# 3. Logical Values

Boolean Functions; the Type `bool`; logical and relational operators; shortcut evaluation

## Our Goal

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Behavior depends on the value of a Boolean expression

## Boolean Values in Mathematics

Boolean expressions can take on one of two values:

*0* or *1*

- *0* corresponds to *“false”*
- *1* corresponds to *“true”*

## The Type `bool` in C++

- represents *logical values*
- Literals `false` and `true`
- Domain $\{$*false*, *true*$\}$

```
bool b = true; // Variable with value true
```

## Relational Operators

`a < b` (smaller than)
`a >= b` (greater than)
`a == b` (equals)
`a != b` (not equal)

arithmetic type × arithmetic type → `bool`

R-value × R-value → R-value

## Table of Relational Operators

| | Symbol | Arity | Precedence | Associativity |
|---|---|---|---|---|
| **smaller** | `<` | 2 | 11 | left |
| **greater** | `>` | 2 | 11 | left |
| **smaller equal** | `<=` | 2 | 11 | left |
| **greater equal** | `>=` | 2 | 11 | left |
| **equal** | `==` | 2 | 10 | left |
| **unequal** | `!=` | 2 | 10 | left |

arithmetic type × arithmetic type → `bool`

R-value × R-value → R-value

## Boolean Functions in Mathematics

- Boolean function

$$f : \{0,1\}^2 \to \{0,1\}$$

- $0$ corresponds to “false”.
- $1$ corresponds to “true”.

## $\mathrm{AND}(x, y)$ $x \wedge y$

- “logical And”

$$f : \{0,1\}^2 \to \{0,1\}$$

- $0$ corresponds to “false”.
- $1$ corresponds to “true”.

| $x$ | $y$ | $\mathrm{AND}(x,y)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## Logical Operator `&&`

`a && b` (logical and)

`bool` × `bool` → `bool`

R-value × R-value → R-value

```
int n = −1;
int p = 3;
bool b = (n < 0) && (0 < p); // b = true
```

---

## $\mathrm{OR}(x, y)$ $x \vee y$

- “logical Or”

$$f : \{0,1\}^2 \to \{0,1\}$$

- $0$ corresponds to “false”.
- $1$ corresponds to “true”.

| $x$ | $y$ | $\mathrm{OR}(x,y)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

---

## Logical Operator `||`

`a || b` (logical or)

`bool` × `bool` → `bool`

R-value × R-value → R-value

```
int n = 1;
int p = 0;
bool b = (n < 0) || (0 < p); // b = false
```

---

## $\mathrm{NOT}(x)$ $\neg x$

- “logical Not”

$$f : \{0,1\} \to \{0,1\}$$

- $0$ corresponds to “false”.
- $1$corresponds to “true”.

| $x$ | $\mathrm{NOT}(x)$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

## Logical Operator `!`

`!b` (logical not)

`bool` → `bool`

R-value → R-value

```
int n = 1;
bool b = !(n < 0); // b = true
```

## Precedences

```
!b && a
   ⇕
(!b) && a

a && b || c && d
       ⇕
(a && b) || (c && d)

a || b   &&   c || d
       ⇕
a || (b && c) || d
```

## Table of Logical Operators

| | Symbol | Arity | Precedence | Associativity |
|---|---|---|---|---|
| **Logical and (AND)** | `&&` | 2 | 6 | left |
| **Logical or (OR)** | `\|\|` | 2 | 5 | left |
| **Logical not (NOT)** | `!` | 1 | 16 | right |

## Precedences

*The unary logical* operator `!`

binds more strongly than

*binary arithmetic* operators. These

bind more strongly than

*relational* operators,

and these bind more strongly than

*binary logical* operators.

```
7 + x < y && y != 3 * z || ! b
7 + x < y && y != 3 * z || (!b)
```

# Completeness

- $\mathrm{AND}$, $\mathrm{OR}$ and $\mathrm{NOT}$ are the boolean functions available in $\mathrm{C{+}{+}}$.
- Any other *binary* boolean function can be generated from them.

| $x$ | $y$ | $\mathrm{XOR}(x,y)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

