L-value is a read-only alias* which cannot be used to change the *underlying* L-value.

```
int n = 5;

const int& r = n; // r is read-only alias of n
r = 6;            // Compiler error: read-only reference

int& rw = n;      // rw is read-write alias
rw = 6;           // OK
```

When to use `const` T&?

```
void f_1(T& arg);
```

```
void f_2(const T& arg);
```

- Argument types are references; call arguments are thus not copied, which is efficient
- But only `f_2` "promises" to not modify the argument

Rule

If possible, declare function argument types as `const` T& (*pass by read-only reference*) : efficient *and* safe.

Typically doesn't pay off for fundamental types (`int`, `double`, ...). Types with a larger memory footprint will be introduced later in this course.

12. Vectors I

Vector Types, Sieve of Erathostenes, Memory Layout, Iteration

Vectors: Motivation

- Now we can iterate over numbers

```
for (int i=0; i<n ; ++i) {...}
```

- Often we have to iterate over *data*. (Example: find a cinema in Zurich that shows “C++ Runner 2049” today)
- Vectors allow to store *homogeneous* data (example: schedules of all cinemas in Zurich)

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~	**11**	~~12~~	**13**	~~14~~	~~15~~	~~16~~	**17**	~~18~~	**19**	~~20~~	~~21~~	~~22~~	**23**

at the end of the crossing out process, only prime numbers remain.

- Question: how do we cross out numbers?
- Answer: with a *vector*.

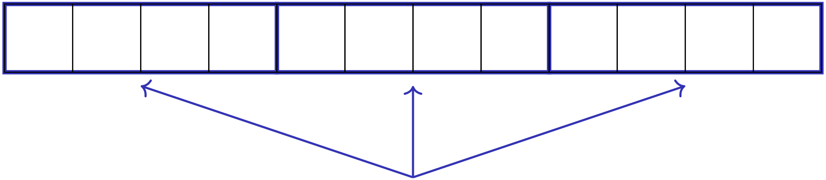
Sieve of Erathostenes with Vectors

```
#include <iostream>
#include <vector> // standard containers with vector functionality
int main() {
  // input
  std::cout << "Compute prime numbers in {2,...,n-1} for n =? ";
  unsigned int n; std::cin >> n;

  // definition and initialization: provides us with Booleans
  // crossed_out[0],..., crossed_out[n-1], initialized to false
  std::vector<bool> crossed_out (n, false);

  // computation and output
  std::cout << "Prime numbers in {2,...," << n-1 << "}:\n";
  for (unsigned int i = 2; i < n; ++i)
    if (!crossed_out[i]) { // i is prime
      std::cout << i << " ";
      // cross out all proper multiples of i
      for (unsigned int m = 2*i; m < n; m += i) crossed_out[m] = true;
    }
  std::cout << "\n";
  return 0;
}
```

Memory Layout of a Vector

A vector occupies a *contiguous* memory area

Example: a vector with 3 elements of type **`T`**



Memory segments for a value of type **`T`** each
(**`T`** occupies e.g. 4 bytes)

Random Access

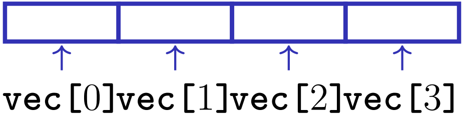
Given

- vector **`vec`** with **`T`** elements
- **`int`** expression **`exp`** with value $i \geq 0$

Then the expression

`vec[exp]`

- is an *L-value* of type **`T`**
- that refers to the ith element **`vec`** (counting from 0!)



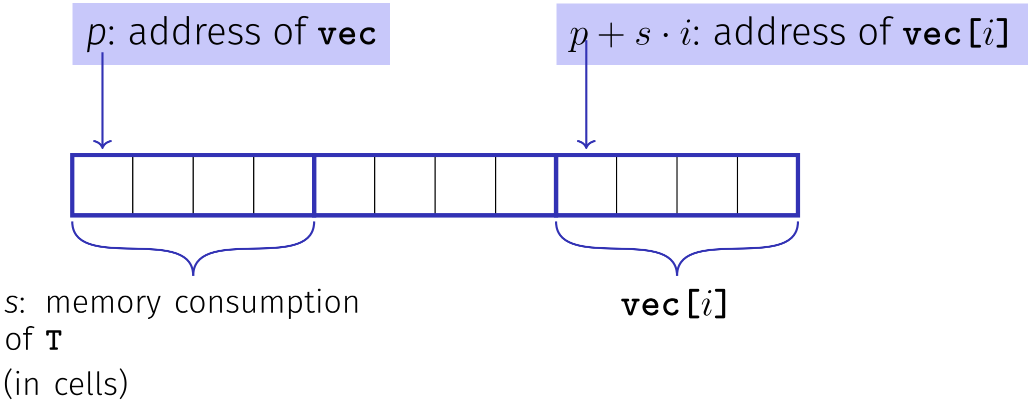
Random Access

```
vec [ exp ]
```

- The value i of **`exp`** is called *index*
- **`[]`** is the *index operator* (also *subscript operator*)

Random Access

Random access is very efficient:



Vector Initialization

- **`std::vector<int> vec(5);`**
  The five elements of **`vec`** are intialized with zeros)
- **`std::vector<int> vec(5, 2);`**
  the 5 elements of **`vec`** are initialized with 2
- **`std::vector<int> vec{4, 3, 5, 2, 1};`**
  the vector is initialized with an *initialization list*
- **`std::vector<int> vec;`**
  An initially empty vector is initialized

Attention

Accessing elements outside the valid bounds of a vector leads to *undefined behavior*

```
std::vector vec(10);
for (unsigned int i = 0; i <= 10; ++i)
  vec[i] = 30; // Runtime error: accessing vec[10]
```

Attention

Bound Checks

When using a subscript operator on a vector, it is the sole *responsibility of the programmer* to check the validity of element accesses.

Vectors Offer Great Functionality

Here a few example functions, additional follow later in the course.

```
std::vector<int> v(10);
std::cout << v.at(10);
  // Access with index check → runtime error
  // Ideal for homework
```

```
v.push_back(-1); // -1 is appended (added at end)
std::cout << v.size(); // outputs 11
std::cout << v.at(10); // outputs -1
```

13. Characters and Texts I

Characters and Texts, ASCII, UTF-8, Caesar Code

Characters and Texts

- We have seen texts before:

$$\texttt{std::cout << } \underbrace{\texttt{"Prime numbers in \{2,...,999\}:\textbackslash n"}}_{\text{String-Literal}}\texttt{;}$$

- can we really work with texts? Yes!

Character:	Value of the fundamental type **`char`**
Text:	**`std::string`** ≈ vector of **`char`** elements

The type **`char`** (“character”)

Represents printable characters (e.g. **`'a'`**) and *control characters* (e.g. **`'\n'`**)

```
char c = 'a';
```

Declares and initialises variable **`c`** of type **`char`** with value **`'a'`**

literal of type **`char`**

The type `char` (“character”)

Is formally an integer type

- values convertible to `int` / `unsigned int`
- all arithmetic operators are available (with dubious use: what is `'a'/'b'` ?)
- values typically occupy 8 Bit

domain:
$\{-128, \ldots, 127\}$ or $\{0, \ldots, 255\}$

The ASCII-Code

- Defines concrete conversion rules **char** $\longrightarrow$ **(unsigned) int**

Zeichen $\longrightarrow \{0, \ldots, 127\}$

'A', 'B', ... , 'Z' $\longrightarrow 65, 66, ..., 90$
'a', 'b', ... , 'z' $\longrightarrow 97, 98, ..., 122$
'0', '1', ... , '9' $\longrightarrow 48, 49, ..., 57$

- Is supported on all common computer systems
- Enables arithmetic over characters

```
for (char c = 'a'; c <= 'z'; ++c)
  std::cout << c; // abcdefghijklmnopqrstuvwxyz
```

Extension of ASCII: Unicode, UTF-8

- Internationalization of Software $\Rightarrow$ large character sets required. Thus common today:
  - Character set *Unicode*: 150 scripts, ca. 137,000 characters
  - encoding standard *UTF-8*: mapping characters $\leftrightarrow$ numbers
- UTF-8 is a *variable-width encoding*: Commonly used characters (e.g. Latin alphabet) require only one byte, other characters up to four
- Length of a character's byte sequence is encoded via bit patterns

Useable Bits	Bit patterns
7	`0xxxxxxx`
11	`110xxxxx 10xxxxxx`

Some Unicode characters in UTF-8

Symbol	Codierung (jeweils 16 Bit)
ﯹ	11101111 10101111 10111001
☠	11100010 10011000 10100000
☃	11100010 10011000 10000011
☙	11100010 10011000 10011001
A	01000001

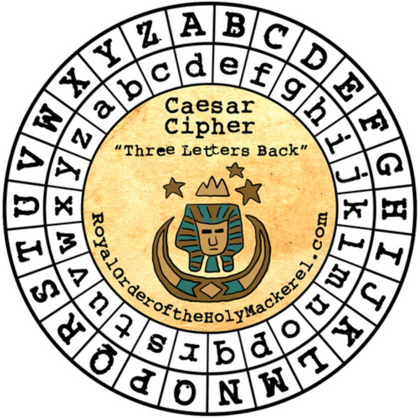
P.S.: Search for `apple "unicode of death"` P.S.: Unicode & UTF-8 are not relevant for the exam

http://t-a-w.blogspot.ch/2008/12/funny-characters-in-unicode.html

Caesar-Code

Replace every printable character in a text by its pre-pre-predecessor.

' '	(32)	$\to$	'|'	(124)
'!'	(33)	$\to$	'}'	(125)
		...		
'D'	(68)	$\to$	'A'	(65)
'E'	(69)	$\to$	'B'	(66)
		...		
$\sim$	(126)	$\to$	'{'	(123)



Caesar-Code: `shift`-Function

```
// PRE:  divisor > 0
// POST: return the remainder of dividend / divisor
//       with 0 <= result < divisor
int mod(int dividend, int divisor);

// POST: if c is one of the 95 printable ASCII characters, c is
//       cyclically shifted s printable characters to the right
char shift(char c, int s) {
    if (c >= 32 && c <= 126) { // c is printable
      c = 32 + mod(c - 32 + s,95);
    }

    return c;
}
```

"`- 32`" transforms interval $[32, 126]$ to $[0, 94]$
"`mod(`x`, 95)`" computes $x \bmod 95$ in $[0, 94]$

Caesar-Code: `caesar`-Function

```
// POST: Each character read from std::cin was shifted cyclically
//       by s characters and afterwards written to std::cout
void caesar(int s) {
  std::cin >> std::noskipws; // #include <ios>

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

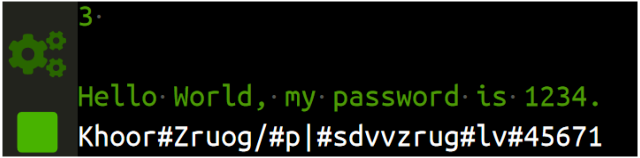
Conversion to **`bool`**: returns *false* if and only if the input is empty

Shift printable characters by **`s`**

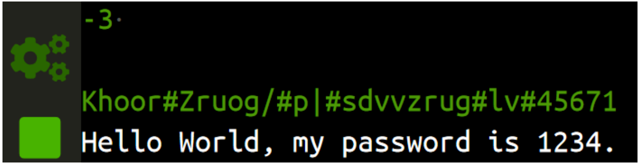
Caesar-Code: Main Program

```
int main() {
  int s;
  std::cin >> s;

  // Shift input by s
  caesar(s);

  return 0;
}
```

Encode: shift by n (here: 3)



Encode: shift by $-n$ (here: -3)

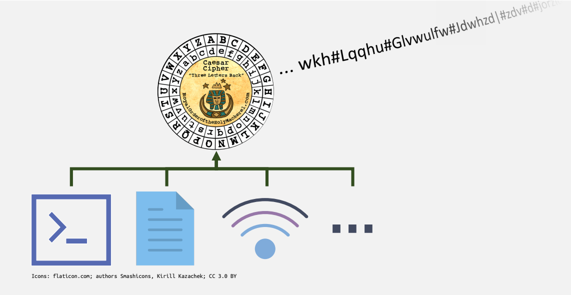


Caesar-Code: Generalisation

```
void caesar(int s) {
  std::cin >> std::noskipws;

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

- Currently only from `std::cin` to `std::cout`

- Better: from arbitrary character source (console, file, ...) to arbitrary character sink (console, ...)



14. Characters and Texts II

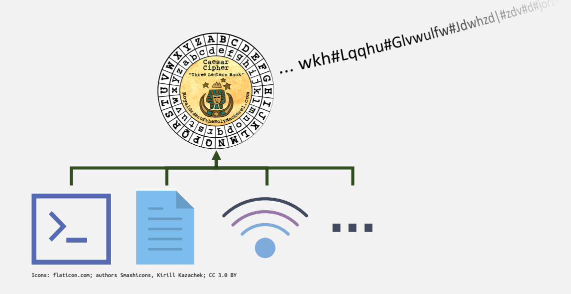
Caesar Code with Streams, Text as Strings, String Operations

Caesar-Code: Generalisation

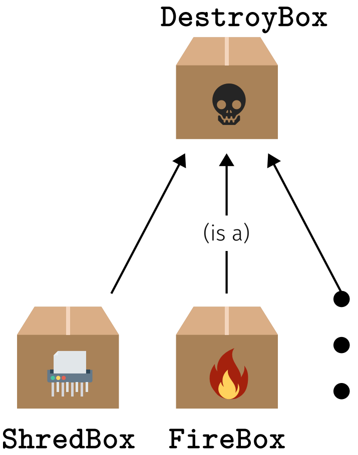
```
void caesar(int s) {
  std::cin >> std::noskipws;

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

- Currently only from `std::cin` to `std::cout`

- Better: from arbitrary character source (console, file, ...) to arbitrary character sink (console, ...)



Interlude: Abstract vs. Concrete Types



(abstract, generic)

```
void move_house(DestroyBox& db) {
  // any destroy box will do
  db.dispose(old_ikea_couch);
  db.dispose(cheap_wine);
  ...
}
```

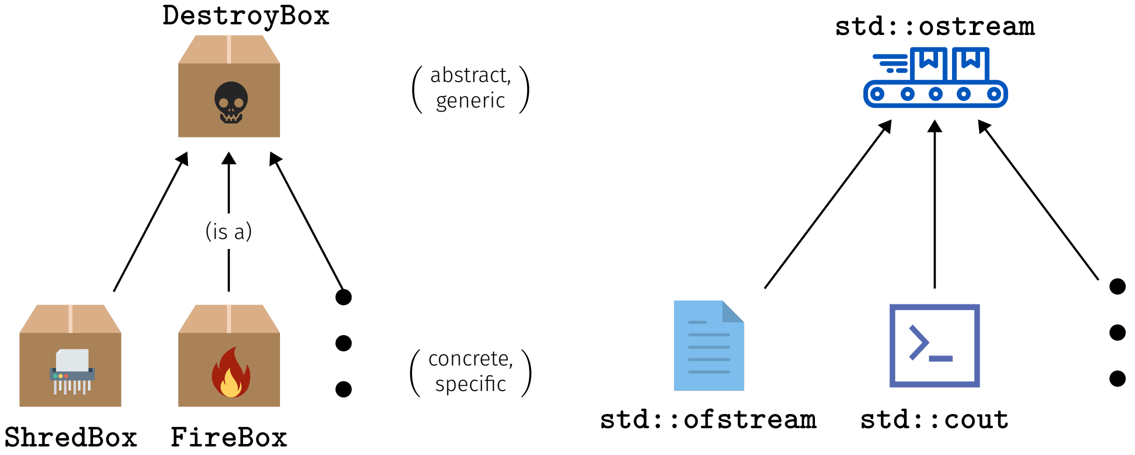
(concrete, specific)

```
FireBox fb(5000°C);
move_house(fb);
```

```
ShredBox sb;
move_house(sb);
```

Icons on current and next slide taken from **`flaticon.com`**. Authors are: DinosoftLabs, Freepik, Kirill Kazachek, Smashicons, Vectors Market, xnimrodx.

Abstract and Concrete Character Streams



Caesar-Code: Generalisation

```
void caesar(std::istream& in,
            std::ostream& out,
            int s) {

  in >> std::noskipws;

  char next;
  while (in >> next) {
    out << shift(next, s);
  }
}
```

- **`std::istream`/`std::ostream`** is an *abstract* *input/output stream* of **`char`**s
- Function is called with *concrete* streams, e.g.:
  - Console: **`std::cin`**/**`cout`**
  - Files: **`std::ifstream`**/**`ofstream`**

Caesar-Code: Generalisation, Example 1

```
#include <iostream>
...

// in void main():
caesar(std::cin, std::cout, s);
```

Calling the generalised **`caesar`** function: from **`std::cin`** to **`std::cout`**

Caesar-Code: Generalisation, Example 2

```
#include <iostream>
#include <fstream>
...

// in void main():
std::string to_file_name = ...; // Name of file to write to
std::ofstream to(to_file_name); // Output file stream

caesar(std::cin, to, s);
```

Calling the generalised **`caesar`** function: from **`std::cin`** to file

Caesar-Code: Generalisation, Example 3

```
#include <iostream>
#include <fstream>
...

// in void main():
std::string from_file_name = ...; // Name of file to read from
std::string to_file_name = ...; // Name of file to write to
std::ifstream from(from_file_name); // Input file stream
std::ofstream to(to_file_name); // Output file stream

caesar(from, to, s);
```

Calling the generalised **caesar** function: from file to file

Caesar-Code: Generalisation, Example 4

```
#include <iostream>
#include <sstream>
...

// in void main():
std::string plaintext = "My password is 1234";
std::istringstream from(plaintext);

caesar(from, std::cout, s);
```

Calling the generalised `caesar` function: from a string to `std::cout`

Streams: Final Words

Note: You only need to be able to *use* streams

- *User knowledge*, on the level of the previous slides, suffices for exercises and exam
- I.e. you do not need to know how streams work internally
- At the end of this course, you'll hear how you can define *abstract*, and corresponding *concrete*, types yourself

Texts

- Text "`to be or not to be`" could be represented as `vector<char>`
- Texts are ubiquitous, however, and thus have their own typ in the standard library: `std::string`
- Requires `#include <string>`

Using `std::string`

- Declaration, and initialisation with a literal:

```
std::string text = "Essen ist fertig!"
```

- Initialise with variable length:

```
std::string text(n, 'a')
```

`text` is filled with n `'a'`s

- Comparing texts:

```
if (text1 == text2) ...
```

`true` if character-wise equal

Using `std::string`

- Querying size:

```
for (unsigned int i = 0; i < text.size(); ++i) ...
```

Size not equal to text length if multi-byte encoding is used, e.g. UTF-8

- Reading single characters:

```
if (text[0] == 'a') ... // or text.at(0)
```

`text[0]` does not check index bounds, whereas `text.at(0)` does

- Writing single characters:

```
text[0] = 'b'; // or text.at(0)
```

Using `std::string`

- Concatenate strings:

```
text = ":-";
text += ")";
assert(text == ":-)");
```

- Many more operations; if interested, see `https://en.cppreference.com/w/cpp/string`

15. Vectors II

Multidimensional Vector/Vectors of Vectors, Shortest Paths, Vectors as Function Arguments

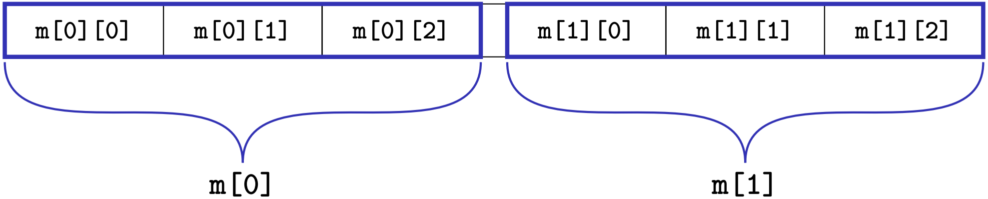
Multidimensional Vectors

- For storing multidimensional structures such as tables, matrices, ...
- ...*vectors of vectors* can be used:

```
std::vector<std::vector<int>> m; // An empty matrix
```

Multidimensional Vectors

In memory: flat



in our head: matrix

columns

rows

	0	1	2
0	`m[0][0]`	`m[0][1]`	`m[0][2]`
1	`m[1][0]`	`m[1][1]`	`m[1][2]`

Multidimensional Vectors: Initialisation

Using initialisation lists:

```
// A 3-by-5 matrix
std::vector<std::vector<std::string>> m = {
  {"ZH", "BE", "LU", "BS", "GE"},
  {"FR", "VD", "VS", "NE", "JU"},
  {"AR", "AI", "OW", "IW", "ZG"}
};

assert(m[1][2] == "VS");
```

Multidimensional Vectors: Initialisation

Fill to specific size:

```
unsigned int a = ...;
unsigned int b = ...;

