Computer Science

Course at D-MATH/D-PHYS of ETH Zurich

Malte Schwerhoff, Felix Friedrich

AS 2018

Welcome

to the Course Informatik

at the MATH/PHYS departement of ETH Zürich.

Place and time:

Tuesday 13:15 - 15:00, ML D28, ML E12.
Pause 14:00 - 14:15, slight shift possible.

Course web page

`http://lec.inf.ethz.ch/ifmp`

Team

chef assistant	Vytautas Astrauskas	
back office	Inna Grijnevitch	
	Martin Clochard	
	Pavol Bielik	
assistants	Eliza Wszola	Moritz Schneider
	Alexander Hedges	Patrik Hadorn
	Viera Klasovita	Philippe Schlattner
	Max Egli	Yannik Ammann
	Christopher Lehner	Adrian Langenbach
	Orhan Saeedi	David Baur
	Maximillian Holst	Corminboeuf Etienne
	Benjamin Rothenberger	Tobias Klenze
	David Sommer	Sefidgar Seyed Reza
lecturers	Dr. Malte Schwerhoff / Dr. Felix Friedrich	

Registration for Exercise Sessions

- Registration via web page
- Registration already open

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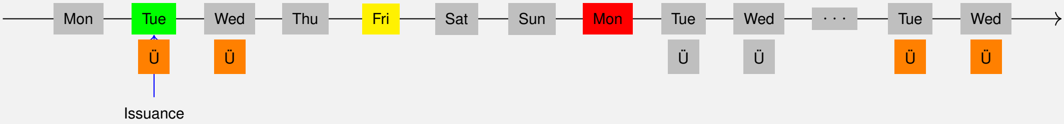
- Registration already open
- 19 groups in total: 9 Tuesday 3-5pm, 10 Wednesday 10-12am
- 16 groups in German, 3 groups in English

Procedure



Mon Tue Wed Thu Fri Sat Sun Mon Tue Wed · · · Tue Wed

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Issuance

- Exercises availabe at lectures
- Preliminary discussion in the following exercise session (on the same/next day)
- StudyCenter (`studycenter.ethz.ch`)
- Solution must be submitted at latest one day before the next lecture (23:59h)
- Discussion of the exercise in the session one week after the submission. Feedback will be provided in the week after the submission.

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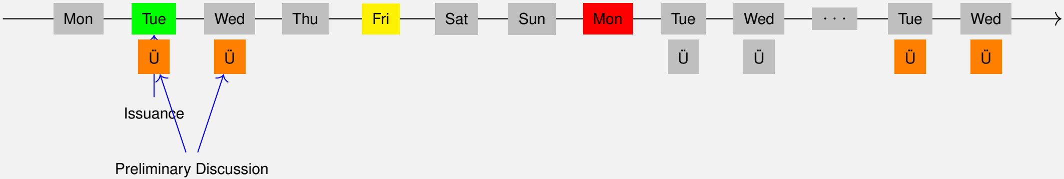

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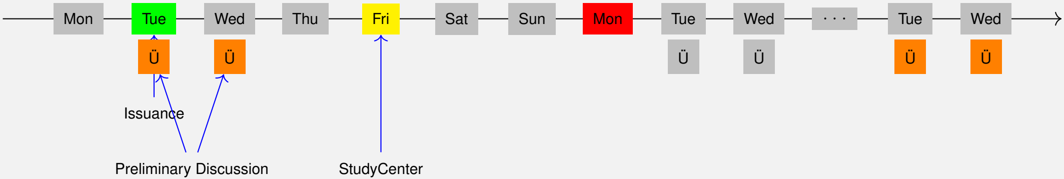

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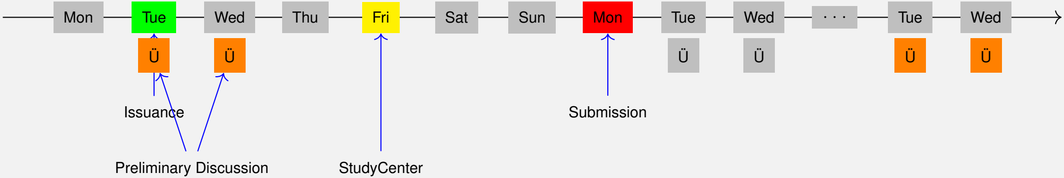

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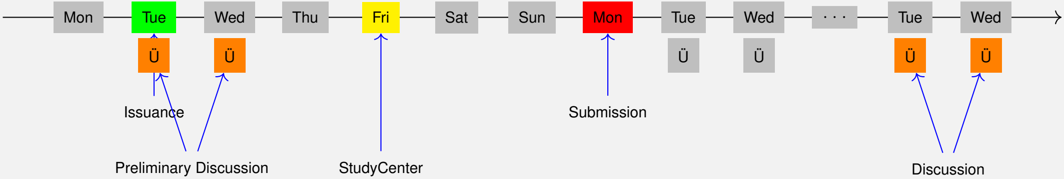

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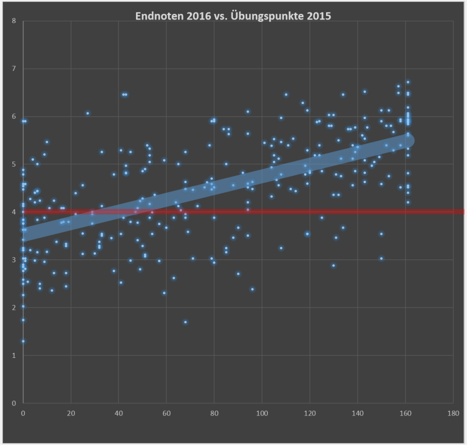


Exams

The exam (in examination period 2018) will cover

- Lectures content (lectures, handouts)
- Exercise content (exercise sessions, exercises).

Exams

Written exam.

We will test your practical skills (programming skills) and theoretical knowledge (background knowledge, systematics).

Offer (VVZ)

- During the semester we offer weekly programming exercises that are graded. Points achieved will be taken as a bonus to the exam.
- The bonus is proportional to the score achieved in specially marked bonus tasks, where a full score equals a bonus of 0.25. The admission to specially marked bonus depends on the successful completion of other exercises. The achieved mark bonus expires as soon as the lecture is given anew.

Offer (Concretely)

- 3 bonus exercises in total; 2/3 of the points suffice for the exam bonus of 0.25 marks
- You can, e.g. fully solve 2 bonus exercises, or solve 3 bonus exercises to 66% each, or ...
- Bonus exercises must be unlocked ($\rightarrow$ experience points) by successfully completing the weekly exercises
- It is again not necessary to solve all weekly exercises completely in order to unlock a bonus exercise
- Details: course website, exercise sessions, online exercise system (Code Expert)

Academic integrity

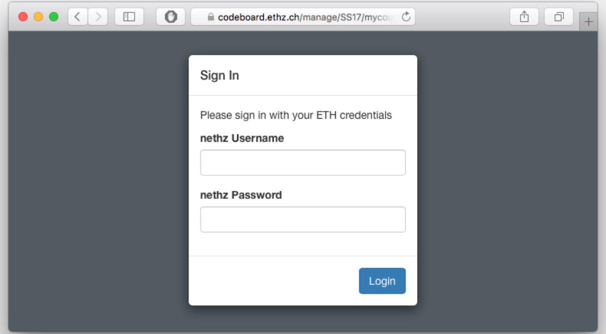
Rule: You submit solutions that you have written yourself and that you have understood.

We check this (partially automatically) and reserve our rights to invite you to interviews.

Should you be invited to an interview: don’t panic. Primary we presume your innocence and want to know if you understood what you have submitted.

Exercise group registration I

- Visit `http://expert.ethz.ch/enroll/AS18/ifmp`
- Log in with your nethz account.



Sign In

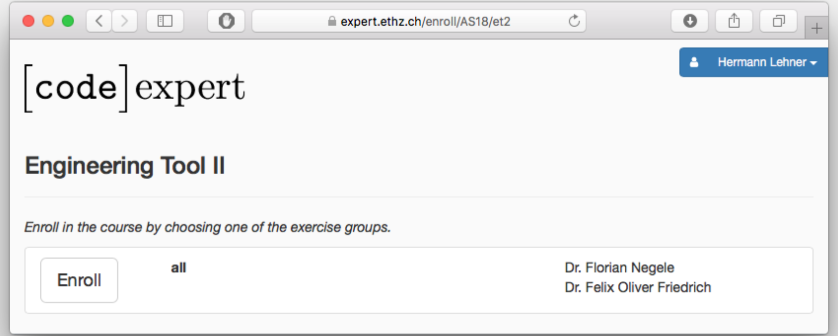
Please sign in with your ETH credentials

nethz Username

nethz Password

Exercise group registration II

Register with the subsequent dialog for an exercise group.

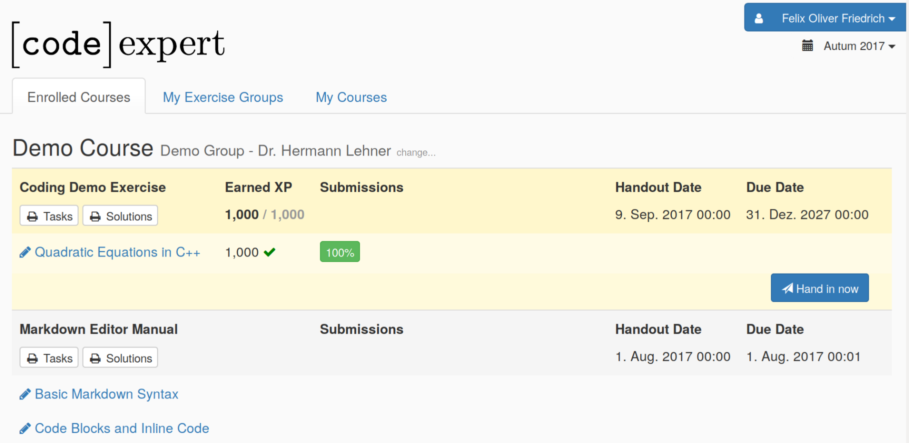


Engineering Tool II

Enroll in the course by choosing one of the exercise groups.

Enroll	all	Dr. Florian Negele Dr. Felix Oliver Friedrich

Overview

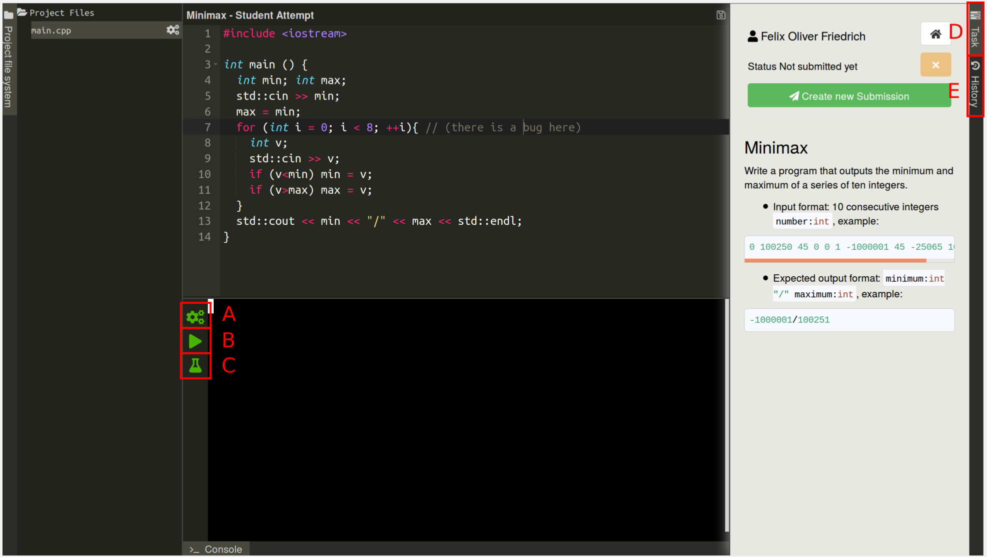


Demo Course Demo Group - Dr. Hermann Lehner change...

Coding Demo Exercise	Earned XP	Submissions	Handout Date	Due Date
Tasks Solutions	1,000 / 1,000		9. Sep. 2017 00:00	31. Dez. 2027 00:00
Quadratic Equations in C++	1,000 ✓	100%		

Markdown Editor Manual	Submissions	Handout Date	Due Date
Tasks Solutions		1. Aug. 2017 00:00	1. Aug. 2017 00:01
Basic Markdown Syntax			
Code Blocks and Inline Code			

Programming Exercise



```
#include <iostream>

int main () {
  int min; int max;
  std::cin >> min;
  max = min;
  for (int i = 0; i < 8; ++i){ // (there is a bug here)
    int v;
    std::cin >> v;
    if (v<min) min = v;
    if (v>max) max = v;
  }
  std::cout << min << "/" << max << std::endl;
}
```

Felix Oliver Friedrich

Status Not submitted yet

Minimax

Write a program that outputs the minimum and maximum of a series of ten integers.

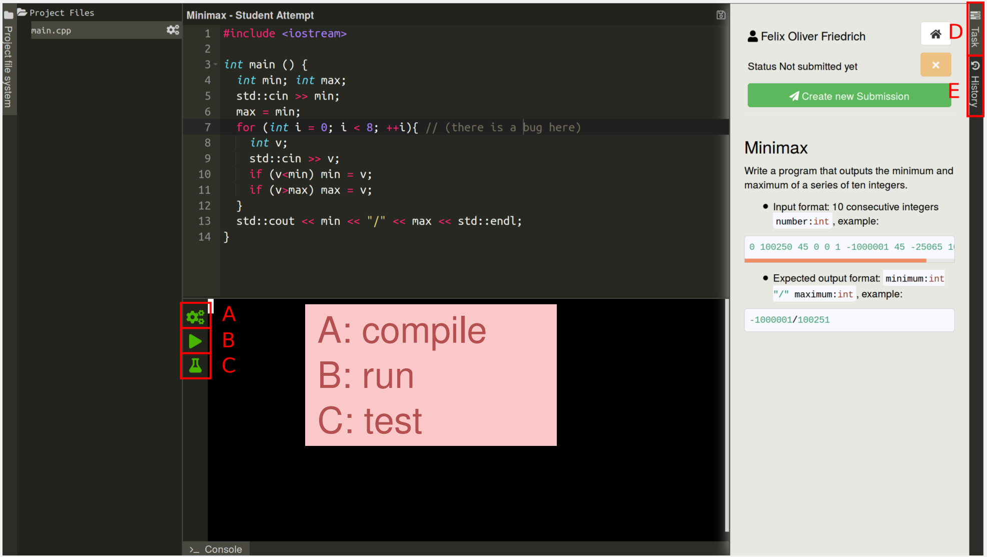
- Input format: 10 consecutive integers `number:int`, example:

```
0 100250 45 0 0 1 -1000001 45 -25065 1
```

- Expected output format: `minimum:int` `"/"` `maximum:int`, example:

```
-1000001/100251
```

Programming Exercise



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Create new Submission

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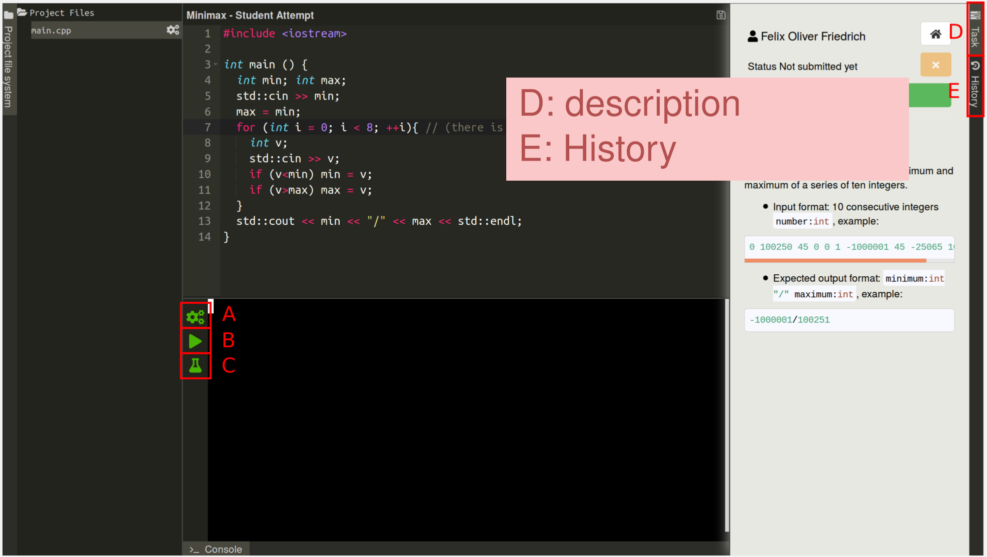
-1000001/100251
```

A: compile
B: run
C: test

Programming Exercise



Minimax - Student Attempt

```cpp
#include <iostream>

int main () {
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Felix Oliver Friedrich

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D: description

E: History

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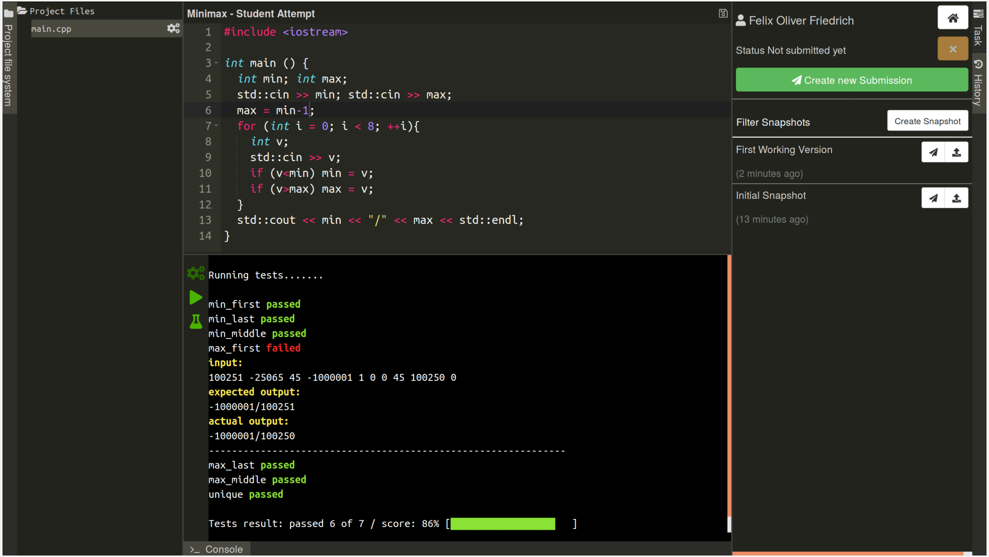
Test and Submit



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int main () {
  int min; int max;
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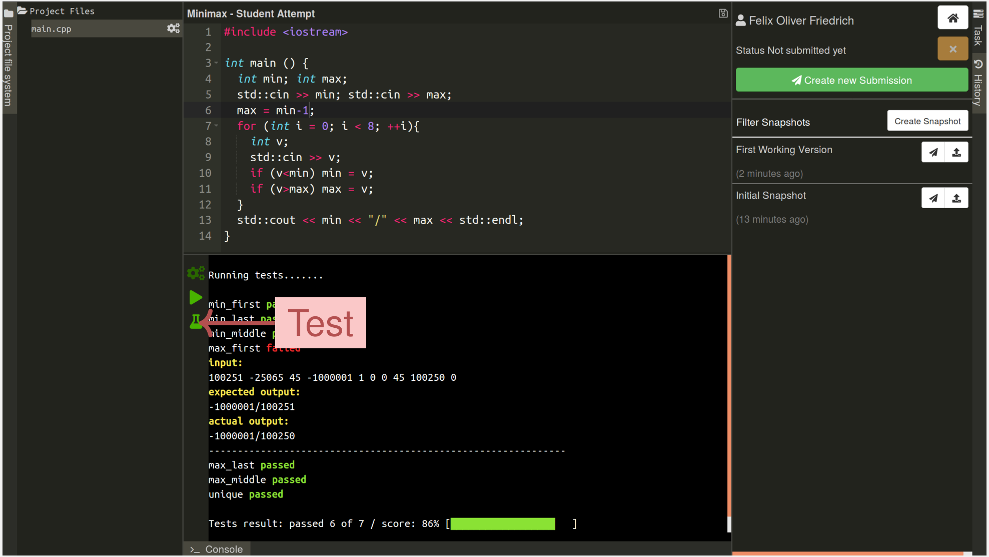
```

```
Running tests.......

min_first passed
min_last passed
min_middle passed
max_first failed
input:
100251 -25065 45 -1000001 1 0 0 45 100250 0
expected output:
-1000001/100251
actual output:
-1000001/100250
----------------------------------------------------------------
max_last passed
max_middle passed
unique passed

Tests result: passed 6 of 7 / score: 86% [████████████   ]
```

Test and Submit



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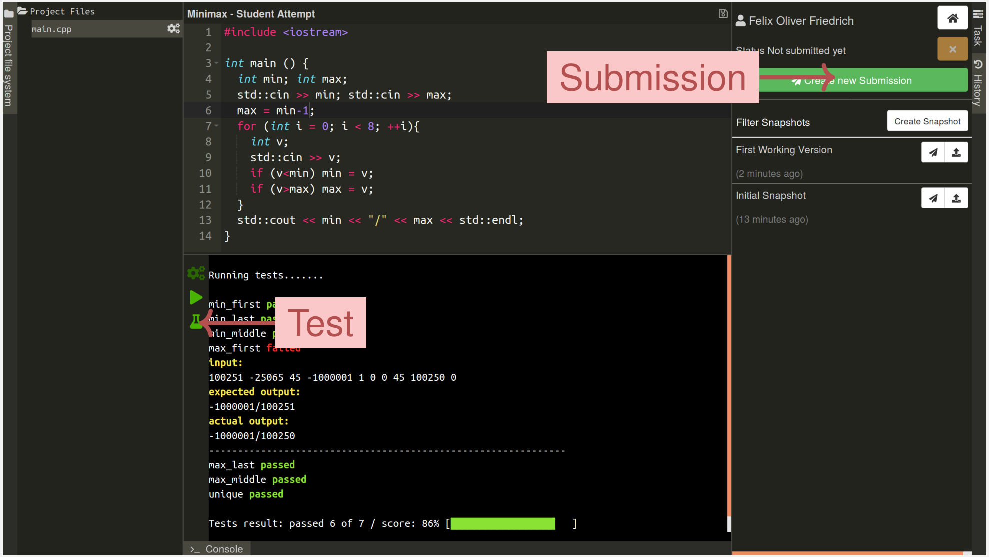

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Create new Submission

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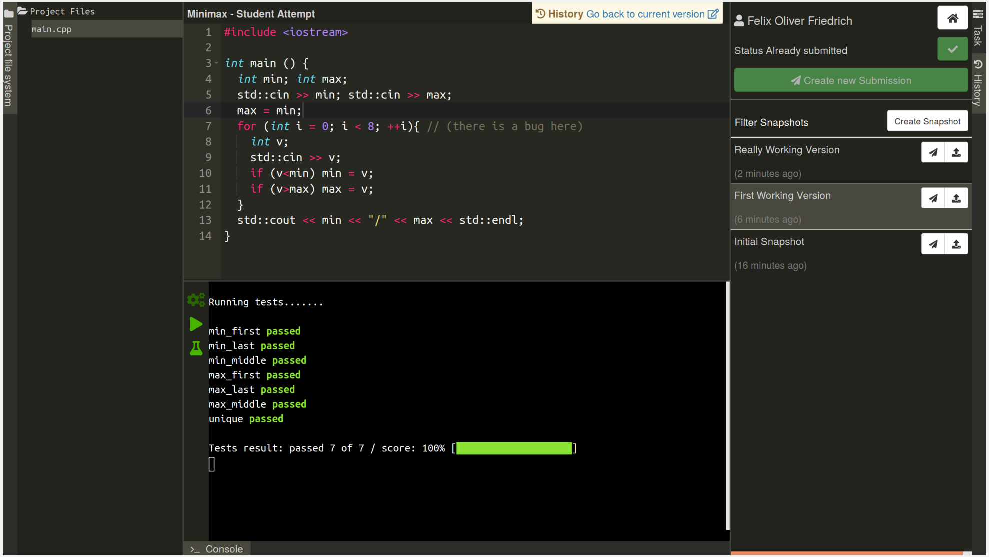
Where is the Save Button?

- The file system is transaction based and is saved permanently (“autosave”). When opening a project it is found in the most recent observed state.
- The current state can be saved as (named) *snaphot*. It is always possible to return to saved snapshot.
- The current state can be submitted (as snapshot). Additionally, each saved named snapshot can be submitted.

Snapshots



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```

```
Running tests.......