---

# Completeness: $\mathrm{XOR}(x,y)$ $x \oplus y$

$$\mathrm{XOR}(x,y) = \mathrm{AND}(\mathrm{OR}(x,y), \mathrm{NOT}(\mathrm{AND}(x,y))).$$

$$x \oplus y = (x \vee y) \wedge \neg(x \wedge y).$$

```
(x || y) && !(x && y)
```

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# Completeness Proof

- Identify binary boolean functions with their characteristic vector.

| $x$ | $y$ | $\mathrm{XOR}(x,y)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

characteristic vector: 0110

$$\mathrm{XOR} = f_{0110}$$

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# Completeness Proof

- Step 1: generate the *fundamental* functions $f_{0001}$, $f_{0010}$, $f_{0100}$, $f_{1000}$

$$\begin{aligned}
f_{0001} &= \mathrm{AND}(x,y)\\
f_{0010} &= \mathrm{AND}(x,\mathrm{NOT}(y))\\
f_{0100} &= \mathrm{AND}(y,\mathrm{NOT}(x))\\
f_{1000} &= \mathrm{NOT}(\mathrm{OR}(x,y))
\end{aligned}$$

## Completeness Proof

- Step 2: generate all functions by applying logical or

$$f_{1101} = \mathrm{OR}(f_{1000}, \mathrm{OR}(f_{0100}, f_{0001}))$$

- Step 3: generate $f_{0000}$

$$f_{0000} = 0.$$

---

## `bool` vs `int`: Conversion

- `bool` can be used whenever `int` is expected – and vice versa.
- Many existing programs use `int` instead of `bool`
  *This is bad style originating from the language $C$ .*

| `bool` | → | `int` |
|---|---|---|
| *true* | → | 1 |
| *false* | → | 0 |

| `int` | → | `bool` |
|---|---|---|
| ≠0 | → | *true* |
| 0 | → | *false* |

```
bool b = 3; // b=true
```

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## DeMorgan Rules

- `!(a && b) == (!a || !b)`
- `!(a || b) == (!a && !b)`

! (rich *and* beautiful) == (poor *or* ugly)

---

## Application: either ... or (XOR)

| | |
|---|---|
| `(x \|\| y)    && !(x && y)` | x or y, and not both |
| `(x \|\| y)    && (!x \|\| !y)` | x or y, and one of them not |
| `!(!x && !y)  && !(x && y)` | not none and not both |
| `!(!x && !y \|\| x && y)` | not: both or none |

## Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

```
x != 0 && z / x > y
```

⇒ No division by 0

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# 4. Defensive Programming

Constants and Assertions

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## Sources of Errors

- Errors that the compiler can find:
  syntactical and some semantical errors
- Errors that the compiler cannot find:
  runtime errors (always semantical)

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## The Compiler as Your Friend: Constants

Constants

- are variables with immutable value

```
const int speed_of_light = 299792458;
```

- Usage: `const` before the definition

# The Compiler as Your Friend: Constants

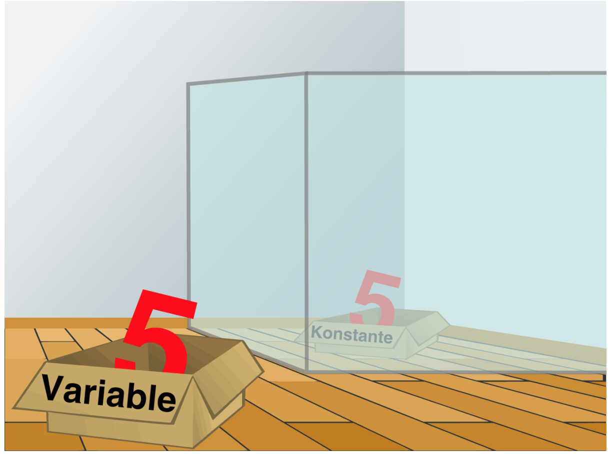
- Compiler checks that the `const`-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

**compiler: error**

- Tool to avoid errors: constants guarantee the promise :*“value does not change”*

# Constants: Variables behind Glass



# The `const`-guideline

> **`const`-guideline**
> For *each variable*, think about whether it will change its value in the lifetime of a program. If not, use the keyword `const` in order to make the variable a constant.