// An a-by-b matrix with all ones
std::vector<std::vector<int>>
  m(a, std::vector<int>(b, 1));
```

`m` (type `std::vector<std::vector<int>>`) is a vector of length `a`, whose elements (type `std::vector<int>`) are vectors of length `b`, whose Elements (type `int`) are all ones

(Many further ways of initialising a vector exist)

Multidimensional Vectors and Type Aliases

- Also possible: vectors of vectors of vectors of ...:
  `std::vector<std::vector<std::vector<...>>>`
- Type names can obviously become looooooong
- The declaration of a *type alias* helps here:

using *Name* = *Typ*;

Name that can now be used to access the type

existing type

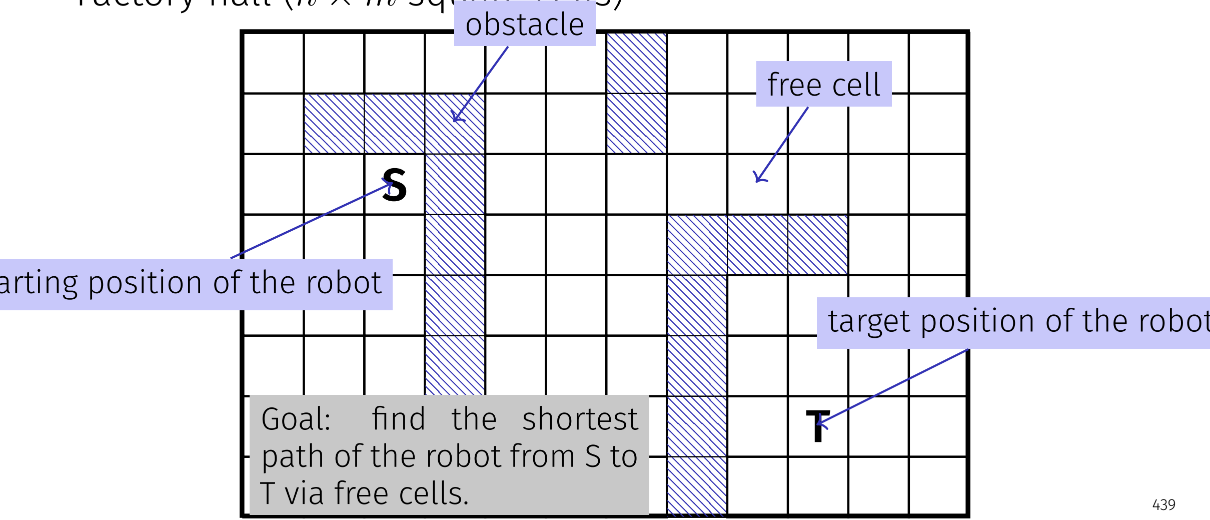
Type Aliases: Example

```
#include <iostream>
#include <vector>
using imatrix = std::vector<std::vector<int>>;

// POST: Matrix 'm' was output to stream 'out'
void print(const imatrix& m, std::ostream& out);

int main() {
  imatrix m = ...;
  print(m, std::cout);
}
```

Recall: `const` reference for enfficiency (no copy) and safety (immutable)

Application: Shortest Paths

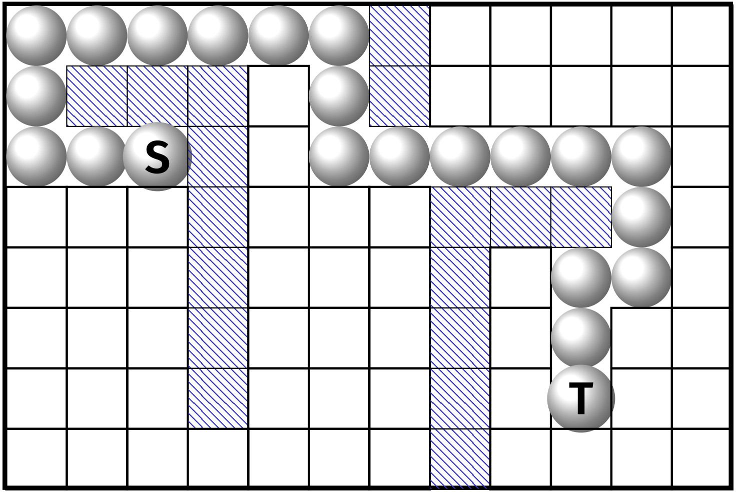
Factory hall ($n \times m$ square cells)



Goal: find the shortest path of the robot from S to T via free cells.

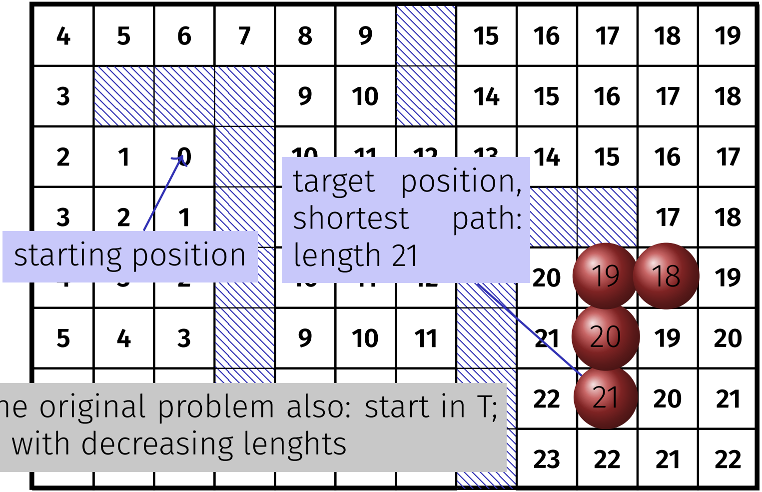
Application: shortest paths

Solution



This problem appears to be different

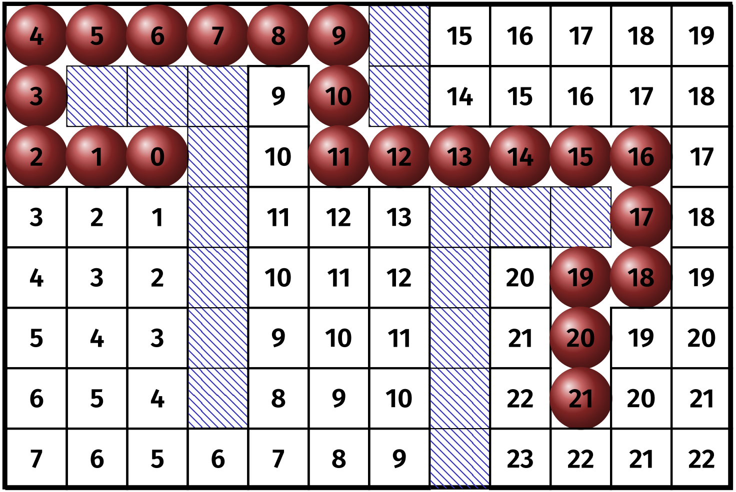
Find the *lengths* of the shortest paths to *all* possible targets.



This solves the original problem also: start in T; follow a path with decreasing lenghts

This problem appears to be different

Find the *lengths* of the shortest paths to *all* possible targets.



Preparation: Input Format

rows columns

```
8 12
------X-----
-XXX--X-----
--SX--------
---X---XXX--
---X---X----
---X---X----
---X---X-T--
-------X----
```

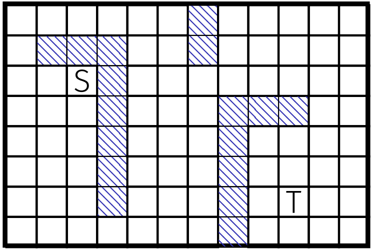
obstacle

free cell

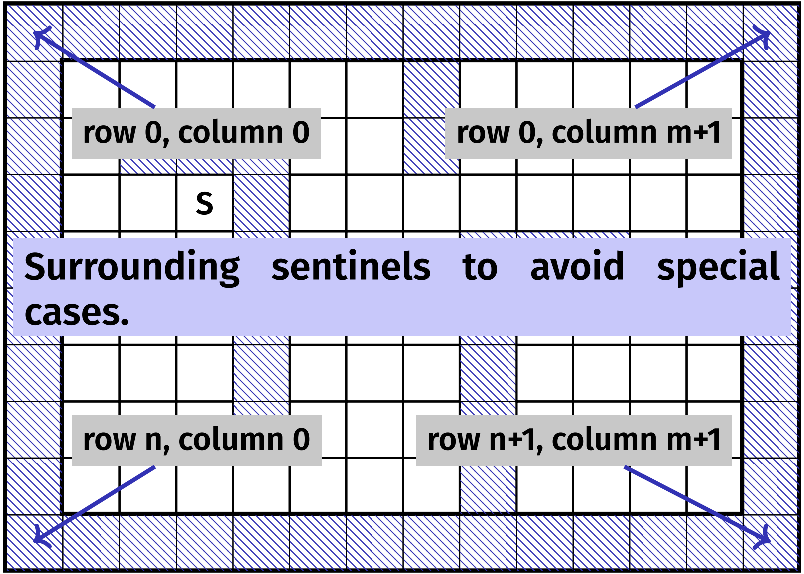
start position

target position

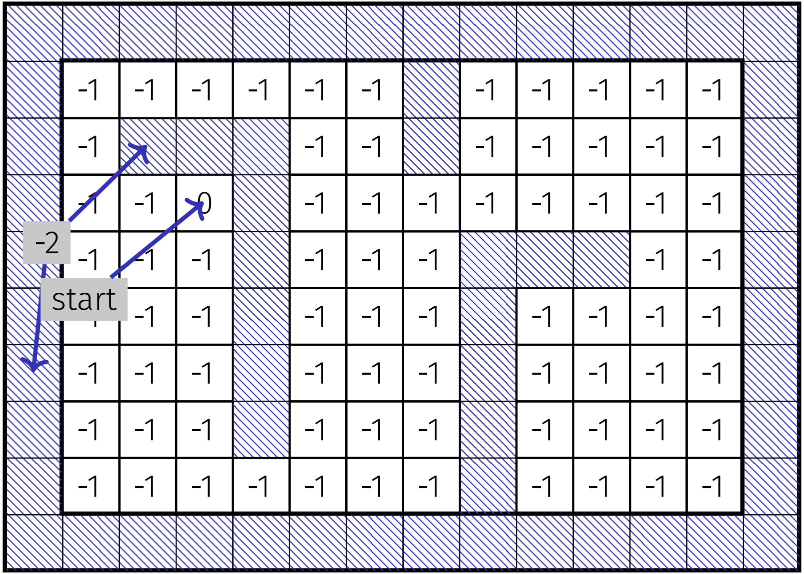
$\Rightarrow$



Preparation: Sentinels



Preparation: Initial Marking



The Shortest Path Program

- Read in dimensions and provide a two dimensional array for the path lengths

```
#include<iostream>
#include<vector>

int main()
{
  // read floor dimensions
  int n; std::cin >> n; // number of rows
  int m; std::cin >> m; // number of columns

  // define a two-dimensional
  // array of dimensions
  // (n+2) x (m+2) to hold the floor plus extra walls around
  std::vector<std::vector<int» floor (n+2, std::vector<int>(m+2));
```

Sentinel

The Shortest Path Program

- Input the assignment of the hall and intialize the lengths

```
int tr = 0;
int tc = 0;
for (int r=1; r<n+1; ++r)
  for (int c=1; c<m+1; ++c) {
    char entry = '-';
    std::cin >> entry;
    if      (entry == 'S') floor[r][c] = 0;
    else if (entry == 'T') floor[tr = r][tc = c] = -1;
    else if (entry == 'X') floor[r][c] = -2;
    else if (entry == '-') floor[r][c] = -1;
  }
```

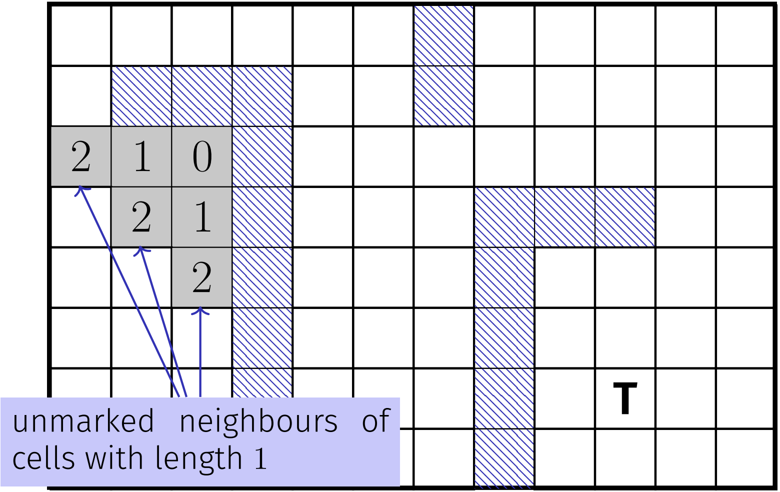
Das Kürzeste-Wege-Programm

- Add the surrounding walls

```
for (int r=0; r<n+2; ++r)
  floor[r][0] = floor[r][m+1] = -2;

for (int c=0; c<m+2; ++c)
  floor[0][c] = floor[n+1][c] = -2;
```

Mark all Cells with their Path Lengths

Step 2: all cells with path length 2



Main Loop

Find and mark all cells with path lengths $i = 1, 2, 3...$

```
for (int i=1;; ++i) {
  bool progress = false;
  for (int r=1; r<n+1; ++r)
     for (int c=1; c<m+1; ++c) {
      if (floor[r][c] != -1) continue;
      if (floor[r-1][c] == i-1 || floor[r+1][c] == i-1 ||
          floor[r][c-1] == i-1 || floor[r][c+1] == i-1 ) {
        floor[r][c] = i; // label cell with i
        progress = true;
      }
    }
  if (!progress) break;
}
```

The Shortest Paths Program

Mark the shortest path by walking backwards from target to start.

```
2int r = tr; int c = tc;2
3while (floor[r][c] > 0)3 {
  4const int d = floor[r][c] - 1;4
  5floor[r][c] = -3;5
  6if      (floor[r-1][c] == d) --r;
  else if (floor[r+1][c] == d) ++r;
  else if (floor[r][c-1] == d) --c;
  else ++c; // (floor[r][c+1] == d)
6}
```

Finish

-3	-3	-3	-3	-3	-3		15	16	17	18	19
-3				9	-3		14	15	16	17	18
-3	-3	0		10	-3	-3	-3	-3	-3	-3	17
3	2	1		11	12	13				-3	18
4	3	2		10	11	12		20	-3	-3	19
5	4	3		9	10	11		21	-3	19	20
6	5	4		8	9	10		22	-3	20	21
7	6	5	6	7	8	9		23	22	21	22

The Shortest Path Program: output

Output

```
for (int r=1; r<n+1; ++r) {
  for (int c=1; c<m+1; ++c)
    if (floor[r][c] == 0)
        std::cout << 'S';
    else if (r == tr && c == tc)
        std::cout << 'T';
    else if (floor[r][c] == -3)
        std::cout << 'o';
    else if (floor[r][c] == -2)
        std::cout << 'X';
    else
        std::cout << '-';
  std::cout << "\n";
}
```

$\Rightarrow$

```
ooooooX-----
oXXX-oX-----
ooSX-oooooo-
---X---XXXo-
---X---X-oo-
---X---X-o--
---X---X-T--
-------X----
```

The Shortest Paths Program

- Algorithm: *Breadth-First Search* (Breadth-first vs. *depth-first* search is typically discussed in lectures on algorithms)
- The program can become pretty slow because for each i all cells are traversed
- Improvement: for marking with i, traverse only the neighbours of the cells marked with $i-1$.
- Improvement: stop once the goal has been reached

16. Recursion 1

Mathematical Recursion, Termination, Call Stack, Examples, Recursion vs. Iteration, n-Queen Problem, Lindenmayer Systems

Mathematical Recursion

- Many mathematical functions can be naturally defined *recursively*
- This means, the function appears in its own definition

$$n! = \begin{cases} 1, & \text{if } n \leq 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

Recursion in C++: In the same Way!

$$n! = \begin{cases} 1, & \text{if } n \leq 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

```
// POST: return value is n!
unsigned int fac(unsigned int n) {
 if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}
```

Infinite Recursion

- is as bad as an infinite loop ...
- ...but even worse: it burns time *and* memory

```
void f() {
  f() // f() → f() →  ... → stack overflow
}
```

Recursive Functions: Termination

As with loops we need **guaranteed progress towards an exit condition ($\approx$ base case)**

Example `fac(n)`:

- Recursion ends if $n \leq 1$
- Recursive call with new argument $< n$
- Exit condition will thus be reached eventually

```
unsigned int fac(
    unsigned int n) {

  if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}
```

Recursive Functions: Evaluation

```
int fac(int n) {
  if (n <= 1)
    return 1;
  else
    return n * fac(n-1);
}

...
std::cout << fac(4);
```

$\texttt{fac(4)} \rightsquigarrow \texttt{int n = 4}$

$\hookrightarrow \texttt{fac(n - 1)} \rightsquigarrow \texttt{int n = 3}$

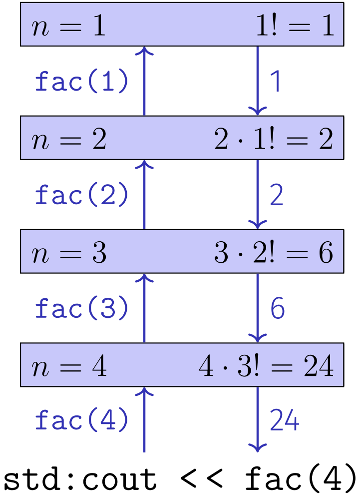
$\vdots$

*Every call of **`fac`** operates on its own **`n`***

The Call Stack

For each function call:

- push value of the call argument onto the stack
- always work with the top value
- at the end of the call the top value is removed from the stack



Euclidean Algorithm

- finds the greatest common divisor $\gcd(a, b)$ of two natural numbers a and b
- is based on the following mathematical recursion (proof in the lecture notes):

$$\gcd(a, b) = \begin{cases} a, & \text{if } b = 0 \\ \gcd(b, a \bmod b), & \text{otherwise} \end{cases}$$

Euclidean Algorithm in C++

$$\gcd(a,b) = \begin{cases} a, & \text{if } b = 0 \\ \gcd(b, a \bmod b), & \text{otherwise} \end{cases}$$

```
unsigned int gcd(unsigned int a, unsigned int b) {
 if (b == 0)
    return a;
  else
    return gcd(b, a % b);
}
```

Termination: $a \bmod b < b$, b thus gets smaller in each recursive call

Fibonacci Numbers

$$F_n := \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{if } n > 1 \end{cases}$$

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots$$

Fibonacci Numbers in C++

Laufzeit

`fib(50)` takes “forever” because it computes
F_{48} two times, F_{47} 3 times, F_{46} 5 times, F_{45} 8 times, F_{44} 13 times,
F_{43} 21 times … F_1 ca. 10^9 times (!)