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min_last passed
min_middle passed
max_first passed
max_last passed
max_middle passed
unique passed

Tests result: passed 7 of 7 / score: 100% [                    ]
```

Felix Oliver Friedrich

Status Already submitted

Really Working Version

(2 minutes ago)

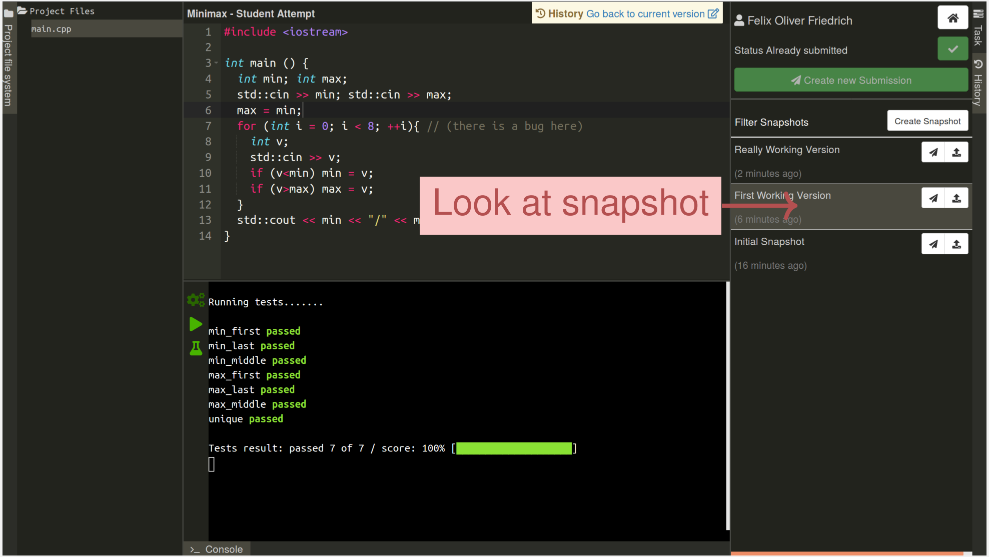
First Working Version

(6 minutes ago)

Initial Snapshot

(16 minutes ago)

Snapshots



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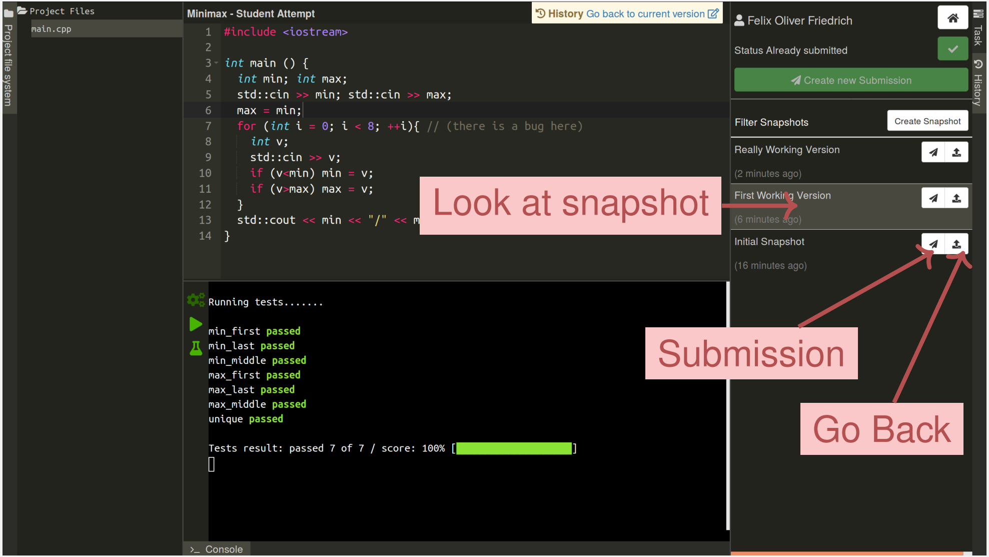

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Submission

Go Back

1. Introduction

Computer Science: Definition and History, Algorithms, Turing Machine, Higher Level Programming Languages, Tools, The first C++Program and its Syntactic and Semantic Ingredients

What is Computer Science?

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- The science of **systematic processing of informations**,. . .

What is Computer Science?

- The science of **systematic processing of informations**,. . .
- . . . particularly the automatic processing using digital computers.

(Wikipedia, according to “Duden Informatik”)

Computer Science vs. Computers

Computer science is not about machines, in the same way that astronomy is not about telescopes.

Mike Fellows, US Computer Scientist (1991)

Computer Science vs. Computers

- Computer science is also concerned with the development of fast computers and networks. . .

Computer Science vs. Computers

- Computer science is also concerned with the development of fast computers and networks. . .
- . . . but not as an end in itself but for the **systematic processing of informations**.

Computer Science $\neq$ Computer Literacy

Computer literacy: *user knowledge*

- Handling a computer
- Working with computer programs for text processing, email, presentations . . .

Computer Science $\neq$ Computer Literacy

Computer Science *Fundamental knowledge*

- How does a computer work?
- How do you write a computer program?

Back from the past: This course

- Systematic problem solving with algorithms and the programming language $\mathrm{C}{+}{+}$.
- Hence:

  not only

  but also programming course.

Algorithm: Fundamental Notion of Computer Science

Algorithm:

- Instructions to solve a problem step by step

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Algorithm:

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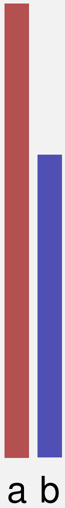
- according to *Muhammed al-Chwarizmi*, author of an arabic computation textbook (about 825)

“Dixit algorizmi. . . ” (Latin translation)

Oldest Nontrivial Algorithm

Euclidean algorithm (from the *elements* from Euklid, 3. century B.C.)



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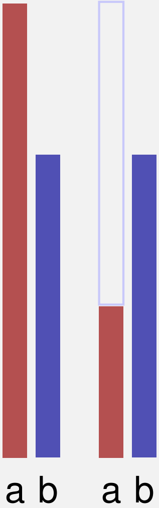

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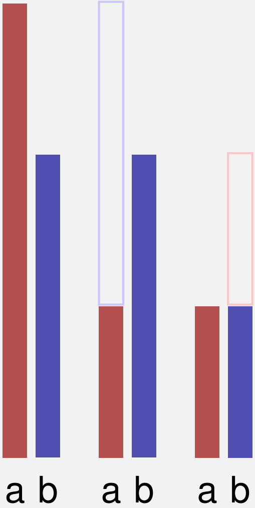

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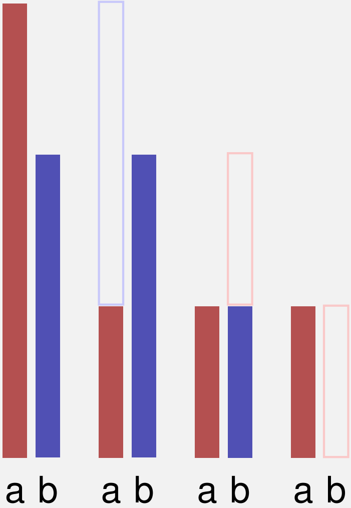

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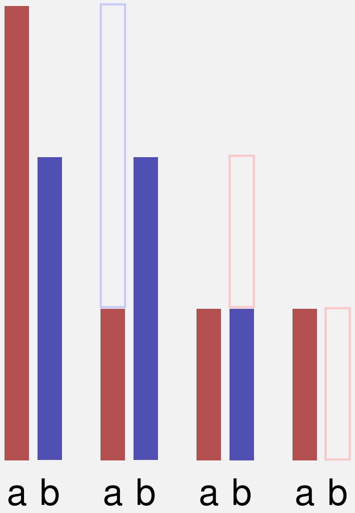

- Input: integers $a > 0$, $b > 0$
- Output: gcd of a und b

```
While $b \neq 0$
    If $a > b$ then
        $a \leftarrow a - b$
    else:
        $b \leftarrow b - a$
Result: $a$.
```

Algorithms: 3 Levels of Abstractions

1. **Core idea** (abstract)**:**
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3. **Implementation** (very detailed)**:**
   made for humans & computers (read- & executable, specific programming language, various implementations possible)

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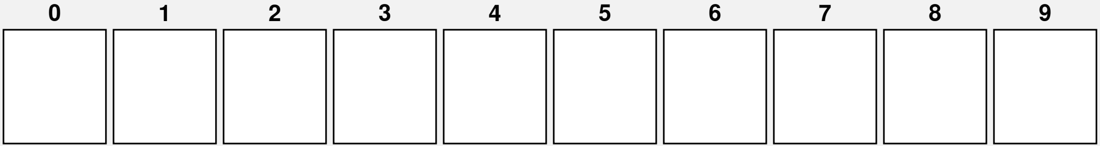
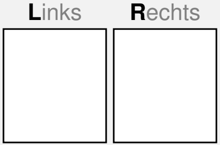
Euclid: Core idea and pseudo code shown, implementation yet missing

Euklid in the Box

Speicher


0 1 2 3 4 5 6 7 8 9


Links **R**echts

Register

Euklid in the Box

Speicher

0	1	2	3	4	5	6	7	8	9
[8] → **L**	[9] → **R**	**L** = 0? *stop*	**R** > **L**? *springe* *zu* **6**	**L** − **R** → [8]	*springe* *zu* 0	**R** − **L** → [9]	*springe* *zu* 0		

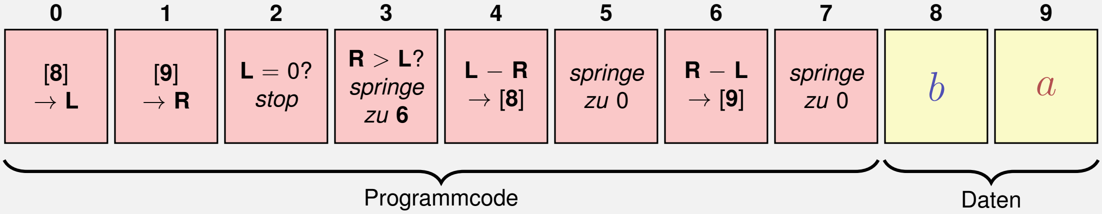
$\underbrace{0 \ldots 7}_{\text{Programmcode}}$

Links	**R**echts

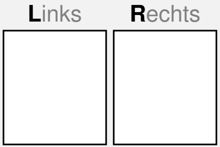
Register

Euklid in the Box

Speicher



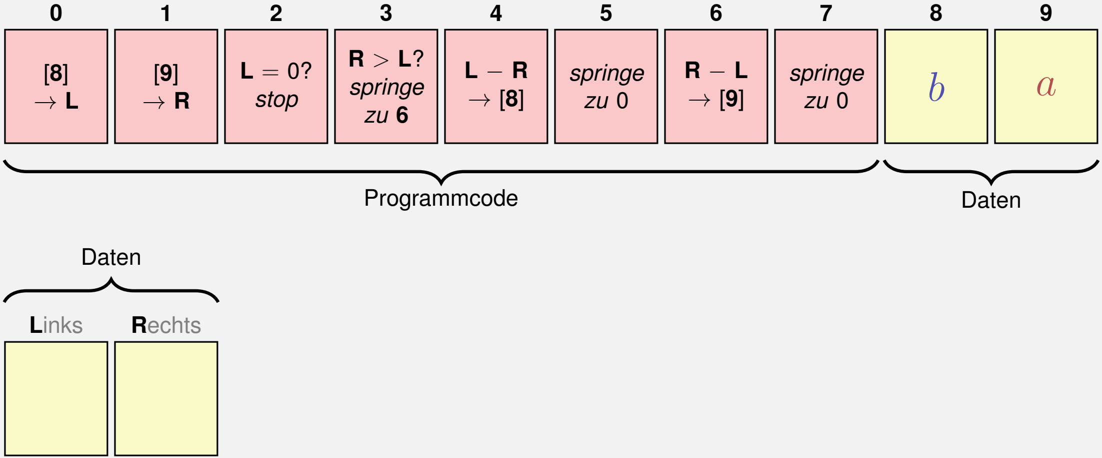
0	1	2	3	4	5	6	7	8	9
[8] → **L**	[9] → **R**	**L** = 0? *stop*	**R** > **L**? *springe zu* **6**	**L** − **R** → [8]	*springe zu* 0	**R** − **L** → [9]	*springe zu* 0	b	a



Register

Euklid in the Box

Speicher



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Programmcode

Daten

Daten

Links	**R**echts

Register

Euklid in the Box

Speicher

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[8] $\rightarrow$ **L**	[9] $\rightarrow$ **R**	**L** $=$ 0? *stop*	**R** $>$ **L**? *springe* *zu* **6**	**L** $-$ **R** $\rightarrow$ [8]	*springe* *zu* 0	**R** $-$ **L** $\rightarrow$ [9]	*springe* *zu* 0	b	a

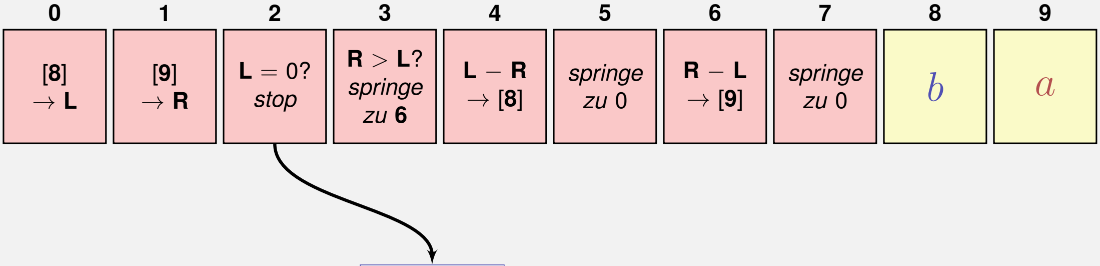
Links	**R**echts
b	a

Register

```
While $b \neq 0$
        If $a > b$ then
                $a \leftarrow a - b$
        else:
                $b \leftarrow b - a$
Ergebnis: $a$.
```

Euklid in the Box

Speicher



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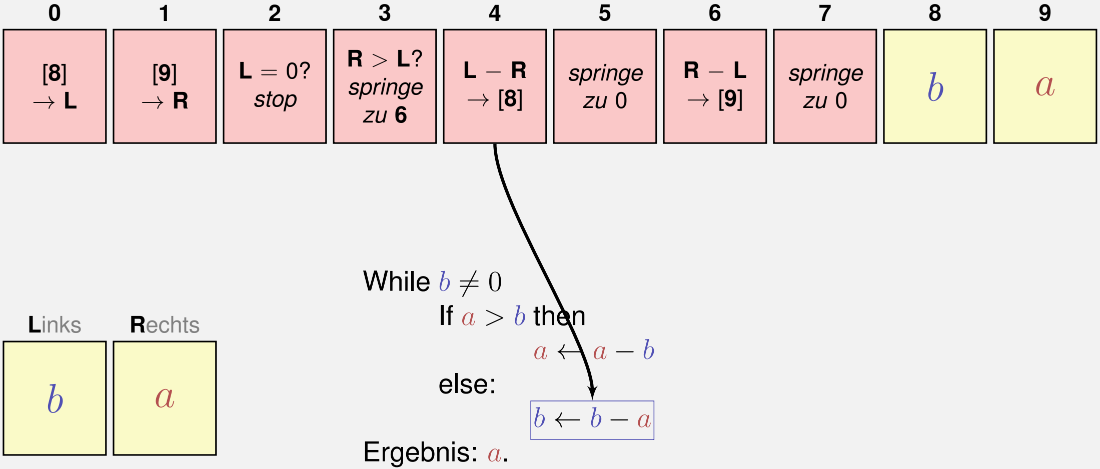
Euklid in the Box

Speicher



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Register

Euklid in the Box

Speicher

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[8] → **L**	[9] → **R**	**L** = 0? *stop*	**R** > **L**? *springe* *zu* **6**	**L** − **R** → [8]	*springe* *zu* 0	**R** − **L** → [9]	*springe* *zu* 0	b	a

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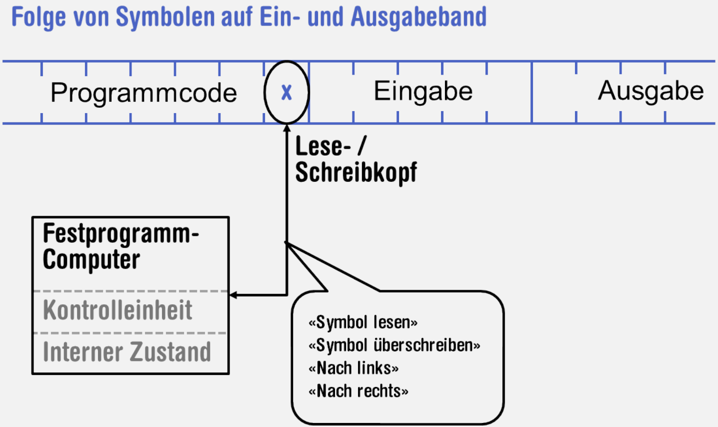
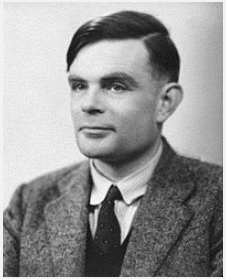
Register

```
While $b \neq 0$
    If $a > b$ then
        $a \leftarrow a - b$
    else:
        $b \leftarrow b - a$
Ergebnis: $a$.
```

Computers – Concept

A bright idea: universal Turing machine (Alan Turing, 1936)

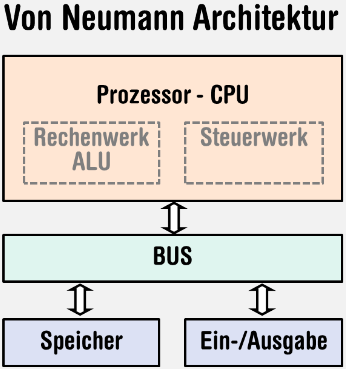




Alan Turing

Computer – Implementation

- Z1 – Konrad Zuse (1938)
- ENIAC – John Von Neumann (1945)



Konrad Zuse

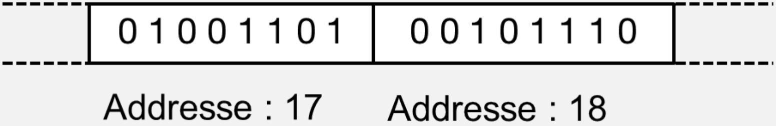
John von Neumann

Memory for data *and* program

- Sequence of bits from $\{0, 1\}$.
- Program state: value of all bits.
- Aggregation of bits to memory cells (often: 8 Bits = 1 Byte)

Memory for data *and* program

- Every memory cell has an address.
- Random access: access time to the memory cell is (nearly) independent of its address.



0 1 0 0 1 1 0 1

0 0 1 0 1 1 1 0

Addresse : 17

Addresse : 18

Programming

- With a *programming language* we issue commands to a computer such that it does exactly what we want.
- The sequence of instructions is the *(computer) program*

The Harvard Computers, human computers, ca.1890

Computing speed

In the time, on average, that the sound takes to travel from from my mouth to you ...

[1] Uniprocessor computer at 1 GHz.

Computing speed

In the time, on average, that the sound takes to travel from from my mouth to you ...

a contemporary desktop PC can process more than 100

[1] Uniprocessor computer at 1 GHz.

Computing speed

In the time, on average, that the sound takes to travel from from my mouth to you ...

30 m $\widehat{=}$ more than $100.000.000$ instructions

a contemporary desktop PC can process more than 100 millions instructions [1]

[1] Uniprocessor computer at 1 GHz.

Why programming?

- Do I study computer science or what ...

Why programming?

- Do I study computer science or what ...
- There are programs for everything ...

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- because computer science is a mandatory subject here, unfortunately...

Why programming?

- Do I study computer science or what ...
- There are programs for everything ...
- I am not interested in programming ...
- because computer science is a mandatory subject here, unfortunately...
- . . .

Mathematics used to be the lingua franca of the natural sciences on all universities. Today this is computer science.
Lino Guzzella, president of ETH Zurich, NZZ Online, 1.9.2017

((BTW: Lino Guzzella is not a computer scientist, he is a mechanical engineer and prof. for thermotronics ☺))

This is why programming!

- Any understanding of modern technology requires knowledge about the fundamental operating principles of a computer.
- Programming (with the computer as a tool) is evolving a cultural technique like reading and writing (using the tools paper and pencil)

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- Programming is *the* interface between engineering and computer science – the interdisciplinary area is growing constantly.

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- Any understanding of modern technology requires knowledge about the fundamental operating principles of a computer.
- Programming (with the computer as a tool) is evolving a cultural technique like reading and writing (using the tools paper and pencil)
- Programming is *the* interface between engineering and computer science – the interdisciplinary area is growing constantly.
- Programming is fun (and is useful)!

Programming Languages

- The language that the computer can understand (machine language) is very primitive.
- Simple operations have to be subdivided into (extremely) many single steps
- The machine language varies between computers.

Higher Programming Languages

can be represented as program text that

- can be *understood* by humans
- is *independent* of the computer model

  → Abstraction!

Why $\mathrm{C}{+}{+}$?

Other popular programming languages: Java, C#, Python, Javascript, Swift, Kotlin, Go, ... $\ldots$

Why C++?

Other popular programming languages: Java, C#, Python, Javascript, Swift, Kotlin, Go, ...

General consensus:

- „The” programming language for systems programming: C
- C has a fundamental weakness: missing (type) safety

Why C++?

Over the years, C++'s greatest strength and its greatest weakness has been its C-Compatibility – B. Stroustrup

Syntax and Semantics

- Like our language, programs have to be formed according to certain rules.
  - Syntax: Connection rules for elementary symbols (characters)
  - Semantics: interpretation rules for connected symbols.

Syntax and Semantics

- Like our language, programs have to be formed according to certain rules.
  - Syntax: Connection rules for elementary symbols (characters)
  - Semantics: interpretation rules for connected symbols.
- Corresponding rules for a computer program are simpler but also more strict because computers are relatively stupid.

Deutsch vs. C++

Deutsch

Alleen sind nicht gefährlich, Rasen ist gefährlich!
(Wikipedia: Mehrdeutigkeit)

C++

```
// computation
int b = a * a; // b = a^2
b = b * b;     // b = a^4
```

Syntax and Semantics of $\mathrm{C{+}{+}}$

Syntax:

- When is a text a $C{+}{+}$ *program?*
- I.e. is it *grammatically* correct?
- $\rightarrow$ Can be checked by a computer

Semantics:

- What does a program *mean*?
- Which algorithm does a program *implement*?
- $\rightarrow$ Requires human understanding

Programming Tools

- **Editor:** Program to modify, edit and store $\mathrm{C}{+}{+}$program texts
- **Compiler:** program to translate a program text into machine language

Programming Tools

- **Editor:** Program to modify, edit and store $\mathrm{C}{+}{+}$program texts
- **Compiler:** program to translate a program text into machine language
- **Computer:** machine to execute machine language programs
- **Operating System:** program to organize all procedures such as file handling, editor-, compiler- and program execution.

The first C++ program

```
// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

Most important ingredients... Statements

```
// Program: power8.cpp
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int main() {
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    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

← Do something (read in a)!

Most important ingredients… Expressions

```cpp
// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
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    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

← Compute a value (a^2)!

“Accessories:” Comments

```
// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
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    std::cout << "Compute a^8 for a =? ";
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    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

comments

Comments and Layout

The compiler does not care...

```
#include <iostream>
int main(){std::cout << "Compute a^8 for a =? ";
int a; std::cin >> a; int b = a * a; b = b * b;
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Comments and Layout

The compiler does not care...

```
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int main(){std::cout << "Compute a^8 for a =? ";
int a; std::cin >> a; int b = a * a; b = b * b;
std::cout << a << "^8 = " << b*b << "\n";return 0;}
```

... but we do!

“Accessories:” Include and Main Function

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// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
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“Accessories:” Include and Main Function

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// Program: power8.cpp
// Raise a number to the eighth power.
#include <iostream>  ←—————— include directive
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
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    std::cin >> a;
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    int b = a * a; // b = a^2
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    return 0;
}
```

declaration of the main function

Statements: Do something!

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int main() {
    // input
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```

expression statements

Statements: Do something!

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    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

← return statement

Statements – Effects

```
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4

    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

effect: output of the string `Compute ...`

Effect: input of a number stored in a

Effect: saving the computed value of a*a into b

Effect: saving the computed value of b*b into b

Effect: return the value 0

Effect: output of the value of a and the computed value o

Statements – Variable Definitions

```
int main() {
    // input
    std::cout << "Compute a^8 for a =? ";
    int a;
    std::cin >> a;
    // computation
    int b = a * a; // b = a^2
    b = b * b;     // b = a^4
    // output b * b, i.e., a^8
    std::cout << a << "^8 = " << b * b << "\n";
    return 0;
}
```

declaration statement

type names

Literals

- represent constant values
- have a fixed *type* and *value*
- are "syntactical values"

Examples:

- `0` has type **`int`**, value 0.
- **`1.2e5`** has type **`double`**, value $1.2 \cdot 10^5$.

Variables

- represent (varying) values
- have
  - *name*
  - *type*
  - *value*
  - *address*

Variables

- represent (varying) values
- have
  - *name*
  - *type*
  - *value*
  - *address*

Example

`int a;` defines a variable with

- name: `a`
- type: `int`
- value: (initially) undefined
- Address: determined by compiler

Expressions: compute a value!

- represent *Computations*

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- are either primary (`b`)

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- represent *Computations*
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- represent *Computations*
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- . . . from different expressions, using operators

Expressions: compute a value!

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- are either primary (`b`)
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- . . . from different expressions, using operators
- have a type and a value

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- represent *Computations*
- are either primary (`b`)
- or composed (`b*b`). . .
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- have a type and a value

Analogy: building blocks

Expressions

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;    // b = a^4

// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";

return 0;
```

Expressions

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;
```

variable name, primary expression (+ name and address)

```
// computation
int b = a * a;  // b = a^2
b = b * b;      // b = a^4
```

variable name, primary expression (+ name and address)

```
// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";

return 0;
```

literal, primary expression

Expressions Building Blocks

composite expression

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;      // b = a^4

// outp
std::cout << a<< "^8 = " << b * b << ".\ n";

return 0;
```

composite expression

Expressions

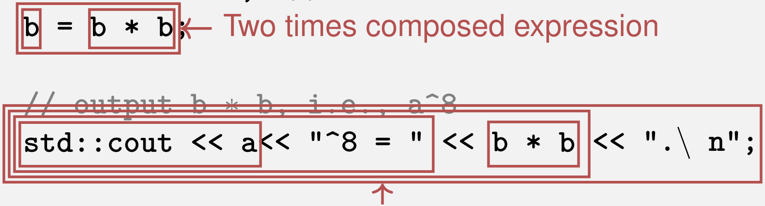
Building Blocks

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;

// output b * b, i.e., a^8
std::cout << a << "^8 = " << b * b << ".\ n";

return (
```



Two times composed expression

Four times composed expression

L-Values and R-Values

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;    // b = a^4

// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";
return 0;
```

L-Values and R-Values

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;
```

L-value (expression + address)

```
// computation
int b = a * a;  // b = a^2
b = b * b;      // b = a^4
```

L-value (expression + address)

```
// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";
return 0;
```

R-Value (expression that is not an L-value)

L-Values and R-Values

R-Value

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;      // b = a^4
```

R-Value

```
// output b * b, i.e., a^8
std::cout << a<< "^8 = " << b * b << ".\ n";
return 0;
```

L-Values and R-Values

L-Wert (“**L**eft of the assignment operator”)

- Expression with *address*
- *Value* is the content at the memory location according to the type of the expression.

L-Values and R-Values

L-Wert (“**L**eft of the assignment operator”)

- Expression with *address*
- *Value* is the content at the memory location according to the type of the expression.
- L-Value can change its value (e.g. via assignment)

Example: variable name

L-Values and R-Values

R-Wert (“**R**ight of the assignment operator”)

- Expression that is no L-value

Example: literal `0`

L-Values and R-Values

R-Wert (“**R**ight of the assignment operator”)

- Expression that is no L-value

Example: literal `0`

- Any L-Value can be used as R-Value (but not the other way round)

L-Values and R-Values

R-Wert ("**R**ight of the assignment operator")

- Expression that is no L-value

  Example: literal `0`

- Any L-Value can be used as R-Value (but not the other way round)

  Every E-Bike can be used as normal bike, but not the other way round

L-Values and R-Values

R-Wert (“**R**ight of the assignment operator”)

- Expression that is no L-value

  Example: literal `0`

- Any L-Value can be used as R-Value (but not the other way round)
- An R-Value *cannot change* its value

Operators and Operands

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;    // b = a^4

// output b * b, i.e., a^8
std::cout << a << "^8 = " << b * b << "\n";
return 0;
```

Operators and Operands

left operand (output stream)

output operator

right operand (string)

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
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std::cin >> a;

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int b = a
b = b * b;      // b = a^4

// output b * b, i.e., a^8
std::cout << a << "^8 = " << b * b << "\n";
return 0;
```

right operand (variable name)

input operator

left operand (input stream)

Operators and Operands

```
// input
std::cout << "Compute a^8 for a =? ";
int a;
std::cin >> a;

// computation
int b = a * a;  // b = a^2
b = b * b;      // b = a^4

// ou                        a^8
std::cout << a << "^8 = " << b * b << "\n";
return 0;
```

assignment operator

multiplication operator

2. Integers

Evaluation of Arithmetic Expressions, Associativity and Precedence, Arithmetic Operators, Domain of Types `int`, `unsigned int`

Celsius to Fahrenheit

```
// Program: fahrenheit.cpp
// Convert temperatures from Celsius to Fahrenheit.
#include <iostream>

int main() {
  // Input
  std::cout << "Temperature in degrees Celsius =? ";
  int celsius;
  std::cin >> celsius;

  // Computation and output
  std::cout << celsius << " degrees Celsius are "
           << 9 * celsius / 5 + 32  << " degrees Fahrenheit.\n";
  return 0;
}
```

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           << 9 * celsius / 5 + 32  << " degrees Fahrenheit.\n";
  return 0;
}
```

`9 * celsius / 5 + 32`

- Arithmetic expression,

```
9 * celsius / 5 + 32
```

- Arithmetic expression,
- three literals, one variable, three operator symbols

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`9 * celsius / 5 + 32`

- Arithmetic expression,
- three literals, one variable, three operator symbols

How to put the expression in parentheses?