A program that adheres to this guideline is called `const`-correct.

# Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior

   ≫ It’s not a bug, it’s a feature! ≪

2. Check at many places in the code if the program is still on track
3. Question the (seemingly) obvious, there could be a typo in the code

# Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression `expr` is false
- requires `#include <cassert>`
- can be switched off (potential performance gain)

# Assertions for the $gcd(x, y)$

Check if the program is on track . . .

```
// Input x and y
std::cout << "x =? ";
std::cin >> x;
std::cout << "y =? ";
std::cin >> y;
```

Input arguments for calculation

```
// Check validity of inputs
assert(x > 0 && y > 0);
```

← Precondition for the ongoing computation

```
... // Compute gcd(x,y), store result in variable a
```

# Assertions for the $gcd(x, y)$

... and question the obvious! . . .

```
...
assert(x > 0 && y > 0);
```

← Precondition for the ongoing computation

```
... // Compute gcd(x,y), store result in variable a

assert (a >= 1);
assert (x % a == 0 && y % a == 0);
for (int i = a+1; i <= x && i <= y; ++i)
  assert(!(x % i == 0 && y % i == 0));
```

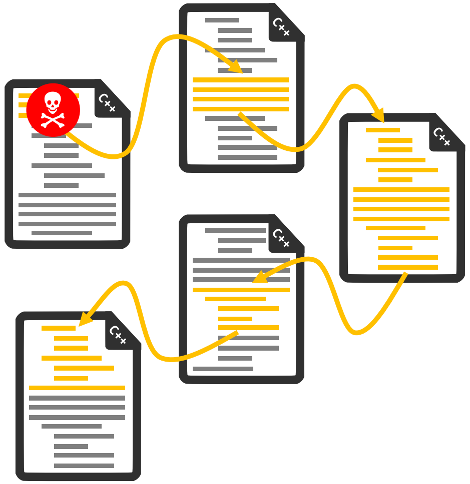
Properties of the gcd

# Switch off Assertions

```
#define NDEBUG // To ignore assertions
#include<cassert>

...
assert(x > 0 && y > 0); // Ignored

... // Compute gcd(x,y), store result in variable a

assert(a >= 1); // Ignored
...
```

## Fail-Fast with Assertions

- Real software: many C++ files, complex control flow
- Errors surface late(r) → impedes error localisation
- Assertions: Detect errors early



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# 5. Control Structures I

Selection Statements, Iteration Statements, Termination, Blocks

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## Control Flow

- Up to now: *linear* (from top to bottom)
- Interesting programs require “branches” and “jumps”

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| [8] → **L** | [9] → **R** | **L** $=$ 0? *stop* | **R** $>$ **L**? *springe zu* **6** | **L** $-$ **R** → [8] | *springe zu* 0 | **R** $-$ **L** → [9] | *springe zu* 0 | $b$ | $a$ |

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## Selection Statements

implement branches

- `if` statement
- `if-else` statement

## `if`-Statement

`if` ( *condition* )
    *statement*

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

If *condition* is true then *statement* is executed

- *statement*: arbitrary statement (*body* of the `if`-Statement)
- *condition*: convertible to `bool`

---

## `if-else`-statement

`if` ( *condition* )
    *statement1*
else
    *statement2*

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

If *condition* is true then *statement1* is executed, otherwise *statement2* is executed.

- *condition*: convertible to `bool`.
- *statement1*: *body* of the `if`-branch
- *statement2*: *body* of the `else`-branch

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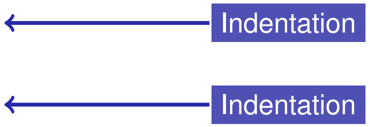
## Layout!