```
unsigned int fib(unsigned int n) {
  if (n == 0) return 0;
  if (n == 1) return 1;
  return fib(n-1) + fib(n-2); // n > 1
}
```

Fast Fibonacci Numbers

Idea:

- Compute each Fibonacci number only once, in the order $F_0, F_1, F_2, \ldots, F_n$
- Memorize the most recent two Fibonacci numbers (variables `a` and `b`)
- Compute the next number as a sum of `a` and `b`

Can be implemented recursively and iteratively, the latter is easier/more direct

Fast Fibonacci Numbers in C++

```
unsigned int fib(unsigned int n) {
  if (n == 0) return 0;
  if (n == 1) return 1;

  unsigned int a = 0; // F_0
  unsigned int b = 1; // F_1

  for (unsigned int i = 2; i <= n; ++i) {
    unsigned int a_old = a; // F_{i-2}
    a = b; // a becomes F_{i-1}
    b += a_old; // b becomes F_{i-1} + F_{i-2}, i.e. F_i
  }

  return b;
}
```

very fast, also for **`fib(50)`**

$$(F_{i-2}, F_{i-1}) \longrightarrow (F_{i-1}, F_i)$$

Recursion and Iteration

Recursion can *always* be simulated by

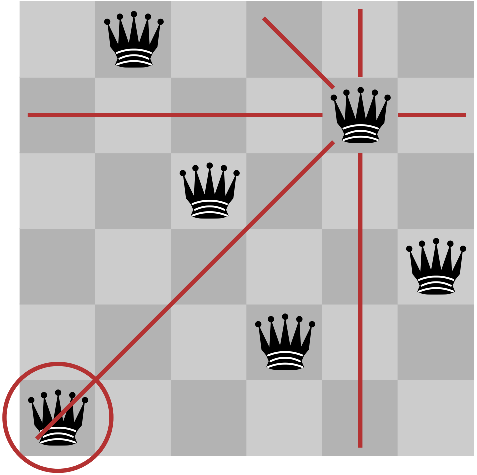
- Iteration (loops)
- explicit “call stack” (e.g. via a vector)

Often recursive formulations are simpler, but sometimes also less efficient.

The Power of Recursion

- Some problems appear to be hard to solve without recursion. With recursion they become significantly simpler.
- Examples: *The n-Queens-Problem*, The towers of Hanoi, Sudoku-Solver, Expression Parsers, Reversing In- or Output, Searching in Trees, Divide-And-Conquer (e.g. sorting) , ...
- ...and the 2. bonus exercise: Nonograms

The n-Queens Problem

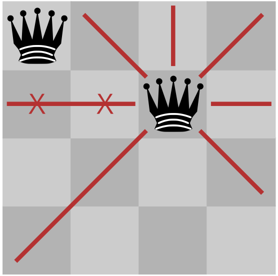


- Provided is a $n \ timesn$ chessboard
- For example $n = 6$
- Question: is it possiblt to position n queens such that no two queens threaten each other?
- If yes, how many solutions are there?

Solution?

- Try all possible placements?
- $\binom{n^2}{n}$ possibilities. Too many!
- Only ne queen per row: n^n possibilities. Better – but still too many.
- Idea: don't proceed with futile attempts, retract incorrect moves instead $\Rightarrow$ *Backtracking*

Solution with Backtracking

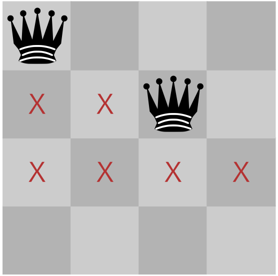


Second Queen in next row (no collision)

queens

0
2
0
0

Solution with Backtracking

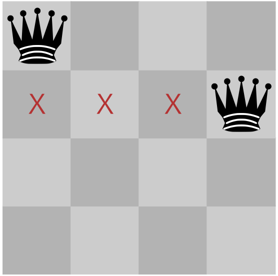


All squares in next row forbidden. Track back !

queens

0
2
4
0

Solution with Backtracking

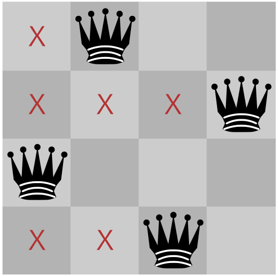


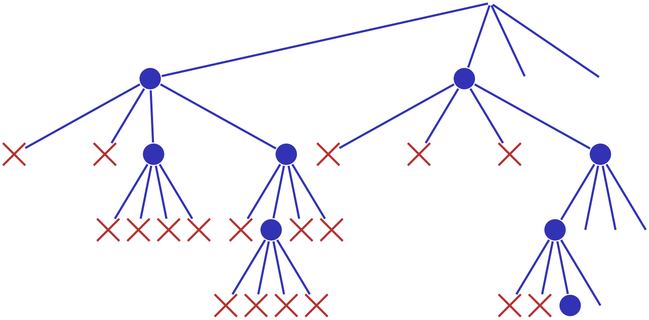
Move queen one step further and try again

queens

0
3
0
0

Search Strategy Visualized as a Tree





Check Queen

```
using Queens = std::vector<unsigned int>;

// post: returns if queen in the given row is valid, i.e.
//       does not share a common row, column or diagonal
//       with any of the queens on rows 0 to row-1
bool valid(const Queens& queens, unsigned int row) {
  unsigned int col = queens[row];
  for (unsigned int r = 0; r != row; ++r) {
    unsigned int c = queens[r];
    if (col == c || col - row == c - r || col + row == c + r)
      return false; // same column or diagonal
  }
  return true; // no shared column or diagonal
}
```

Recursion: Find a Solution

```
// pre: all queens from row 0 to row-1 are valid,
//      i.e. do not share any common row, column or diagonal
// post: returns if there is a valid position for queens on
//      row .. queens.size(). if true is returned then the
//      queens vector contains a valid configuration.
bool solve(Queens& queens, unsigned int row) {
  if (row == queens.size())
    return true;
  for (unsigned int col = 0; col != queens.size(); ++col) {
    queens[row] = col;
    if (valid(queens, row) && solve(queens,row+1))
      return true; // (else check next position)
  }
  return false; // no valid configuration found
}
```

Recursion: Count all Solutions

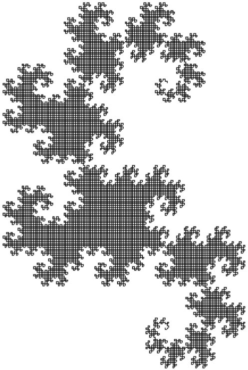
```
// pre: all queens from row 0 to row-1 are valid,
//   i.e. do not share any common row, column or diagonal
// post: returns the number of valid configurations of the
//   remaining queens on rows row ... queens.size()
int nSolutions(Queens& queens, unsigned int row) {
  if (row == queens.size())
    return 1;
  int count = 0;
  for (unsigned int col = 0; col != queens.size(); ++col) {
    queens[row] = col;
    if (valid(queens, row))
      count += nSolutions(queens,row+1);
  }
  return count;
}
```

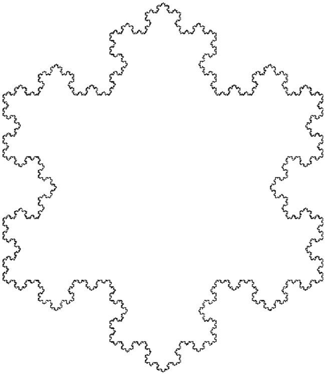
Main Program

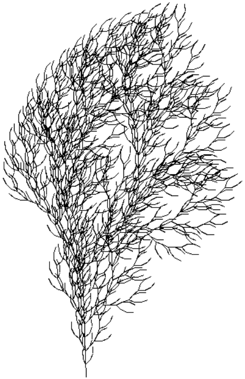
```
// pre: positions of the queens in vector queens
// post: output of the positions of the queens in a graphical way
void print(const Queens& queens);

int main() {
  int n;
  std::cin >> n;
  Queens queens(n);
  if (solve(queens,0)) {
    print(queens);
    std::cout << "# solutions:" << nSolutions(queens,0) << std::endl
  } else
    std::cout << "no solution" << std::endl;
  return 0;
}
```

Lindenmayer-Systems (L-Systems)

Fractals from Strings and Turtles







- L-Systems have been invented by the Hungarian biologist Aristid Lindenmayer (1925–1989) to model the growth of plants.
- Recursion is of course relevant for the exam, but L-Systems themselves are not

Definition and Example

- alphabet Σ
- Σ^*: finite words over Σ
- production $P : \Sigma \to \Sigma^*$
- initial word $s_0 \in \Sigma^*$

- $\{\mathrm{F}, +, -\}$
-
c	$P(c)$
F	$\mathrm{F} + \mathrm{F} +$
$+$	$+$
$-$	$-$

- F

Definition

The triple $\mathcal{L} = (\Sigma, P, s_0)$ is an L-System.

The Language Described

Wörter $w_0, w_1, w_2, \ldots \in \Sigma^*$:

$$P(\mathrm{F}) = \mathrm{F} + \mathrm{F} +$$

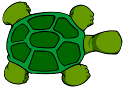
$$\begin{array}{lcl}
w_0 & := & s_0 \\
w_1 & := & P(w_0) \\
w_2 & := & P(w_1) \\
 & \vdots &
\end{array}
\qquad
\begin{array}{lcl}
w_0 & := & \mathrm{F} \\
 & & \mathrm{F} + \mathrm{F} + \\
w_1 & := & \boxed{\mathrm{F}}\boxed{+}\boxed{\mathrm{F}}\boxed{+} \\
w_2 & := & \boxed{\mathrm{F} + \mathrm{F} +}\boxed{+}\boxed{\mathrm{F} + \mathrm{F} +}\boxed{+} \\
 & & P(\mathrm{F})P(+)P(\mathrm{F})P(+) \\
 & \vdots &
\end{array}$$

Definition

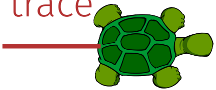
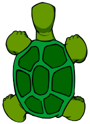
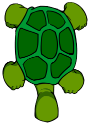
$$P(c_1 c_2 \ldots c_n) := P(c_1)P(c_2)\ldots P(c_n)$$

Turtle Graphics

Turtle with position and direction

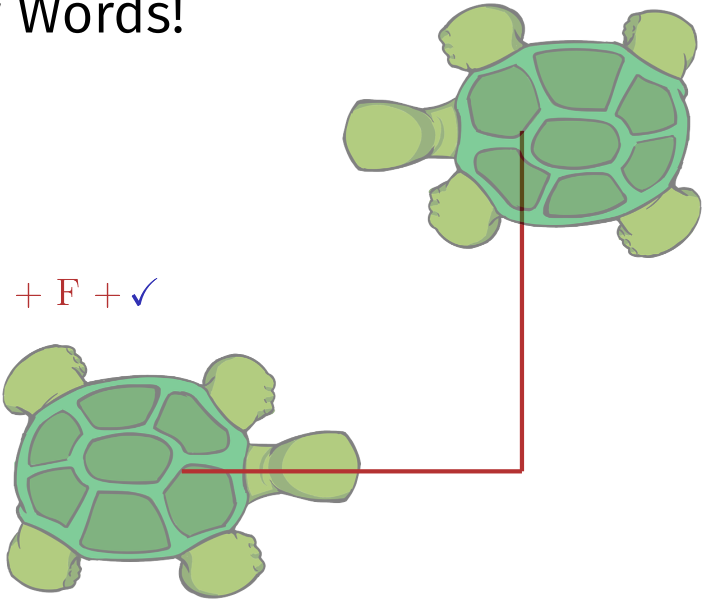


Turtle understands 3 commands:

F: move one step forwards ✓	$+$: rotate by 90 degrees ✓	$-$: rotate by -90 degrees ✓
 		

Draw Words!

$$w_1 = \mathrm{F} + \mathrm{F} + \checkmark$$



`lindenmayer`: Main Program

word $w_0 \in \Sigma^*$:

```
int main() {
  std::cout << "Maximal Recursion Depth =? ";
  unsigned int n;
  std::cin >> n;

  std::string w = "F"; // w_0
  produce(w,n);

  return 0;
}
```

$w = w_0 = \mathrm{F}$

lindenmayer: production

```
// POST: recursively iterate over the production of the characters
//       of a word.
//       When recursion limit is reached, the word is "drawn"
void produce(std::string word, int depth) {
  if (depth > 0) {
    for (unsigned int k = 0; k < word.length(); ++k)
      produce(replace(word[k]), depth-1);
  } else {
    draw_word(word);
  }
}
```

$w = w_i \to w = w_{i+1}$

draw $w = w_n$

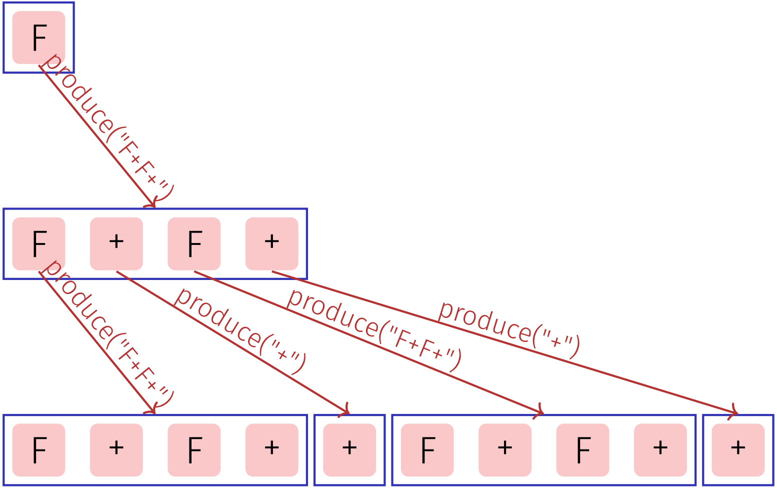
lindenmayer: replace

```
// POST: returns the production of c
std::string replace(const char c) {
  switch (c) {
    case 'F':
      return "F+F+";
    default:
      return std::string (1, c); // trivial production c -> c
  }
}
```

lindenmayer: draw

```
// POST: draws the turtle graphic interpretation of word
void draw_word(const std::string& word) {
  for (unsigned int k = 0; k < word.length(); ++k)
    switch (word[k]) {
      case 'F':
        turtle::forward(); // move one step forward
        break;
      case '+':
        turtle::left(90); // turn counterclockwise by 90 degrees
        break;
      case '-':
        turtle::right(90); // turn clockwise by 90 degrees
    }
}
```

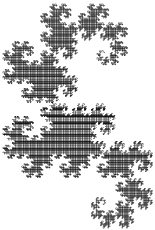
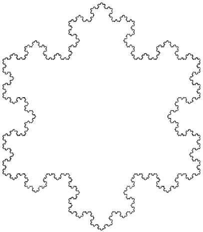
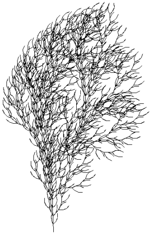
The Recursion



(Implementation above proceeds *depth-first*)

L-Systeme: Erweiterungen

- arbitrary symbols without graphical interpetation
- arbitrary angles (snowflake)
- saving and restoring the state of the turtle $\rightarrow$ plants (bush)

17. Recursion 2

Building a Calculator, Formal Grammars, Extended Backus Naur Form (EBNF), Parsing Expressions

Motivation: Calculator

Goal: we build a command line calculator

```
Input: 3 + 5
Output: 8
Input: 3 / 5
Output: 0.6
Input: 3 + 5 * 20
Output: 103
Input: (3 + 5) * 20
Output: 160
Input: -(3 + 5) + 20
Output: 12
```

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses
- unary operator `-`

Naive Attempt (without Parentheses)

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != '=') {
    double rval;
    std::cin >> rval;

    if (op == '+')
        lval += rval;
    else if (op == '*')
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 2 + 3 * 3 =
Result 15
```

Analyzing the Problem

Input:

$$\underbrace{13 + 4 * (15 - 7*}_{\text{Needs to be stored such that evaluation can be performed}} 3) =$$

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!
We will store symbols elegantly using recursion.
We need a new formal tool (that is independent of $\mathrm{C{+}{+}}$).

Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

A formal grammar defines which strings are valid.

To describe the formal grammar, we use:

Extended Backus Naur Form (EBNF)

What Can We Do about the Unnecessary Diversity of Notation for Syntactic Definitions?

Niklaus Wirth
Federal Institute of Technology (ETH), Zürich, and
Xerox Palo Alto Research Center

Key Words and Phrases: syntactic description language, extended BNF
CR Categories: 4.20

The population of programming languages is steadily growing, and there is no end of this growth in sight. Many language definitions appear in journals, many are found in technical reports, and perhaps an even greater number remains confined to proprietory circles. After frequent exposure to these definitions, one cannot fail to notice the lack of "common denominators." The only widely accepted fact is that the language structure is defined by a syntax. But even notation for syntactic description eludes any commonly agreed standard form, although the underlying ancestor is invariably the Backus-Naur Form of the Algol 60 report. As variations are often only slight, they become annoying for their very lack of an apparent motivation.

Out of sympathy with the troubled reader who is weary of adapting to a new variant of BNF each time another language definition appears, and without any claim for originality, I venture to submit a simple notation that has proven valuable and satisfactory in use. It has the following properties to recommend it:

Author's present address: Xerox Corporation, Palo Alto Research Center, 3333 Coyote Hill Road, Palo Alto, CA 94304.

1. The notation distinguishes clearly between meta-, terminal, and nonterminal symbols.
2. It does not exclude characters used as metasymbols from use as symbols of the language (as e.g. "|" in BNF).
3. It contains an explicit iteration construct, and thereby avoids the heavy use of recursion for expressing simple repetition.
4. It avoids the use of an explicit symbol for the empty string (such as ⟨empty⟩ or ϵ).
5. It is based on the ASCII character set.

This meta language can therefore conveniently be used to define its own syntax, which may serve here as an example of its use. The word *identifier* is used to denote *nonterminal symbol*, and *literal* stands for *terminal symbol*. For brevity, *identifier* and *character* are not defined in further detail.

```
syntax     = {production}.
production = identifier "=" expression ".".
expression = term {"|" term}.
term       = factor {factor}.
factor     = identifier | literal | "(" expression ")" |
             "[" expression "]" | "{" expression "}".
literal    = """" character {character} """".
```

Repetition is denoted by curly brackets, i.e. {a} stands for ϵ | a | aa | aaa | . . . . Optionality is expressed by square brackets, i.e. [a] stands for a | ϵ. Parentheses merely serve for grouping, e.g. (a|b)c stands for ac | bc. Terminal symbols, i.e. literals, are enclosed in quote marks (and, if a quote mark appears as a literal itself, it is written twice), which is consistent with common practice in programming languages.

Received January 1977; revised February 1977

Number

An integer is a sequence of digits. A sequence of digits ist

- a digit or 2
- a digit followed by a sequence of digits 2 0 1 9

```
unsigned_integer = digits .
digit  = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' .
digits = digit | digit digits .
```

alternative

terminal symbo

non-terminal symbol

Number (non-recursive)

An integer is a sequence of digits. A sequence of digits ist

- a digit, or 2
- a digit followed by an **arbitrary number of digits** 2 0 1 9

unsigned_integer = digits .

digit = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9' .

digits = digit { digit } .

optional repetition

Number, extended

A floating point number is

- a sequence of digits, or
- a sequence of digits followed by `.` followed by digits

```
Float = Digits | Digits "." Digits.
```

Expressions

```
-(3-(4-5))*(3+4*5)/6
```

What do we need in a grammar?

- Number , (Expression)
  -Number, -(Expression)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term - Term, ...

Factor

Term

Expression

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses or
- a negated factor.

non-terminal symbol

$$
\begin{aligned}
\text{factor} \quad &= \text{unsigned\_number} \\
&\mid \texttt{"("}\ \text{expression}\ \texttt{")"} \\
&\mid \texttt{"-"}\ \text{factor}\,.
\end{aligned}
$$

terminal symbol

alternative

The EBNF for Expressions

```
factor = unsigned_number
       | "(" expression ")"
       | "-" factor.
```

Implication: a factor starts with

- a digit, or
- with “(” , or
- with "-"”.

The EBNF for Expressions

A term is

- factor,
- factor $*$ factor, factor / factor,
- factor $*$ factor $*$ factor, factor / factor $*$ factor, ...
- ...

term = factor { "*" factor | "/" factor }.

optional repetition

The EBNF for Expressions

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term | "-" term }.
```

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.
- **Parser:** A program for parsing.
- **Useful:** From the EBNF we can automatically generate a parser:
  - Rules become functions
  - Alternatives and options become `if`–statements.
  - Nonterminial symbols on the right hand side become function calls
  - Optional repetitions become `while`–statements

Rules

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term | "-" term }.
```

Functions (Parser)

Expression is read from an input stream.

```
// POST: returns true if and only if in_stream = factor ...
//       and in this case extracts factor from in_stream
bool factor (std::istream& in_stream);

// POST: returns true if and only if in_stream = term ...,
//       and in this case extracts all factors from in_stream
bool term (std::istream& in_stream);

// POST: returns true if and only if in_stream = expression ...,
//       and in this case extracts all terms from in_stream
bool expression (std::istream& in_stream);
```

Functions (Parser with Evaluation)

Expression is read from an input stream.

```
// POST: extracts a factor from in_stream
//       and returns its value
double factor (std::istream& in_stream);

// POST: extracts a term from in_stream
//       and returns its value
double term (std::istream& in_stream);

// POST: extracts an expression from in_stream
//       and returns its value
double expression (std::istream& in_stream);
```

One Character Lookahead...

...to find the right alternative.

```
// POST: the next character at the stream is returned
//       without being consumed. returns 0 if stream ends.
char peek (std::istream& input){
  if (input.eof()) return 0; // end of stream
  return input.peek();  // next character in input
}

// POST: leading whitespace characters are extracted from input
//       and the first non-whitespace character on input returned
char lookahead (std::istream& input) {
  input >> std::ws;       // skip whitespaces
  return peek(input);
}
```

Parse numbers

```
bool isDigit(char ch){
  return ch >= '0' && ch <= '9';
}
// POST: returns an unsigned integer consumed from the stream
// number = digit {digit}.
unsigned int unsigned_number (std::istream& input){
  char ch = lookahead(input);
  assert(isDigit(ch));
  unsigned int num = 0;
  while(isDigit(ch) && input >> ch){ // read remaining digits
    num = num * 10 + ch - '0';
    ch = peek(input
  }
  return num;
}
```

unsigned_number = digit { digit }.
digit = '0'|'1'|'2'|'3'|'4'|'5'|'6'|'7'|'8

Cherry-Picking

…to extract the desired character.

```
// POST: if expected matches the next lookahead then consume it
//       and return true; return false otherwise
bool consume (std::istream& in_stream, char expected)
{
    if (lookahead(in_stream) == expected){
        in_stream >> expected; // consume one character
        return true;
    }
    return false;
}
```

Evaluating Factors

```
double factor (std::istream& in_stream)
{
  double value;
  if (consume(in_stream, '(')) {
    value = expression (in_stream);
    consume(in_stream, ')');
  } else if (consume(in_stream, '-')) {
    value = -factor (in_stream);
  } else {
    value = unsigned_number(in_stre
  }
  return value;
}
```

```
factor = "(" expression ")"
       | "-" factor
       | unsigned_number.
```

Evaluating Terms

```
double term (std::istream& in_stream)
{
  double value = factor (in_stream);
  while(true){
    if (consume(in_stream, '*'))
      value *= factor(in_stream);
    else if (consume(in_stream, '/'))
      value /= factor(in_stream)
    else
      return value;
  }
}
```

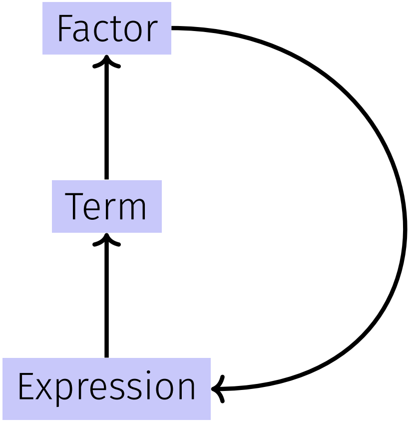
term = factor { "*" factor | "/" factor }.