Precedence

Multiplication/Division before Addition/Subtraction

```
9 * celsius / 5 + 32
```

bedeutet

```
(9 * celsius / 5) + 32
```

Precedence

Rule 1: precedence

Multiplicative operators (*, /, %) have a higher precedence ("bind more strongly") than additive operators (+, -)

Associativity

From left to right

```
9 * celsius / 5 + 32
```

bedeutet

```
((9 * celsius) / 5) + 32
```

Associativity

Rule 2: Associativity

Arithmetic operators (`*`, `/`, `%`, `+`, `-`) are left associative: operators of same precedence evaluate from left to right

Arity

Rule 3: Arity

Unary operators +, − first, then binary operators +, −.

```
-3 - 4
```

means

```
(-3) - 4
```

Parentheses

Any expression can be put in parentheses by means of

- associativities
- precedences
- arities

of the operands in an unambiguous way.

Expression Trees

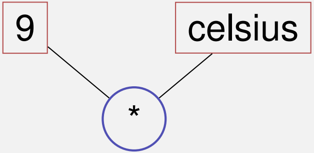
Parentheses yield the expression tree

```
9 * celsius / 5 + 32
```

Expression Trees

Parentheses yield the expression tree

```
(9 * celsius) / 5 + 32
```



Expression Trees

Parentheses yield the expression tree

```
((9 * celsius) / 5) + 32
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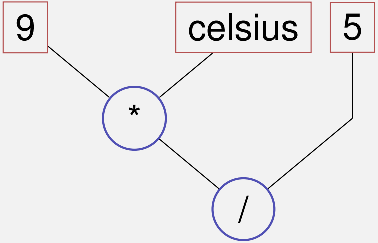

Expression Trees

Parentheses yield the expression tree

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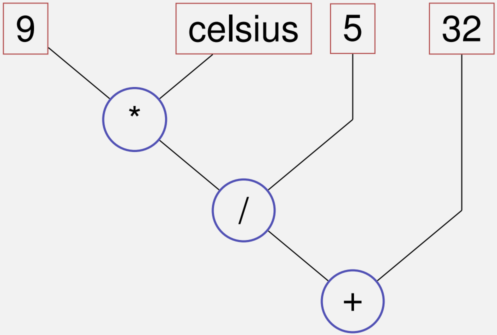
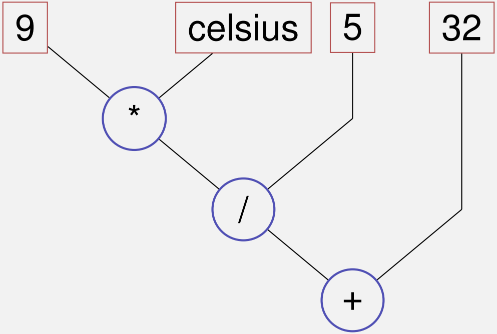
(((9 * celsius) / 5) + 32)
```



Evaluation Order

"From top to bottom" in the expression tree

```
9 * celsius / 5 + 32
```



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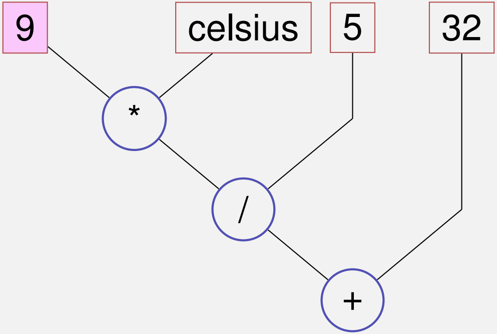

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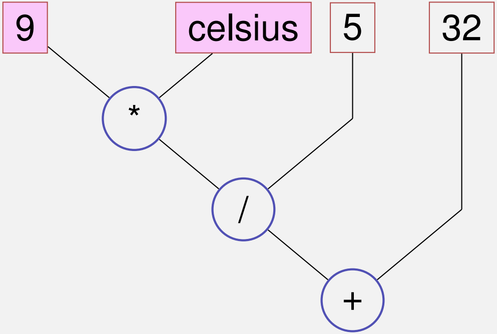

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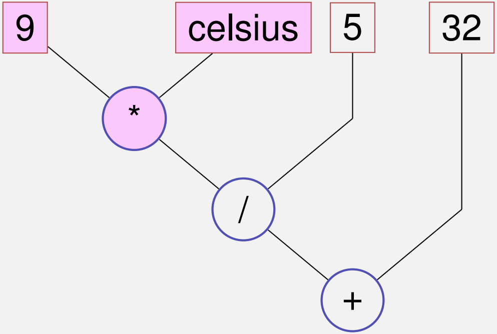

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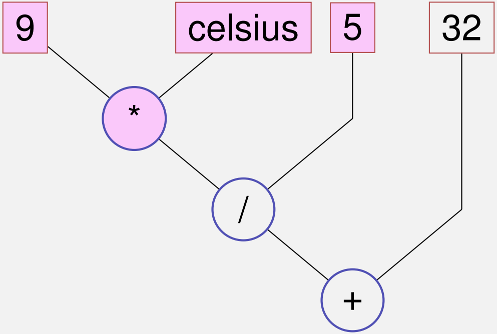

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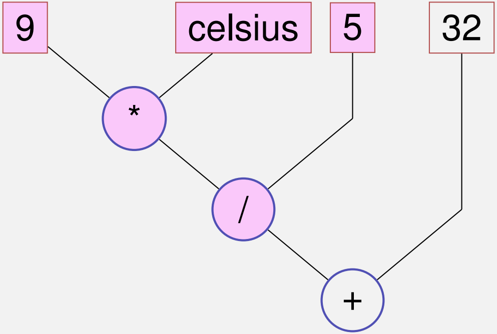

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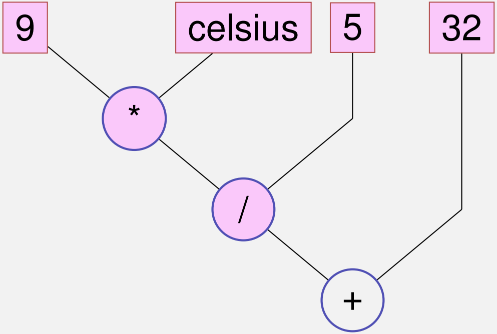

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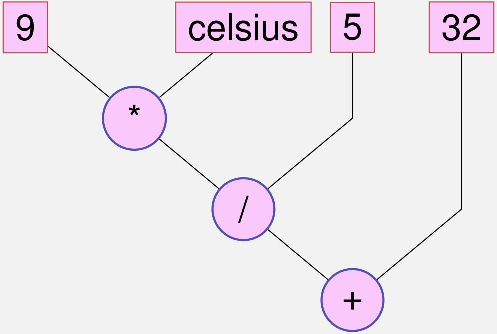
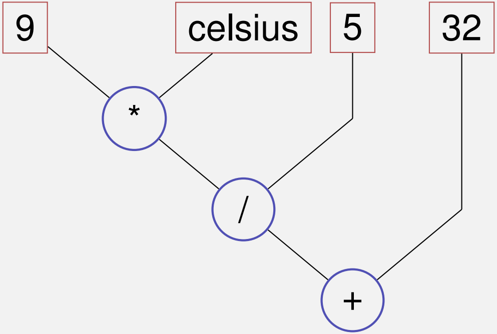


Evaluation Order

Order is not determined uniquely:

```
9 * celsius / 5 + 32
```



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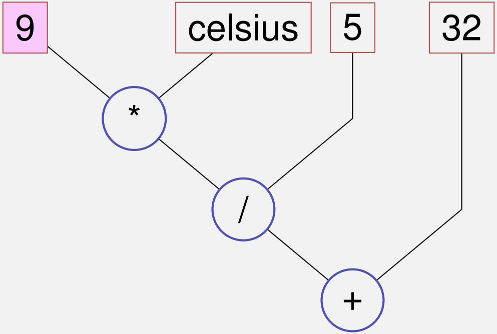

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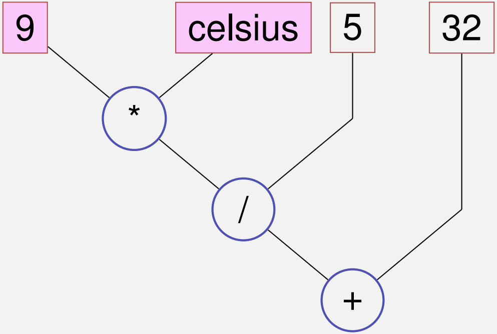

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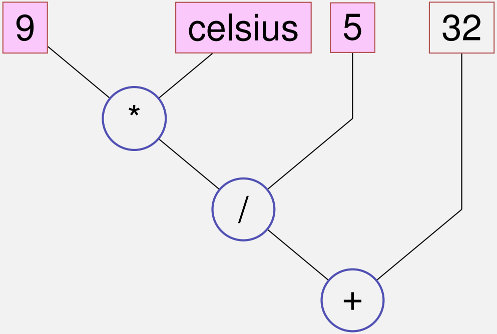

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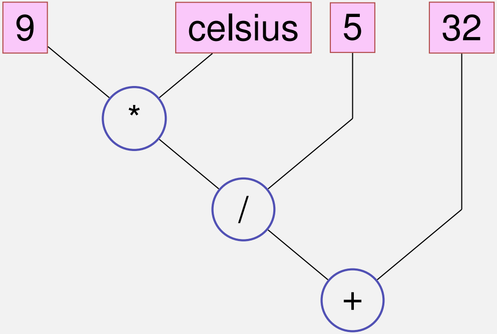

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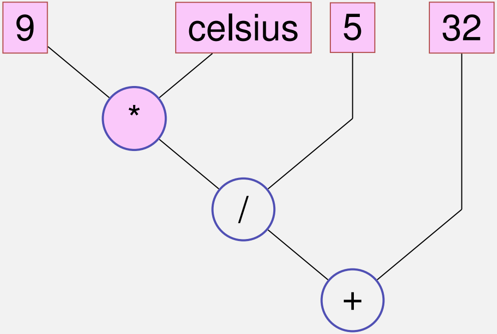

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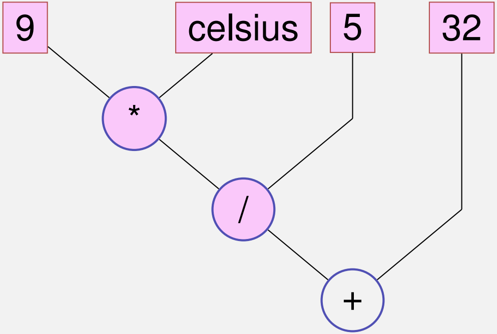

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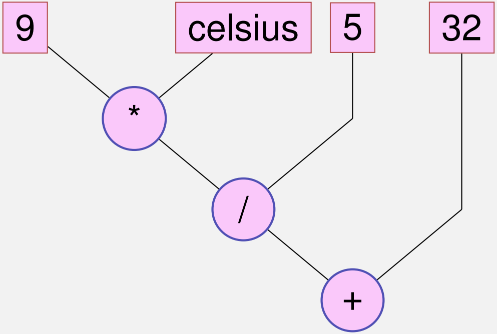
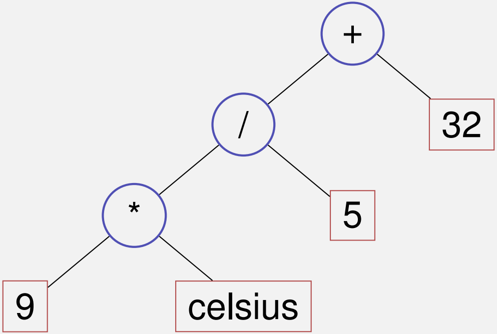


Expression Trees – Notation

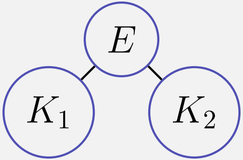
Common notation: root on top

```
9 * celsius / 5 + 32
```



Evaluation Order – more formally

- Valid order: any node is evaluated *after* its children



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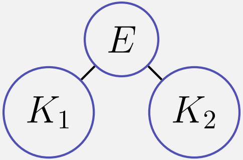
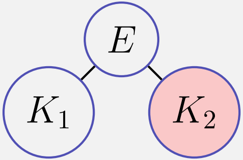


C++: the valid order to be used is not defined.

Evaluation Order – more formally

- Valid order: any node is evaluated *after* its children



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Evaluation Order – more formally

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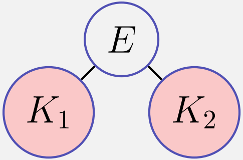
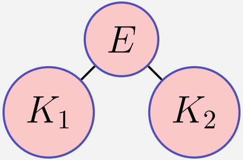


$\mathrm{C{+}{+}}$: the valid order to be used is not defined.

Evaluation Order – more formally

- Valid order: any node is evaluated *after* its children



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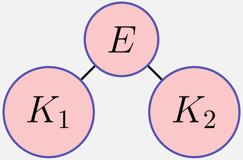
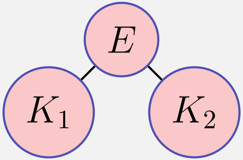
Evaluation Order – more formally

- 

C++: the valid order to be used is not defined.

- "Good expression": any valid evaluation order leads to the same result.

Evaluation Order – more formally

- 

C++: the valid order to be used is not defined.

- Example for a “bad expression”: `a*(a=2)`

Evaluation order

Guideline

Avoid modifying variables that are used in the same expression more than once.

Arithmetic operations

	Symbol	Arity	Precedence	Associativity
Unary +	+	1	16	right
Negation	−	1	16	right
Multiplication	*	2	14	left
Division	/	2	14	left
Modulo	%	2	14	links
Addition	+	2	13	left
Subtraction	−	2	13	left

Interlude: Assignment expression – in more detail

- Already known: `a = b` means
  Assignment of `b` (R-value) to `a` (L-value).
  Returns: L-value

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Interlude: Assignment expression – in more detail

- Already known: `a = b` means
  Assignment of `b` (R-value) to `a` (L-value).
  Returns: L-value
- What does `a = b = c` mean?
- Answer: assignment is right-associative

`a = b = c` $\Longleftrightarrow$ `a = (b = c)`

Interlude: Assignment expression – in more detail

`a = b = c` $\Longleftrightarrow$ `a = (b = c)`

Example multiple assignment:

`a = b = 0` $\Longrightarrow$ `b=0; a=0`

Division

- Operator / implements integer division

`5 / 2` has value 2

Division

- Operator / implements integer division

`5 / 2` has value 2

- In `fahrenheit.cpp`

```
9 * celsius / 5 + 32

15 degrees Celsius are 59 degrees Fahrenheit
```

Division

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Division

- In `fahrenheit.cpp`

```
9 * celsius / 5 + 32
```

```
15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent…

```
9 / 5 * celsius + 32
```

Division

- In `fahrenheit.cpp`

```
9 * celsius / 5 + 32


15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent…

```
1 * celsius + 32
```

Division

- In `fahrenheit.cpp`

```
9 * celsius / 5 + 32
```

```
15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent…

```
15 + 32
```

Division

- In `fahrenheit.cpp`

```
9 * celsius / 5 + 32

15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent. . .

```
47
```

Division

- In `fahrenheit.cpp`

```
9 * celsius / 5 + 32
```

```
15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent... but not in C++!

```
9 / 5 * celsius + 32
```

```
15 degrees Celsius are 47 degrees Fahrenheit
```

Loss of Precision

Guideline

- Watch out for potential loss of precision
- Postpone operations with potential loss of precision to avoid “error escalation”

Division and Modulo

- Modulo-operator computes the rest of the integer division

  `5 / 2` has value 2, `5 % 2` has value 1.

Division and Modulo

- Modulo-operator computes the rest of the integer division

  `5 / 2` has value 2, `5 % 2` has value 1.

- It holds that:

  `(a / b) * b + a % b` has the value of `a`.

Increment and decrement

- Increment / Decrement a number by one is a frequent operation
- works like this for an L-value:

```
expr = expr + 1.
```

Increment and decrement

```
expr = expr + 1.
```

Disadvantages

- relatively long

Increment and decrement

```
expr = expr + 1.
```

Disadvantages

- relatively long
- `expr` is evaluated twice
    - Later: L-valued expressions whose evaluation is “expensive”

Increment and decrement

```
expr = expr + 1.
```

Disadvantages

- relatively long
- `expr` is evaluated twice
  - Later: L-valued expressions whose evaluation is “expensive”
  - `expr` could have an effect (but should not, cf. guideline)

In-/Decrement Operators

Post-Increment

```
expr++
```

Value of `expr` is increased by one, the *old* value of `expr` is returned (as R-value)

In-/Decrement Operators

Pre-increment

```
++expr
```

Value of `expr` is increased by one, the *new* value of `expr` is returned (as L-value)

In-/Decrement Operators

Post-Dekrement

```
expr--
```

Value of `expr` is decreased by one, the *old* value of `expr` is returned (as R-value)

In-/Decrement Operators

Prä-Dekrement

```
--expr
```

Value of `expr` is increased by one, the *new* value of `expr` is returned (as L-value)

In-/Decrement Operators

Example

```
int a = 7;
std::cout << ++a << "\n";
std::cout << a++ << "\n";
std::cout << a << "\n";
```

In-/Decrement Operators

Example

```
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n";
std::cout << a << "\n";
```

In-/Decrement Operators

Example

```
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n"; // 8
std::cout << a << "\n";
```

In-/Decrement Operators

Example

```
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n"; // 8
std::cout << a << "\n"; // 9
```

`C++` vs. `++C`

Strictly speaking our language should be named `++C` because

- it is an advancement of the language `C`

`C++` vs. `++C`

Strictly speaking our language should be named `++C` because

- it is an advancement of the language `C`
- while `C++` returns the old `C`.

Arithmetic Assignments

```
a += b
```

$$\Leftrightarrow$$

```
a = a + b
```

Arithmetic Assignments

```
a += b
```

$$\Leftrightarrow$$

```
a = a + b
```

analogously for `-`, `*`, `/` and `%`

Binary Number Representations

Binary representation (Bits from $\{0, 1\}$)

$$b_n b_{n-1} \dots b_1 b_0$$

corresponds to the number $b_n \cdot 2^n + \dots + b_1 \cdot 2^1 + b_0 \cdot 2^0$

Binary Number Representations

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Example: `101011`

Binary Number Representations

Binary representation (Bits from $\{0, 1\}$)

$$b_n b_{n-1} \dots b_1 b_0$$

corresponds to the number $b_n \cdot 2^n + \cdots + b_1 \cdot 2 + b_0$

Example: `101011` corresponds to `32+8+2+1`.

Binary Number Representations

Binary representation (Bits from $\{0,1\}$)

$$b_n b_{n-1} \ldots b_1 b_0$$

corresponds to the number $b_n \cdot 2^n + \cdots + b_1 \cdot 2 + b_0$

Example: `101011` corresponds to `43`.

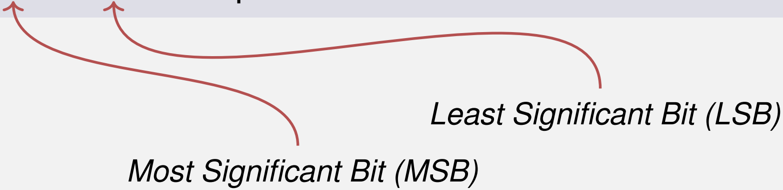
Binary Number Representations

Binary representation (Bits from $\{0, 1\}$)

$$b_n b_{n-1} \dots b_1 b_0$$

corresponds to the number $b_n \cdot 2^n + \cdots + b_1 \cdot 2 + b_0$

Example: 101011 corresponds to 43.



Least Significant Bit (LSB)

Most Significant Bit (MSB)

Computing Tricks

- Estimate the orders of magnitude of powers of two.[2]:

$$2^{10} = 1024 = 1\mathrm{Ki} \approx 10^3.$$
$$2^{32} = 4 \cdot (1024)^3 = 4\mathrm{Gi}.$$
$$2^{64} = 16\mathrm{Ei} \approx 16 \cdot 10^{18}.$$

[2] Decimal vs. binary units: MB - Megabyte vs. MiB - Megabibyte (etc.)
kilo (K, Ki) – mega (M, Mi) – giga (G, Gi) – tera(T, Ti) – peta(P, Pi) – exa (E, Ei)

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$$2^{64} = 16\mathrm{Ei} \approx 16 \cdot 10^{18}.$$

[2] Decimal vs. binary units: MB - Megabyte vs. MiB - Megabibyte (etc.)
kilo (K, Ki) – mega (M, Mi) – giga (G, Gi) – tera(T, Ti) – peta(P, Pi) – exa (E, Ei)

Computing Tricks

- Estimate the orders of magnitude of powers of two.[2]:

$$2^{10} = 1024 = 1\mathrm{Ki} \approx 10^3.$$
$$2^{32} = 4 \cdot (1024)^3 = 4\mathrm{Gi}.$$
$$2^{64} = 16\mathrm{Ei} \approx 16 \cdot 10^{18}.$$

[2] Decimal vs. binary units: MB - Megabyte vs. MiB - Megabibyte (etc.)
kilo (K, Ki) – mega (M, Mi) – giga (G, Gi) – tera(T, Ti) – peta(P, Pi) – exa (E, Ei)

Hexadecimal Numbers

Numbers with base 16

$$h_n h_{n-1} \dots h_1 h_0$$

corresponds to the number

$$h_n \cdot 16^n + \dots + h_1 \cdot 16 + h_0.$$

notation in C++: prefix `0x`

Example: `0xff` corresponds to 255.

Hex Nibbles

hex	bin	dec
0	0000	0
1	**0001**	**1**
2	**0010**	**2**
3	0011	3
4	**0100**	**4**
5	0101	5
6	0110	6
7	0111	7
8	**1000**	**8**
9	1001	9
a	1010	10
b	1011	11
c	1100	12
d	1101	13
e	1110	14
f	1111	15

Why Hexadecimal Numbers?

- A Hex-Nibble requires exactly 4 bits.

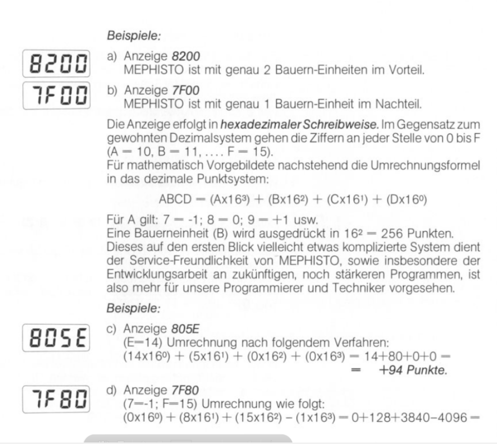
Why Hexadecimal Numbers?

- A Hex-Nibble requires exactly 4 bits.
- “compact representation of binary numbers”

Why Hexadecimal Numbers?

“For programmers and technicians”

(user manual chess computer *Mephisto II*, 1981)



Beispiele:

8200

a) Anzeige ***8200***
MEPHISTO ist mit genau 2 Bauern-Einheiten im Vorteil.

7F00

b) Anzeige ***7F00***
MEPHISTO ist mit genau 1 Bauern-Einheit im Nachteil.

Die Anzeige erfolgt in ***hexadezimaler Schreibweise.*** Im Gegensatz zum gewohnten Dezimalsystem gehen die Ziffern an jeder Stelle von 0 bis F (A = 10, B = 11, ... F = 15).
Für mathematisch Vorgebildete nachstehend die Umrechnungsformel in das dezimale Punktsystem:

$$ABCD = (Ax16^3) + (Bx16^2) + (Cx16^1) + (Dx16^0)$$

Für A gilt: 7 = -1; 8 = 0; 9 = +1 usw.
Eine Bauerneinheit (B) wird ausgedrückt in 16^2 = 256 Punkten.
Dieses auf den ersten Blick vielleicht etwas komplizierte System dient der Service-Freundlichkeit von MEPHISTO, sowie insbesondere der Entwicklungsarbeit an zukünftigen, noch stärkeren Programmen, ist also mehr für unsere Programmierer und Techniker vorgesehen.

Beispiele:

805E

c) Anzeige ***805E***
(E=14) Umrechnung nach folgendem Verfahren:
$(14x16^0) + (5x16^1) + (0x16^2) + (0x16^3) = 14+80+0+0 =$
$= \quad$ ***+94 Punkte.***

7F80

d) Anzeige ***7F80***
(7=-1; F=15) Umrechnung wie folgt:
$(0x16^0) + (8x16^1) + (15x16^2) - (1x16^3) = 0+128+3840-4096 =$

Example: Hex-Colors

$$\#\underbrace{00}_{r}\underbrace{FF}_{g}\underbrace{00}_{b}$$

Example: Hex-Colors

$$\#\underbrace{\text{FF}}_{r}\underbrace{\text{FF}}_{g}\underbrace{\text{00}}_{b}$$

Example: Hex-Colors

$$\#\underbrace{80}_{r}\underbrace{80}_{g}\underbrace{80}_{b}$$

Example: Hex-Colors

$$\#\underbrace{\mathrm{FF}}_{r}\underbrace{\mathrm{00}}_{g}\underbrace{\mathrm{50}}_{b}$$

Domain of Type int

```
// Output the smallest and the largest value of type int.
#include <iostream>
#include <limits>

int main() {
  std::cout << "Minimum int value is "
           << std::numeric_limits<int>::min() << ".\n"
           << "Maximum int value is "
           << std::numeric_limits<int>::max() << ".\n";
  return 0;
}
```

Domain of Type int

```
// Output the smallest and the largest value of type int.
#include <iostream>
#include <limits>

int main() {
  std::cout << "Minimum int value is "
            << std::numeric_limits<int>::min() << ".\n"
            << "Maximum int value is "
            << std::numeric_limits<int>::max() << ".\n";
  return 0;
}
```

```
Minimum int value is -2147483648.
Maximum int value is 2147483647.
```

Domain of Type int

```cpp
// Output the smallest and the largest value of type int.
#include <iostream>
#include <limits>

int main() {
  std::cout << "Minimum int value is "
            << std::numeric_limits<int>::min() << ".\n"
            << "Maximum int value is "
            << std::numeric_limits<int>::max() << ".\n";
  return 0;
}
```

```
Minimum int value is -2147483648.
Maximum int value is 2147483647.
```

Where do these numbers come from?

Domain of the Type `int`

- Representation with B bits. Domain

$$\{-2^{B-1}, \ldots, -1, 0, 1, \ldots, 2^{B-1}-2, 2^{B-1}-1\}$$

Domain of the Type `int`

- Representation with B bits. Domain

$$\{-2^{B-1}, \ldots, -1, 0, 1, \ldots, 2^{B-1} - 2, 2^{B-1} - 1\}$$

  Where does this partitioning come from?

- On most platforms $B = 32$

Domain of the Type `int`

- Representation with B bits. Domain

$$\{-2^{B-1}, \ldots, -1, 0, 1, \ldots, 2^{B-1} - 2, 2^{B-1} - 1\}$$

Where does this partitioning come from?

- For the type `int` $\mathrm{C}{+}{+}$ guarantees $B \geq 16$

Over- and Underflow

- Arithmetic operations (`+,-,*`) can lead to numbers outside the valid domain.
- Results can be incorrect!

`power8.cpp`: $15^8 = -1732076671$

`power20.cpp`: $3^{20} = -808182895$

- There is *no error message!*

The Type `unsigned int`

- Domain

$$\{0, 1, \ldots, 2^B - 1\}$$

- All arithmetic operations exist also for `unsigned int`.
- Literals: `1u, 17u` . . .

Mixed Expressions

- Operators can have operands of different type (e.g. `int` and `unsigned int`).