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```



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## Iteration Statements

implement “loops”

- `for`-statement
- `while`-statement
- `do`-statement

## Compute $1 + 2 + ... + n$

```
// Program: sum_n.cpp
// Compute the sum of the first n natural numbers.

#include <iostream>

int main()
{
  // input
  std::cout << "Compute the sum 1+...+n for n =? ";
  unsigned int n;
  std::cin >> n;

  // computation of sum_{i=1}^n i
  unsigned int s = 0;
  for (unsigned int i = 1; i <= n; ++i) s += i;

  // output
  std::cout << "1+...+" << n << " = " << s << ".\n";
  return 0;
}
```

---

## `for`-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

| `i` | | `s` |
|---|---|---|
| `i==1` | wahr | `s == 1` |
| `i==2` | wahr | `s == 3` |
| `i==3` | falsch | |
| | | `s == 3` |

---

## `for`-Statement: Syntax

`for` (*init statement*; *condition*; *expression*)
  *body statement*

- *init statement*: expression statement, declaration statement, null statement
- *condition*: convertible to `bool`
- *expression*: any expression
- *body statement*: any statement (*body* of the for-statement)

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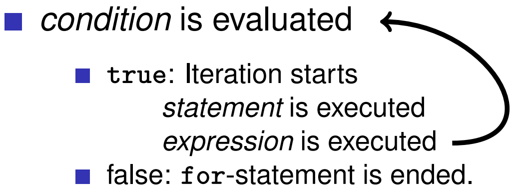
## `for`-Statement: semantics

`for` ( *init statement*  *condition* ; *expression* )
  *statement*

- *init-statement* is executed



- *condition* is evaluated
  - `true`: Iteration starts
    *statement* is executed
    *expression* is executed
  - false: `for`-statement is ended.

# Gauß as a Child (1777 - 1855)

- Math-teacher wanted to keep the pupils busy with the following task:

> *Compute the sum of numbers from 1 to 100!*

- Gauß finished after one minute.

# The Solution of Gauß

- The requested number is

$$1 + 2 + 3 + \cdots + 98 + 99 + 100.$$

- This is half of

$$\begin{array}{rrcrcrcrcr} & 1 & + & 2 & + & \cdots & + & 99 & + & 100 \\ + & 100 & + & 99 & + & \cdots & + & 2 & + & 1 \\ \hline = & 101 & + & 101 & + & \cdots & + & 101 & + & 101 \end{array}$$

- Answer: $100 \cdot 101/2 = 5050$

# `for`-Statement: Termination

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

Here and in most cases:

- *expression* changes its value that appears in *condition* .
- After a finite number of iterations *condition* becomes false: *Termination*

# Infinite Loops

- Infinite loops are easy to generate:

```
for ( ; ; ) ;
```

  - Die *empty condition* is true.
  - Die *empty expression* has no effect.
  - Die *null statement* has no effect.

- ... but can in general not be automatically detected.

```
for (init; cond; expr) stmt;
```

# Halting Problem

**Undecidability of the Halting Problem**

There is no $\mathrm{C}{+}{+}$ program that can determine for each $\mathrm{C}{+}{+}$-Program $P$ and each input $I$ if the program $P$ terminates with the input $I$.

This means that the correctness of programs can in general *not* be automatically checked.[4]

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[4] Alan Turing, 1936. Theoretical questions of this kind were the main motivation for Alan Turing to construct a computing machine.

# Example: Prime Number Test

**Def.:** a natural number $n \geq 2$ is a prime number, if no $d \in \{2, \ldots, n-1\}$ divides $n$ .

A loop that can test this:

```
unsigned int d;
for (d=2; n%d != 0; ++d);
```

# Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value `d=2`, then plus 1 in every iteration (`++d`)
- Exit: `n%d != 0` evaluates to `false` as soon as a divisor is found — at the latest, once `d == n`
- Progress guarantees that the exit condition will be reached

# Example: Correctness

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

Every potential divisor 2 `<= d <= n` will be tested. If the loop terminates with `d == n` then and only then is `n` prime.

# Blocks

- Blocks group a number of statements to a new statement

  `{statement1 statement2 ... statementN}`
- Example: body of the main function

```
int main() {
     ...
}
```

- Example: loop body

```
for (unsigned int i = 1; i <= n; ++i) {
    s += i;
    std::cout << "partial sum is " << s << "\n";
}
```