Evaluating Expressions

```
double expression (std::istream& in_stream)
{
  double value = term(in_stream);
  while(true){
    if (consume(in_stream, '+'))
      value += term (in_stream);
    else if (consume(in_stream, '-'))
      value -= term(in_stream)
    else
      return value;
  }
}
```

expression = term { "+" term | "-" term }.

Recursion!



EBNF — and it works!

EBNF (`calculator.cpp`, Evaluation from left to right):

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term | "-" term }.
```

```
std::stringstream input ("1-2-3");
std::cout << expression (input) << "\n"; // -4
```

18. Structs

Rational Numbers, Struct Definition

Calculating with Rational Numbers

- Rational numbers ($\mathbb{Q}$) are of the form $\frac{n}{d}$ with n and d in $\mathbb{Z}$
- C++does not provide a built-in type for rational numbers

Goal

We build a C++-type for rational numbers ourselves! ☺

Vision

How it could (will) look like

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;
std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

A First Struct

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

Invariant: specifies valid value combinations (informal).

member variable (**d**enominator)

- **`struct`** defines a new **type**
- formal range of values: *cartesian product* of the value ranges of existing types
- real range of values: **`rational`** $\subsetneq$ **`int`** $\times$ **`int`**.

Accessing Member Variables

```
struct rational {
    int n;
    int d; // INV: d != 0
};

rational add (rational a, rational b){
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
```

$$\frac{r_n}{r_d} := \frac{a_n}{a_d} + \frac{b_n}{b_d} = \frac{a_n \cdot b_d + a_d \cdot b_n}{a_d \cdot b_d}$$

A First Struct: Functionality

A **struct** defines a new *type*, not a *variable*!

```
// new type rational
struct rational {
    int n;
    int d; // INV: d != 0
};
```

Meaning: every object of the new type is represented by two objects of type **int** the objects are called **n** and **d** .

```
// POST: return value is the sum of a and b
rational add (const rational a, const rational b)
{
  rational result;
  result.n =  a.n * b.d +  a.d * b.n;
  result.d = a.d * b.d;
  return result;
}
```

member access to the **int** objects of **a**.

Input

```
// Input r
rational r;
std::cout << "Rational number r:\n";
std::cout << " numerator =? ";
std::cin >> r.n;
std::cout << " denominator =? ";
std::cin >> r.d;

// Input s the same way
rational s;
...
```

Vision comes within Reach ...

```
// computation
const rational t = add (r, s);

// output
std::cout << "Sum is " << t.n << "/" << t.d << ".\n";
```

Struct Definitions

name of the new type (identifier)

names of the underlying types

names of the *member variables*

$$
\begin{array}{l}
\texttt{struct } T \texttt{ \{} \\
\quad T_1 \; name_1; \\
\quad T_2 \; name_2; \\
\quad \vdots \qquad \vdots \\
\quad T_n \; name_n; \\
\texttt{\};}
\end{array}
$$

Range of Values of T: $T_1 \times T_2 \times \ldots \times T_n$

Struct Defintions: Examples

```
struct rational_vector_3 {
  rational x;
  rational y;
  rational z;
};
```

underlying types can be fundamental or *user defined*

Struct Definitions: Examples

```
struct extended_int {
  // represents value if is_positive==true
  // and -value otherwise
  unsigned int value;
  bool is_positive;
};
```

the underlying types can be *different*

Structs: Accessing Members

expression of struct-type T

name of a member-variable of type T.

$$\mathit{expr}.\mathit{name}_k$$

expression of type T_k; value is the value of the object designated by name_k

member access operator .

Structs: Initialization and Assignment

Default Initialization:

```
rational t;
```

- Member variables of `t` are default-initialized
- for member variables of fundamental types nothing happens (values remain undefined)

Structs: Initialization and Assignment

Initialization:

```
rational t = \{5, 1\};
```

- Member variables of **t** are initialized with the values of the list, according to the declaration order.

Structs: Initialization and Assignment

Assignment:

```
rational s;
...
rational t = s;
```

- The values of the member variables of **`s`** are assigned to the member variables of **`t`**.

Structs: Initialization and Assignment

```
t.n                  .n
      = add (r, s)      ;
t.d                  .d
```

Initialization:

```
rational t = add (r, s);
```

- `t` is initialized with the values of `add(r, s)`

Structs: Initialization and Assignment

Assignment:

```
rational t;
t = add (r, s);
```

- `t` is default-initialized
- The value of `add (r, s)` is assigned to `t`

Structs: Initialization and Assignment

`rational s;` ⟵ member variables are uninitialized

`rational t = {1,5};` ⟵ *member-wise* initialization:
`t.n = 1, t.d = 5`

`rational u = t;` ⟵ member-wise copy

`t = u;` ⟵ member-wise copy

`rational v = add (u,t);` ⟵ member-wise copy

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

- member-wise comparison does not make sense in general...
- ...otherwise we had, for example, $\frac{2}{3} \neq \frac{4}{6}$

Structs as Function Arguments

```
void increment(rational dest, const rational src)
{
    dest = add (dest, src); // modifies local copy only
}
```

Call by Value !

```
rational a;
rational b;
a.d = 1; a.n = 2;
b = a;
increment (b, a); // no effect!
std::cout << b.n << "/" << b.d; // 1 / 2
```

Structs as Function Arguments

```
void increment(rational & dest, const rational src)
{
    dest = add (dest, src);
}
```

Call by Reference

```
rational a;
rational b;
a.d = 1; a.n = 2;
b = a;
increment (b, a);
std::cout << b.n << "/" << b.d; // 2 / 2
```

User Defined Operators

Instead of

```
rational t = add(r, s);
```

we would rather like to write

```
rational t = r + s;
```

This can be done with *Operator Overloading* ($\rightarrow$ *next week*).

19. Classes

Overloading Functions and Operators, Encapsulation, Classes, Member Functions, Constructors

Overloading Functions

- Functions can be addressed by name in a scope
- It is even possible to declare and to defined several functions with the same name
- the “correct” version is chosen according to the *signature* of the function.

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }        // f1
int sq (int x) { ... }              // f2
int pow (int b, int e) { ... }      // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call (we do not go into details)

```
std::cout << sq (3);   // compiler chooses f2
std::cout << sq (1.414); // compiler chooses f1
std::cout << pow (2);  // compiler chooses f4
std::cout << pow (3,3); // compiler chooses f3
```

Operator Overloading

- Operators are special functions and can be overloaded
- Name of the operator *op*:

```
operatorop
```

- we already know that, for example, `operator+` exists for different types

Adding `rational` Numbers – Before

```
// POST: return value is the sum of a and b
rational add (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = add (r, s);
```

Adding `rational` Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = r + s;
```

infix notation

Other Binary Operators for Rational Numbers

```
// POST: return value is difference of a and b
rational operator- (rational a, rational b);

// POST: return value is the product of a and b
rational operator* (rational a, rational b);

// POST: return value is the quotient of a and b
// PRE: b != 0
rational operator/ (rational a, rational b);
```

Unary Minus

has the same symbol as the binary minus but only one argument:

```
// POST: return value is -a
rational operator- (rational a)
{
    a.n = -a.n;
    return a;
}
```

Comparison Operators

are not built in for structs, but can be defined

```
// POST: returns true iff a == b
bool operator== (rational a, rational b)
{
    return a.n * b.d == a.d * b.n;
}
```

$$\frac{2}{3} = \frac{4}{6} \quad \checkmark$$

Arithmetic Assignment

We want to write

```
rational r;
r.n = 1; r.d = 2;                     // 1/2

rational s;
s.n = 1; s.d = 3;                     // 1/3

r += s;
std::cout << r.n << "/" << r.d;       // 5/6
```

Operator+= First Trial

```
rational operator+= (rational a, rational b)
{
    a.n = a.n * b.d + a.d * b.n;
    a.d *= b.d;
    return a;
}
```

does not work. Why?

- The expression `r += s` has the desired value, but because the arguments are R-values (call by value!) it does not have the desired effect of modifying `r`.
- The result of `r += s` is, against the convention of C++ no L-value.

Operator +=

```
rational& operator+= (rational& a, rational b)
{
    a.n = a.n * b.d + a.d * b.n;
    a.d *= b.d;
    return a;
}
```

this works

- The L-value **a** is increased by the value of **b** and returned as L-value

  `r += s;` now has the desired effect.

In/Output Operators

can also be overloaded.

- Before:

```
std::cout << "Sum is " << t.n << "/" << t.d << "\n";
```

- After (desired):

```
std::cout << "Sum is " << t << "\n";
```

In/Output Operators

can be overloaded as well:

```
// POST: r has been written to out
std::ostream& operator<< (std::ostream& out, rational r)
{
    return out << r.n << "/" << r.d;
}
```

writes **r** to the output stream
and returns the stream as L-value.

Input

```
// PRE: in starts with a rational number of the form "n/d"
// POST: r has been read from in
std::istream& operator>> (std::istream& in, rational& r){
    char c; // separating character '/'
    return in >> r.n >> c >> r.d;
}
```

reads **r** from the input stream
and returns the stream as L-value.

Goal Attained!

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;

std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

operator >>

operator +

operator<<

A new Type with Functionality...

```
struct rational {
    int n;
    int d; // INV: d != 0
};

// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
```

...should be in a Library!

rational.h

- Definition of a struct **`rational`**
- Function declarations

rational.cpp

- arithmetic operators (**`operator+`**, **`operator+=`**, ...)
- relational operators (**`operator==`**, **`operator>`**, ...)
- in/output (**`operator >>`**, **`operator <<`**, ...)

Thought Experiment

The three core missions of ETH:

- research
- education
- technology transfer

We found a startup: RAT PACK®!

- Selling the `rational` library to customers
- ongoing development according to customer's demands

The Customer is Happy

…and programs busily using `rational`.

- output as `double`-value $(\frac{3}{5} \rightarrow 0.6)$

```
// POST: double approximation of r
double to_double (rational r)
{
  double result = r.n;
  return result / r.d;
}
```

The Customer Wants More

“Can we have rational numbers with an extended value range?”

- Sure, no problem, e.g.:

```
struct rational {
  int n;
  int d;
};
```

$\Rightarrow$

```
struct rational {
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

New Version of RAT PACK®

It sucks, nothing works any more!

- What is the problem?

$-\frac{3}{5}$ is sometimes 0.6, this cannot be true!

- That is your fault. Your conversion to `double` is the problem, our library is correct.

Up to now it worked, therefore the new version is to blame!

Liability Discussion

```
// POST: double approximation of r
double to_double (rational r){
  double result = r.n;
  return result / r.d;
}
```

`r.is_positive` and `result.is_positive` do not appear.

correct using…

```
struct rational {
  int n;
  int d;
};
```

…not correct using

```
struct rational {
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

We are to Blame!!

- Customer sees and uses our **representation** of rational numbers (initially `r.n, r.d`)
- When we change it (`r.n, r.d, r.is_positive`), the customer's programs do not work anymore.
- No customer is willing to adapt the programs when the version of the library changes.

⇒ RAT PACK® is history...

Idea of Encapsulation (Information Hiding)

- A type is uniquely defined by its *value range* and its *functionality*
- The *representation* should *not be visible*.
- $\Rightarrow$ The customer is not provided with *representation* but with **functionality**!

`str.length()`,
`v.push_back(1)`,...

Classes

- provide the concept for **encapsulation** in C++
- are a variant of structs
- are provided in many *object oriented programming languages*

Encapsulation: `public` / `private`

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

is used instead of **`struct`** if anything at all shall be "hidden"

only difference

- **`struct`**: by default **nothing** is hidden
- **`class`** : by default **everything** is hidden

Encapsulation: `public` / `private`

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

Good news: `r.d = 0` cannot happen any more by accident.

Bad news: the customer cannot do anything any more ...

Application Code

... and we can't, either. (no `operator+`, ...)

```
rational r;
r.n = 1;      // error: n is private
r.d = 2;      // error: d is private
int i = r.n; // error: n is private
```

Member Functions: Declaration

public area

```
class rational {
public:
    // POST: return value is the numerator of this instance
    int numerator () const {
      return n;
    }
    // POST: return value is the denominator of this instance
    int denominator () const {
      return d;
    }
private:
    int n;
    int d; // INV: d!= 0
};
```

member function

member functions have access to private data

the scope of members in a class is the whole class, independent of the declaration order

Member Functions: Call

```
// Definition des Typs
class rational {
    ...
};
...
// Variable des Typs
rational r;
```

member access

```
int n = r.numerator();   // Zaehler
int d = r.denominator(); // Nenner
```

Member Functions: Definition

```
// POST: returns  numerator of this instance
int numerator ()  const
{
  return n;
}
```

- A member function is called for an expression of the class. in the function, `this` is the name of this *implicit argument*. `this` itself is a pointer to it.
- *const* refers to the instance `this`, i.e., it promises that the value associated with the implicit argument cannot be changed
- `n` is the shortcut in the member function for `this->n` (precise explanation of "`->`" next week)

const and Member Functions

```
class rational {
public:
  int numerator () const
  { return n; }
  void set_numerator (int N)
  { n = N;}
...
}
```

```
rational x;
x.set_numerator(10); // ok;
const rational y = x;
int n = y.numerator(); // ok;
y.set_numerator(10); // error;
```

The **const** at a member function is to promise that an instance cannot be changed via this function.

const items can only call **const** member functions.

Comparison

Roughly like this it were ...

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return this->n;
    }
};

rational r;
...
std::cout << r.numerator();
```

... without member functions

```
struct bruch {
    int n;
    ...
};

int numerator (const bruch& dieser)
{
    return dieser.n;
}

bruch r;
..
std::cout << numerator(r);
```

Member-Definition: In-Class vs. Out-of-Class

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return n;
    }
    ....
};
```

- No separation between declaration and definition (bad for libraries)

```
class rational {
    int n;
    ...
public:
    int numerator () const;
    ...
};

int rational::numerator () const
{
  return n;
}
```

- This also works.

Constructors

- are special member functions of a class that are named like the class
- can be overloaded like functions, i.e. can occur multiple times with varying *signature*
- are called like a function when a variable is declared. The compiler chooses the “closest” matching function.
- if there is no matching constructor, the compiler emits an *error message*.

Initialisation? Constructors!

```
class rational
{
public:
    rational (int num, int den)
        : n (num), d (den)      ← Initialization of the member variables
    {
        assert (den != 0);      ← function body.
    }
...
};
...
rational r (2,3); // r = 2/3
```

Constructors: Call

- directly

```
rational r (1,2); \small // initialisiert r mit 1/2
```

- indirectly (copy)

```
rational r = rational (1,2);
```

Initialisation “rational = int”?

```
class rational
{
public:
    rational (int num)
       : n (num), d (1)
    {}    ⟵ empty function body
...
};
...
rational r (2); // explicit initialization with 2
rational s = 2; // implicit conversion
```

The Default Constructor

```
class rational
{
public:
    ...
    rational ()
        : n (0), d (1)
    {}
...
};
...
rational r;   // r = 0
```

empty list of arguments

⇒ There are no uninitiatlized variables of type rational any more!

Alterantively: Deleting a Default Constructor

```
class rational
{
public:
    ...
    rational () = delete;
...
};
...
rational r;   // error: use of deleted function ’rational::rational
```

⇒ There are no uninitiatlized variables of type rational any more!

User Defined Conversions

are defined via constructors with exactly *one* argument

```
rational (int num)
    : n (num), d (1)
  {}
```

User defined conversion from **int** to **rational**. values of type **int** can now be converted to **rational**.

```
rational r = 2; // implizite Konversion
```

The Default Constructor

- is automatically called for declarations of the form
  `rational r;`
- is the unique constructor with empty argmument list (if existing)
- must exist, if **`rational r;`** is meant to compile
- if in a struct there are no constructors at all, the default constructor is automatically generated

RAT PACK® Reloaded ...

Customer's program now looks like this:

```
// POST: double approximation of r
double to_double (const rational r)
{
  double result = r.numerator();
  return result / r.denominator();
}
```

- We can adapt the member functions together with the representation ✓

RAT PACK® Reloaded ...

before

```
class rational {
...
private:
  int n;
  int d;
};
```

```
int numerator () const
{
  return n;
}
```

after

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

RAT PACK® Reloaded ?

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const
{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

- value range of nominator and denominator like before
- possible overflow in addition

Encapsulation still Incompleete

Customer's point of view (`rational.h`):

```
class rational {
public:
    // POST: returns numerator of *this
    int numerator () const;
    ...
private:
   // none of my business
};
```

- We determined denominator and nominator type to be **`int`**
- Solution: encapsulate not only data but alsoe types.

Fix: “our” type `rational::integer`

Customer’s point of view (`rational.h`):

```
public:
    using integer = long int; // might change
    // POST: returns numerator of *this
    integer numerator () const;
```

- We provide an additional type!
- Determine only **Functionality**, e.g:
  - implicit conversion `int` $\rightarrow$ `rational::integer`
  - function `double to_double (rational::integer)`

RAT PACK® Revolutions

Finally, a customer program that remains stable

```
// POST: double approximation of r
double to_double (const rational r)
{
  rational::integer n = r.numerator();
  rational::integer d = r.denominator();
  return to_double (n) / to_double (d);
}
```

Separate Declaration and Definition

```
class rational {
public:
    rational (int num, int denum);
    using integer = long int;
    integer numerator () const;
    ...
private:
  ...
};
```

rational.h

```
rational::rational (int num, int den):
     n (num), d (den) {}
rational::integer rational::numerator () const
{
     return n;
}
```

class name :: member name

rational.cpp

20. Dynamic Data Structures I

Dynamic Memory, Addresses and Pointers, Const-Pointer Arrays, Array-based Vectors

Recap: `vector<`T`>`

- Can be initialised with arbitrary size **`n`**
- Supports various operations:

```
e = v[i];           // Get element
v[i] = e;           // Set element
l = v.size();       // Get size
v.push_front(e);    // Prepend element
v.push_back(e);     // Append element
...
```

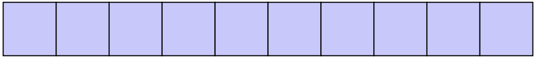
- A vector is a *dynamic data structure*, whose size may change at runtime

Our Own Vector!

- Today, we'll implement our own vector: **`vec`**
- Step 1: **`vec<int>`** (today)
- Step 2: **`vec<`**T**`>`** (later, only superficially)

Vectors in Memory

Already known: A vector has a *contiguous* memory layout



Question: How to *allocate* a chunk of memory of *arbitrary* size during runtime, i.e. *dynamically*?

`new` for Arrays

underlying type

```
new T[expr]
```

new-O type `int`, value n

- **Effect**: new contiguous chunk of memory n elements of type *T* is allocated

- This chunk of memory is called an *array* (of length n)

`new` for Arrays

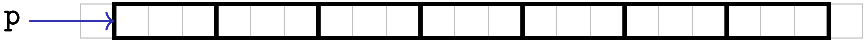
underlying type

`p = new`

T`[`*expr*`]`

new-O type **`int`**, value n

- **Value**: the starting address of the memory chunk



- **Type**: A pointer T`*` (more soon)

Outlook: `new` and `delete`

`new` *T*`[`*expr*`]`

- So far: memory (local variables, function arguments) “lives” only inside a function call
- But now: memory chunk inside vector must not “die” before the vector itself
- Memory allocated with **`new`** is *not* automatically *deallocated* (= released)
- Every **`new`** must have a matching **`delete`** that releases the memory explicitly $\rightarrow$ **in two weeks**

`new` (Without Arrays)

underlying type

`new` T`(...)`

new-Operator

constructor arguments

- **Effect**: memory for a new object of type T is allocated ...
- ...and initialized by means of the matching constructor
- **Value**: address of the new T object, **Type**: Pointer T`*`
- Also true here: object “lives” until deleted explicitly (usefulness will become clearer later)

Pointer Types

`T*` Pointer type for base type **`T`**

An expression of type **`T*`** is called *pointer (to* **`T`***)*

```
int* p; // Pointer to an int
std::string* q; // Pointer to a std::string
```

Pointer Types

T*   Pointer type for base type **T**

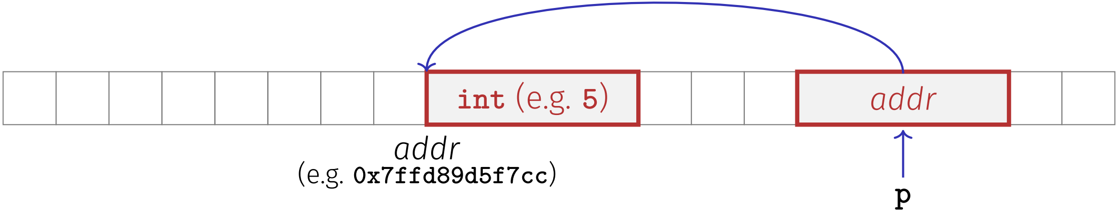
A T* must actually point to a T

```
int* p = ...;
std::string* q = p; // compiler error!
```

Pointer Types

Value of a pointer to **T** is the *address* of an object of type **T**

```
int* p = ...;
std::cout << p; // e.g. 0x7ffd89d5f7cc
```



Address Operator

Question: How to obtain an object's address?