```
17 + 17u
```

- Such mixed expressions are of the “more general” type `unsigned int`.
- `int`-operands are *converted* to `unsigned int`.

Conversion

`int` Value	Sign	`unsigned int` Value
x	≥ 0	x
x	< 0	$x + 2^B$

Conversion

`int` Value	Sign	`unsigned int` Value
x	≥ 0	x
x	< 0	$x + 2^B$

Using two's complement representation (to come), nothing happens internally

Computing with Binary Numbers (4 digits)

Simple Addition

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array} \qquad \begin{array}{r} 0010 \\ +0011 \\ \hline 0101 \end{array}$$

Computing with Binary Numbers (4 digits)

Simple Subtraction

$$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array} \qquad \begin{array}{r} 0101 \\ -0011 \\ \hline 0010 \end{array}$$

Computing with Binary Numbers (4 digits)

Addition with Overflow

$$\begin{array}{r} 7 \\ +9 \\ \hline 16 \end{array} \qquad \begin{array}{r} 0111 \\ +1001 \\ \hline (1)0000 \end{array}$$

Computing with Binary Numbers (4 digits)

Negative Numbers?

$$\begin{array}{r} 5 \\ +(-5) \\ \hline 0 \end{array} \qquad \begin{array}{r} 0101 \\ ???? \\ \hline (1)0000 \end{array}$$

Computing with Binary Numbers (4 digits)

Simpler -1

$$\begin{array}{rcr} 1 & \qquad & 0001 \\ +(-1) & & 1111 \\ \hline 0 & & (1)0000 \end{array}$$

Computing with Binary Numbers (4 digits)

Utilize this:

$$\begin{array}{r} 3 \\ +? \\ \hline -1 \end{array} \qquad\qquad \begin{array}{r} 0011 \\ +???? \\ \hline 1111 \end{array}$$

Computing with Binary Numbers (4 digits)

Invert!

$$\begin{array}{r} 3 \\ +(-4) \\ \hline -1 \end{array} \qquad \begin{array}{r} 0011 \\ +1100 \\ \hline 1111 \end{array} \widehat{=}\ 2^B - 1$$

Computing with Binary Numbers (4 digits)

$$\begin{array}{r} a \\ +(-a-1) \\ \hline -1 \end{array} \qquad \begin{array}{r} a \\ \bar{a} \\ \hline 1111 \end{array} \mathrel{\widehat{=}} 2^B - 1$$

Computing with Binary Numbers (4 digits)

- Negation: inversion and addition of 1

$$-a \quad \widehat{=} \quad \bar{a}+1$$

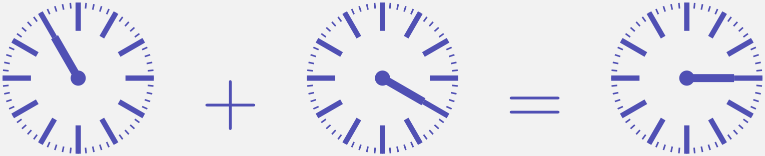
Computing with Binary Numbers (4 digits)

- Wrap around semantics (calculating modulo 2^B

$$-a \quad \widehat{=} \quad 2^B - a$$

Why this works

Modulo arithmetics: Compute on a circle[3]



$11 \equiv 23 \equiv -1 \equiv \ldots \mod 12$

$4 \equiv 16 \equiv \ldots \mod 12$

$3 \equiv 15 \equiv \ldots \mod 12$

[3] The arithmetics also work with decimal numbers (and for multiplication).

Negative Numbers (3 Digits)

	a	$-a$
0	000	
1	001	
2	010	
3	011	
4	100	
5	101	
6	110	
7	111	

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001		
2	010		
3	011		
4	100		
5	101		
6	110		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	**111**	-1
2	010		
3	011		
4	100		
5	101		
6	110		
7	**111**		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	**110**	-2
3	011		
4	100		
5	101		
6	**110**		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	**101**	-3
4	100		
5	**101**		
6	110		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	101	-3
4	**100**	**100**	-4
5	101		
6	110		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	101	-3
4	100	100	-4
5	101		
6	110		
7	111		

Negative Numbers (3 Digits)

	a	$-a$	
0	000	000	0
1	001	111	-1
2	010	110	-2
3	011	101	-3
4	100	100	-4
5	101		
6	110		
7	111		

The most significant bit decides about the sign *and* it contributes to the value.

3. Logical Values

Boolean Functions; the Type `bool`; logical and relational operators; shortcut evaluation

Our Goal

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

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Boolean Values in Mathematics

Boolean expressions can take on one of two values:

0 or *1*

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Boolean expressions can take on one of two values:

0 or *1*

- *0* corresponds to *“false”*
- *1* corresponds to *“true”*

The Type `bool` in C++

- represents *logical values*

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- Literals `false` and `true`

The Type `bool` in C++

- represents *logical values*
- Literals `false` and `true`
- Domain $\{$*false*, *true*$\}$

```
bool b = true; // Variable with value true
```

Relational Operators

`a < b` (smaller than)

arithmetic type × arithmetic type → `bool`

R-value × R-value → R-value

Relational Operators

`a < b` (smaller than)

```
bool b = (1 < 3); // b =
```

Relational Operators

`a < b` (smaller than)

```
bool b = (1 < 3); // b = true
```

Relational Operators

`a >= b` (greater than)

```
int a = 0;
bool b = (a >= 3); // b =
```

Relational Operators

`a >= b` (greater than)

```
int a = 0;
bool b = (a >= 3); // b = false
```

Relational Operators

`a == b` (equals)

```
int a = 4;
bool b = (a % 3 == 1); // b =
```

Relational Operators

`a == b` (equals)

```
int a = 4;
bool b = (a % 3 == 1); // b = true
```

Relational Operators

`a != b` (not equal)

```
int a = 1;
bool b = (a != 2*a-1); // b =
```

Relational Operators

`a != b` (not equal)

```
int a = 1;
bool b = (a != 2*a-1); // b = false
```

Boolean Functions in Mathematics

- Boolean function

$$f : \{0,1\}^2 \to \{0,1\}$$

- 0 corresponds to “false”.
- 1 corresponds to “true”.

$\mathrm{AND}(x, y)$ $x \wedge y$

- “logical And”

$$f : \{0, 1\}^2 \to \{0, 1\}$$

- 0 corresponds to “false”.
- 1 corresponds to “true”.

x	y	$\mathrm{AND}(x, y)$
0	0	0
0	1	0
1	0	0
1	1	1

Logical Operator &&

`a && b` (logical and)

$$\texttt{bool} \times \texttt{bool} \to \texttt{bool}$$

$$\text{R-value} \times \text{R-value} \to \text{R-value}$$

Logical Operator &&

`a && b` (logical and)

```
int n = -1;
int p = 3;
bool b = (n < 0) && (0 < p); //
```

Logical Operator &&

`a && b` (logical and)

```
int n = -1;
int p = 3;
bool b = (n < 0) && (0 < p); // b = true
```

$\mathrm{OR}(x, y)$ $x \vee y$

- "logical Or"

$$f : \{0, 1\}^2 \to \{0, 1\}$$

- 0 corresponds to "false".
- 1 corresponds to "true".

x	y	$\mathrm{OR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	1

Logical Operator `||`

`a || b` (logical or)

$$\texttt{bool} \times \texttt{bool} \rightarrow \texttt{bool}$$

$$\text{R-value} \times \text{R-value} \rightarrow \text{R-value}$$

Logical Operator ||

`a || b` (logical or)

```
int n = 1;
int p = 0;
bool b = (n < 0) || (0 < p); //
```

Logical Operator ||

`a || b` (logical or)

```
int n = 1;
int p = 0;
bool b = (n < 0) || (0 < p); // b = false
```

$\mathrm{NOT}(x)$ $\neg x$

- "logical Not"

$$f : \{0,1\} \to \{0,1\}$$

- 0 corresponds to "false".
- 1corresponds to "true".

x	$\mathrm{NOT}(x)$
0	1
1	0

Logical Operator `!`

`!b` (logical not)

`bool` → `bool`

R-value → R-value

Logical Operator !

`!b` (logical not)

```
int n = 1;
bool b = !(n < 0); //
```

Logical Operator !

`!b` (logical not)

```
int n = 1;
bool b = !(n < 0); // b = true
```

Precedences

```
!b && a
```

Precedences

```
!b && a
```

$$\Updownarrow$$

```
(!b) && a
```

Precedences

```
a && b || c && d
```

Precedences

```
  a && b || c && d
         ⇕
(a && b) || (c && d)
```

Precedences

```
a || b   &&   c || d
```

Precedences

```
a || b   &&   c || d
```

$$\Updownarrow$$

```
a || (b && c) || d
```

Precedences

```
7 + x < y && y != 3 * z || ! b
```

Precedences

The unary logical operator `!`

binds more strongly than

```
7 + x < y && y != 3 * z || (!b)
```

Precedences

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binary arithmetic operators. These

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(7 + x) < y && y != (3 * z) || (!b)
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Precedences

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Precedences

The unary logical operator `!`

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binary arithmetic operators. These

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and these bind more strongly than

binary logical operators.

```
((7 + x) < y) && (y != (3 * z)) || (!b)
```

Completeness

- AND, OR and NOT are the boolean functions available in $\mathrm{C}{+}{+}$.

Completeness: $\mathrm{XOR}(x, y)$ $x \oplus y$

- AND, OR and NOT are the boolean functions available in $\mathrm{C{+}{+}}$.
- Any other *binary* boolean function can be generated from them.

x	y	$\mathrm{XOR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	0

Completeness: $\mathrm{XOR}(x, y)$ $x \oplus y$

$$\mathrm{XOR}(x, y) = \mathrm{AND}(\mathrm{OR}(x, y), \mathrm{NOT}(\mathrm{AND}(x, y))).$$

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$$x \oplus y = (x \vee y) \wedge \neg(x \wedge y).$$

Completeness: $\mathrm{XOR}(x, y)$ $x \oplus y$

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$$x \oplus y = (x \vee y) \wedge \neg(x \wedge y).$$

```
(x || y) && !(x && y)
```

Completeness Proof

- Identify binary boolean functions with their characteristic vector.

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characteristic vector: 0110

Completeness Proof

- Identify binary boolean functions with their characteristic vector.

x	y	$\mathrm{XOR}(x, y)$
0	0	0
0	1	1
1	0	1
1	1	0

characteristic vector: 0110

$$\mathrm{XOR} = f_{0110}$$

Completeness Proof

- Step 1: generate the *fundamental* functions f_{0001}, f_{0010}, f_{0100}, f_{1000}

$$\begin{aligned}
f_{0001} &= \mathrm{AND}(x, y) \\
f_{0010} &= \mathrm{AND}(x, \mathrm{NOT}(y)) \\
f_{0100} &= \mathrm{AND}(y, \mathrm{NOT}(x)) \\
f_{1000} &= \mathrm{NOT}(\mathrm{OR}(x, y))
\end{aligned}$$

Completeness Proof

- Step 2: generate all functions by applying logical or

$$f_{1101} = \mathrm{OR}(f_{1000}, \mathrm{OR}(f_{0100}, f_{0001}))$$

Completeness Proof

- Step 2: generate all functions by applying logical or

$$f_{1101} = \mathrm{OR}(f_{1000}, \mathrm{OR}(f_{0100}, f_{0001}))$$

- Step 3: generate f_{0000}

$$f_{0000} = 0.$$

`bool` vs `int`: Conversion

- `bool` can be used whenever `int` is expected

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`bool`	→	`int`
true	→	1
false	→	0

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- `bool` can be used whenever `int` is expected – and vice versa.

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`int`	→	`bool`
≠0	→	*true*
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≠0	→	*true*
0	→	*false*

```
bool b = 3; // b=true
```

`bool` vs `int`: Conversion

- `bool` can be used whenever `int` is expected – and vice versa.
- Many existing programs use `int` instead of `bool`
  This is bad style originating from the language C .

`bool`	→	`int`
true	→	1
false	→	0

`int`	→	`bool`
≠0	→	*true*
0	→	*false*

```
bool b = 3; // b=true
```

DeMorgan Rules

- `!(a && b) == (!a || !b)`

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`!` (rich *and* beautiful) == (poor *or* ugly)

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- `!(a || b) == (!a && !b)`

`!` (rich *and* beautiful) == (poor *or* ugly)

Application: either ... or (XOR)

```
(x || y)     && !(x && y)
```

Application: either ... or (XOR)

```
(x || y)    && !(x && y)
```

x or y, and not both

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`(x || y)      && !(x && y)`	x or y, and not both
`(x || y)      && (!x || !y)`	x or y, and one of them not

Application: either ... or (XOR)

`(x || y)      && !(x && y)` x or y, and not both

`(x || y)      && (!x || !y)` x or y, and one of them not

`!(!x && !y)   && !(x && y)`

Application: either ... or (XOR)

`(x || y)      && !(x && y)`	x or y, and not both
`(x || y)      && (!x || !y)`	x or y, and one of them not
`!(!x && !y)   && !(x && y)`	not none and not both

Application: either ... or (XOR)

```
(x || y)     && !(x && y)    x or y, and not both

(x || y)     && (!x || !y)   x or y, and one of them not

!(!x && !y)  && !(x && y)    not none and not both

!(!x && !y || x && y)
```

Application: either ... or (XOR)

```
(x || y)       && !(x && y)    x or y, and not both

(x || y)       && (!x || !y)   x or y, and one of them not

!(!x && !y)    && !(x && y)    not none and not both

!(!x && !y || x && y)          not: both or none
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

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x has value 6 ⇒

```
x != 0 && z / x > y
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 6 ⇒

```
true && z / x > y
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 6 ⇒

```
true && z / x > y
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
x != 0 && z / x > y
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
false && z / x > y
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
false
```

Short circuit Evaluation

- Logical operators `&&` and `||` evaluate the *left operand first*.
- If the result is then known, the right operand will *not be* evaluated.

x has value 0 ⇒

```
x != 0 && z / x > y
```

⇒ No division by 0

4. Defensive Programming

Constants and Assertions

Sources of Errors

- Errors that the compiler can find:
  syntactical and some semantical errors

Sources of Errors

- Errors that the compiler can find:
  syntactical and some semantical errors
- Errors that the compiler cannot find:
  runtime errors (always semantical)

The Compiler as Your Friend: Constants

Constants

- are variables with immutable value

```
const int speed_of_light = 299792458;
```

- Usage: `const` before the definition

The Compiler as Your Friend: Constants

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Constants

- are variables with immutable value

```
const int speed_of_light = 299792458;
```

- Usage: `const` before the definition

The Compiler as Your Friend: Constants

- Compiler checks that the `const`-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise :*“value does not change”*

The Compiler as Your Friend: Constants

- Compiler checks that the `const`-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise : *“value does not change”*

The Compiler as Your Friend: Constants

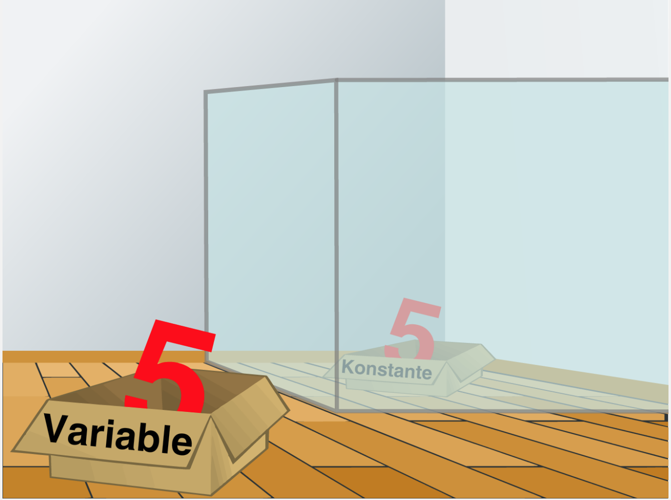
- Compiler checks that the `const`-promise is kept

```
const int speed_of_light = 299792458;
...
speed_of_light = 300000000;
```

compiler: error

- Tool to avoid errors: constants guarantee the promise :*“value does not change”*

Constants: Variables behind Glass



The `const`-guideline

`const`-guideline

For *each variable*, think about whether it will change its value in the lifetime of a program. If not, use the keyword `const` in order to make the variable a constant.

A program that adheres to this guideline is called `const`-correct.

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior

$\gg$ It’s not a bug, it’s a feature! $\ll$

Avoid Sources of Bugs

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2. Check at many places in the code if the program is still on track

Avoid Sources of Bugs

1. Exact knowledge of the wanted program behavior
2. Check at many places in the code if the program is still on track
3. Question the (seemingly) obvious, there could be a typo in the code

Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression `expr` is false

Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression `expr` is false
- requires `#include <cassert>`

Against Runtime Errors: *Assertions*

```
assert(expr)
```

- halts the program if the boolean expression `expr` is false
- requires `#include <cassert>`
- can be switched off (potential performance gain)

Assertions for the $gcd(x, y)$

Check if the program is on track . . .

```
// Input x and y
std::cout << "x =? ";
std::cin >> x;
std::cout << "y =? ";
std::cin >> y;

// Check validity of inputs
assert(x > 0 && y > 0);

... // Compute gcd(x,y), store result in variable a
```

Input arguments for calculation

Assertions for the $gcd(x, y)$

Check if the program is on track . . .

```
// Input x and y
std::cout << "x =? ";
std::cin >> x;
std::cout << "y =? ";
std::cin >> y;

// Check validity of inputs
assert(x > 0 && y > 0);

... // Compute gcd(x,y), store result in variable a
```

Precondition for the ongoing computation

Assertions for the $gcd(x, y)$

... and question the obvious! . . .

```
...
assert(x > 0 && y > 0);
```

Precondition for the ongoing computation

```
... // Compute gcd(x,y), store result in variable a

assert (a >= 1);
assert (x % a == 0 && y % a == 0);
for (int i = a+1; i <= x && i <= y; ++i)
  assert(!(x % i == 0 && y % i == 0));
```

Assertions for the $gcd(x, y)$

... and question the obvious! . . .

```
...
assert(x > 0 && y > 0);

... // Compute gcd(x,y), store result in variable a

assert (a >= 1);
assert (x % a == 0 && y % a == 0);
for (int i = a+1; i <= x && i <= y; ++i)
  assert(!(x % i == 0 && y % i == 0));
```

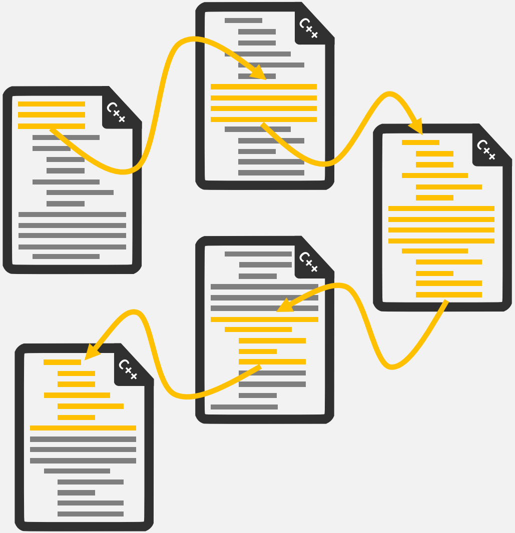
Properties of the gcd

Switch off Assertions

```
#define NDEBUG // To ignore assertions
#include<cassert>

...
assert(x > 0 && y > 0); // Ignored

... // Compute gcd(x,y), store result in variable a

assert(a >= 1); // Ignored
...
```

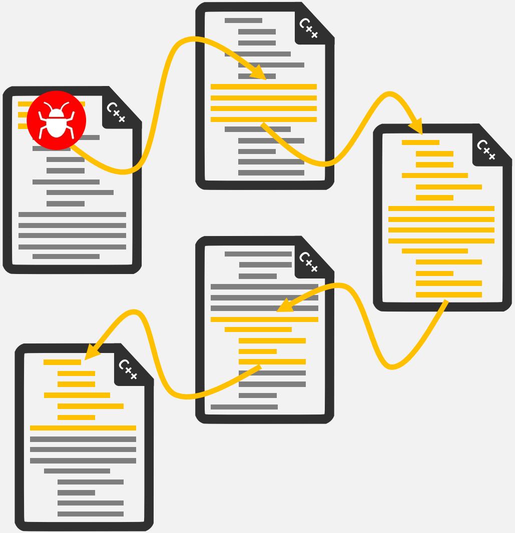
Fail-Fast with Assertions

- Real software: many C++ files, complex control flow



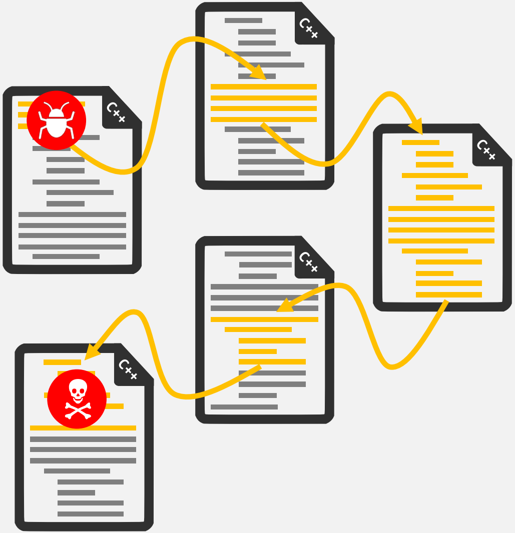
Fail-Fast with Assertions

- Real software: many C++ files, complex control flow



Fail-Fast with Assertions

- Real software: many C++ files, complex control flow
- Errors surface late(r) → impedes error localisation



Fail-Fast with Assertions

- Real software: many C++ files, complex control flow
- Errors surface late(r) → impedes error localisation
- Assertions: Detect errors early

5. Control Structures I

Selection Statements, Iteration Statements, Termination, Blocks

Control Flow

- Up to now: *linear* (from top to bottom)
- Interesting programs require “branches” and “jumps”

0	1	2	3	4	5	6	7	8	9
[8] → **L**	[9] → **R**	**L** = 0? *stop*	**R** > **L**? *springe* *zu* **6**	**L** − **R** → [8]	*springe* *zu* 0	**R** − **L** → [9]	*springe* *zu* 0	b	a

Selection Statements

implement branches

- `if` statement
- `if-else` statement

`if`-Statement

`if` (*condition*)
    statement

`if`-Statement

`if` (*condition*)
    statement

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

`if`-Statement

`if` (*condition*)
    statement

If *condition* is true then *statement* is executed

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

`if`-Statement

`if` (*condition*)
    statement

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
```

- *statement*: arbitrary statement (*body* of the `if`-Statement)
- *condition*: convertible to `bool`

if-else-statement

```
if ( condition )
    statement1
else
    statement2
```

if-else-statement

```
if ( condition )
    statement1
else
    statement2
```

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

if-else-statement

```
if ( condition )
    statement1
else
    statement2
```

If *condition* is true then *statement1* is executed, otherwise *statement2* is executed.

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

`if-else`-statement

```
if ( condition )
    statement1
else
    statement2
```

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

- *condition*: convertible to `bool`.
- *statement1*: *body* of the `if`-branch
- *statement2*: *body* of the `else`-branch

Layout!

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Layout!

```
int a;
std::cin >> a;
if (a % 2 == 0)
    std::cout << "even";
else
    std::cout << "odd";
```

Indentation

Indentation

Iteration Statements

implement “loops”

- `for`-statement
- `while`-statement
- `do`-statement

Compute $1 + 2 + ... + n$

```
// input
std::cout << "Compute the sum 1+...+n for n=?";
unsigned int n;
std::cin >> n;

// computation of sum_{i=1}^n i
unsigned int s = 0;
for (unsigned int i = 1; i <= n; ++i)
    s += i;

// output
std::cout << "1+...+" << n << " = " << s << ".\n";
```

Compute $1 + 2 + ... + n$

```
// input
std::cout << "Compute the sum 1+...+n for n=?";
unsigned int n;
std::cin >> n;

// computation of sum_{i=1}^n i
unsigned int s = 0;
for (unsigned int i = 1; i <= n; ++i)
    s += i;

// output
std::cout << "1+...+" << n << " = " << s << ".\n";
```

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: n == 2, s == 0

i	s

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i	s
i==1	

for-Statement Example

```
for ( unsigned int i=1; i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	i <= 2?	

for-Statement Example

```
for ( unsigned int i=1; i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	wahr	`s == 1`

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
`i==1`	wahr	`s == 1`
`i==2`		

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	i <= 2?	

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	wahr	

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	wahr	s == 3

for-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	wahr	s == 3
i==3		

`for`-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

`i`		`s`
`i==1`	wahr	`s == 1`
`i==2`	wahr	`s == 3`
`i==3`	`i <= 2`?	

for-Statement Example

```
for ( unsigned int i=1; i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

i		s
i==1	wahr	s == 1
i==2	wahr	s == 3
i==3	falsch	

`for`-Statement Example

```
for ( unsigned int i=1;  i <= n ; ++i )
    s += i;
```

Assumptions: `n == 2`, `s == 0`

`i`		`s`
`i==1`	wahr	`s == 1`
`i==2`	wahr	`s == 3`
`i==3`	falsch	

`s == 3`

`for`-Statement: Syntax

`for` (*init statement*`;` *condition*`;` *expression*)
    body statement

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- *init statement*: expression statement, declaration statement, null statement

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- *expression*: any expression

`for`-Statement: Syntax

`for` (*init statement*; *condition*; *expression*)
    body statement

- *init statement*: expression statement, declaration statement, null statement
- *condition*: convertible to `bool`
- *expression*: any expression
- *body statement*: any statement (*body* of the for-statement)

Gauß as a Child (1777 - 1855)

- Math-teacher wanted to keep the pupils busy with the following task:

Gauß as a Child (1777 - 1855)

- Math-teacher wanted to keep the pupils busy with the following task:

Compute the sum of numbers from 1 to 100!

Gauß as a Child (1777 - 1855)

- Math-teacher wanted to keep the pupils busy with the following task:

Compute the sum of numbers from 1 to 100!

- Gauß finished after one minute.

The Solution of Gauß

- The requested number is

$$1 + 2 + 3 + \cdots + 98 + 99 + 100.$$

The Solution of Gauß

- The requested number is

$$1 + 2 + 3 + \cdots + 98 + 99 + 100.$$

- This is half of

$$\begin{array}{rrrrrrrrrr}
 & 1 & + & 2 & + & \cdots & + & 99 & + & 100 \\
+ & 100 & + & 99 & + & \cdots & + & 2 & + & 1 \\
\hline
= & 101 & + & 101 & + & \cdots & + & 101 & + & 101
\end{array}$$

The Solution of Gauß

- The requested number is

$$1 + 2 + 3 + \cdots + 98 + 99 + 100.$$

- This is half of

$$\begin{array}{cccccccccc}
 & 1 & + & 2 & + & \cdots & + & 99 & + & 100 \\
+ & 100 & + & 99 & + & \cdots & + & 2 & + & 1 \\
\hline
= & 101 & + & 101 & + & \cdots & + & 101 & + & 101
\end{array}$$

- Answer: $100 \cdot 101/2 = 5050$

for-Statement: Termination

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

Here and in most cases:

- *expression* changes its value that appears in *condition* .

for-Statement: Termination

```
for (unsigned int i = 1; i <= n; ++i)
    s += i;
```

Here and in most cases:

- After a finite number of iterations *condition* becomes false: *Termination*

Infinite Loops

- Infinite loops are easy to generate:

```
for ( ; ; ) ;
```

  - Die *empty condition* is true.
  - Die *empty expression* has no effect.
  - Die *null statement* has no effect.

Infinite Loops

- Infinite loops are easy to generate:

```
for ( ; ; ) ;
```

  - Die *empty condition* is true.
  - Die *empty expression* has no effect.
  - Die *null statement* has no effect.

- ... but can in general not be automatically detected.

```
for (init; cond; expr) stmt;
```

Halting Problem

Undecidability of the Halting Problem

There is no $\mathrm{C{+}{+}}$ program that can determine for each $\mathrm{C{+}{+}}$-Program P and each input I if the program P terminates with the input I.

[4] Alan Turing, 1936. Theoretical questions of this kind were the main motivation for Alan Turing to construct a computing machine.

Halting Problem

Undecidability of the Halting Problem

There is no $\mathrm{C}{+}{+}$ program that can determine for each $\mathrm{C}{+}{+}$-Program P and each input I if the program P terminates with the input I.

This means that the correctness of programs can in general *not* be automatically checked.[4]

[4] Alan Turing, 1936. Theoretical questions of this kind were the main motivation for Alan Turing to construct a computing machine.

Example: Prime Number Test

Def.: a natural number $n \geq 2$ is a prime number, if no $d \in \{2, \ldots, n-1\}$ divides n .

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A loop that can test this:

```
unsigned int d;
for (d=2; n%d != 0; ++d);
```

Example: Prime Number Test

Def.: a natural number $n \geq 2$ is a prime number, if no $d \in \{2, \ldots, n-1\}$ divides n .

A loop that can test this:

```
unsigned int d;
for (d=2; n%d != 0; ++d);
```

(body is the null statement)

Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value `d=2`, then plus 1 in every iteration (`++d`)

Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value `d=2`, then plus 1 in every iteration (`++d`)
- Exit: `n%d != 0` evaluates to `false` as soon as a divisor is found — at the latest, once `d == n`

Example: Termination

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

- Progress: Initial value `d=2`, then plus 1 in every iteration (`++d`)
- Exit: `n%d != 0` evaluates to `false` as soon as a divisor is found — at the latest, once `d == n`
- Progress guarantees that the exit condition will be reached

Example: Correctness

```
unsigned int d;
for (d=2; n%d != 0; ++d); // for n >= 2
```

Every potential divisor `2 <= d <= n` will be tested. If the loop terminates with `d == n` then and only then is `n` prime.

Blocks

- Blocks group a number of statements to a new statement

```
{statement1 statement2 ... statementN}
```

Blocks

- Blocks group a number of statements to a new statement
- Example: body of the main function

```
int main() {
     ...
}
```

Blocks

- Blocks group a number of statements to a new statement

- Example: loop body

```
for (unsigned int i = 1; i <= n; ++i) {
    s += i;
    std::cout << "partial sum is " << s << "\n";
}
```

Blocks

- Blocks group a number of statements to a new statement
- Beispiel: if / else

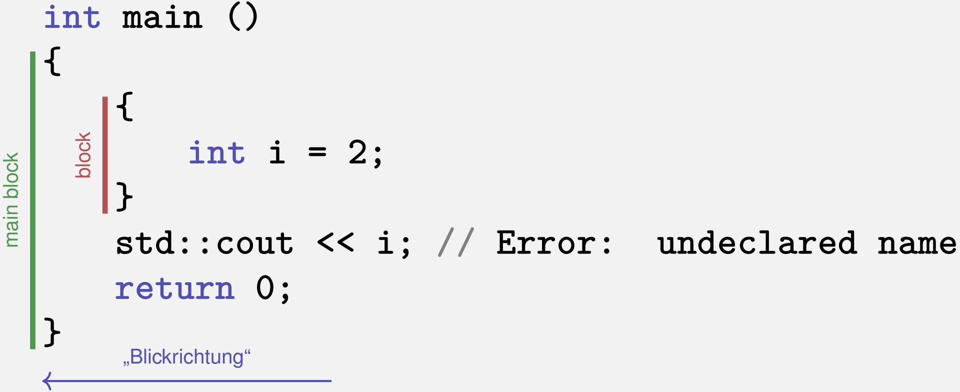
```
if (d < n) // d is a divisor of n in {2,...,n−1}
    std::cout << n << " = " << d << " * " << n / d << ".\n";
else {
    assert (d == n);
    std::cout << n << " is prime.\n";
}
```

6. Control Statements II

Visibility, Local Variables, While Statement, Do Statement, Jump Statements

Visibility

Declaration in a block is not *visible* outside of the block.



```
int main ()
{
    {
        int i = 2;
    }
    std::cout << i; // Error:  undeclared name
    return 0;
}
```

Potential Scope

in the block

```
{
    int i = 2;
    ...
}
```

in function body

```
int main() {
    int i = 2;
    ...
    return 0;
}
```

in control statement

```
for ( int i = 0; i < 10; ++i) {s += i; ... }
```

Potential Scope

in the block

scope

```
{
    int i = 2;
    ...
}
```

in function body

scope

```
int main() {
    int i = 2;
    ...
    return 0;
}
```

in control statement

```
for ( int i = 0; i < 10; ++i) {s += i; ... }
```

scope

Scope

```
int main()
{
   int i = 2;
   for (int i = 0; i < 5; ++i)
      // outputs 0,1,2,3,4
      std::cout << i;
    // outputs 2
    std::cout << i;
   return 0;
}
```

Potential Scope

```
int main()
{
  int i = 2;
  for (int i = 0; i < 5; ++i)
      // outputs 0,1,2,3,4
      std::cout << i;
   // outputs 2
   std::cout << i;
  return 0;
}
```

Real Scope

```
int main()
{
  int i = 2;
  for (int i = 0; i < 5; ++i)
      // outputs 0,1,2,3,4
      std::cout << i;
   // outputs 2
   std::cout << i;
  return 0;
}
```

Local Variables

```
int main()
{
    int i = 5;
    for (int j = 0; j < 5; ++j) {
        std::cout << ++i; // outputs
        int k = 2;
        std::cout << --k; // outputs
    }
}
```

Local Variables

```
int main()
{
    int i = 5;
    for (int j = 0; j < 5; ++j) {
        std::cout << ++i; // outputs 6, 7, 8, 9, 10
        int k = 2;
        std::cout << --k; // outputs 1, 1, 1, 1, 1
    }
}
```

Local Variables

```
int main()
{
    int i = 5;
    for (int j = 0; j < 5; ++j) {
        std::cout << ++i; // outputs
        int k = 2;
        std::cout << --k; // outputs
    }
}
```

Local variables (declaration in a block) have *automatic storage duration*.

`while` Statement

`while` (*condition*)
  statement

while Statement

```
while ( condition )
  statement
```

is equivalent to

```
for ( ; condition ; )
  statement
```

Example: The Collatz-Sequence $(n \in \mathbb{N})$

- $n_0 = n$
- $n_i = \begin{cases} \frac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1} + 1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

The Collatz-Sequence

- $n_0 = n$
- $n_i = \begin{cases} \frac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1}+1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

n=5: 5

The Collatz-Sequence

- $n_0 = n$
- $n_i = \begin{cases} \frac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1} + 1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

n=5: 5, 16

The Collatz-Sequence

- $n_0 = n$
- $n_i = \begin{cases} \dfrac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1}+1 & \text{, if } n_{i-1} \text{ odd} \end{cases} , i \geq 1.$

n=5: 5, 16, 8

The Collatz-Sequence

- $n_0 = n$
- $n_i = \begin{cases} \frac{n_{i-1}}{2} & \text{, if } n_{i-1} \text{ even} \\ 3n_{i-1} + 1 & \text{, if } n_{i-1} \text{ odd} \end{cases}, i \geq 1.$

n=5: 5, 16, 8, 4

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n=5: 5, 16, 8, 4, 2, 1, 4, 2, 1, ... (repetition at 1)

`do` Statement

```
do
  statement
while ( expression );
```

`do` Statement

```
do
  statement
while ( expression );
```

is equivalent to

```
statement
while ( expression )
  statement
```

`break` and `continue` in practice

- Advantage: Can avoid nested `if-else`blocks (or complex disjunctions)

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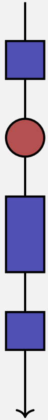
- Advantage: Can avoid nested `if-else`blocks (or complex disjunctions)
- But they result in additional jumps (for- and backwards) and thus potentially complicate the control flow
- Their use is thus controversial, and should be carefully considered

Control Flow `for`

`for` (*init statement* *condition* ; *expression*)
    statement



init-statement

condition

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expression

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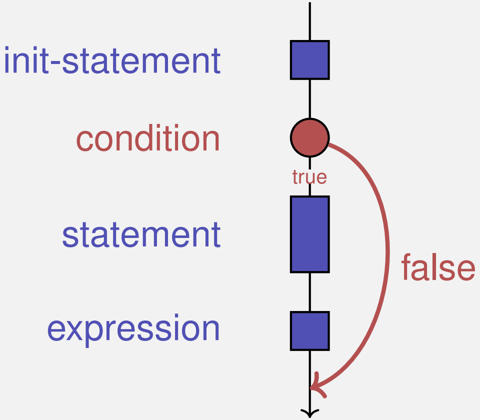

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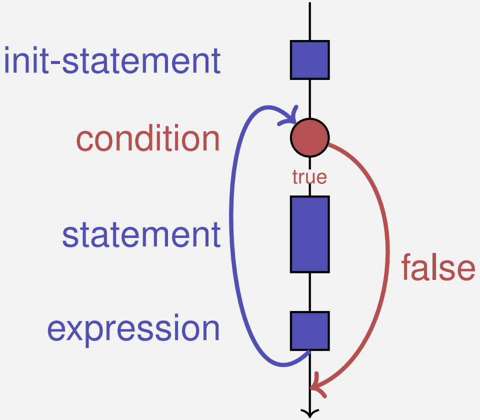
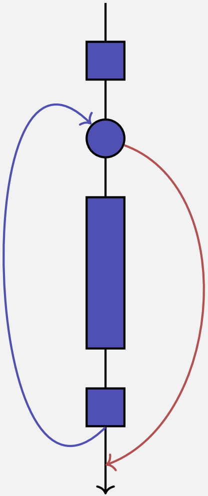


Control Flow `break` and `continue` in `for`



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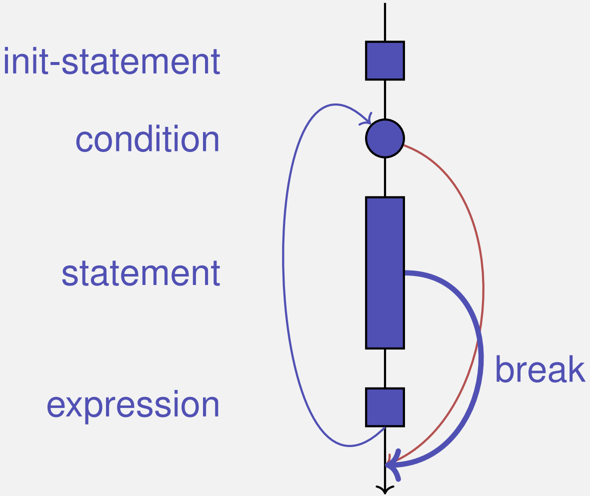

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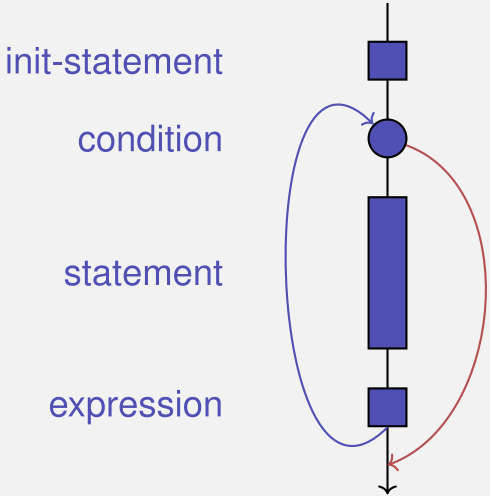

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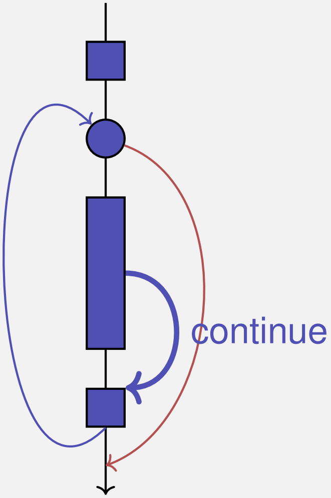

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Control Flow: the Good old Times?

Observation

Actually, we only need `if` and jumps to arbitrary places in the program (`goto`).

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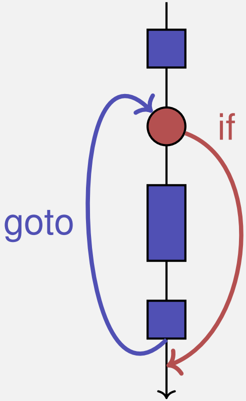

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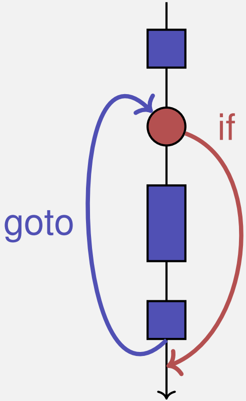
Languages based on them:

- Machine Language



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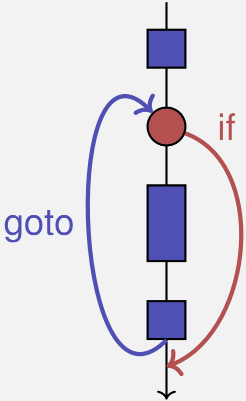

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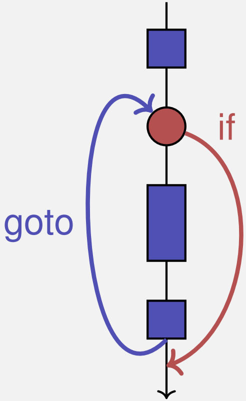
- Machine Language
- Assembler ("higher" machine language)
- BASIC, the first prorgamming language for the general public (1964)



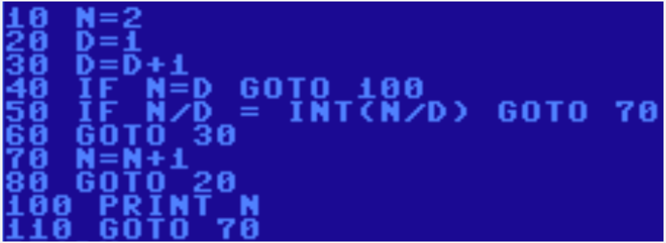
BASIC and home computers...

...allowed a whole generation of young adults to program.

Home-Computer Commodore C64 (1982)

Spaghetti-Code with `goto`

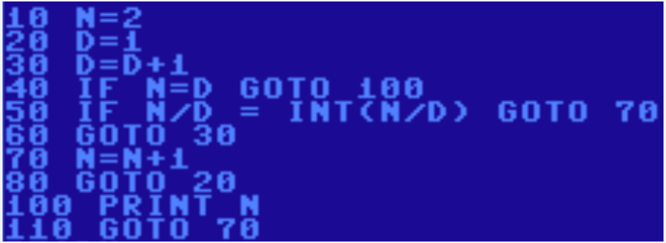
Output of of ???????????
using the programming language BASIC:



```
10  N=2
20  D=1
30  D=D+1
40  IF  N=D  GOTO  100
50  IF  N/D  =  INT(N/D)  GOTO  70
60  GOTO  30
70  N=N+1
80  GOTO  20
100  PRINT  N
110  GOTO  70
```

Spaghetti-Code with `goto`

Output of all prime numbers
using the programming language BASIC:



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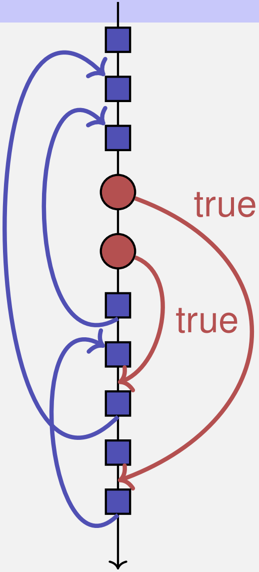

The “right” Iteration Statement

Goals: readability, conciseness, in particular

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- few lines of code
- simple control flow
- simple expressions

Often not all goals can be achieved simultaneously.

Odd Numbers in $\{0, \ldots, 100\}$

First (correct) attempt:

```cpp
for (unsigned int i = 0; i < 100; ++i)
{
    if (i % 2 == 0)
        continue;
    std::cout << i << "\n";
}
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, *less* lines:

```
for (unsigned int i = 0; i < 100; ++i)
{
    if (i % 2 != 0)
        std::cout << i << "\n";
}
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, *simpler* control flow:

```
for (unsigned int i = 1; i < 100; i += 2)
      std::cout << i << "\n";
```

Odd Numbers in $\{0, \ldots, 100\}$

Less statements, *simpler* control flow:

```
for (unsigned int i = 1; i < 100; i += 2)
        std::cout << i << "\n";
```

This is the “right” iteration statement

Outputting Grades

1. Functional requirement:

$$
\begin{aligned}
6 &\rightarrow \texttt{"Excellent ... You passed!"} \\
5, 4 &\rightarrow \texttt{"You passed!"} \\
3 &\rightarrow \texttt{"Close, but ... You failed!"} \\
2, 1 &\rightarrow \texttt{"You failed!"} \\
\textit{otherwise} &\rightarrow \texttt{"Error!"}
\end{aligned}
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\end{aligned}$$

2. Moreover: Avoid duplication of text and code

Outputting Grades with `if` Statements

```
int grade;
...
if (grade == 6) std::cout << "Excellent ... ";
if (4 <= grade && grade <= 6) {
    std::cout << "You passed!";
} else if (1 <= grade && grade < 4) {
    if (grade == 3) std::cout << "Close, but ... ";
    std::cout << "You failed!";
} else std::cout << "Error!";
```

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int grade;
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} else std::cout << "Error!";
```

Disadvantage: Control flow – and thus program behaviour – not quite obvious

Outputting Grades with `switch` Statement

```
switch (grade) {
  case 6: std::cout << "Excellent ... ";
  case 5:
  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
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  default: std::cout << "Error!";
}
```

Jump to matching case

Outputting Grades with `switch` Statement

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Fall-through

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Fall-through

Exit `switch`

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Exit switch

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}
```

← In all other cases

Outputting Grades with `switch` Statement

```cpp
switch (grade) {
  case 6: std::cout << "Excellent ... ";
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  case 4: std::cout << "You passed!";
    break;
  case 3: std::cout << "Close, but ... ";
  case 2:
  case 1: std::cout << "You failed!";
    break;
  default: std::cout << "Error!";
}
```

Advantage: Control flow clearly recognisable

The `switch`-Statement

`switch` (*condition*)
statement

- *condition*: Expression, convertible to integral type
- *statement* : arbitrary statemet, in which **`case`** and **`default`**-lables are permitted, **`break`** has a special meaning.

The `switch`-Statement

```
switch (condition)
         statement
```

- *condition*: Expression, convertible to integral type
- *statement* : arbitrary statemet, in which `case` and `default`-lables are permitted, `break` has a special meaning.
- Use of fall-through property is controversial and should be carefully considered (corresponding compiler warning can be enabled)

7. Floating-point Numbers I

Types `float` and `double`; Mixed Expressions and Conversion; Holes in the Value Range

“Proper” Calculation

```
// Input
std::cout << "Temperature in degrees Celsius =? ";
int celsius;
std::cin >> celsius;

// Computation and output
std::cout << celsius << " degrees Celsius are "
         << 9 * celsius / 5 + 32 << " degrees Fahrenheit.\\n";
```

```
28 degrees Celsius are 82 degrees Fahrenheit.
```

“Proper” Calculation

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```

```
28 degrees Celsius are 82 degrees Fahrenheit.
```

richtig wäre 82.4

“Proper” Calculation

```
// Input
std::cout << "Temperature in degrees Celsius =? ";
float celsius; // Enable fractional numbers
std::cin >> celsius;

// Computation and output
std::cout << celsius << " degrees Celsius are "
         << 9 * celsius / 5 + 32 << " degrees Fahrenheit.\\n";
```

```
28 degrees Celsius are 82.4 degrees Fahrenheit.
```

Fixed-point numbers

- fixed number of integer places (e.g. 7)
- fixed number of decimal places (e.g. 3)

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```
    82.4 = 0000082.400
```

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- fixed number of decimal places (e.g. 3)

```
     82.4 = 0000082.400
```

Disadvantages

- Value range is getting *even* smaller than for integers.

Fixed-point numbers

- fixed number of integer places (e.g. 7)
- fixed number of decimal places (e.g. 3)

`0.0824 = 0000000.082` ←— third place truncated

Disadvantages

- Representability depends on the position of the decimal point.

Floating-point numbers

- Observation: same number, different representations with varying “efficiency”, e.g.

  $$\begin{aligned} 0.0824 &= 0.00824 \cdot 10^{1} &&= 0.824 \cdot 10^{-1} \\ &= 8.24 \cdot 10^{-2} &&= 824 \cdot 10^{-4} \end{aligned}$$

  Number of *significant digits* remains constant

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  Number of *significant digits* remains constant

- Floating-point number representation thus:

  - Fixed number of significant places (e.g. 10),
  - Plus position of the decimal point via exponent
  - Number is $\mathit{Mantissa} \times 10^{\mathit{Exponent}}$

Types `float` and `double`

- are the fundamental C++ types for floating point numbers
- approximate the field of real numbers $(\mathbb{R}, +, \times)$ from mathematics

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- have a big value range, sufficient for many applications:
  - **`float`**: approx. 7 digits, exponent up to ± 38
  - **`double`**: approx. 15 digits, exponent up to ± 308
- are fast on most computers (hardware support)

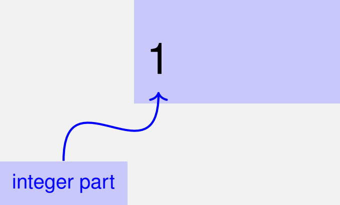
Arithmetic Operators

Analogous to `int`, but . . .

- Division operator / models a “proper” division (real-valued, not integer)
- No modulo operator, i.e. no `%`

Literals

are different from integers



1

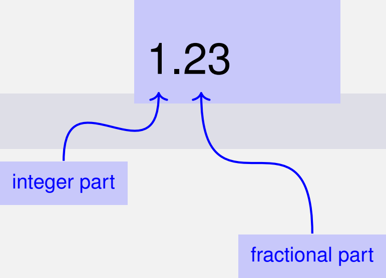
integer part

Literals

are different from integers by providing

- decimal point

`1.0` : type **`double`**, value 1



1.23

integer part

fractional part

Literals

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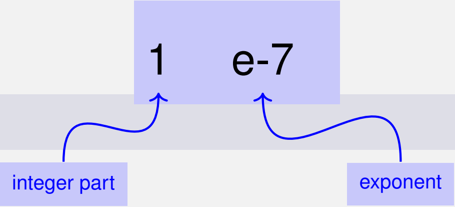

- or exponent.

  `1e3` : type **`double`**, value 1000

Literals

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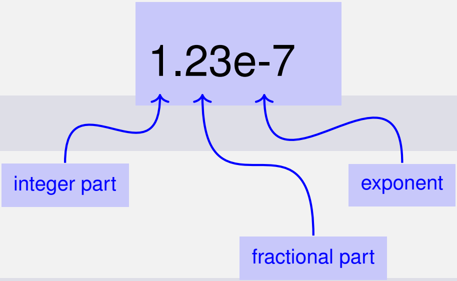

1.23e-7

integer part

fractional part

exponent

- and / or exponent.

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  `1.23e-7` : type **`double`**, value $1.23 \cdot 10^{-7}$

Literals

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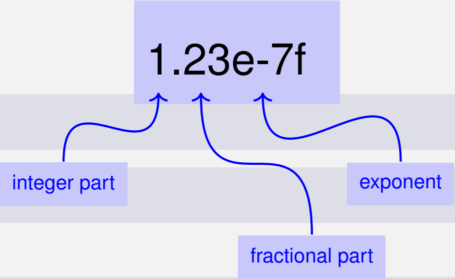
  `1.0` : type **`double`**, value 1

  `1.27f` : type **`float`**, value 1.27



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  `1.23e-7f` : type **`float`**, value $1.23 \cdot 10^{-7}$

Computing with `float`: Example

Approximating the Euler-Number

$$e = \sum_{i=0}^{\infty} \frac{1}{i!} \approx 2.71828\ldots$$

using the first 10 terms.

Computing with `float`: Euler Number

```
std::cout << "Approximating the Euler number... \n";

// values for i-th iteration, initialized for i = 0
float t = 1.0f; // term 1/i!
float e = 1.0f; // i-th approximation of e

// iteration 1, ..., n
for (unsigned int i = 1; i < 10; ++i) {
    t /= i;    // 1/(i-1)! -> 1/i!
    e += t;
    std::cout << "Value after term " << i << ": "
              << e << "\n";
}
```

Computing with `float`: Euler Number

```
Value after term 1: 2
Value after term 2: 2.5
Value after term 3: 2.66667
Value after term 4: 2.70833
Value after term 5: 2.71667
Value after term 6: 2.71806
Value after term 7: 2.71825
Value after term 8: 2.71828
Value after term 9: 2.71828
```

Mixed Expressions, Conversion

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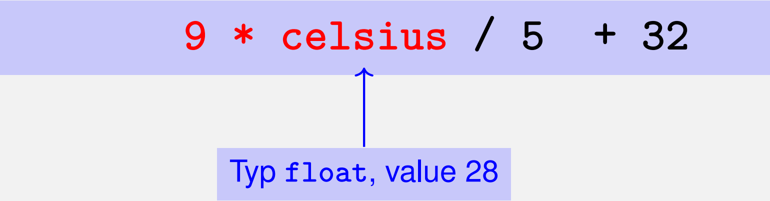

```
9 * celsius / 5  + 32
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Typ `float`, value 28

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.
- In mixed expressions integers are converted to floating point numbers.

```
9 * 28.0f / 5  + 32
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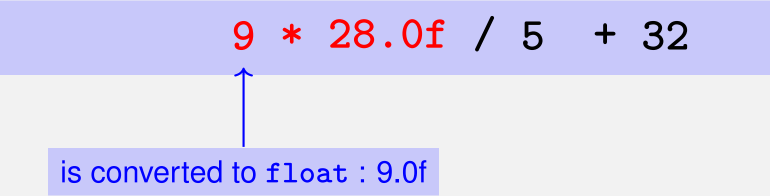

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is converted to `float` : 9.0f

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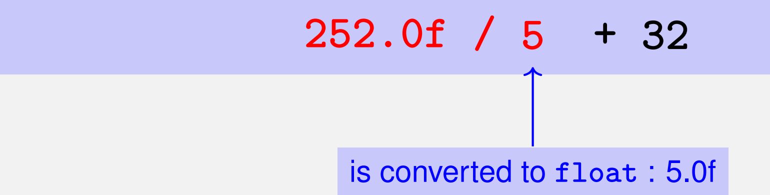

```
252.0f / 5  + 32
```

is converted to `float` : 5.0f

Mixed Expressions, Conversion

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- In mixed expressions integers are converted to floating point numbers.