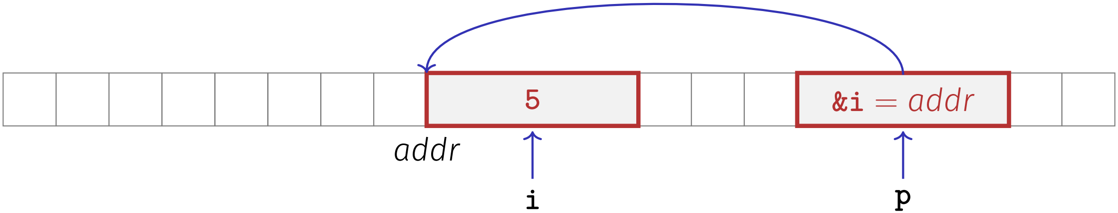
1. Directly, when creating a new object via **`new`**
2. For existing objects: via the *address operator* **`&`**

&expr ← expr: l-value of type T

- **Value** of the expression: the *address* of object (l-value) *expr*
- **Type** of the expression: A pointer $T*$ (of type T)

Address Operator

```
int i = 5; // i initialised with 5
int* p = &i; // p initialised with address of i
```



Next question: How to "follow" a pointer?

Dereference Operator

Answer: by using the *dereference operator* $*$

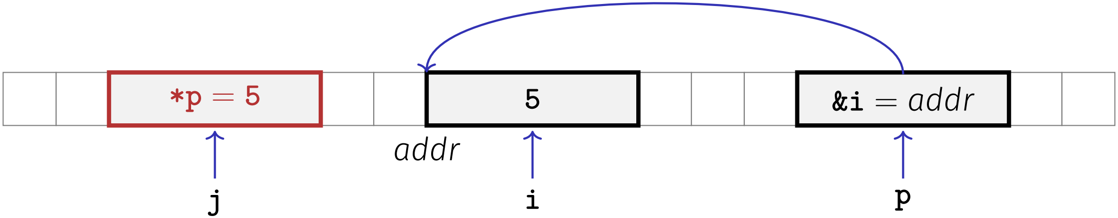
$$*expr$$

expr: r-value of type $T*$

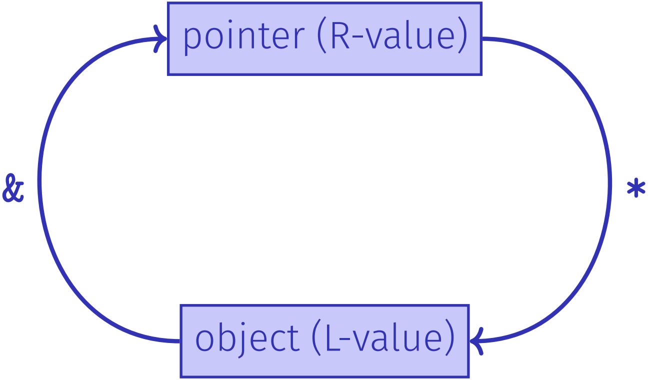
- **Value** of the expression: the *value* of the object located at the address denoted by *expr*
- **Type** of the expression: T

Dereference Operator

```
int i = 5;
int* p = &i; // p = address of i
int j = *p; // j = 5
```



Address and Dereference Operator



Mnenmonic Trick

The declaration

```
T* p;       // p is of the type "pointer to T"
```

can be read as

```
T *p;       // *p is of type T
```

Although this is legal, we do not write it like this!

Null-Pointer

- Special pointer value that signals that no object is pointed to
- represented b the literal **`nullptr`** (convertible to **`T*`**)

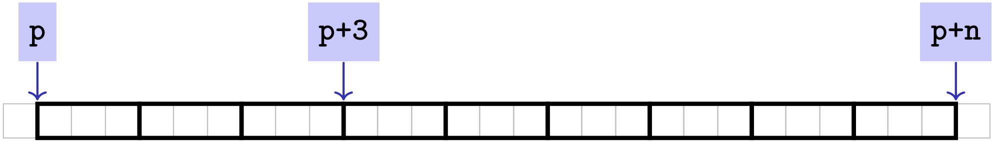
```
int* p = nullptr;
```

- Cannot be dereferenced (runtime error)
- Exists to avoid undefined behaviour

```
int* p; // Accessing p is undefined behaviour
int* q = nullptr; // q explicitly points nowhere
```

Pointer Arithmetic: Pointer plus `int`

```
T* p = new T[n]; // p points to first array element
```

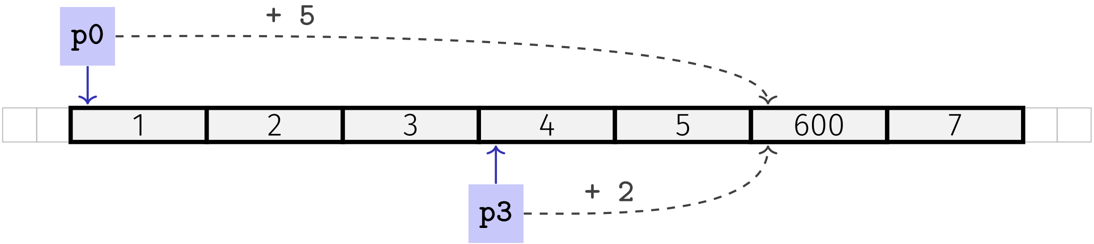


Question: How to point to rear elements? $\rightarrow$ via *Pointer arithmetic*:

- `p` yields the *value* of the *first* array element, `*p` its *value*
- `*(p + i)` yields the value of the `i`*th* array element, for $0 \leq$ `i` $<$ `n`
- `*p` is equivalent to `*(p + 0)`

Pointer Arithmetic: Pointer plus `int`

```
int* p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
int* p3 = p0 + 3; // p3 points to 4th element
*(p3 + 2) = 600; // set value of 6th element to 600
std::cout << *(p0 + 5); // output 6th element's value (i.e. 600)
```



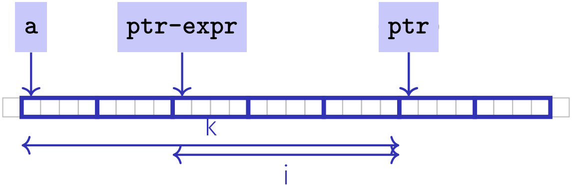
Pointer Arithmetic: Pointer minus `int`

- If *ptr* is a pointer to the element with index k in an array a with length n
- and the value of *expr* is an integer i, $0 \leq k - i \leq n$,

then the expression

ptr - expr

provides a pointer to an element of a with index $k - i$.



Pointer Subtraction

- If *p1* and *p2* point to elements of the same array **`a`** with length n
- and $0 \leq k_1, k_2 \leq n$ are the indices corresponding to *p1* and *p2*, then

  p1 - *p2* has value k_1 - k_2

  $\uparrow$

  Only valid if *p1* and p2 point into the same array.

- The pointer difference describes how far apart the elements are from each other in memory

Pointer Operators

Description	**Op**	**Arity**	**Precedence**	**Associativity**	**Assignment**
Subscript	`[]`	2	17	left	R-value → L-value
Dereference	`*`	1	16	right	R-Wert → L-Wert
Address	`&`	1	16	rechts	L-value → R-value

Precedences and associativities of **+**, **−**, **++** (etc.) as in Chapter 2

Pointers are not Integers!

- Addresses can be interpreted as house numbers of the memory, that is, integers
- But integer and pointer arithmetic behave differently.

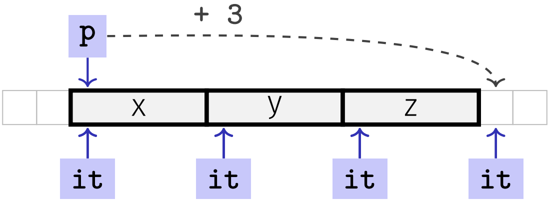
`ptr + 1` is **not** the next house number but the s-next, where s is the memory requirement of an object of the type behind the pointer `ptr`.

- Integers and pointers are not compatible

```
int* ptr = 5; // error: invalid conversion from int to int*
int a = ptr;  // error: invalid conversion from int* to int
```

Sequential Pointer Iteration

```cpp
char* p = new char[3]{'x', 'y', 'z'};
```




```cpp
for (char* it = p;
        it != p + 3;
        ++it) {

    std::cout << *it << ' '; //
}
```

`it` points to first element

Abort if end reached

Advance pointer element-wise

Output current element: 'x'

Random Access to Arrays

```
char* p = new char[3]{'x', 'y', 'z'};
```

- The expression `*(p + i)`
- can also be written as `p[i]`
- E.g. `p[1] == *(p + 1) == 'y'`

Random Access to Arrays

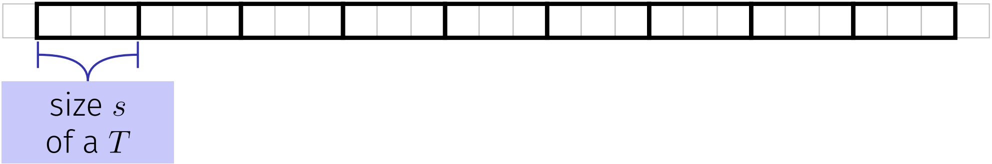
iteration over an array via indices and *random access*:

```
char* p = new char[3]{'x', 'y', 'z'};

for (int i = 0; i < 3; ++i)
  std::cout << p[i] << ' ';
```

But: this is less *efficient* than the previously shown *sequential* access via pointer iteration

Random Access to Arrays

```
T* p = new T[n];
```



- Access `p[i]`, i.e. `*(p + i)`, “costs” computation $p + i \cdot s$
- Iteration via *random access* (`p[0]`, `p[1]`, …) costs one addition and one multiplication per access
- Iteration via *sequentiall access* (`++p`, `++p`, …) costs only one addition per access
- Sequential access is thus to be preferred for iterations

Reading a book ...with random access ...with sequential access

Random Access

- open book on page 1
- close book
- open book on pages 2-3
- close book
- open book on pages 4-5
- close book
- ....

Sequential Access

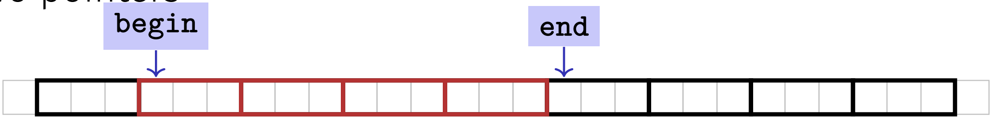
- open book on page 1
- turn the page
- turn the page
- turn the page
- turn the page
- turn the page
- ...

Static Arrays

- `int* p = new int[`$expr$`]` creates a dynamic array of size $expr$
- C++has inherited *static* arrays from its predecessor language C: `int a[`$cexpr$`]`
- Static arrays have, among others, the disadvantage that their size $cexpr$ must be a constant. I.e. $cexpr$ can, e.g. be `5` or `4*3+2`, but kein von der Tastatur eingelesener Wert `n`.
- A static array variable `a` can be used just like a pointer
- Rule of thumb: Vectors are better than dynamic arrays, which are better than static arrays

Arrays in Functions

C++ *covention*: arrays (or a segment of it) are passed using two pointers



- **`begin`**: Pointer to the first element
- **`end`**: Pointer *past* the last element
- **`[begin, end)`** Designates the elements of the segment of the array
- **`[begin, end)`** is empty if **`begin == end`**
- **`[begin, end)`** must be a *valid range*, i.e. a (pot. empty) array segment

Arrays in (mutating) Functions: `fill`

```
// PRE: [begin, end) is a valid range
// POST: Every element within [begin, end) was set to value
void fill(int* begin, int* end, int value) {
  for (int* p = begin; p != end; ++p)
    *p = value;
}
```

```
int* p = new int[5];
fill(p, p+5, 1); // Array at p becomes {1, 1, 1, 1, 1}
```

Functions with/without Effect

- Pointers can (like references) be used for functions with effect. Example: **`fill`**
- But many functions don't have an effect, they only read the data
- $\Rightarrow$ Use of **`const`**
- So far, for example:

```
const int zero = 0;
const int& nil = zero;
```

Positioning of Const

Where does the **const**-modifier belong to?
const T is equivalent to T **const** (and can be written like this):

```
const int zero = ...   ⟺   int const zero = ...
const int& nil = ...   ⟺   int const& nil = ...
```

Both keyword orders are used in praxis

Const and Pointers

Read the declaration from right to left

`int const p1;`	**p1** is a constant integer
`int const* p2;`	**p2** is a pointer to a constant integer
`int* const p3;`	**p3** is a constant pointer to an integer
`int const* const p4;`	**p4** is a constant pointer to a constant integer

Non-mutating Functions: **print**

There are also *non*-mutating functions that access elements of an array only in a read-only fashion

```
// PRE: [begin, end) is a valid range
// POST: The values in [begin, end) were printed
void print(
    int const* const begin,
    const int* const end) {

  for (int const* p = begin; p != end; ++p)
    std::cout << *p << ' ';
}
```

Const pointer to const int

Likewise (but different keyword order)

Pointer, *not const*, to const int

Pointer **p** may itself not be **const** since it is mutated (**++p**)

`const` is not absolute

- The value at an address can change even if a **`const`**-pointer stores this address.

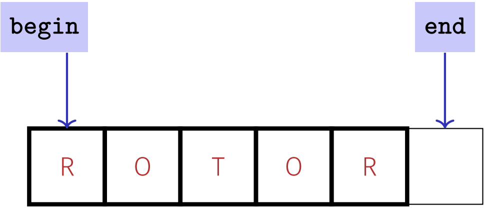
Beispiel

```
int a[5];
const int* begin1 = a;
int*       begin2 = a;
*begin1 = 1;     // error *begin1 is const
*begin2 = 1;     // ok, although *begin will be modified
```

- **`const`** is a promise from the point of view of the **`const`**-pointer, not an absolute guarantee

Wow – Palindromes!

```
// PRE: [begin end) is a valid range of characters
// POST: returns true if the range forms a palindrome
bool is_palindrome (const char* begin, const char* end) {
  while (begin < end)
    if (*(begin++) != *(--end)) return false;
  return true;
}
```



Arrays, `new`, Pointer: Conclusion

- Arrays are contiguous chunks of memory of statically unknown size
- `new` T`[`n`]` allocates a T-array of size n
- T`* p = new` T`[`n`]`: pointer `p` points to the first array element
- Pointer arithmetic enables accessing rear array elements
- Sequentially iterating over arrays via pointers is more efficient than random access
- `new` T allocates memory for (and initialises) a single T-object, and yields a pointer to it
- Pointers can point to something (not) `const`, and they can be (not) `const` themselves
- Memory allocated by `new` is *not* automatically released (more on this soon)
- Pointers and references are related, both “link” to objects in memory. See also additional the slides `pointers.pdf`)

Array-based Vector

- Vectors …that somehow rings a bell 🤔
- Now we know how to allocate memory chunks of arbitrary size …
- …we can implement a vector, based on such a chunk of memory
- **`avec`** – an array-based vector of **`int`** elements

Unser eigener Vektor!

- Wir implementieren unseren eigenen Vektor: `vec`
- Schritt 1: `vec<int>` (heute)
- Schritt 2: `vec<`T`>` (später, nur kurz angeschnitten)

Array-based Vector `avec`: Class Signature

```
class avec {
  // Private (internal) state:
  int* elements; // Pointer to first element
  unsigned int count; // Number of elements

public: // Public interface:
  avec(unsigned int size); // Constructor
  unsigned int size() const; // Size of vector
  int& operator[](int i); // Access an element
  void print(std::ostream& sink) const; // Output elems.
}
```

Constructor `avec::avec()`

```
avec::avec(unsigned int size)
        : count(size) {

  elements = new int[size];
}
```

Save size

Allocate memory

Side remark: vector is not initialised with a default value

Excursion: Accessing Member Variables

```
avec::avec(unsigned int size): count(size) {
  this->elements = new int[size];
}
```

- `elements` is a member variable of our `avec` instance
- That instance can be accessed via the *pointer* `this`
- `elements` is a shorthand for `(*this).elements`
- Dereferencing a pointer (`*this`) followed by a member access (`.elements`) is such a common operation that it can be written more concisely as `this->elements`
- Mnemonic trick: “Follow the pointer to the member variable”

Function `avec::size()`

```
int avec::size() const {
  return this->count;
}
```

Doesn't modify the vector

Return size

Usage example:

```
avec v = avec(7);
assert(v.size() == 7); // ok
```

Function `avec::operator[]`

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Return `i`th element

Element access with index check:

```
int& avec::at(int i) const {
  assert(0 <= i && i < this->count);

  return this->elements[i];
}
```

Function `avec::operator[]`

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Usage example:

```
avec v = avec(7);
std::cout << v[6]; // Outputs a "random" value
v[6] = 0;
std::cout << v[6]; // Outputs 0
```

Function `avec::operator[]` is needed twice

```
int& avec::operator[](int i) { return elements[i]; }
const int& avec::operator[](int i) const { return elements[i]; }
```

- The first member function is *not const* and returns a *non-const* reference

  ```
  avec v = ...; // A non-const vector
  std::cout << v.get[0]; // Reading elements is allowed
  v.get[0] = 123; // Modifying elements is allowed
  ```

- It is called on non-const vectors

Function `avec::operator[]` is needed twice

```
int& avec::operator[](int i) { return elements[i]; }
const int& avec::operator[](int i) const { return elements[i]; }
```

- The second member function *is const* and returns a *const* reference

```
const avec v = ...; // A const vector
std::cout << v.get[0]; // Reading elements is allowed
v.get[0] = 123; // Compiler error: modifications are not allowed
```

- It is called on const vectors

Also see the example getters_and_const.cpp attached to this PDF

Function `avec::print()`

Output elements using sequential access:

```
void avec::print(std::ostream& sink) const {
  for (int* p = this->elements;
       p != this->elements + this->count;
       ++p)
  {
    sink << *p << ' ';
  }
}
```

Pointer to first element

Abort iteration if past last element

Advance pointer element-wis

Output current element

Function `avec::print()`

Finally: overload output operator:

```
_________________ operator<<(_________________ sink,
                             _________________ vec) {
  vec.print(sink);
  return _________________;
}
```

```
std::ostream& operator<<(std::ostream& sink,
                         const avec& vec) {
  vec.print(sink);
  return sink;
}
```

Observations:

- Constant reference to `vec`, since unchanged

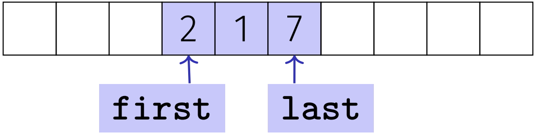
Further Functions?

```
class avec {
  ...
  void push_front(int e)       // Prepend e to vector
  void push_back(int e)        // Append e to vector
  void remove(unsigned int i) // Cut out ith element
  ...
}
```

Commonalities: such operations need to change the vector's *size*

Resizing arrays

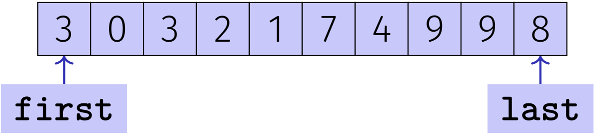
An allocated block of memory (e.g. **`new int[3]`**) cannot be resized later on



Possibility:

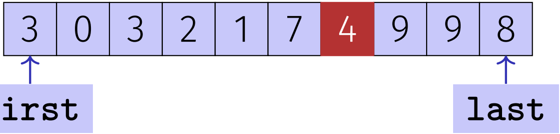
- Allocate more memory than initially necessary
- Fill from inside out, with pointers to first and last element

Resizing arrays

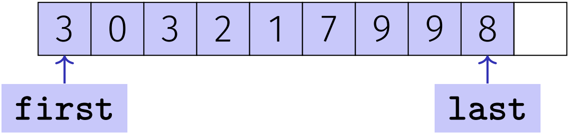


- But eventually, all slots will be in use
- Then unavoidable: Allocate larger memory block and copy data over