```
50.4f + 32
```

is converted to `float` : 32.0f

Mixed Expressions, Conversion

- Floating point numbers are more general than integers.
- In mixed expressions integers are converted to floating point numbers.

```
                              82.4f
```

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;

float n2;
std::cout << "Second number =? ";
std::cin >> n2;

float d;
std::cout << "Their difference =? ";
std::cin >> d;

std::cout << "Computed difference - input difference = "
         << n1 - n2 - d << "\n";
```

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;
```

input 1.5

```
float n2;
std::cout << "Second number =? ";
std::cin >> n2;
```

input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.5

```
std::cout << "Computed difference - input difference = "
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input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.5

```
std::cout << "Computed difference - input difference = "
         << n1 - n2 - d << "\n";
```

output 0

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;
```

input 1.1

```
float n2;
std::cout << "Second number =? ";
std::cin >> n2;
```

input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.1

```
std::cout << "Computed difference - input difference = "
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```

Holes in the value range

```
float n1;
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std::cin >> n1;
```

input 1.1

```
float n2;
std::cout << "Second number =? ";
std::cin >> n2;
```

input 1.0

```
float d;
std::cout << "Their difference =? ";
std::cin >> d;
```

input 0.1

```
std::cout << "Computed difference - input difference = "
          << n1 - n2 - d << "\n";
```

output 2.23517e-8

Holes in the value range

```
float n1;
std::cout << "First number  =? ";
std::cin >> n1;

float n2;
std::cout << "Second number =? ";
std::cin >> n2;

float d;
std::cout << "Their difference =? ";
std::cin >> d;

std::cout << "Computed difference - input difference = "
         << n1 - n2 - d << "\n";
```

input 1.1

input 1.0

input 0.1

output 2.23517e-8

What is going on here?

Value range

Integer Types:

- Over- and Underflow relatively frequent, but ...
- the value range is contiguous (no holes): $\mathbb{Z}$ is “discrete”.

Value range

Integer Types:

- Over- and Underflow relatively frequent, but ...
- the value range is contiguous (no holes): $\mathbb{Z}$ is “discrete”.

Floating point types:

- Overflow and Underflow seldom, but ...
- there are holes: $\mathbb{R}$ is “continuous”.

8. Floating-point Numbers II

Floating-point Number Systems; IEEE Standard; Limits of Floating-point Arithmetics; Floating-point Guidelines; Harmonic Numbers

Floating-point Number Systems

A Floating-point number system is defined by the four natural numbers:

- $\beta \geq 2$, the base,
- $p \geq 1$, the precision (number of places),
- $e_{\min}$, the smallest possible exponent,
- $e_{\max}$, the largest possible exponent.

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Notation:

$$F(\beta, p, e_{\min}, e_{\max})$$

Floating-point number Systems

$F(\beta, p, e_{\min}, e_{\max})$ contains the numbers

$$\pm \sum_{i=0}^{p-1} d_i \beta^{-i} \cdot \beta^e,$$

$$d_i \in \{0, \ldots, \beta - 1\}, \quad e \in \{e_{\min}, \ldots, e_{\max}\}.$$

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represented in base β:

$$\pm\, d_0 \bullet d_1 \ldots d_{p-1} \times \beta^e,$$

Floating-point Number Systems

Representations of the decimal number 0.1 (with $\beta = 10$):

$$1.0 \cdot 10^{-1}, \quad 0.1 \cdot 10^{0}, \quad 0.01 \cdot 10^{1}, \quad \ldots$$

Different representations due to choice of exponent

Normalized representation

Normalized number:

$$\pm\, d_0 \bullet d_1 \ldots d_{p-1} \times \beta^e, \qquad d_0 \neq 0$$

Normalized representation

Normalized number:

$$\pm\, d_0{\bullet}d_1 \ldots d_{p-1} \times \beta^e, \qquad d_0 \neq 0$$

Remark 1

The normalized representation is unique and therefore prefered.

Normalized representation

Normalized number:

$$\pm\, d_0{\bullet}d_1 \ldots d_{p-1} \times \beta^{e}, \qquad d_0 \neq 0$$

Remark 2

The number 0, as well as all numbers smaller than $\beta^{e_{\min}}$, have no normalized representation (we will come back to this later)

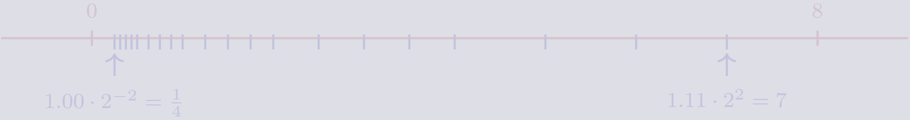
Set of Normalized Numbers

$$F^*(\beta, p, e_{\min}, e_{\max})$$

Normalized Representation

Example $F^*(2, 3, -2, 2)$ (only positive numbers)

$d_0{\bullet}d_1d_2$	$e = -2$	$e = -1$	$e = 0$	$e = 1$	$e = 2$
1.00_2	0.25	0.5	1	2	4
1.01_2	0.3125	0.625	1.25	2.5	5
1.10_2	0.375	0.75	1.5	3	6
1.11_2	0.4375	0.875	1.75	3.5	7



Normalized Representation

Example $F^*(2, \mathbf{3}, -2, 2)$ (only positive numbers)

$d_{0\bullet}d_1d_2$	$e=-2$	$e=-1$	$e=0$	$e=1$	$e=2$
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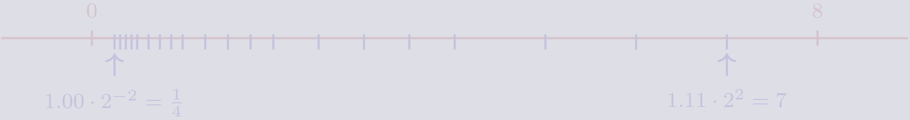

Normalized Representation

Example $F^*(2, 3, \mathbf{-2}, 2)$ (only positive numbers)

$d_0{\bullet}d_1d_2$	$\mathbf{e = -2}$	$e = -1$	$e = 0$	$e = 1$	$e = 2$
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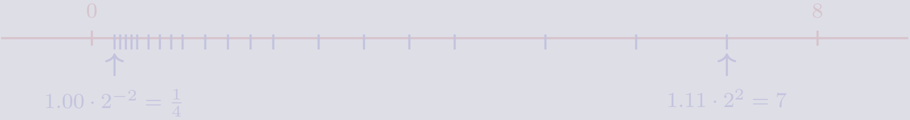

Normalized Representation

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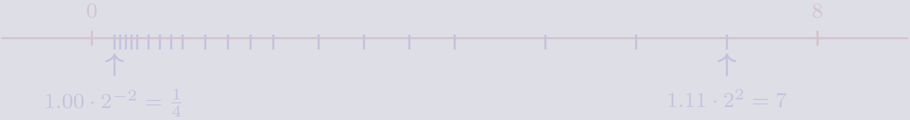

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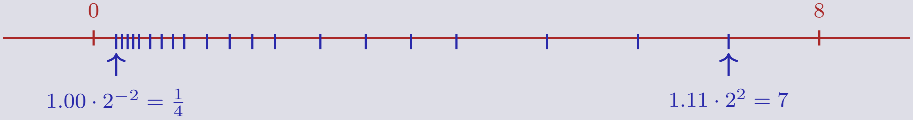

Binary and Decimal Systems

- Internally the computer computes with $\beta = 2$ (binary system)
- Literals and inputs have $\beta = 10$ (decimal system)

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Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$x = \sum_{i=0}^{\infty} b_i 2^{-i}$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$x = b_0 \bullet b_1 b_2 b_3 \ldots$$

Conversion $(0 < x < 2)$

Computation of the *binary representation*:

$$\begin{aligned} x &= b_0{\bullet}b_1b_2b_3\ldots \\ &= b_0 + 0{\bullet}b_1b_2b_3\ldots \end{aligned}$$

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$$\begin{aligned} x &= b_0{\bullet}b_1b_2b_3\ldots \\ &= b_0 + 0{\bullet}b_1b_2b_3\ldots \\ &\Longrightarrow \\ (x - b_0) &= 0{\bullet}b_1b_2b_3b_4\ldots \end{aligned}$$

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```
for (int b_0; x != 0; x = 2 * (x - b_0)) {
  b_0 = (x >= 1);
  std::cout << b_0;
}
```

Example (binary)

$$
\begin{aligned}
x &= 1_{\bullet}01011 \\
&= 1 + 0_{\bullet}01011 \\
&\Longrightarrow \\
2 \cdot (x - 1) &= 0_{\bullet}1011
\end{aligned}
$$

Example (binary)

$$\begin{aligned} x &= 1_{\bullet}\mathbf{01011} \\ &= 1 + 0_{\bullet}01011 \\ &\Longrightarrow \\ 2\cdot(x-1) &= \mathbf{0_{\bullet}1011} \end{aligned}$$

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Example (binary)

$$\begin{aligned} x &= 0{\bullet}\mathbf{1011} \\ &= 0 + 0{\bullet}1011 \\ &\Longrightarrow \\ 2 \cdot (x - 0) &= \mathbf{1{\bullet}011} \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= \mathbf{1}_\bullet 011 \\ &= \mathbf{1} + 0_\bullet 011 \\ &\Longrightarrow \\ 2 \cdot (x - \mathbf{1}) &= 0_\bullet 11 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 1{\bullet}\mathbf{011} \\ &= 1 + 0{\bullet}011 \\ &\Longrightarrow \\ 2 \cdot (x - 1) &= \mathbf{0{\bullet}11} \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= \mathbf{0}{\bullet}11 \\ &= \mathbf{0} + 0{\bullet}11 \\ &\Longrightarrow \\ 2 \cdot (x - \mathbf{0}) &= 1{\bullet}1 \end{aligned}$$

Example (binary)

$$\begin{aligned} x &= 0\bullet 11 \\ &= 0 + 0\bullet 11 \\ &\Longrightarrow \\ 2 \cdot (x - 0) &= 1\bullet 1 \end{aligned}$$

Example (binary)

$$
\begin{aligned}
x &= 1{\bullet}1 \\
&= 1 + 0{\bullet}1 \\
&\Longrightarrow \\
2 \cdot (x - 1) &= 1
\end{aligned}
$$

Example (binary)

$$
\begin{aligned}
x &= 1\bullet\mathbf{1} \\
&= 1 + 0\bullet 1 \\
&\Longrightarrow \\
2\cdot(x-1) &= \mathbf{1}
\end{aligned}
$$

Example (binary)

$$\begin{aligned} x &= 1 \\ &= 1 + 0 \\ &\implies \\ 2 \cdot (x - 1) &= 0 \end{aligned}$$

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Binary representation of 1.1_{10}

x	b_i	$x - b_i$	$2(x - b_i)$
1.1	$b_0 = \mathbf{1}$		

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0.4	$b_2 = \mathbf{0}$	0.4	0.8

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1.6	$b_4 = \mathbf{1}$	0.6	1.2
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Binary representation of 1.1_{10}

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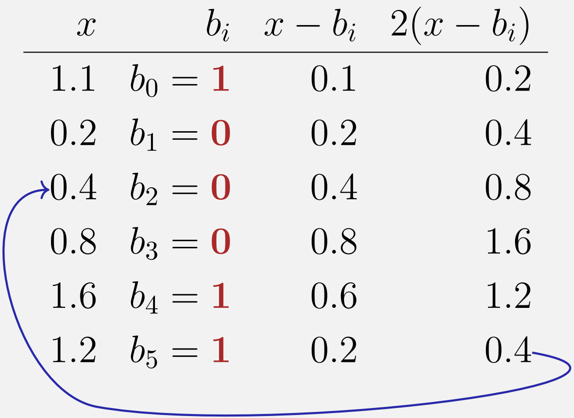

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Binary representation of 1.1_{10}

x	b_i	$x - b_i$	$2(x - b_i)$
1.1	$b_0 = 1$	0.1	0.2
0.2	$b_1 = 0$	0.2	0.4
0.4	$b_2 = 0$	0.4	0.8
0.8	$b_3 = 0$	0.8	1.6
1.6	$b_4 = 1$	0.6	1.2
1.2	$b_5 = 1$	0.2	0.4

$\Rightarrow$ $1.0\overline{0011}$, periodic, *not* finite

Binary Number Representations of 1.1 and 0.1

- are not finite $\Rightarrow$ conversion errors
- `1.1f` und `0.1f`: *Approximations* of 1.1 and 0.1
- In `diff.cpp`: $1.1 - 1.0 \neq 0.1$

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- In `diff.cpp`: $1.1 - 1.0 \neq 0.1$

Binary Number Representations of 1.1 and 0.1

on my computer:

$$
\begin{aligned}
\texttt{1.1} &= \underline{1.1000000000000000}888178\ldots \\
\texttt{1.1f} &= \underline{1.1000000}238418\ldots
\end{aligned}
$$

Computing with Floating-point Numbers

is nearly as simple as with integers.

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$
\begin{array}{rr}
 & 1.111 \cdot 2^{-2} \\
+ & 1.011 \cdot 2^{-1}
\end{array}
$$

1. adjust exponents by denormalizing one number

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \checkmark \end{array}$$

1. adjust exponents by denormalizing one number

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

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2. binary addition of the significands

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline & = 100.101 \cdot 2^{-2} \checkmark \end{array}$$

2. binary addition of the significands

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline = & 100.101 \cdot 2^{-2} \end{array}$$

3. renormalize

Computing with Floating-point Numbers

Example ($\beta = 2, p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline & = 1.00101 \cdot 2^{0} \checkmark \end{array}$$

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Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

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\begin{array}{rr}
 & 1.111 \cdot 2^{-2} \\
+ & 10.110 \cdot 2^{-2} \\
\hline
 & = 1.00101 \cdot 2^{0}
\end{array}
$$

4. round to p significant places, if necessary

Computing with Floating-point Numbers

Example ($\beta = 2$, $p = 4$):

$$\begin{array}{rr} & 1.111 \cdot 2^{-2} \\ + & 10.110 \cdot 2^{-2} \\ \hline & = 1.001 \cdot 2^{0} \checkmark \end{array}$$

4. round to p significant places, if necessary

The IEEE Standard 754

- defines floating-point number systems and their rounding behavior
- is used nearly everywhere

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The IEEE Standard 754

- Single precision (**`float`**) numbers:

$F^*(2, 24, -126, 127)$ (32 bit) plus 0, ∞, $\ldots$

- Double precision (**`double`**) numbers:

$F^*(2, 53, -1022, 1023)$ (64 bit) plus 0, ∞, $\ldots$

- All arithmetic operations round the *exact* result to the next representable number

The IEEE Standard 754

- Single precision (`float`) numbers:

  $F^*(2, 24, -126, 127)$ (32 bit) plus $0, \infty, \ldots$

- Double precision (`double`) numbers:

  $F^*(2, 53, -1022, 1023)$ (64 bit) plus $0, \infty, \ldots$

- All arithmetic operations round the *exact* result to the next representable number

Example: 32-bit Representation of a Floating Point Number

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

$\pm$ Exponent Mantisse

$$\pm \begin{matrix} 2^{-126}, \ldots, 2^{127} \\ 0, \infty, \ldots \end{matrix}$$

$$\begin{matrix} 1.00000000000000000000000 \\ \cdots \\ 1.11111111111111111111111 \end{matrix}$$

Floating-point Rules

Rule 1

Rule 1

Do not test rounded floating-point numbers for equality.

```
for (float i = 0.1; i != 1.0; i += 0.1)
    std::cout << i << "\n";
```

Floating-point Rules Rule 1

Rule 1

Do not test rounded floating-point numbers for equality.

```
for (float i = 0.1; i != 1.0; i += 0.1)
    std::cout << i << "\n";
```

Floating-point Rules Rule 1

Rule 1

Do not test rounded floating-point numbers for equality.

```
for (float i = 0.1; i != 1.0; i += 0.1)
    std::cout << i << "\n";
```

endless loop because i never becomes exactly 1

Floating-point Rules Rule 2

Rule 2

Do not add two numbers of very different orders of magnitude!

Floating-point Rules Rule 2

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Do not add two numbers of very different orders of magnitude!

$$\begin{array}{r} 1.000 \cdot 2^5 \\ +1.000 \cdot 2^0 \end{array}$$

Floating-point Rules

Rule 2

Do not add two numbers of very different orders of magnitude!

$$\begin{aligned} &1.000 \cdot 2^5 \\ &+1.000 \cdot 2^0 \\ &= 1.00001 \cdot 2^5 \end{aligned}$$

Floating-point Rules

Rule 2

Do not add two numbers of very different orders of magnitude!

$$
\begin{aligned}
&1.000 \cdot 2^5 \\
&+1.000 \cdot 2^0 \\
&= 1.00001 \cdot 2^5 \\
&\text{“}{=}\text{”}\ 1.000 \cdot 2^5 \quad \text{(Rounding on 4 places)}
\end{aligned}
$$

Floating-point Rules Rule 2

Rule 2

Do not add two numbers of very different orders of magnitude!

$$\begin{aligned} & 1.000 \cdot 2^5 \\ & +1.000 \cdot 2^0 \\ = {} & 1.00001 \cdot 2^5 \\ \text{“}{=}\text{”} {} & 1.000 \cdot 2^5 \end{aligned}$$

(Rounding on 4 places)

Addition of 1 does not have any effect!

Harmonic Numbers

Rule 2

- The n-the harmonic number is

$$H_n = \sum_{i=1}^{n} \frac{1}{i}$$

Harmonic Numbers

Rule 2

- The n-the harmonic number is

$$H_n = \sum_{i=1}^{n} \frac{1}{i} \approx \ln n.$$

Harmonic Numbers

Rule 2

- The n-the harmonic number is

$$H_n = \sum_{i=1}^{n} \frac{1}{i} \approx \ln n.$$

- This sum can be computed in forward or backward direction, which is mathematically clearly equivalent

Harmonic Numbers Rule 2

```
std::cout << "Compute H_n for n =? ";
unsigned int n;
std::cin >> n;

float fs = 0;
for (unsigned int i = 1; i <= n; ++i)
    fs += 1.0f / i;
std::cout << "Forward sum = " << fs << "\n";

float bs = 0;
for (unsigned int i = n; i >= 1; --i)
    bs += 1.0f / i;
std::cout << "Backward sum = " << bs << "\n";
```

Harmonic Numbers Rule 2

```
std::cout << "Compute H_n for n =? ";
unsigned int n;
std::cin >> n;

float fs = 0;
for (unsigned int i = 1; i <= n; ++i)
    fs += 1.0f / i;
std::cout << "Forward sum = " << fs << "\n";

float bs = 0;
for (unsigned int i = n; i >= 1; --i)
    bs += 1.0f / i;
std::cout << "Backward sum = " << bs << "\n";
```

Input: **10000000**

forwards: **15.4037**

backwards: **16.686**

Harmonic Numbers Rule 2

```
std::cout << "Compute H_n for n =? ";
unsigned int n;
std::cin >> n;

float fs = 0;
for (unsigned int i = 1; i <= n; ++i)
    fs += 1.0f / i;
std::cout << "Forward sum = " << fs << "\n";

float bs = 0;
for (unsigned int i = n; i >= 1; --i)
    bs += 1.0f / i;
std::cout << "Backward sum = " << bs << "\n";
```

Input: **100000000**

forwards: **15.4037**

backwards: **18.8079**

Harmonic Numbers Rule 2

Observation:

- The forward sum stops growing at some point and is “really” wrong.
- The backward sum approximates H_n well.

Harmonic Numbers

Rule 2

Observation:

- The forward sum stops growing at some point and is “really” wrong.
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Harmonic Numbers

Rule 2

Observation:

- The forward sum stops growing at some point and is “really” wrong.
- The backward sum approximates H_n well.

Explanation:

- For $1 + 1/2 + 1/3 + \cdots$, later terms are too small to actually contribute
- Problem similar to $2^5 + 1$ “$=$” 2^5

Harmonic Numbers Rule 2

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- The backward sum approximates H_n well.

Explanation:

- For $1 + 1/2 + 1/3 + \cdots$, later terms are too small to actually contribute
- Problem similar to $2^5 + 1$ “$=$” 2^5

Floating-point Guidelines Rule 3

Rule 4

Do not subtract two numbers with a very similar value.

Cancellation problems, cf. lecture notes.

Literature

David Goldberg: What Every Computer Scientist Should Know About Floating-Point Arithmetic (1991)

Randy Glasbergen, 1996

9. Functions I

Defining and Calling Functions, Evaluation of Function Calls, the Type `void`

Computing Powers

```
double a;
int n;
std::cin >> a; // Eingabe a
std::cin >> n; // Eingabe n

double result = 1.0;
if (n < 0) { // a^n = (1/a)^(-n)
  a = 1.0/a;
  n = -n;
}
for (int i = 0; i < n; ++i)
  result *= a;

std::cout << a << "^" << n << " = " << result << ".\n";
```

Computing Powers

```
double a;
int n;
std::cin >> a; // Eingabe a
std::cin >> n; // Eingabe n
```

```
double result = 1.0;
if (n < 0) { // a^n = (1/a)^(-n)
  a = 1.0/a;
  n = -n;
}
for (int i = 0; i < n; ++i)
  result *= a;
```

```
std::cout << a << "^" << n << " = " << result << ".\n";
```

Computing Powers

```
double a;
int n;
std::cin >> a; // Eingabe a
std::cin >> n; // Eingabe n

double result = 1.0;
if (n < 0) { // a^n = (1/a)^(-n)
  a = 1.0/a;
  n = -n;
}
for (int i = 0; i < n; ++i)
  result *= a;

std::cout << a << "^" << n << " = " << pow(a,n) << ".\n";
```

"Funktion `pow`"

Function to Compute Powers

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Function to Compute Powers

```
double pow(double b, int e){...}
```

Function to Compute Powers

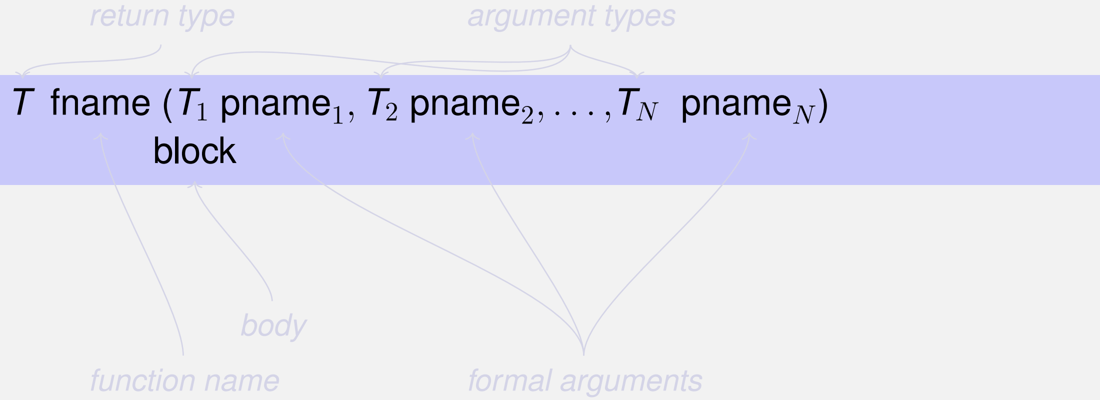
```
// Prog: callpow.cpp
// Define and call a function for computing powers.
#include <iostream>

  double pow(double b, int e){...}

int main()
{
  std::cout << pow( 2.0, -2) << "\n"; // outputs 0.25
  std::cout << pow( 1.5, 2) << "\n"; // outputs 2.25
  std::cout << pow(-2.0, 9) << "\n"; // outputs -512

  return 0;
}
```

Function Definitions



return type

argument types

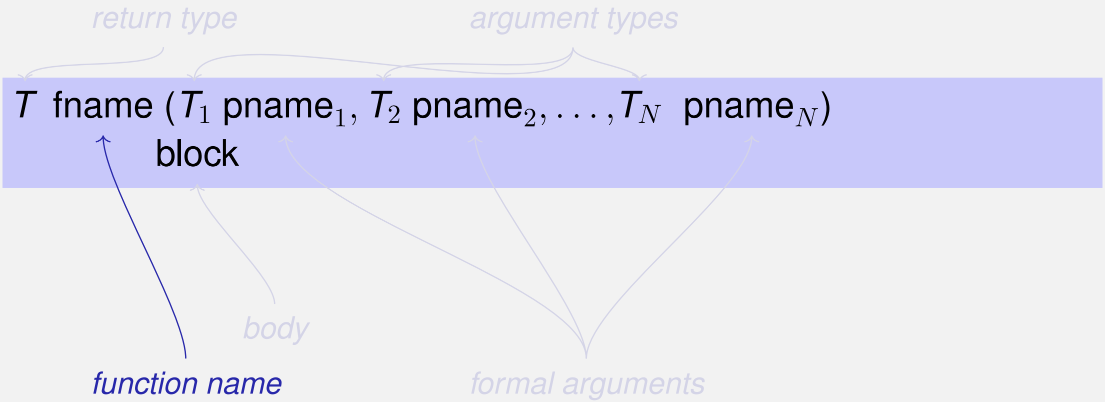
T fname (T_1 pname$_1$, T_2 pname$_2$, $\ldots$, T_N pname$_N$)
block

body

function name

formal arguments

Function Definitions



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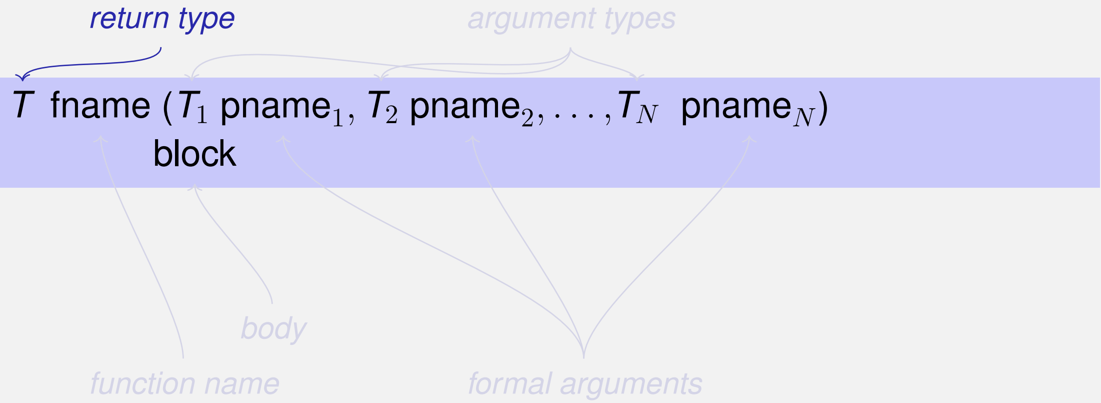
Function Definitions



return type

argument types

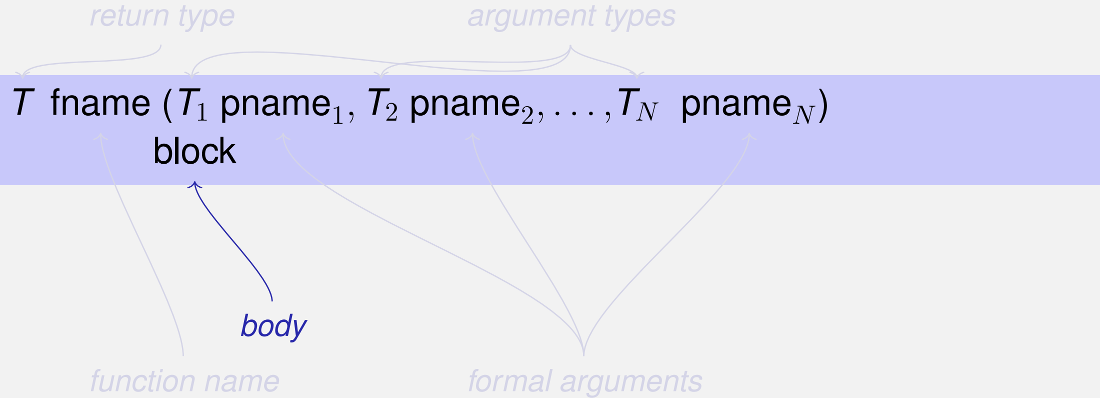
T fname (T_1 pname$_1$, T_2 pname$_2$, $\ldots$, T_N pname$_N$)
block

body

function name

formal arguments

Function Definitions



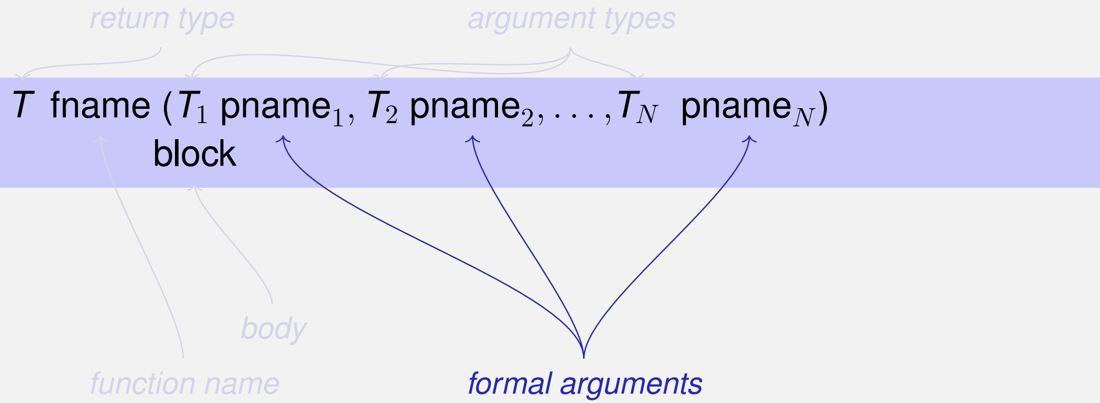
return type *argument types*

T fname (T_1 pname$_1$, T_2 pname$_2$, $\ldots$, T_N pname$_N$)
    block

body

function name *formal arguments*

Function Definitions



return type

argument types

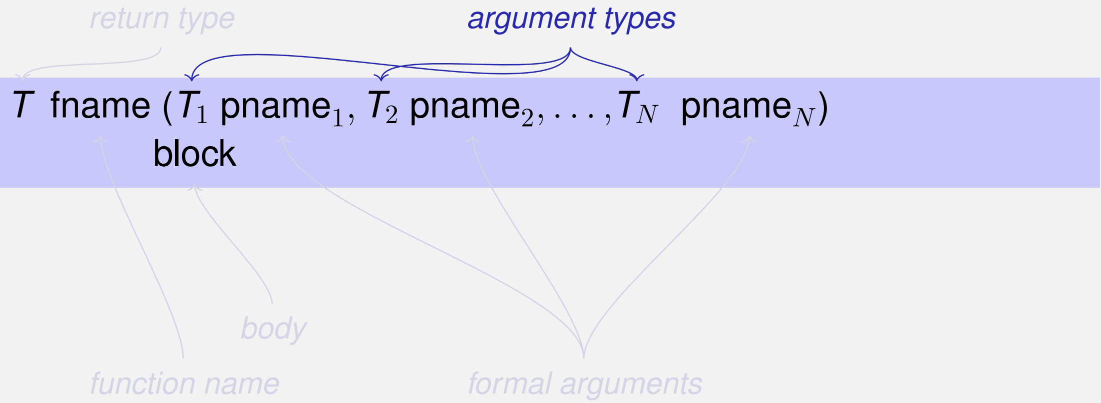
T fname (T_1 pname$_1$, T_2 pname$_2$, $\ldots$, T_N pname$_N$)
block

body

function name

formal arguments

Function Definitions



return type

argument types

T fname $(T_1$ pname$_1$, T_2 pname$_2, \ldots, T_N$ pname$_N)$
block

body

function name

formal arguments

Xor

```
// post: returns l XOR r
bool Xor(bool l, bool r)
{
    return l && !r || !l && r;
}
```

Harmonic

```
// PRE: n >= 0
// POST: returns nth harmonic number
//       computed with backward sum
float Harmonic(int n)
{
    float res = 0;
    for (unsigned int i = n; i >= 1; --i)
        res += 1.0f / i;
    return res;
}
```

min

```
// POST: returns the minimum of a and b
int min(int a, int b)
{
    if (a<b)
        return a;
    else
        return b;
}
```

Function Calls

fname (*expression*$_1$, *expression*$_2$, . . . , *expression*$_N$)

- All call arguments must be convertible to the respective formal argument types.
- The function call is an expression of the return type of the function.

Example: `pow(a,n)`: Expression of type `double`

Function Calls

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Example: `pow(a,n)`: Expression of type `double`

Function Calls

fname (*expression*$_1$, *expression*$_2$, . . . , *expression*$_N$)

- All call arguments must be convertible to the respective formal argument types.
- The function call is an expression of the return type of the function.

Example: `pow(a,n)`: Expression of type `double`

Function Calls

For the types we know up to this point it holds that:

- Call arguments are R-values
  ↪ *call-by-value* (also *pass-by-value*), more on this soon
- The function call is an R-value.

Function Calls

For the types we know up to this point it holds that:

- Call arguments are R-values
  $\hookrightarrow$ *call-by-value* (also *pass-by-value*), more on this soon
- The function call is an R-value.

fname: R-value $\times$ R-value $\times \cdots \times$ R-value $\longrightarrow$ R-value

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

Call of pow

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}


...
pow (2.0, -2)
```

b=2.0,e=-2

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

b=2.0,e=-2

// ok

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

result=1.0

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

e == -2

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

b=0.5

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

e=2

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=0

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=0

result=0.5

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i) → i=1
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=1

result=0.25

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

i=2

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

result=0.25

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

result=0.25

Return

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

Return

value: 0.25

Evaluation Function Call

```
double pow(double b, int e){
    assert (e >= 0 || b != 0);
    double result = 1.0;
    if (e<0) {
        // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        result * = b;
    return result;
}

...
pow (2.0, -2)
```

→ value: 0.25

Scope of Formal Arguments

```
int main(){
    double b = 2.0;
    int e = -2;
    double z = pow(b, e);

    std::cout << z; // 0.25
    std::cout << b; // 2
    std::cout << e; // -2
    return 0;
}
```

Scope of Formal Arguments

```
double pow(double b, int e){
    double r = 1.0;
    if (e<0) {
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        r * = b;
    return r;
}
```

```
int main(){
    double b = 2.0;
    int e = -2;
    double z = pow(b, e);

    std::cout << z; // 0.25
    std::cout << b; // 2
    std::cout << e; // -2
    return 0;
}
```

Scope of Formal Arguments

```cpp
double pow(double b, int e){
    double r = 1.0;
    if (e<0) {
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e ; ++i)
        r *= b;
    return r;
}
```

```cpp
int main(){
    double b = 2.0;
    int e = -2;
    double z = pow(b, e);

    std::cout << z; // 0.25
    std::cout << b; // 2
    std::cout << e; // -2
    return 0;
}
```

Not the formal arguments `b` and `e` of pow but the variables defined here locally in the body of `main`

The type `void`

```
// POST: "(i, j)" has been written to standard output
???? print_pair(int i, int j) {
    std::cout << "(" << i << ", " << j << ")\n";
}

int main() {
    print_pair(3,4); // outputs (3, 4)
    return 0;
}
```

The type `void`

```
// POST: "(i, j)" has been written to standard output
void print_pair(int i, int j) {
    std::cout << "(" << i << ", " << j << ")\n";
}

int main() {
    print_pair(3,4); // outputs (3, 4)
    return 0;
}
```

The type `void`

- Fundamental type with empty value range

The type `void`

- Fundamental type with empty value range
- Usage as a return type for functions that do *only* provide an effect

`void`-Functions

- do not require `return`.
- execution ends when the end of the function body is reached or if
- `return;` is reached

10. Functions II

Pre- and Postconditions Stepwise Refinement, Scope, Libraries and Standard Functions

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

Preconditions

precondition:

- what is required to hold when the function is called?
- defines the *domain* of the function

0^e is undefined for $e < 0$

```
// PRE: e >= 0 || b != 0.0
```

Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Postconditions

postcondition:

- What is guaranteed to hold after the function call?
- Specifies *value* and *effect* of the function call.

Here only value, no effect.

```
// POST: return value is b^e
```

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Funktion `pow`: works for all numbers $b \neq 0$

Pre- and Postconditions

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- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Funktion `pow`: works for all numbers $b \neq 0$

Pre- and Postconditions

- should be correct:
- *if* the precondition holds when the function is called *then* also the postcondition holds after the call.

Funktion `pow`: works for all numbers $b \neq 0$

White Lies...

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect:

- Overflow if e or b are too large
- b^e potentially not representable as a double (holes in the value range!)

White Lies…

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

is formally incorrect:

- Overflow if e or b are too large
- b^e potentially not representable as a double (holes in the value range!)

White Lies are Allowed

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
```

Mathematical conditions as a compromise between formal correctness and lax practice

Checking Preconditions…

- Preconditions are only comments.

Checking Preconditions. . .

- Preconditions are only comments.
- How can we ensure that they hold when the function is called?

. . . with assertions

```
#include <cassert>
...
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e) {
    assert (e >= 0 || b != 0);
    double result = 1.0;
    ...
}
```

Postconditions with Asserts

- The result of “complex” computations is often easy to check.

Postconditions with Asserts

- The result of “complex” computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

Postconditions with Asserts

- The result of “complex” computations is often easy to check.
- Then the use of asserts for the postcondition is worthwhile.

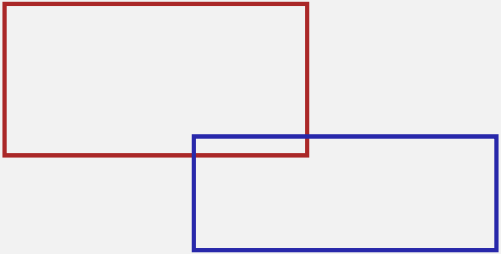
```
// PRE: the discriminant p*p/4 - q is nonnegative
// POST: returns larger root of the polynomial x^2 + p x + q
double root(double p, double q)
{
    assert(p*p/4 >= q); // precondition
    double x1 = - p/2 + sqrt(p*p/4 - q);
    assert(equals(x1*x1+p*x1+q,0)); // postcondition
    return x1;
}
```

Stepwise Refinement

- A simple *technique* to solve complex problems

Example Problem

Find out if two rectangles intersect!



Top-Down Approach

- Formulate a coarse solution using
  - comments
  - ficticious functions
- Repeated refinement:
  - comments ⟶ program text
  - ficticious functions ⟶ function definitions

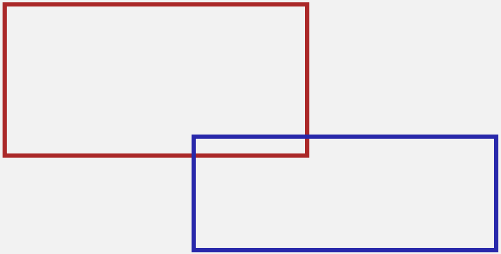
Top-Down Approach

- Formulate a coarse solution using
  - comments
  - ficticious functions
- Repeated refinement:
  - comments $\longrightarrow$ program text
  - ficticious functions $\longrightarrow$ function definitions

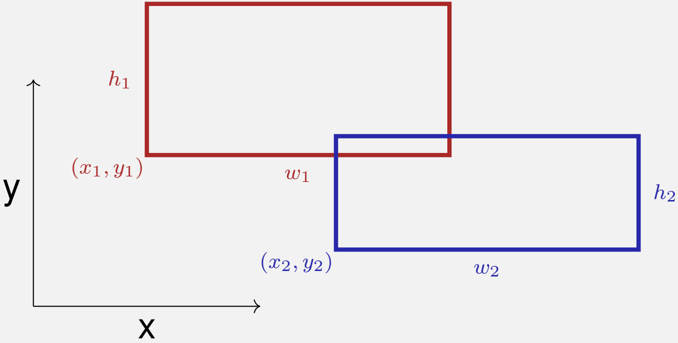
Coarse Solution

```
int main()
{
    // input rectangles

    // intersection?

    // output solution

    return 0;
}
```

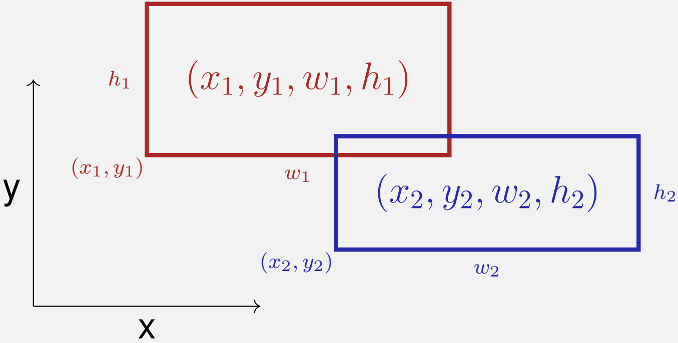
Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

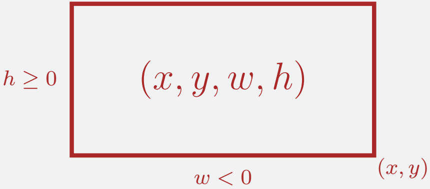


Refinement 1: Input Rectangles



Refinement 1: Input Rectangles

Width w and height h may be negative.



Refinement 1: Input Rectangles

```
int main()
{
    std::cout << "Enter two rectangles [x y w h each] \n";
    int x1, y1, w1, h1;
    std::cin >> x1 >> y1 >> w1 >> h1;
    int x2, y2, w2, h2;
    std::cin >> x2 >> y2 >> w2 >> h2;

    // intersection?

    // output solution

    return 0;
}
```

Refinement 2: Intersection? and Output

```
int main()
{
    input rectangles ✓

    bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);

    if (clash)
        std::cout << "intersection!\n";
    else
        std::cout << "no intersection!\n";

    return 0;
}
```

Refinement 3: Intersection Function…

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}

int main() {
    input rectangles ✓

    intersection? ✓

    output solution ✓

    return 0;
}
```

Refinement 3: Intersection Function. . .

```
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

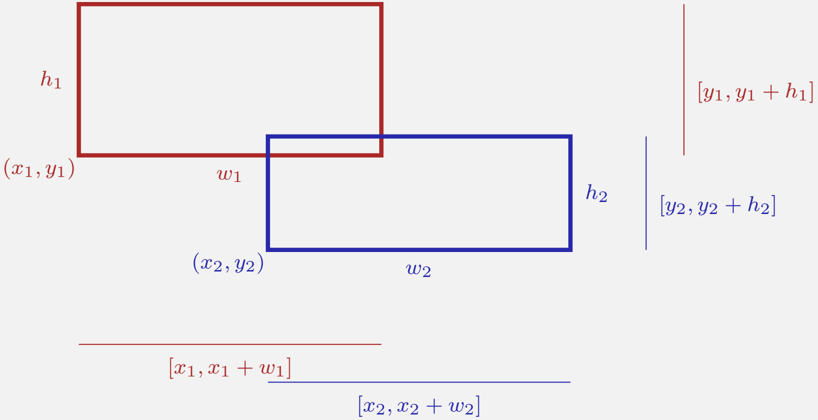
Function main ✓

Refinement 3: ...with PRE and POST

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles,
//      where w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1) and
//      (x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                          int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

Refinement 4: Interval Intersection

Two rectangles intersect if and only if their x and y-intervals intersect.



Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                         int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

Refinement 4: Interval Intersections

```
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                         int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2); ✓
}
```

Refinement 4: Interval Intersections

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return false; // todo
}
```

`Function rectangles_intersect` ✓

`Function main` ✓

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
       && min(a1, b1) <= max(a2, b2);
}
```

Refinement 5: Min and Max

```
// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
   return max(a1, b1) >= min(a2, b2)
      && min(a1, b1) <= max(a2, b2); ✓
}
```

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

Function intervals_intersect ✓

Function rectangles_intersect ✓

Function main ✓

Refinement 5: Min and Max

```
// POST: the maximum of x and y is returned
int max(int x, int y){
    if (x>y) return x; else return y;
}
```

already exists in the standard library

```
// POST: the minimum of x and y is returned
int min(int x, int y){
    if (x<y) return x; else return y;
}
```

Function intervals_intersect ✓

Function rectangles_intersect ✓

Function main ✓

Back to Intervals

```
// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//      with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
    return std::max(a1, b1) >= std::min(a2, b2)
       && std::min(a1, b1) <= std::max(a2, b2); ✓
}
```

Look what we have achieved step by step!

```cpp
#include <iostream>
#include <algorithm>

// PRE: [a1, b1], [a2, h2] are (generalized) intervals,
//     with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect(int a1, int b1, int a2, int b2)
{
  return std::max(a1, b1) >= std::min(a2, b2)
      && std::min(a1, b1) <= std::max(a2, b2);
}

// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
//      w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect(int x1, int y1, int w1, int h1,
                         int x2, int y2, int w2, int h2)
{
    return intervals_intersect(x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect(y1, y1 + h1, y2, y2 + h2);
}
```

```cpp
int main ()
{
  std::cout << "Enter two rectangles [x y w h each]\n";
  int x1, y1, w1, h1;
  std::cin >> x1 >> y1 >> w1 >> h1;
  int x2, y2, w2, h2;
  std::cin >> x2 >> y2 >> w2 >> h2;
  bool clash = rectangles_intersect(x1,y1,w1,h1,x2,y2,w2,h2);
  if (clash)
    std::cout << "intersection!\n";
  else
    std::cout << "no intersection!\n";
  return 0;
}
```

Result

- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

  `rectangles_intersect`

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- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

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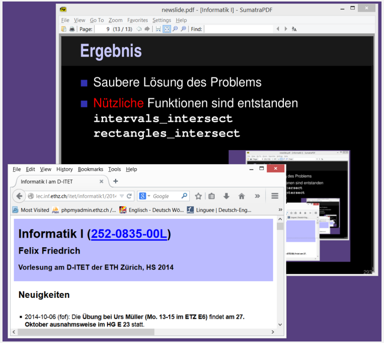
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Ergebnis

- Saubere Lösung des Problems
- Nützliche Funktionen sind entstanden

  `intervals_intersect`

  `rectangles_intersect`

Informatik I (252-0835-00L)

Felix Friedrich

Vorlesung am D-ITET der ETH Zürich, HS 2014

Neuigkeiten

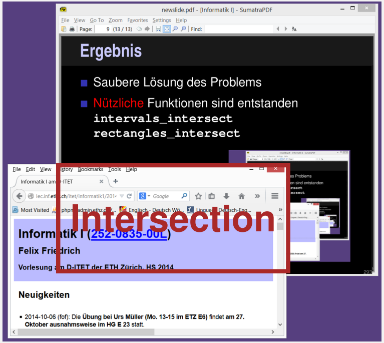
- 2014-10-06 (fof): Die **Übung bei Urs Müller (Mo. 13-15 im ETZ E6)** findet **am 27. Oktober ausnahmsweise im HG E 23** statt.

Result

- Clean solution of the problem
- Useful functions have been implemented

  `intervals_intersect`

  `rectangles_intersect`



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Intersection
293

Ergebnis

- Saubere Lösung des Problems
- Nützliche Funktionen sind entstanden

  `intervals_intersect`

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Inform atik I (252-0835-00L)

Felix Frie drich

Vorlesung a n D-ITET der ETH Zürich, HS 2014

Neuigkeiten

- 2014-10-06 (fof): Die **Übung bei Urs Müller (Mo. 13-15 im ETZ E6)** findet **am 27. Oktober ausnahmsweise im HG E 23** statt.

Where can a Function be Used?

```
#include <iostream>

int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

Scope of a Function

- is the part of the program where a function can be called

Scope of a Function

- is the part of the program where a function can be called

Extension by *declaration* of a function: like the definition but without `{...}`.

```
double pow(double b, int e);
```

This does not work. . .

```
#include <iostream>



int main()
{
    std::cout << f(1); // Error:  f undeclared
    return 0;
}

int f(int i) // Scope of f starts here
{
    return i;
}
```

Gültigkeit f

. . . but this works!

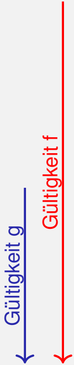
```
#include <iostream>
int f(int i); // Gueltigkeitsbereich von f ab hier

int main()
{
    std::cout << f(1);
    return 0;
}

int f(int i)
{
    return i;
}
```

Forward Declarations, why?

Functions that mutually call each other:

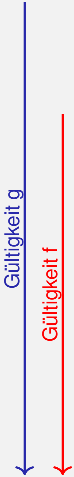


```
int f(...) // f valid from here
{
    g(...) // g undeclared
}

int g(...) // g valid from here!
{
    f(...) // ok
}
```

Forward Declarations, why?

Functions that mutually call each other:



```
int g(...); // g valid from here

int f(...) // f valid from here
{
    g(...) // ok
}


int g(...)
{
    f(...) // ok
}
```

Reusability

- Functions such as `rectangles_intersect` and `pow` are useful in many programs.

Reusability

- Functions such as `rectangles_intersect` and `pow` are useful in many programs.
- “Solution”: copy-and-paste the source code

Level 1: Outsource the Function

```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Level 1: Outsource the Function

`double pow(double b, int e);` in **separate file** `mymath.cpp`

Level 1: Include the Function

```
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"

int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

Level 1: Include the Function

```
// Prog: callpow2.cpp
// Call a function for computing powers.

#include <iostream>
#include "mymath.cpp"

int main()
{
  std::cout << pow( 2.0, -2) << "\n";
  std::cout << pow( 1.5, 2) << "\n";
  std::cout << pow( 5.0, 1) << "\n";
  std::cout << pow(-2.0, 9) << "\n";

  return 0;
}
```

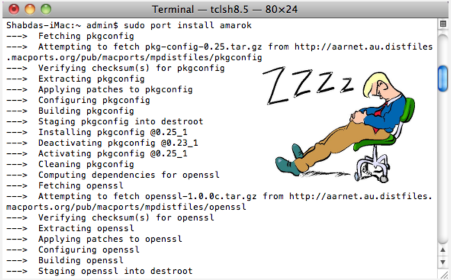
in working directory

Disadvantage of Including

- `#include` copies the file (`mymath.cpp`) into the main program (`callpow2.cpp`).

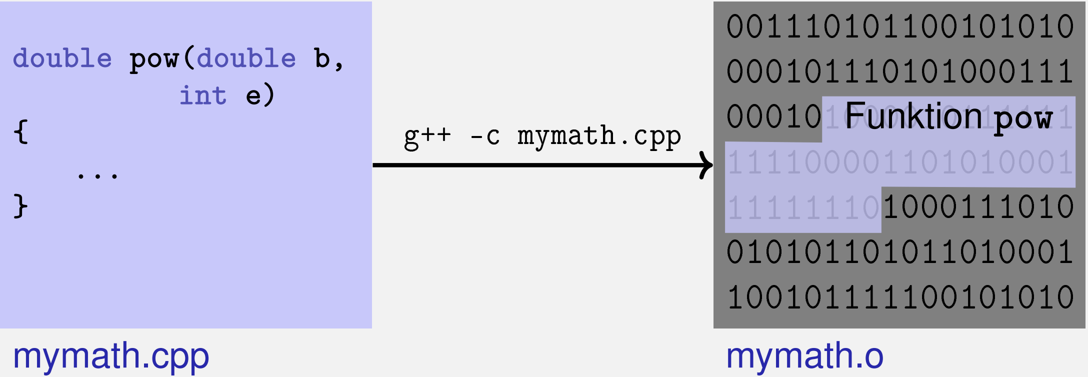
Disadvantage of Including

- `#include` copies the file (`mymath.cpp`) into the main program (`callpow2.cpp`).
- The compiler has to (re)compile the function definition for each program



```
Shabdas-iMac:~ admin$ sudo port install amarok
--->  Fetching pkgconfig
--->  Attempting to fetch pkg-config-0.25.tar.gz from http://aarnet.au.distfiles
.macports.org/pub/macports/mpdistfiles/pkgconfig
--->  Verifying checksum(s) for pkgconfig
--->  Extracting pkgconfig
--->  Applying patches to pkgconfig
--->  Configuring pkgconfig
--->  Building pkgconfig
--->  Staging pkgconfig into destroot
--->  Installing pkgconfig @0.25_1
--->  Deactivating pkgconfig @0.23_1
--->  Activating pkgconfig @0.25_1
--->  Cleaning pkgconfig
--->  Computing dependencies for openssl
--->  Fetching openssl
--->  Attempting to fetch openssl-1.0.0c.tar.gz from http://aarnet.au.distfiles.
macports.org/pub/macports/mpdistfiles/openssl
--->  Verifying checksum(s) for openssl
--->  Extracting openssl
--->  Applying patches to openssl
--->  Configuring openssl
--->  Building openssl
--->  Staging openssl into destroot
```

Level 2: Separate Compilation



```
double pow(double b,
           int e)
{
    ...
}
```

mymath.cpp

```
g++ -c mymath.cpp
```

```
001110101100101010
000101110101000111
00010 Funktion pow
111100001101010001
11111110 1000111010
010101101011010001
100101111100101010
```

mymath.o

Level 2: Separate Compilation

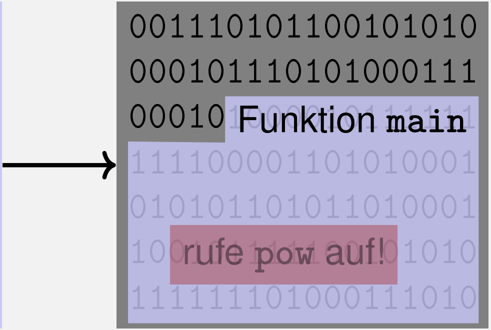
```
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e);
```

mymath.h

Level 2: Separate Compilation

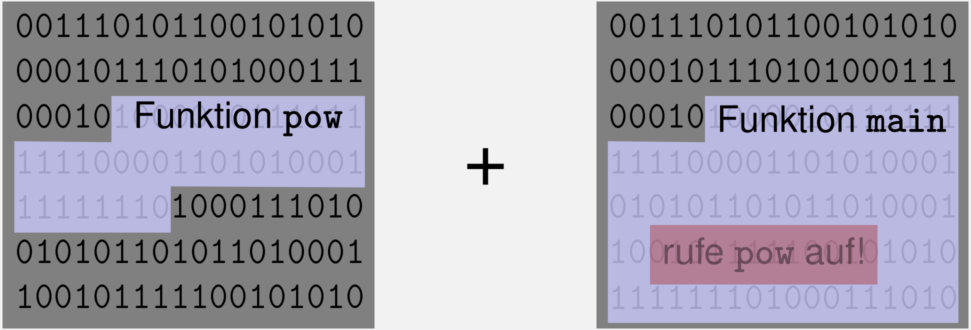
```
#include <iostream>
#include "mymath.h"
int main()
{
  std::cout << pow(2,−2) << "\n";
  return 0;
}
```

callpow3.cpp



callpow3.o

The linker unites...