Resizing arrays



Deleting elements requires shifting (by copying) all preceding or following elements



Similar: inserting at arbitrary position

21. Dynamic Data Structures II

Linked Lists, Vectors as Linked Lists

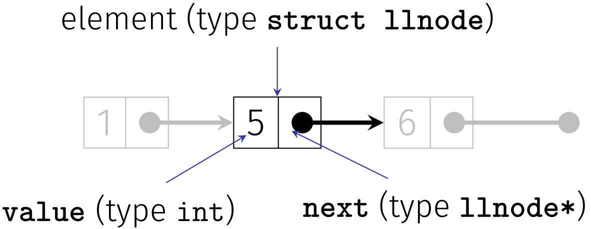
Different Memory Layout: Linked List

- **No** contiguous area of memory and **no** random access
- Each element points to its successor
- Insertion and deletion of **arbitrary** elements is simple

$1 \rightarrow 5 \rightarrow 6 \xrightarrow[\text{pointer}]{} 3 \rightarrow 8 \rightarrow 8 \rightarrow 9$

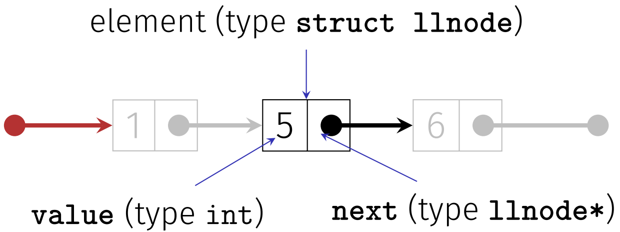
$\Rightarrow$ Our vector can be implemented as a linked list

Linked List: Zoom




```
struct llnode {
  int value;
  llnode* next;

  llnode(int v, llnode* n): value(v), next(n) {} // Constructor
};
```

Vector = Pointer to the First Element




```
class llvec {
  llnode* head;
public: // Public interface identical to avec's
  llvec(unsigned int size);
  unsigned int size() const;
  ...
};
```

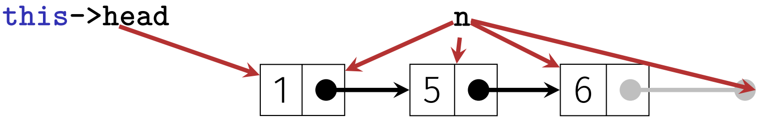
Function `llvec::print()`

```
struct llnode {
  int value;
  llnode* next;
  ...
};
```

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;      ← Pointer to first element
       n != nullptr;                ← Abort if end reached
       n = n->next)                 ← Advance pointer element-wise
  {
    sink << n->value << ' ';        ← Output current element
  }
}
```

Function `llvec::print()`

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';  // 1 5 6
  }
}
```



Function `llvec::operator[]`

Accessing ith Element is implemented similarly to **`print()`**:

```
int& llvec::operator[](unsigned int i) {
  llnode* n = this->head;

  for (; 0 < i; --i)
    n = n->next;

  return n->value;
}
```

Pointer to first element

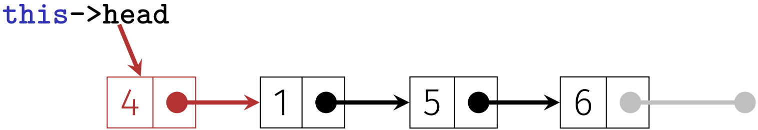
Step to `i`th element

Return `i`th element

Function `llvec::push_front()`

Advantage **`llvec`**: Prepending elements is very easy:

```
void llvec::push_front(int e) {
  this->head =
    new llnode{e, this->head};
}
```



Attention: If the new **`llnode`** weren't allocated *dynamically*, then it would be deleted (= memory deallocated) as soon as **`push_front`** terminates

Function `llvec::llvec()`

Constructor can be implemented using **`push_front()`**:

```
llvec::llvec(unsigned int size) {
  this->head = nullptr;

  for (; 0 < size; --size)
    this->push_front(0);
}
```

`head` initially points to nowhere

Prepend **`0`** **`size`** times

Use case:

```
llvec v = llvec(3);
std::cout << v; // 0 0 0
```

Function `llvec::push_back()`

Simple, but inefficient: traverse linked list to its end and append new element

```
void llvec::push_back(int e) {
  llnode* n = this->head;

  for (; n->next != nullptr; n = n->next);

  n->next =
    new llnode{e, nullptr};
}
```

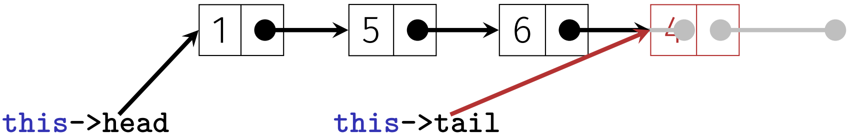
Start at first e

… and go to the last element

Append new element to currently last

Function `llvec::push_back()`

- More efficient, but also slightly more complex:
  1. Second pointer, pointing to the last element: `this->tail`
  2. Using this pointer, it is possible to append to the end directly



- But: Several corner cases, e.g. vector still empty, must be accounted for

Function `llvec::size()`

Simple, but inefficient: *compute* size by counting

```
unsigned int llvec::size() const {
  unsigned int c = 0;

  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
    ++c;

  return c;
}
```

Count initially `0`

Count linked-list length

Return count

Function `llvec::size()`

More efficient, but also slightly more complex: *maintain* size as member variable

1. Add member variable **`unsigned int count`** to class **`llvec`**
2. **`this->count`** must now be updated *each* time an operation (such as **`push_front`**) affects the vector's size

Efficiency: Arrays vs. Linked Lists

- Memory: our **`avec`** requires roughly n ints (vector size n), our **`llvec`** roughly $3n$ ints (a pointer typically requires 8 byte)

- Runtime (with **`avec`** = **`std::vector`**, **`llvec`** = **`std::list`**):

```
prepending (insert at front) [100,000x]:
    ► avec:     675 ms
    ► llvec:     10 ms
appending (insert at back) [100,000x]:
    ► avec:       2 ms
    ► llvec:      9 ms
removing first [100,000x]:
    ► avec:     675 ms
    ► llvec:      4 ms
removing last [100,000x]:
    ► avec:       0 ms
    ► llvec:      4 ms
```

```
removing randomly [10,000x]:
    ► avec:       3 ms
    ► llvec:    113 ms
inserting randomly [10,000x]:
    ► avec:      16 ms
    ► llvec:    117 ms
fully iterate sequentially (5000 elements) [5,000x]:
    ► avec:     354 ms
    ► llvec:    525 ms
```

22. Containers, Iterators and Algorithms

Containers, Sets, Iterators, const-Iterators, Algorithms, Templates

Vectors are Containers

- Viewed abstractly, a vector is
  1. A collection of elements
  2. Plus operations on this collection
- In C++, `vector<T>` and similar data structures are called *container*
- Called *collections* in some other languages, e.g. Java

Container properties

- Each container has certain *characteristic properties*
- For an array-based vector, these include:
  - Efficient index-based access (`v[i]`)
  - Efficient use of memory: Only the elements themselves require space (plus element count)
  - Inserting at/removing from arbitrary index is potentially inefficient
  - Looking for a specific element is potentially inefficient
  - Can contain the same element more than once
  - Elements are in insertion order (ordered but not sorted)

Containers in C++

- Nearly every application requires maintaining and manipulating arbitrarily many data records
- But with different requirements (e.g. only append elements, hardly ever remove, often search elements, . . .)
- That's why C++'s standard library includes several containers with different properties, see `https://en.cppreference.com/w/cpp/container`
- Many more are available from 3rd-party libraries, e.g. `https://www.boost.org/doc/libs/1_68_0/doc/html/container.html`, `https://github.com/abseil/abseil-cpp`

Example Container: `std::unordered_set`<T>

- A *mathematical set* is an unordered, duplicate-free collection of elements:

$$\{1,2,1\} = \{1,2\} = \{2,1\}$$

- In C++: `std::unordered_set`<T>
- Properties:
  - Cannot contain the same element twice
  - Elements are not in any particular order
  - Does not provide index-based access (`s[i]` undefined)
  - Efficient “element contained?” check
  - Efficient insertion and removal of elements
- Side remark: implemented as a hash table

Use Case `std::unordered_set`<T>

Problem:

- given a sequence of pairs (*name, percentage*) of Code Expert submissions ...

```
// Input: file submissions.txt
Friedrich 90
Schwerhoff 10
Lehner 20
Schwerhoff 11
```

- ... determine the submitters that achieved at least 50%

```
// Output
Friedrich
```

Use Case `std::unordered_set<T>`

```
std::ifstream in("submissions.txt");
std::unordered_set<std::string> names;

std::string name;
unsigned int score;

while (in >> name >> score) {
  if (50 <= score)
    names.insert(name);
}

std::cout << "Unique submitters: "
          << names << '\n';
```

Open **`submissions.txt`**

Set of names, initially empty

Pair *(name, score)*

Input next pair

Record name if score suffices

Output recorded names

Example Container: **`std::set`**$<T>$

- Nearly equivalent to **`std::unordered_set`**$<T>$, but the elements are *ordered*

$$\{1, 2, 1\} = \{1, 2\} \neq \{2, 1\}$$

- Element look-up, insertion and removal are still efficient (better than for **`std::vector`**$<T>$), but less efficient than for **`std::unordered_set`**$<T>$
- That's because maintaining the order does not come for free
- Side remark: implemented as a red-black tree

Use Case std::set<T>

```
std::ifstream in("submissions.txt");
std::set<std::string> names;

std::string name;
unsigned int score;

while (in >> name >> score) {
  if (50 <= score)
    names.insert(name);
}

std::cout << "Unique submitters: "
          << names << '\n';
```

set instead of **unsorted_set** . . .

. . . and the output is in alphabetical order

Printing Containers

- Recall: **`avec::print()`** and **`llvec::print()`**
- What about printing **`set`**, **`unordered_set`**, ...?
- Commonality: iterate over container elements and print them

Similar Functions

- Lots of other useful operations can be implemented by iterating over a container:
- `contains(c, e)`: true iff container `c` contains element `e`
- `min/max(c)`: Returns the smallest/largest element
- `sort(c)`: Sorts `c`'s elements
- `replace(c, e1, e2)`: Replaces each `e1` in `c` with `e2`
- `sample(c, n)`: Randomly chooses `n` elements from `c`
- ...

Recall: Iterating With Pointers

- Iteration over an *array*:
  - Point to start element: `p = this->arr`
  - Access current element: `*p`
  - Check if end reached:
    `p == this->arr + size`
  - Advance pointer: `p = p + 1`

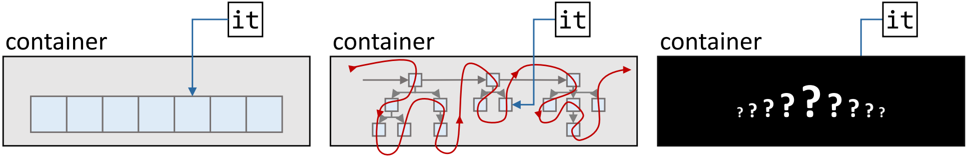
- Iteration over a *linked list*:
  - Point to start element: `p = this->head`
  - Access current element: `p->value`
  - Check if end reached: `p == nullptr`
  - Advance pointer: `p = p->next`

Iterators

- Iteration requires only the previously shown four operations
- But their implementation depends on the container
- $\Rightarrow$ Each C++container implements their own *Iterator*
- Given a container `c`:
  - `it = c.begin()`: Iterator pointing to the first element
  - `it = c.end()`: Iterator pointing *behind* the last element
  - `*it`: Access current element
  - `++it`: Advance iterator by one element
- Iterators are essentially pimped pointers

Iterators

- Iterators allow accessing different containers in a *uniform* way: `*it`, `++it`, etc.
- Users remain independent of the container implementation
- Iterator knows how to iterate over the elements of “its” container
- Users don’t need to and also shouldn’t know internal details
- ⇒



Example: Iterate over `std::vector`

`it` is an iterator specific to `std::vector<int>`

`it` initially points to the first element

```
std::vector<int> v = {1, 2, 3};

for (std::vector<int>::iterator it = v.begin();
     it != v.end();
     ++it) {

  *it = -*it;
}

std::cout << v; // -1 -2 -3
```

Abort if `it` reached the end

Advance `it` element-wise

Negate current element ($e \to -e$)

Example: Iterate over `std::vector`

Recall: type aliases can be used to shorten often-used type names

```
using ivit = std::vector<int>::iterator; // int-vector iterator

for (ivit it = v.begin();
     ...
```

Negate as a Function

As before: passing a *range (interval)* to work on

```
void neg(std::vector<int>::iterator begin;
         std::vector<int>::iterator end) {

  for (std::vector<int>::iterator it = begin;
       it != end;
       ++it) {

    *it = -*it;
  }
}
```

Negate elements in interval [**begin**, **end**)

Negate as a Function

As before: passing a *range (interval)* to work on

```
void neg(std::vector<int>::iterator begin;
         std::vector<int>::iterator end);
```

```
// in main():
std::vector<int> v = {1, 2, 3};
neg(v.begin(), v.begin() + (v.size() / 2));
```

Negate first half

Algorithms Library in C++

- The C++standard library includes lots of useful algorithms (functions) that work on iterator-defined intervals [*begin*, *end*)
- For example **`find`**, **`fill`** and **`sort`**; see also `https://en.cppreference.com/w/cpp/algorithm`
- Thanks to iterators, these $\geq$ 100 (!) algorithms can be applied to any* container: the 17 (!) C++standard container, our **`avec`** and **`llvec`** (discussed next), etc.
- Without this uniform access to container elements, we would have to duplicate *lots* of code

Not every algorithm can be applied to every container. It is, e.g. Not possible to sort a **`std::set`**.

An iterator for `llvec`

We need:

1. An `llvec`-specific iterator with at least the following functionality:
   - Access current element: `operator*`
   - Advance iterator: `operator++`
   - End-reached check: `operator!=` (or `operator==`)
2. Member functions `begin()` and `end()` for `llvec` to get an iterator to the beginning and past the end, respectively

Iterator `llvec::iterator` (Step 1/2)

```
class llvec {
  ...
public:
  class iterator {
    ...
  };


  ...
}
```

- The iterator belongs to our vector, that's why **`iterator`** is a public *inner class* of **`llvec`**
- Instances of our iterator are of type **`llvec::iterator`**

Iterator `llvec::iterator` (Step 1/2)

```
class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
  bool operator!=(const iterator& other) const;
};
```

Pointer to current vector element

Create iterator to specific element

Advance iterator by one element

Access current element

Compare with other iterator

Iterator `llvec::iterator` (Step 1/2)

```
// Constructor
llvec::iterator::iterator(llnode* n): node(n) {}
```

Let iterator point to **n** initially

```
// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
  assert(this->node != nullptr);

  this->node = this->node->next;

  return *this;
}
```

Advance iterator by one element

Return reference to advanced iterator

Iterator `llvec::iterator` (Step 1/2)

```
// Element access
int& llvec::iterator::operator*() const {
  return this->node->value;
}
```

Access current element

```
// Comparison: when are two iterators not equal?
bool llvec::iterator::operator!=(
    const llvec::iterator& other) const
{
  return this->node != other.node;
}
```

`this` iterator different from **`other`** if they point to different element

An iterator for `llvec` (Repetition)

We need:

1. An `llvec`-specific iterator with at least the following functionality:
   - Access current element: `operator*`
   - Advance iterator: `operator++`
   - End-reached check: `operator!=` (or `operator==`)

2. Member functions `begin()` and `end()` for `llvec` to get an iterator to the beginning and past the end, respectively

Iterator `llvec::iterator` (Step 2/2)

```
class llvec {
  ...
public:
  class iterator {...};

  iterator begin();
  iterator end();


  ...
}
```

`llvec` needs member functions to issue iterators pointing *to the beginning* and *past the end*, respectively, of the vector

Iterator `llvec::iterator` (Step 2/2)

```
llvec::iterator llvec::begin() {
  return llvec::iterator(this->head);
}

llvec::iterator llvec::end() {
  return llvec::iterator(nullptr);
}
```

Iterator to first vector element

Iterator past last vector element

Const-Iterators

- In addition to **iterator**, every container should also provide a *const-iterator* **const_iterator**
- Const-iterators grant only read access to the underlying Container
- For example for **llvec**:

```
llvec::const_iterator llvec::cbegin() const;
llvec::const_iterator llvec::cend() const;

const int& llvec::const_iterator::operator*() const;
...
```

- Therefore not possible (compiler error):

```
*(v.cbegin()) = 0
```

Const-Iterators

Const-Iterator *can* be used to allow only reading:

```
llvec v = ...;
for (llvec::const_iterator it = v.cbegin(); ...)
  std::cout << *it;
```

It would also possible to use the non-const **`iterator`** here

Const-Iterators

Const-Iterator *must* be used if the vector is const:

```
const llvec v = ...;
for (llvec::const_iterator it = v.cbegin(); ...)
  std::cout << *it;
```

It is not possible to use **`iterator`** here (compiler error)

Range-based `for` Loop

- Sequential iteration over an **`llvec`**, using an iterator (const-iterator possible, as are other containers):

```
llvec v(3); // v == {0, 0, 0}
for (llvec::iterator it = v.begin(); it != v.end(); ++it)
  std::cout << *it; // 000
```

- Can alternatively be written as follows:

```
for (int i : v) std::cout << i; // 000
```

  Is then translated to an iterator-based loop.
- Mutating access is possible as well:

```
for (int& i : v) i += 3;
for (int i : v) std::cout << i; // 369
```

Type-generic Container

Type-specific containers

Type-generic container

`https://upload.wikimedia.org/wikipedia/commons/d/df/Container_01_KMJ.jpg` (CC BY-SA 3.0)

P.S.: Templates are not relevant for the exam

Type-generic Container

Class **cell**: a simple, single-element container for **int**

```
class cell {
  int element;

public:
  cell(int e);
  int& value();
};
```

Constructor stores **e** in container

container's element

```
cell::cell(int e)
  : element(e) {}

int& cell::value() {
  return this->element;
}
```

Access the element

Better: generic **cell<E>** for every element type **E** (analogous to **std::vector<E>**)

Type-generic Container with Templates

Templates enable *type-generic* functions and classes:

```
template<typename E>
class cell {
  E element;

public:
  cell(E e);
  E& value();
};
```

Let **E** an arbitrary type …

…then **cell** manages an element of type **E**

- Types can be used as *parameters*
- Type parameters are valid in the “templated” scope

Type-generic Container with Templates

- Signatures and implementations must be “templated”
- For separately provided implementations, the class prefix must be written in generic form

```cpp
template<typename E>
class cell {
  E element;

public:
  cell(E e);
  E& value();
};
```

```cpp
template<typename E>
cell<E>::cell(E e)
    : element(e) {}

template<typename E>
E& cell<E>::value() {
  return this->element;
}
```

Type-generic Container with Templates

```
cell<int> c1(313);
cell<std::string> c2("terrific!")
```

- For *declarations*, e.g. `cell<int>`, type parameters must be provided explicitly …
- … but they are *inferred* by the compiler everywhere else, e.g. for `c1(313)`, i.e. when invoking the generic constructor `cell(E e)` (where type parameter `E` is instantiated by the compiler with `int`)

More Templates: Generic Output Operator

- Goal: A *generic* output operator **`<<`** for *iterable Containers*: **`llvec`**, **`avec`**, **`std::vector`**, **`std::set`**, ...
- I.e. **`std::cout << c << '\n'`** should work for any such container **`c`**

More Templates: Generic Output Operator

- Generic output operator with two type parameters

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container);
```

Intuition: operator works for every output stream **`sink`** of type **`S`** and every container **`container`** of type **`C`**

More Templates: Generic Output Operator

- Generic output operator with two type parameters

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container);
```

- The compiler infers suitable types from the call arguments

```
std::set<int> s = ...;
std::cout << s << '\n';
```

S = std::ostream, C = std::set<int>

More Templates: Generic Output Operator

Implementation of `<<` *constrains* `S` and `C` (Compiler errors if not satisfied):

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container) {
  for (typename C::const_iterator it = container.begin();
       it != container.end();
       ++it) {

    sink << *it << ' ';
  }

  return sink;
}
```

`C` must appropriate iterators – with appropriate functions

More Templates: Generic Output Operator

Implementation of `<<` *constrains* `S` and `C` (Compiler errors if not satisfied):

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container) {
  for (typename C::const_iterator it = container.begin();
       it != container.end();
       ++it) {

    sink << *it << ' ';
  }

  return sink;
}
```

`S` must support outputting elements (`*it`) and characters (`' '`)

Templates: Conclusion

- Templates realise *static code generation*/*static metaprogramming* in C++
- Template code is *copied* per type instantiation. When using `cell<int>` and `cell<std::string>`, the compiler creates two *instantiated copies* of `cell`'s code: conceptually, the two (no longer generic) classes `cell_int` and `cell_stdstring`.
- Templates reduce code duplication and facilitate code reuse
- Compiler errors that refer to templates are unfortunately often even more complex than C++ errors usually already are

23. Dynamic Datatypes and Memory Management

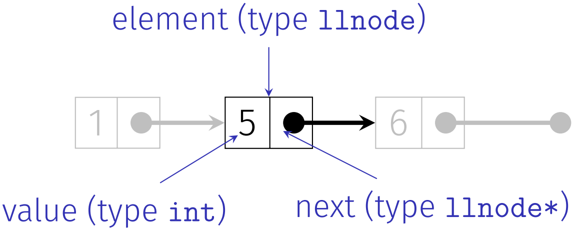
Problem

Last week: dynamic data type
Have allocated dynamic memory, but not released it again. In particular: no functions to remove elements from **`llvec`**.
Today: correct memory management!

Goal: class stack with memory management

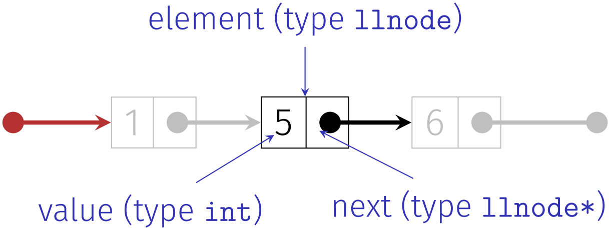
```
class stack{
public:
  // post: Push an element onto the stack
  void push(int value);
  // pre: non-empty stack
  // post: Delete top most element from the stack
  void pop();
  // pre: non-empty stack
  // post: return value of top most element
  int top() const;
  // post: return if stack is empty
  bool empty() const;
  // post: print out the stack
  void print(std::ostream& out) const;
...
```

Recall the Linked List




```
struct llnode {
  int value;
  llnode* next;
  // constructor
  llnode (int v, llnode* n) : value (v), next (n) {}
};
```

Stack = Pointer to the Top Element




```
class stack {
public:
  void push (int value);
  ...
private:
  llnode* topn;
};
```

Recall the `new` Expression

underlying type

```
new T(...)
```

new-Operator   constructor arguments

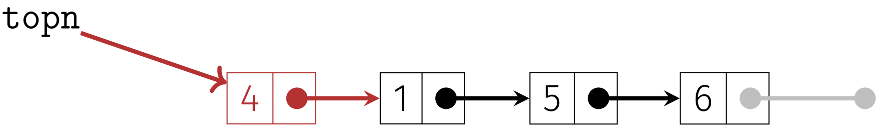
- **Effect**: memory for a new object of type T is allocated …
- …and initialized by means of the matching constructor
- **Value**: address of the new T object, **Type**: Pointer `T*`!

The `new` Expression

`push(4)`

- **Effect:** new object of type *T* is allocated in memory ...
- ...and intialized by means of the matching constructor
- **Value:** address of the new object

```
void stack::push(int value) {
  topn = new llnode(value, topn);
}
```



The `delete` Expression

Objects generated with **`new`** have **dynamic storage duration:** they “live” until they are explicitly *deleted*

`delete` *expr*

type **`void`**

delete-Operator

pointer of type $T*$, pointing to an object that had been created with **`new`**.

- **Effect:** object is **deconstructed** (explanation below)
  ... and **memory is released**.

`delete` for Arrays

`delete[]` *expr*

type `void`

delete-Operator

pointer of type *T**, that points to an array that previously had been allocated using **`new`**

- **Effect:** array is deleted and memory is released

Who is born must die. . .

Guideline "Dynamic Memory"

For each `new` there is a matching `delete`!

Non-compliance leads to memory leaks

- old objects that occupy memory. . .
- . . . until it is full (heap overflow)

Careful with `new` and `delete`!

```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

... and used for releaseing the object

error: memory released!

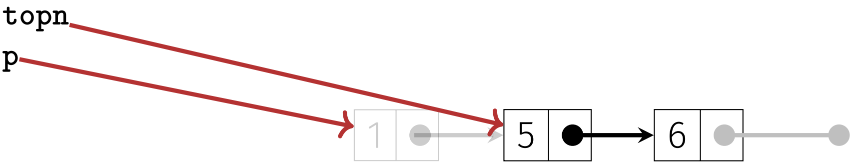
Dereferencing of „dangling pointers”

- Pointer to released objects: **dangling pointers**
- Releasing an object more than once using **`delete`** is a similar severe error

Stack Continued: pop()

```
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```

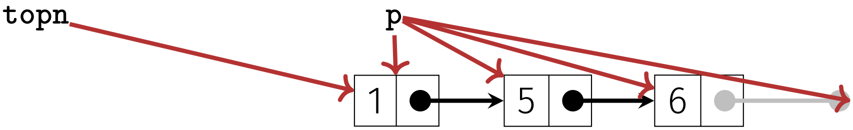
reminder: shortcut for `(*topn).next`



Print the Stack

`print()`

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1 5 6
}
```



Output Stack: operator«

```
class stack {
public:
   void push (int value);
   void pop();
   void print (std::ostream& o) const;
   ...
private:
   llnode* topn;
};

// POST: s is written to o
std::ostream& operator<< (std::ostream& o, const stack& s){
   s.print (o);
   return o;
}
```

empty(), top()

```
bool stack::empty() const {
  return top == nullptr;
}

int stack::top() const {
  assert(!empty());
  return topn->value;
}
```

Empty Stack

```
class stack{
public:
  stack() : topn (nullptr) {} // default constructor

  void push(int value);
  void pop();
  void print(std::ostream& out) const;
  int top() const;
  bool empty() const;
private:
  llnode* topn;
}
```

Zombie Elements

```
{
  stack s1; // local variable
  s1.push (1);
  s1.push (3);
  s1.push (2);
  std::cout << s1 << "\n"; // 2 3 1
}
// s1 has died (become invalid)...
```

- ...but the three elements of the stack `s1` continue to live (memory leak)!
- They should be released together with `s1`.

The Destructor

- The Destructor of class *T* is the unique member function with declaration

$$\sim T\,(\,);$$

- is automatically called when the memory duration of a class object ends – i.e. when **`delete`** is called on an object of type **`T*`** or when the enclosing scope of an object of type **`T`** ends.
- If no destructor is declared, it is automatically generated and calls the destructors for the member variables (pointers `topn`, no effect – reason for zombie elements

Using a Destructor, it Works

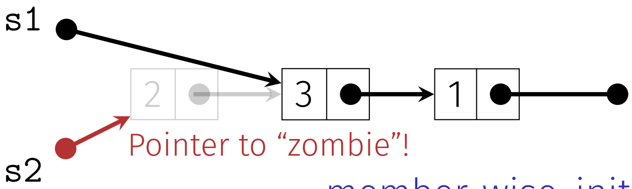
```
// POST: the dynamic memory of *this is deleted
stack::~stack(){
  while (topn != nullptr){
    llnode* t = topn;
    topn = t->next;
    delete t;
  }
}
```

- automatically deletes all stack elements when the stack is being released
- Now our stack class seems to follow the guideline “dynamic memory” (?)

Stack Done? Obviously not...

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop (); // Oops, crash!
```

What has gone wrong?



member-wise initialization: copies the topn pointer only.

```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();  // Oops, crash!
```

The actual problem

Already this goes wrong:

```
{
  stack s1;
  s1.push(1);
  stack s2 = s1;
}
```

When leaving the scope, both stacks are deconstructed. But both stacks try to delete the same data, because both stacks have **access to the same pointer**.

Possible solutions

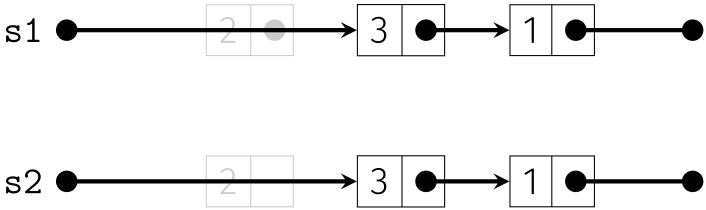
Smart-Pointers (we will not go into details here):

- Count the number of pointers referring to the same objects and delete only when that number goes down to 0 **`std::shared_pointer`**
- Make sure that not more than one pointer can point to an object: **`std::unique_pointer`**.

or:

- We make a real copy of all data – as discussed below.

We make a real copy




```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();  // ok
```

The Copy Constructor

- The copy constructor of a class *T* is the unique constructor with declaration

  $$T\ (\ \texttt{const}\ T\&\ x\ );$$

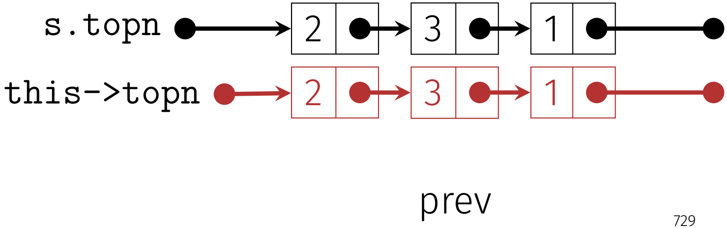
- is automatically called when values of type *T* are initialized with values of type **T**

  ```
  T x = t;
  T x (t);
  ```
  (t of type T)

- If there is no copy-constructor declared then it is generated automatically (and initializes member-wise – reason for the problem above

It works with a Copy Constructor

```
// POST: *this is initialized with a copy of s
stack::stack (const stack& s) : topn (nullptr) {
  if (s.topn == nullptr) return;
  topn = new llnode(s.topn->value, nullptr);
  llnode* prev = topn;
  for(llnode* n = s.topn->next; n != nullptr; n = n->next){
    llnode* copy = new llnode(n->value, nullptr);
    prev->next = copy;
    prev = copy;
  }
}
```



Aside: copy recursively

```
llnode* copy (node* that){
  if (that == nullptr) return nullptr;
  return new llnode(that->value, copy(that->next));
}
```

Elegant, isn't it? Why did we not do it like this?
Reason: linked lists can become very long. `copy` could then lead to stack overflow[6]. Stack memory is usually smaller than heap memory.

[6]not an overflow of the stack that we are implementing but the call stack

Initialization $\neq$ Assignment!

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2;
s2 = s1; // Zuweisung

s1.pop ();
std::cout << s1 << "\n"; // 3 1
s2.pop (); // Oops, Crash!
```

The Assignment Operator

- Overloading **`operator=`** as a member function
- Like the copy-constructor without initializer, but additionally
  - Releasing memory for the “old” value
  - Check for self-assignment (`s1=s1`) that should not have an effect
- If there is no assignment operator declared it is automatically generated (and assigns member-wise – reason for the problem above

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//             copy of s (right operand)
stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
    stack copy = s; // Copy Construction
    std::swap(topn, copy.topn); // now copy has the garbage!
  } // copy is cleaned up -> deconstruction
  return *this; // return as L-Value (convention)
}
```

Cooool trick! ☺

Done

```
class stack{
public:
  stack(); // constructor
  ~stack(); // destructor
  stack(const stack& s); // copy constructor
  stack& operator=(const stack& s); // assignment operator

  void push(int value);
  void pop();
  int top() const;
  bool empty() const;
  void print(std::ostream& out) const;
private:
  llnode* topn;
}
```

Dynamic Datatype

- Type that manages dynamic memory (e.g. our class for a stack)
- Minimal Functionality:
  - Constructors
  - Destructor
  - Copy Constructor
  - Assignment Operator

Rule of Three: if a class defines at least one of them, it must define all three

Trees

Trees are

- Generalized lists: nodes can have more than one successor
- Special graphs: graphs consist of nodes and edges. A tree is a fully connected, directed, acyclic graph.

Trees

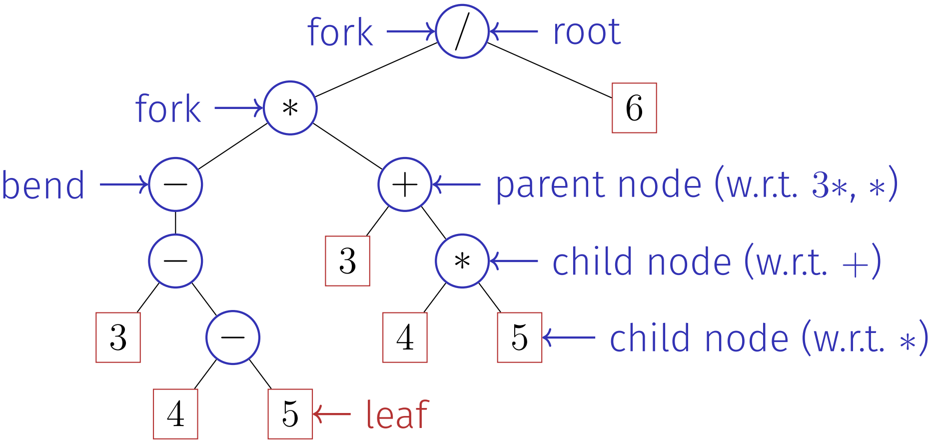
Use

- Decision trees: hierarchic representation of decision rules
- Code tress: representation of a code, e.g. morse alphabet, huffman code
- Search trees: allow efficient searching for an element by value
- syntax trees: parsing and traversing of expressions, e.g. in a compiler

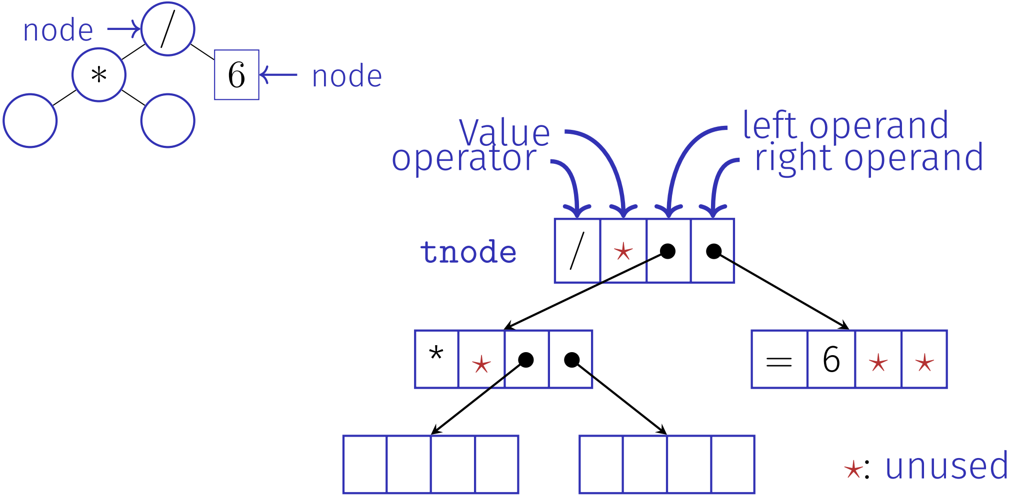
Trees are treated in more detail in other courses (Datastructures and Algorithms (CSE), Algorithms and Complexity (Math Bachelor))

(Expression) Trees

`-(3-(4-5))*(3+4*5)/6`

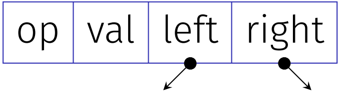


Nodes: Forks, Bends or Leaves



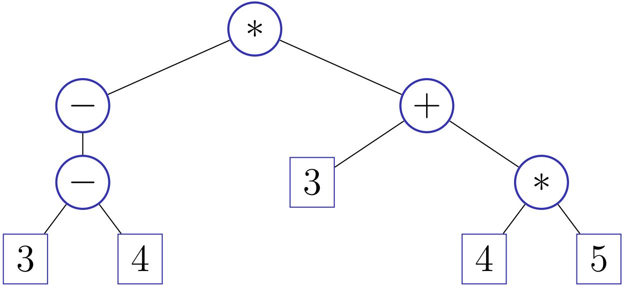
Nodes (`struct tnode`)

tnode




```
struct tnode {
  char op; // leaf node: op is '='
           // internal node: op is '+', '-', '*' or '/'
  double val;
  tnode* left; // == nullptr for unary minus
  tnode* right;

  tnode(char o, double v, tnode* l, tnode* r)
    : op(o), val(v), left(l), right(r) {}
};
```

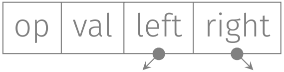
Size = Count Nodes in Subtrees



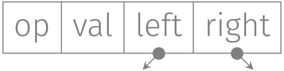
- Size of a leave: 1
- Size of other nodes: 1 + sum of child nodes' size
- E.g. size of the "+"-node is 5

Count Nodes in Subtrees

```
// POST: returns the size (number of nodes) of
//       the subtree with root n
int size (const tnode* n) {
  if (n){ // shortcut for n != nullptr
    return size(n->left) + size(n->right) + 1;
  }
  return 0;
}
```



Evaluate Subtrees

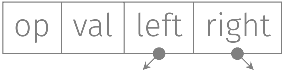



```
// POST: evaluates the subtree with root n
double eval(const tnode* n){
  assert(n);
  if (n->op == '=') return n->val;
  double l = 0;
  if (n->left) l = eval(n->left);
  double r = eval(n->right);
  switch(n->op){
    case '+': return l+r;
    case '-': return l-r;
    case '*': return l*r;
    case '/': return l/r;
    default: return 0;
  }
}
```

← leaf…
…or fork:
← op unary, or left branch
← right branch

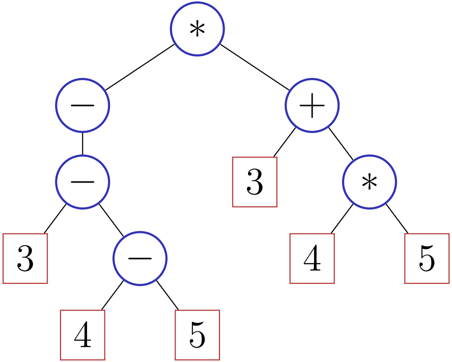
Cloning Subtrees

```
// POST: a copy of the subtree with root n is made
//       and a pointer to its root node is returned
tnode* copy (const tnode* n) {
  if (n == nullptr)
    return nullptr;
  return new tnode (n->op, n->val, copy(n->left), copy(n->right));
}
```



Felling Subtrees

```
// POST: all nodes in the subtree with root n are deleted
void clear(tnode* n) {
  if(n){
    clear(n->left);
    clear(n->right);
    delete n;
  }
}
```



Using Expression Subtrees

```
// Construct a tree for 1 - (-(3 + 7))
tnode* n1 = new tnode('=', 3, nullptr, nullptr);
tnode* n2 = new tnode('=', 7, nullptr, nullptr);
tnode* n3 = new tnode('+', 0, n1, n2);
tnode* n4 = new tnode('-', 0, nullptr, n3);
tnode* n5 = new tnode('=', 1, nullptr, nullptr);
tnode* root = new tnode('-', 0, n5, n4);

// Evaluate the overall tree
std::cout << "1 - (-(3 + 7)) = " << eval(root) << '\n';

// Evaluate a subtree
std::cout << "3 + 7 = " << eval(n3) << '\n';

clear(root); // free memory
```

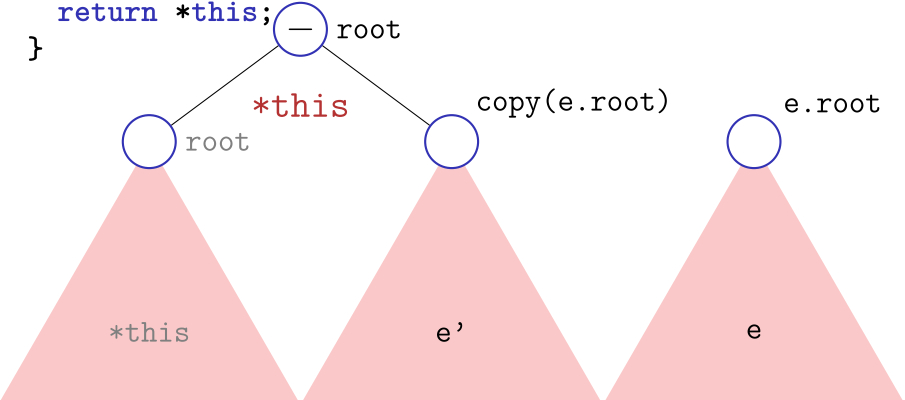
Planting Trees

creates a tree
with one leaf

```
class texpression {
public:
    texpression (double d)
        : root (new tnode ('=', d, 0, 0)) {}
    ...
private:
    tnode* root;
};
```

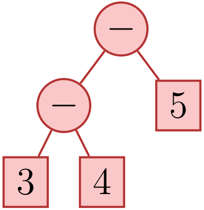
Letting Trees Grow

```
texpression& texpression::operator-= (const texpression& e)
{
  assert (e.root);
  root = new tnode ('-', 0, root, copy(e.root));
  return *this;
}
```



Raising Trees

```
texpression operator- (const texpression& l,
                       const texpression& r){
    texpression result = l;
    return result -= r;
}

texpression a = 3;
texpression b = 4;
texpression c = 5;
texpression d = a-b-c;
```



Rule of three: Clone, reproduce and cut trees

```
texpression::~texpression(){
  clear(root);
}

texpresssion::texpression (const texpression& e)
    : root(copy(e.root)) { }

texpression& texpression::operator=(const texpression& e){
  if (root != e.root){
    texpression cp = e;
    std::swap(cp.root, root);
  }
  return *this;
}
```

Concluded

```
class texpression{
public:
  texpression (double d); // constructor
  ~texpression(); // destructor
  texpression (const texpression& e); // copy constructor
  texpression& operator=(const texpression& e); // assignment op
  texpression operator-();
  texpression& operator-=(const texpression& e);
  texpression& operator+=(const texpression& e);
  texpression& operator*=(const texpression& e);
  texpression& operator/=(const texpression& e);
  double evaluate();
private:
 tnode* root;
};
```

From values to trees!

```
using number_type = texpression ;

// term = factor { "*" factor | "/" factor }
number_type term (std::istream& is){
  number_type value = factor (is);
  while (true) {
    if (consume (is, '*'))
      value *= factor (is);
    else if (consume (is, '/'))
      value /= factor (is);
  else
      return value;
  }
}
```

`double_calculator.cpp`
(expression value)
→
`texpression_calculator.cpp`
(expression tree)

Concluding Remark

- In this lecture, we have intentionally refrained from implementing member functions in the node classes of the list or tree.[7]
- When there is inheritace and polymorphism used, the implementation of the functionality such as evaluate, print, clear (etc:.) is better implemented in member functions.
- In any case it is not a good idea to implement the memory management of the composite data strcuture list or tree within the nodes.

[7] Parts of the implementations are even simpler (because the case `n==nullptr` can be caught more easily

24. Subtyping, Inheritance and Polymorphism

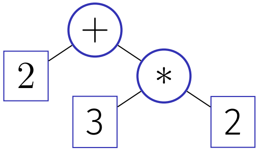
Expression Trees, Separation of Concerns and Modularisation, Type Hierarchies, Virtual Functions, Dynamic Binding, Code Reuse, Concepts of Object-Oriented Programming

Last Week: Expression Trees

- Goal: Represent arithmetic expressions, e.g.

```
2 + 3 * 2
```

- Arithmetic expressions form a *tree structure*

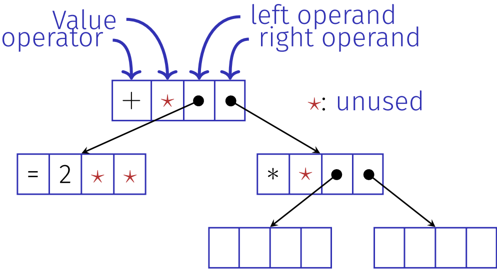


- Expression trees comprise *different* nodes: literals (e.g. 2), binary operators (e.g. $+$), unary operators (e.g. $\sqrt{}$), function applications (e.g. $\cos$), etc.

Disadvantages

Implemented via *a single* node type:

```
struct tnode {
  char op; // Operator ('=' for literals)
  double val; // Literal's value
  tnode* left; // Left child (or nullptr)
  tnode* right; // ...
  ...
};
```



Observation: **`tnode`** is the "sum" of all required nodes (constants, addition, ...) ⇒ memory wastage, inelegant

Disadvantages

Observation: `tnode` is the “sum” of all required nodes – and every function must “dissect” this “sum”, e.g.:

```
double eval(const tnode* n) {
  if (n->op == '=') return n->val; // n is a constant
  double l = 0;
  if (n->left) l = eval(n->left); // n is not a unary operator
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l+r; // n is an addition node
    case '*': return l*r; // ...
    ...
```

⇒ Complex, and therefore error-prone

Disadvantages

```
struct tnode {
  char op;
  double val;
  tnode* left;
  tnode* right;
  ...
};
```

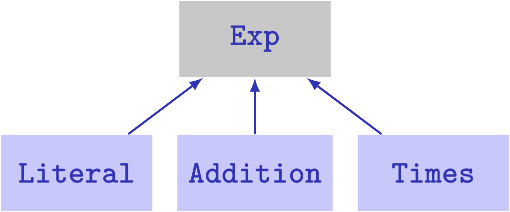
```
double eval(const tnode* n) {
  if (n->op == '=') return n->val;
  double l = 0;
  if (n->left) l = eval(n->left);
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l+r;
    case '*': return l*r;
    ...
```

This code isn't *modular* – we'll change that today!

New Concepts Today

1. Subtyping

- Type hierarchy: **`Exp`** represents general expressions, **`Literal`** etc. are concrete expression
- Every **`Literal`** etc. also is an **`Exp`** (subtype relation)



- That's why a **`Literal`** etc. can be used everywhere, where an **`Exp`** is expected:

```
Exp* e = new Literal(132);
```

New Concepts Today

2. Polymorphism and Dynamic Dispatch

- A variable of *static* type **`Exp`** can “host” expressions of different *dynamic* types:

```
Exp* e = new Literal(2); // e is the literal 2
e = new Addition(e, e); // e is the addition 2 + 2
```

- Executed are the member functions of the *dynamic* type:

```
Exp* e = new Literal(2);
std::cout << e->eval(); // 2

e = new Addition(e, e);
std::cout << e->eval(); // 4
```

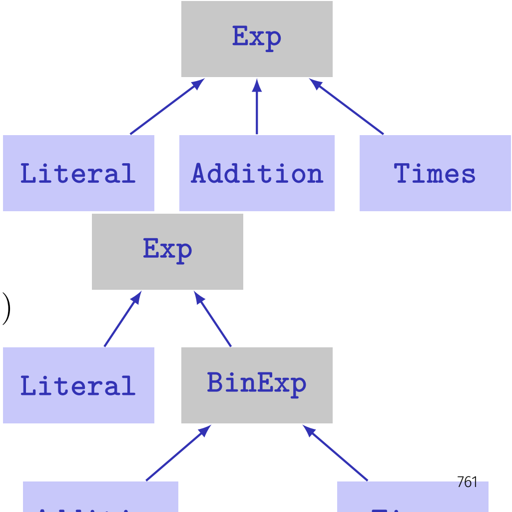
New Concepts Today

3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (**`Addition`**, **`Times`**):

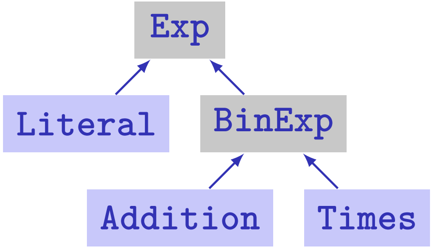
$$1 + size(left\ operand) + size(right\ operand)$$

$\Rightarrow$ Implement functionality once, and let subtypes *inherit* it



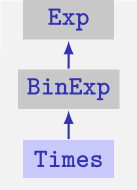
Advantages

- Subtyping, inheritance and dynamic binding enable *modularisation through spezialisation*
- Inheritance enables sharing common code across modules $\Rightarrow$ *avoid code duplication*




```
Exp* e = new Literal(2);
std::cout << e->eval();

e = new Addition(e, e);
std::cout << e->eval();
```

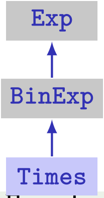
Syntax and Terminology




```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```

Note: Today, we focus on the new concepts (subtyping, ...) and ignore the orthogonal aspect of encapsulation (**`class`**, **`private`** vs. **`public`** member variables)

Syntax and Terminology




```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```

- **`BinExp`** is a *subclass*[1] of **`Exp`**
- **`Exp`** is the *superclass*[2] of **`BinExp`**
- **`BinExp`** *inherits* from **`Exp`**
- **`BinExp`** *publicly* inherits from **`Exp`** (**`public`**), that's why **`BinExp`** is a *subtype* of **`Exp`**
- Analogously: **`Times`** and **`BinExp`**
- Subtype relation is transitive: **`Times`** is also a subtype of **`Exp`**

[1] derived class, child class [2] base class, parent class

Abstract Class `Exp` and Concrete Class `Literal`

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

…that makes **`Exp`** an *abstract* class

Enforces implementation by derived classes …

Activates dynamic dispatch

```
struct Literal : public Exp {
  double val;

  Literal(double v);
  int size() const;
  double eval() const;
};
```

`Literal` inherits from **`Exp`** …

…but is otherwise just a regular class

`Literal`: Implementation

```
Literal::Literal(double v): val(v) {}
```

```
int Literal::size() const {
  return 1;
}
```

```
double Literal::eval() const {
  return this->val;
}
```

Subtyping: A Literal is an Expression

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);
Exp* e = lit; // OK: Literal is a subtype of Exp
```

But not vice versa:

```
Exp* e = ...
Literal* lit = e; // ERROR: Exp is not a subtype of Literal
```

Polymorphie: a Literal Behaves Like a Literal

```
struct Exp {
  ...
  virtual double eval();
};

double Literal::eval() {
  return this->val;
}
```

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

- *virtual* member function: the *dynamic* (here: **`Literal`**) type determines the member function to be executed
  $\Rightarrow$ *dynamic binding*
- Without **`Virtual`** the *static type* (hier: **`Exp`**) determines which function is executed
- We won't go into further details

Further Expressions: `Addition` and `Times`

```cpp
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

```cpp
int Addition::size() const {
  return 1 + left->size()
           + right->size();
}
```

```cpp
struct Times : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

```cpp
int Times::size() const {
  return 1 + left->size()
           + right->size();
}
```

😃 Separation of concerns

🤬 Code duplication

Extracting Commonalities . . . : BinExp

```
struct BinExp : public Exp {
  Exp* left;
  Exp* right;

  BinExp(Exp* l, Exp* r);
  int size() const;
};
```

```
BinExp::BinExp(Exp* l, Exp* r): left(l), right(r) {}
```

```
int BinExp::size() const {
  return 1 + this->left->size() + this->right->size();
}
```

Note: **BinExp** does not implement **eval** and is therefore also an abstract class, just like **Exp**

...Inheriting Commonalities: `Addition`

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

`Addition` inherits member variables (**`left`**, **`right`**) and functions (**`size`**) from **`BinExp`**

```
Addition::Addition(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Addition::eval() const {
  return
    this->left->eval() +
    this->right->eval();
}
```

Calling the *super constructor* (constructor of **`BinExp`**) initialises the member variables **`left`** and **`right`**

...Inheriting Commonalities: `Times`

```
struct Times : public BinExp {
  Times(Exp* l, Exp* r);
  double eval() const;
};
```

```
Times::Times(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Times::eval() const {
  return
    this->left->eval() *
    this->right->eval();
}
```

Observation: `Additon::eval()` and `Times::eval()` are very similar and could also be unified. However, this would require the concept of *functional programming*, which is outside the scope of this course.

Further Expressions and Operations

- Further expressions, as classes derived from **`Exp`**, are possible, e.g. $-$, $/$, $\sqrt{\ }$, $\cos$, $\log$
- A former bonus exercise (included in today's lecture examples on Code Expert) illustrates possibilities: variables, trigonometric functions, parsing, pretty-printing, numeric simplifications, symbolic derivations, ...

Mission: Monolithic → Modular ✓

```
struct tnode {
  char op;
  double val;
  tnode* left;
  tnode* right;
  ...
}
```

```
double eval(const tnode* n) {
  if (n->op == '=') return n->val;
  double l = 0;
  if (n->left != 0) l = eval(n->left);
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l + r;
    case '*': return l - r;
    case '-': return l - r;
    case '/': return l / r;
    default:
      // unknown operator
      assert (false);
  }
}
```

```
int size (const tnode* n) const { ... }

...
```

```
struct Literal : public Exp {
  double val;
  ...
  double eval() const {
    return val;
  }
};
```

```
struct Addition : public Exp {
  ...
  double eval() const {
    return left->eval() + right->eval();
  }
};
```

```
struct Times : public Exp {
  ...
  double eval() const {
    return left->eval() * right->eval();
  }
}
```

+

```
struct Cos : public Exp {
  ...
  double eval() const {
    return std::cos(argument->eval());
  }
}
```

And there is so much more …

Not shown/discussed:

- Private inheritance (`class B :` ~~`public`~~ `A`)
- Subtyping and polymorphism without pointers
- Non-virtuell member functions and static dispatch (~~`virtual`~~ `double eval()`)
- Overriding inherited member functions and invoking overridden implementations
- Multiple inheritance
- …

Object-Oriented Programming

In the last 3rd of the course, several concepts of *object-oriented programming* were introduced, that are briefly summarised on the upcoming slides.

Encapsulation (weeks 10-13):

- Hide the implementation details of types (private section) from users
- Definition of an interface (public area) for accessing values and functionality in a controlled way
- Enables ensuring invariants, and the modification of implementations without affecting user code

Object-Oriented Programming

Subtyping (week 14):

- Type hierarchies, with super- and subtypes, can be created to model relationships between more abstract and more specialised entities
- A subtype supports at least the functionality that its supertype supports – typically more, though, i.e. a subtype extends the interface (public section) of its supertype
- That's why supertypes can be used anywhere, where subtypes are required ...
- ...and functions that can operate on more abstract type (supertypes) can also operate on more specialised types (subtypes)
- The streams introduced in week 7 form such a type hierarchy: **`ostream`** is the abstract supertyp, **`ofstream`** etc. are specialised subtypes

Object-Oriented Programming

Polymorphism and *dynamic binding* (week 14):

- A pointer of static typ T_1 can, at runtime, point to objects of (dynamic) type T_2, if T_2 is a subtype of T_1
- When a virtual member function is invoked from such a pointer, the dynamic type determines which function is invoked
- I.e.: despite having the same static type, a different behaviour can be observed when accessing the common interface (member functions) of such pointers
- In combination with subtyping, this enables adding further concrete types (streams, expressions, ...) to an existing system, without having to modify the latter

Object-Oriented Programming

Inheritance (week 14):

- Derived classes inherit the functionality, i.e. the implementation of member functions, of their parent classes
- This enables sharing common code and thereby avoids code duplication
- An inherited implementation can be overridden, which allows derived classes to behave differently than their parent classes (not shown in this course)

25. Conclusion

Purpose and Format

Name the most important key words to each chapter.
Checklist: “does every notion make some sense for me?”

- M motivating example for each chapter
- C concepts that do not depend from the implementation (language)
- L language (C++): all that depends on the chosen language
- E examples from the lectures

Kapitelüberblick

- 1. Introduction
- 2. Integers
- 3. Booleans
- 4. Defensive Programming
- 5./6. Control Statements
- 7./8. Floating Point Numbers
- 9./10. Functions
- 11. Reference Types
- 12./13. Vectors and Strings
- 14./15. Recursion
- 16. Structs and Overloading
- 17. Classes
- 18./19. Dynamic Datastructures
- 20. Containers, Iterators and Algorithms
- 21. Dynamic Datatypes and Memory Management
- 22. Subtyping, Polymorphism and Inheritance

1. Introduction

M
- Euclidean algorithm

C
- algorithm, Turing machine, programming languages, compilation, syntax and semantics
- values and effects, fundamental types, literals, variables

L
- include directive #include <iostream>
- main function int main(){...}
- comments, layout // Kommentar
- types, variables, L-value a , R-value a+b
- expression statement b=b*b; , declaration statement int a;, return statement return 0;

2. Integers

M

- Celsius to Fahrenheit

C

- associativity and precedence, arity
- expression trees, evaluation order
- arithmetic operators
- binary representation, hexadecimal numbers
- signed numbers, twos complement

L

- arithmetic operators `9 * celsius / 5 + 32`
- increment / decrement `expr++`
- arithmetic assignment `expr1 += expr2`
- conversion `int` ↔ `unsigned int`

E

- Celsius to Fahrenheit, equivalent resistance

3. Booleans

C

- Boolean functions, completeness
- DeMorgan rules

L

- the type `bool`
- logical operators `a && !b`
- relational operators `x < y`
- precedences `7 + x < y && y != 3 * z`
- short circuit evaluation `x != 0 && z / x > y`
- the **assert**-statement, **#include <cassert>**

E

- Div-Mod identity.

4. Definsive Programming

C

- Assertions and Constants

L

- The **`assert`**-statement, **`#include <cassert>`**
- **`const int speed_of_light=2999792458`**

E

- Assertions for the GCD

5./6. Control Statements

M
- linear control flow vs. interesting programs

C
- selection statements, iteration statements
- (avoiding) endless loops, halting problem
- Visibility and scopes, automatic memory
- equivalence of iteration statement

L
- if statements `if (a % 2 == 0) {..}`
- for statements `for (unsigned int i = 1; i <= n; ++i) ...`
- while and do-statements `while (n > 1) {...}`
- blocks and branches `if (a < 0) continue;`
- Switch statement `switch(grade) {case 6: }`

E
- sum computation (Gauss), prime number tests, Collatz sequence, Fibonacci numbers, calculator, output grades

7./8. Floating Point Numbers

M

- correct computation: Celsius / Fahrenheit

C

- fixpoint vs. floating point
- holes in the value range
- compute using floating point numbers
- floating point number systems, normalisation, IEEE standard 754
- *guidelines for computing with floating point numbers*

L

- types `float`, `double`
- floating point literals `1.23e-7f`

E

- Celsius/Fahrenheit, Euler, Harmonic Numbers

9./10. Functions

M

- Computation of Powers

C

- Encapsulation of Functionality
- functions, formal arguments, arguments
- scope, forward declarations
- procedural programming, modularization, separate compilation
- *Stepwise Refinement*

L

- declaration and definition of functions

  `double pow(double b, int e){ ... }`
- function call `pow (2.0, -2)`
- the type `void`

E

- powers, perfect numbers, minimum, calendar

11. Reference Types

M

- Swap

C

- value- / reference- semantics, pass by value, pass by reference, return by reference
- lifetime of objects / temporary objects
- constants

L

- reference type `int& a`
- call by reference, return by reference `int& increment (int& i)`
- const guideline, const references, reference guideline

E

- swap, increment

12./13. Vectors and Strings

M

- Iterate over data: sieve of erathosthenes

C

- vectors, memory layout, random access
- (missing) bound checks
- vectors
- characters: ASCII, UTF8, texts, strings

L

- vector types `std::vector<int> a {4,3,5,2,1};`
- characters and texts, the type char `char c = 'a';`, Konversion nach **int**
- vectors of vectors
- Streams `std::istream, std::ostream`

E

- sieve of Erathosthenes, Caesar-code, shortest paths

14./15. Recursion

M

- recursive math. functions, the n-Queen problem, Lindenmayer systems, a command line calculator

C

- recursion
- call stack, memory of recursion
- correctness, termination,
- recursion vs. iteration
- Backtracking, EBNF, formal grammars, parsing

E

- factorial, GCD, sudoku-solver, command line calcoulator

16. Structs and Overloading

M
- build your own rational number

C
- heterogeneous data types
- function and operator overloading
- encapsulation of data

L
- struct definition `struct rational {int n; int d;};`
- member access `result.n = a.n * b.d + a.d * b.n;`
- initialization and assignment,
- function overloading `pow(2)` vs. `pow(3,3);`, operator overloading

E
- rational numbers, complex numbers

17. Classes

Ⓜ
- rational numbers with encapsulation

Ⓒ
- Encapsulation, Construction, Member Functions

Ⓛ
- classes `class rational { ... };`
- access control `public:` / `private:`
- member functions `int rational::denominator () const`
- The implicit argument of the member functions

Ⓔ
- finite rings, complex numbers

18./19. Dynamic Datastructures

M

- Our own vector

C

- linked list, allocation, deallocation, dynamic data type

L

- The **`new`** statement
- pointer `int* x;`, Null-pointer `nullptr`.
- address and derference operator `int *ip = &i; int j = *ip;`
- pointer and const `const int *a;`

E

- linked list, stack

20. Containers, Iterators and Algorithms

M
- vectors are containers

C
- iteration with pointers
- containers and iterators
- algorithms

L
- Iterators `std::vector<int>::iterator`
- Algorithms of the standard library `std::fill (a, a+5, 1);`
- implement an iterator
- iterators and const

E
- output a vector, a set

21. Dynamic Datatypes and Memory Management

M

- Stack
- Expression Tree

C

- Guideline ”dynamic memory“
- Pointer sharing
- Dynamic Datatype
- Tree-Structure

L

- **`new`** and **`delete`**
- Destructor **`stack::~stack()`**
- Copy-Constructor **`stack::stack(const stack& s)`**
- Assignment operator
  `stack& stack::operator=(const stack& s)`
- Rule of Three

E

- Binary Search Tree

22. Subtyping, Polymorphism and Inheritance

Ⓜ

- extend and generalize expression trees

Ⓒ

- Subtyping
- polymorphism and dynamic binding
- Inheritance

Ⓛ

- base class **`struct Exp{}`**
- derived class **`struct BinExp: public Exp{}`**
- abstract class **`struct Exp{virtual int size() const = 0...}`**
- polymorphie **`virtual double eval()`**

Ⓔ

- expression node and extensions

The End

End of the Course