001110101100101010
000101110101000111
00010 Funktion `pow`
111100001101010001
11111110 1000111010
010101101011010001
100101111100101010

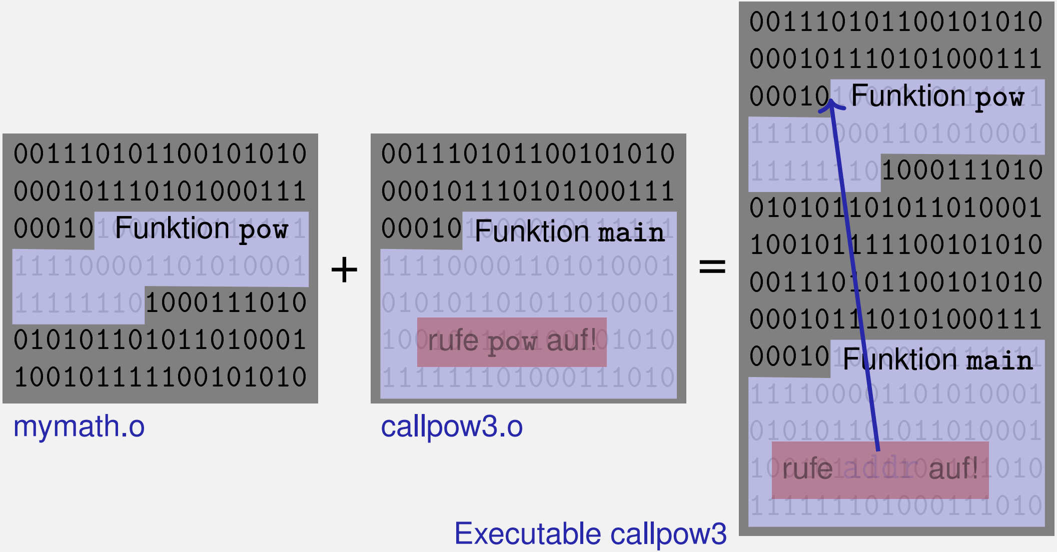
mymath.o

+

001110101100101010
000101110101000111
00010 Funktion `main`
111100001101010001
010101101011010001
10 rufe `pow` auf! 1010
111111101000111010

callpow3.o

... what belongs together



Funktion pow

mymath.o

+

Funktion main

rufe pow auf!

callpow3.o

=

Funktion pow

Funktion main

rufe addr auf!

Executable callpow3

Availability of Source Code?

Observation

`mymath.cpp` (source code) is not required any more when the `mymath.o` (object code) is available.

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Many vendors of libraries do not provide source code.

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`mymath.cpp` (source code) is not required any more when the `mymath.o` (object code) is available.

Many vendors of libraries do not provide source code.

Header files then provide the *only* readable informations.

Open-Source Software

- Source code is generally available.

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- Only this allows the continued development of code by users and dedicated “hackers”.

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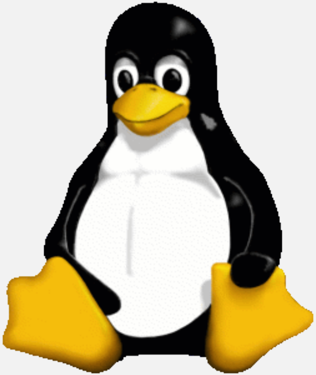
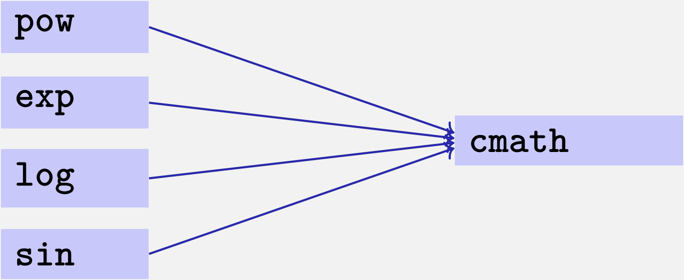


Libraries

- Logical grouping of similar functions



Name Spaces...

```
// cmath
namespace std {

  double pow(double b, int e);

  ....
  double exp(double x);
  ...
}
```

. . . Avoid Name Conflicts

```
#include <cmath>
#include "mymath.h"

int main()
{
    double x = std::pow(2.0, -2); // <cmath>
    double y = pow(2.0, -2); // mymath.h
}
```

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with `std::pow`);
- lead to interesting and efficient programs in a simple way;

Functions from the Standard Library

- help to avoid re-inventing the wheel (such as with `std::pow`);
- lead to interesting and efficient programs in a simple way;
- guarantee a quality standard that cannot easily be achieved with code written from scratch.

Example: Prime Number Test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, n-1\}$ dividing n .

```
unsigned int d;
for (d=2; n % d != 0; ++d);
```

Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \dots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

Prime Number test with `sqrt`

$n \geq 2$ is a prime number if and only if there is no d in $\{2, \ldots, \lfloor\sqrt{n}\rfloor\}$ dividing n .

```
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

- This works because `std::sqrt` rounds to the next representable `double` number (IEEE Standard 754).

Functions Should be More Capable!

```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2);
}
```

Functions Should be More Capable! Swap ?

```
void swap(int x, int y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // fail!  ☹
}
```

Functions Should be More Capable! Swap ?

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2);
}
```

Functions Should be More Capable! Swap ?

```
// POST: values of x and y are exchanged
void swap(int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap(a, b);
    assert(a==1 && b==2); // ok! ☺
}
```

Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.

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- Not a new concept specific to functions, but rather a new class of types

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- Not a new concept specific to functions, but rather a new class of types

Reference types (e.g. `int&`)

11. Reference Types

Reference Types: Definition and Initialization, Pass By Value, Pass by Reference, Temporary Objects, Constants, Const-References

Swap!

```
// POST: values of x and y are exchanged
void swap (int& x, int& y) {
 int t = x;
 x = y;
 y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap (a, b);
    assert (a == 1 && b == 2); // ok!  ☺
}
```

Reference Types

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↑

Reference Types

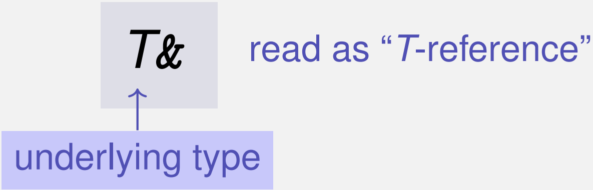
Reference Types: Definition

$T\&$ read as “*T*-reference”

$\uparrow$

underlying type

Reference Types: Definition



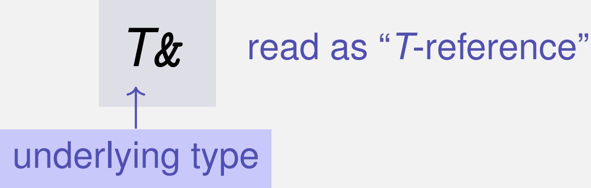
T&

read as "*T*-reference"

underlying type

- *T*& has the same range of values and functionality as *T*, ...

Reference Types: Definition



$T\&$

read as “T-reference”

underlying type

- $T\&$ has the same range of values and functionality as T, ...
- but initialization and assignment work differently.

Anakin Skywalker alias Darth Vader

Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; //  alias
darth_vader = 22;

std::cout << anakin_skywalker;
```

Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; //  alias
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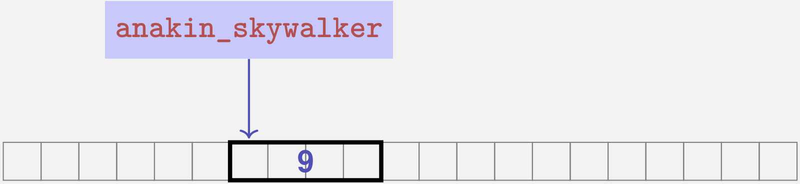


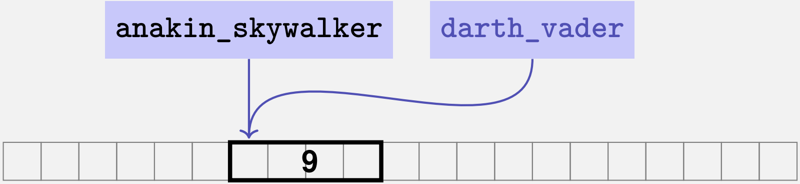
anakin_skywalker

9

Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; //  alias
darth_vader = 22;

std::cout << anakin_skywalker;
```



Anakin Skywalker alias Darth Vader

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std::cout << anakin_skywalker;
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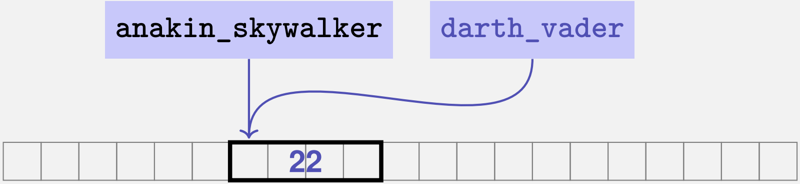
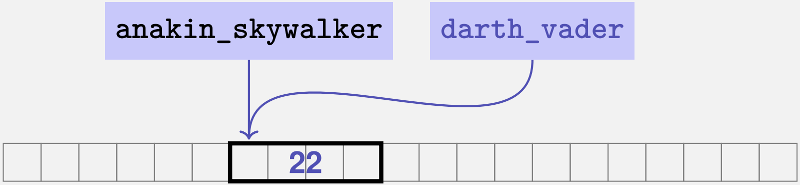


Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; //  alias
darth_vader = 22;
```

assignment to the L-value behind the alias

```
std::cout << anakin_skywalker;
```



anakin_skywalker

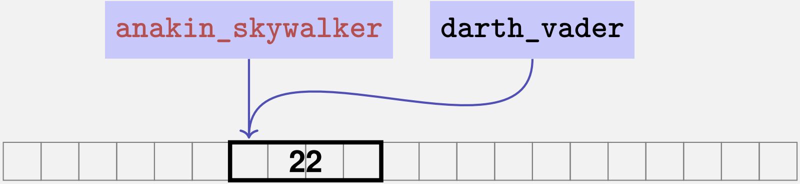
darth_vader

22

Anakin Skywalker alias Darth Vader

```
int anakin_skywalker = 9;
int& darth_vader = anakin_skywalker; //  alias
darth_vader = 22;

std::cout << anakin_skywalker; // 22
```



Anakin Skywalker alias Darth Vader

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int anakin_skywalker = 9;
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std::cout << anakin_skywalker; // 22
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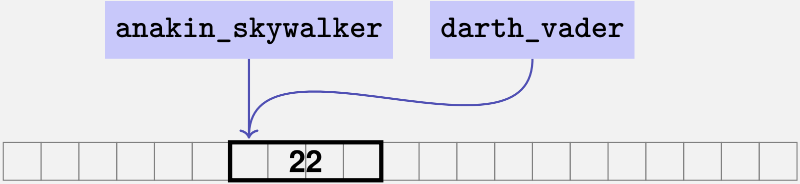

Reference Types: Intialization and Assignment

```
int& darth_vader = anakin_skywalker;
```

- A variable of reference type (a *reference*) can only be initialized with an L-Value .

Reference Types: Intialization and Assignment

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int& darth_vader = anakin_skywalker;
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- A variable of reference type (a *reference*) can only be initialized with an L-Value .
- The variable is becoming an *alias* of the L-value (a different name for the referenced object).

Reference Types: Intialization and Assignment

```
int& darth_vader = anakin_skywalker;
darth_vader = 22; // anakin_skywalker = 22
```

- A variable of reference type (a *reference*) can only be initialized with an L-Value .
- The variable is becoming an *alias* of the L-value (a different name for the referenced object).
- Assignment to the reference is to the object behind the alias.

Reference Types: Implementation

Internally, a value of type *T&* is represented by the address of an object of type *T*.

```
int& j;  // Error: j must be an alias of something
```

Reference Types: Implementation

Internally, a value of type *T&* is represented by the address of an object of type *T*.

```
int& j;  // Error: j must be an alias of something

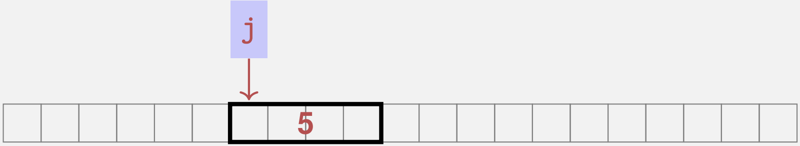
int& k = 5;  // Error: the literal 5 has no address
```

Pass by Reference

```
void increment (int& i)
{
    ++i;
}
...
int j = 5;
increment (j);
std::cout << j << "\n"; // 6
```

Pass by Reference

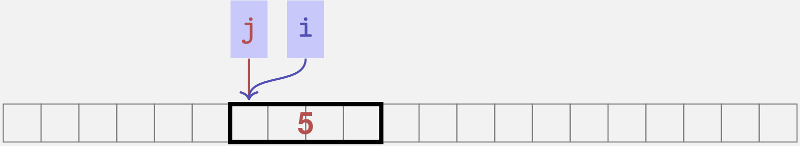
```
void increment (int& i)
{
    ++i;
}
...
int j = 5;
increment (j);
std::cout << j << "\n"; // 6
```



Pass by Reference

```
void increment (int& i)
{ // i becomes an alias of the call argument
    ++i;
}
...
int j = 5;
increment (j);
std::cout << j << "\n"; // 6
```

← initialization of the formal arguments



Pass by Reference

```
void increment (int& i)
{
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}
...
int j = 5;
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std::cout << j << "\n"; // 6
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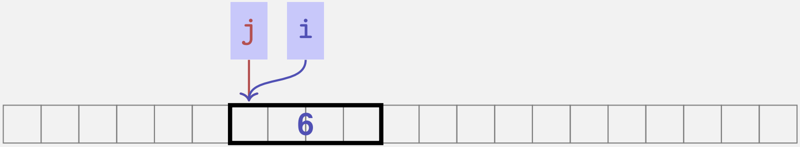
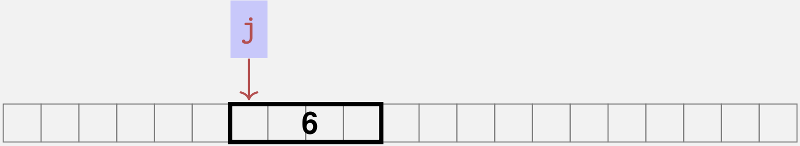


Pass by Reference

```
void increment (int& i)
{
    ++i;
}
...
int j = 5;
increment (j);
std::cout << j << "\n"; // 6
```



Pass by Reference

Formal argument has reference type:

⇒ **Pass by Reference**

Formal argument is (internally) initialized with the *address* of the call argument (L-value) and thus becomes an *alias*.

Pass by Value

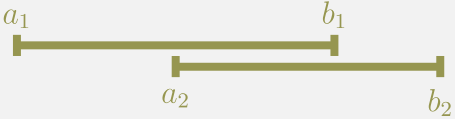
Formal argument does not have a reference type:

$\Rightarrow$ **Pass by Value**

Formal argument is initialized with the *value* of the actual parameter (R-Value) and thus becomes a *copy*.

References in the Context of intervals_intersect

```
// PRE:  [a1, b1], [a2, b2] are (generalized) intervals,
// POST: returns true if [a1, b1], [a2, b2] intersect, in which case
//       [l, h] contains the intersection of [a1, b1], [a2, b2]
bool intervals_intersect (int& l, int& h,
                          int a1, int b1, int a2, int b2) {
  sort (a1, b1);
  sort (a2, b2);
  l = std::max (a1, a2); // Assignments
  h = std::min (b1, b2); // via references
  return l <= h;
}
...
int lo = 0; int hi = 0;
if (intervals_intersect (lo, hi, 0, 2, 1, 3)) // Initialization
    std::cout << "[" << lo << "," << hi << "]" << "\n"; // [1,2]
```



References in the Context of intervals_intersect

```
// POST: a <= b
void sort (int& a, int& b) {
   if (a > b)
     std::swap (a, b); // Initialization ("passing through" a, b
}

bool intervals_intersect (int& l, int& h,
                          int a1, int b1, int a2, int b2) {
  sort (a1, b1); // Initialization
  sort (a2, b2); // Initialization
  l = std::max (a1, a2);
  h = std::min (b1, b2);
  return l <= h;
}
```

Return by Value / Reference

- Even the return type of a function can be a reference type (return by reference)

Return by Value / Reference

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int& increment (int& i)
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    return ++i;
}
```

Return by Value / Reference

- Even the return type of a function can be a reference type (return by reference)
- In this case the function call itself is an L-value

```
int& increment (int& i)
{
    return ++i;
}
```

exactly the semantics of the pre-increment

Temporary Objects

What is wrong here?

```
int& foo (int i)
{
    return i;
}
```

Temporary Objects

What is wrong here?

```
int& foo (int i)
{
    return i; ←
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Return value of type `int&` becomes an alias of the formal argument. But the memory lifetime of i ends after the call!

Temporary Objects

What is wrong here?

```
int& foo (int i)
{
    return i;
}
```

Return value of type `int&` becomes an alias of the formal argument. But the memory lifetime of i ends after the call!

```
int k = 3;
int& j = foo (k); // j is an alias of a zombie
std::cout << j << "\n";  // undefined behavior
```

Temporary Objects

What is wrong here?

```
int& foo (int i)
{
    return i;
}
```

```
int k = 3;
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std::cout << j << "\n";  // undefined behavior
```
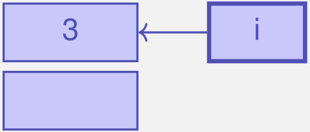

Temporary Objects

What is wrong here?

```
int& foo (int i)
{
    return i;
}
```

value of the actual parameter is pushed onto the *call stack*



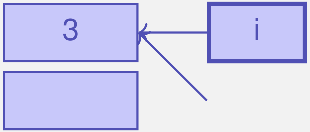
```
int k = 3;
int& j = foo (k); // j is an alias of a zombie
std::cout << j << "\n";  // undefined behavior
```

Temporary Objects

What is wrong here?

i is returned as reference

```
int& foo (int i)
{
    return i;
}
```



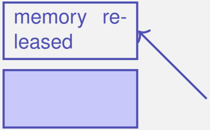
```
int k = 3;
int& j = foo (k); // j is an alias of a zombie
std::cout << j << "\n";  // undefined behavior
```

Temporary Objects

What is wrong here?

```
int& foo (int i)
{
    return i;
}
```

...and disappears from the stack



memory released

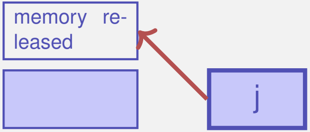
```
int k = 3;
int& j = foo (k); // j is an alias of a zombie
std::cout << j << "\n";  // undefined behavior
```

Temporary Objects

What is wrong here?

```
int& foo (int i)
{
    return i;
}
```

j becomes alias to released memory



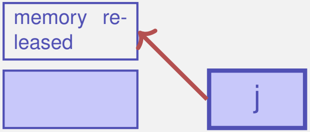
```
int k = 3;
int& j = foo (k); // j is an alias of a zombie
std::cout << j << "\n";  // undefined behavior
```

Temporary Objects

What is wrong here?

value of j is output

```
int& foo (int i)
{
    return i;
}
```



```
int k = 3;
int& j = foo (k); // j is an alias of a zombie
std::cout << j << "\n";  // undefined behavior
```

The Reference Guidline

Reference Guideline

When a reference is created, the object referred to must “stay alive” at least as long as the reference.

Const-References

- have type `const` T `&`
- type can be interpreted as “(`const` T) `&`”
- can be initialized with R-Values (compiler generates a temporary object with sufficient lifetime)

Const-References

- have type const *T* &
- type can be interpreted as “(const *T*) &”
- can be initialized with R-Values (compiler generates a temporary object with sufficient lifetime)

```
const T& r = lvalue;
```

r is initialized with the address of *lvalue* (efficient)

Const-References

- have type `const` *T* `&`
- type can be interpreted as “(`const` *T*) `&`”
- can be initialized with R-Values (compiler generates a temporary object with sufficient lifetime)

`const` *T*`&` `r` = *rvalue*;

r is initialized with the address of a temporary object with the value of the *rvalue* (pragmatic)

When `const` *T&* ?

Rule

Argument type `const` *T* `&` (pass by *read-only* reference) is used for efficiency reasons instead of *T* (pass by value), if the type *T* requires large memory. For fundamental types (`int`, `double`,...) it does not pay off.

When `const` *T*& ?

Rule

Argument type `const` *T* `&` (pass by *read-only* reference) is used for efficiency reasons instead of *T* (pass by value), if the type *T* requires large memory. For fundamental types (`int`, `double`,...) it does not pay off.

Examples will follow later in the course

What exactly does Constant Mean?

Consider an L-value with type `const` *T*

- Case 1: *T* is no reference type

Then the L-value is a constant.

```
const int n = 5;
int& i = n;
i = 6;
```

What exactly does Constant Mean?

Consider an L-value with type `const` *T*

- Case 1: *T* is no reference type

Then the L-value is a constant.

```
const int n = 5;
int& i = n; // error: const-qualification is discarded
i = 6;
```

The compiler detects our attempt to cheat

What exactly does Constant Mean?

Consider L-value of type `const` T

- Case 2: T is reference type.

Then the L-value is a read-only alias which cannot be used to change the value

What exactly does Constant Mean?

Consider L-value of type `const` *T*

- Case 2: *T* is reference type.

Then the L-value is a read-only alias which cannot be used to change the value

```
int n = 5;
const int& i = n;// i: read-only alias of n
int& j = n;      // j: read-write alias
i = 6;           // Error: i is a read-only alias
j = 6;           // ok: n takes on value 6
```

12. Vectors and Strings I

Vector Types, Sieve of Erathostenes, Memory Layout, Iteration, Characters and Texts, ASCII, UTF-8, Caesar-Code

Vectors: Motivation

- Now we can iterate over numbers

```
for (int i=0; i<n ; ++i) ...
```

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- ... but not yet over data!

Vectors: Motivation

- Now we can iterate over numbers

```
for (int i=0; i<n ; ++i) ...
```

- ... but not yet over data!
- Vectors store *homogeneous* data.

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

Vectors: a first Application

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Cross out all real factors of 2 ...

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
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2	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~	11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~	21	~~22~~	23

Cross out all real factors of 2 ...

Vectors: a first Application

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... and go to the next number

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	**3**	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~	11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~	21	~~22~~	23

cross out all real factors of 3 ...

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	**3**	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~	~~21~~	~~22~~	23

cross out all real factors of 3 ...

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
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Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
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2	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~	**11**	~~12~~	**13**	~~14~~	~~15~~	~~16~~	**17**	~~18~~	**19**	~~20~~	~~21~~	~~22~~	**23**

at the end of the crossing out process, only prime numbers remain.

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~	**11**	~~12~~	**13**	~~14~~	~~15~~	~~16~~	**17**	~~18~~	**19**	~~20~~	~~21~~	~~22~~	**23**

- Question: how do we cross out numbers ??

Vectors: a first Application

The Sieve of Erathostenes

- computes all prime numbers $< n$
- method: cross out all non-prime numbers

2	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~	**11**	~~12~~	**13**	~~14~~	~~15~~	~~16~~	**17**	~~18~~	**19**	~~20~~	~~21~~	~~22~~	**23**

- Question: how do we cross out numbers ??
- Answer: with a *vector*.

Erathostenes with Vectors: Initialization

```
...
#include <vector>
...
std::vector<bool> crossed_out (n, false);
```

Initialization with n elements initial value `false`.

element type in triangular brackets

Erathostenes with Vectors: Computation

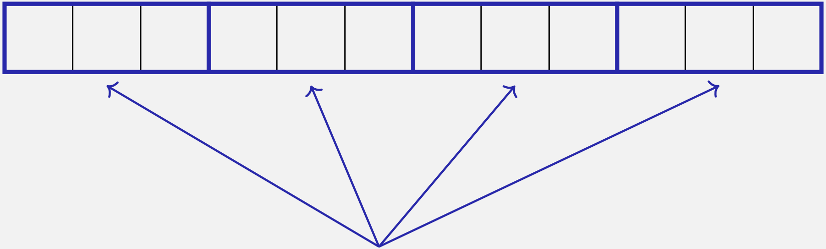
```
for (unsigned int i = 2; i < crossed_out.size(); ++i)
  if (!crossed_out[i]) { // i is prime
    std::cout << i << " ";
    // cross out all proper multiples of i
    for (unsigned int m = 2*i; m < n; m += i)
      crossed_out[m] = true;
  }
```

Memory Layout of a Vector

- A vector occupies a *contiguous* memory area

Memory Layout of a Vector

- A vector occupies a *contiguous* memory area

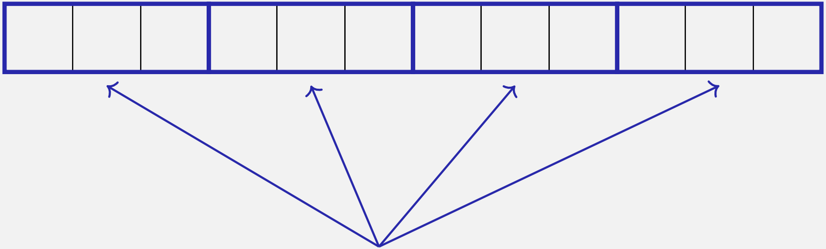


memory cells for a value of type `T` each

Memory Layout of a Vector

- A vector occupies a *contiguous* memory area

example: a vector with 4 elements



memory cells for a value of type `T` each

Random Access

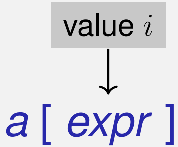
The L-value

$$a\ [\ \overset{\boxed{\text{value } i}}{\overset{\big\downarrow}{expr}}\ ]$$

has type *T* and refers to the i-th element of the vector *a* (counting from 0!)

Random Access

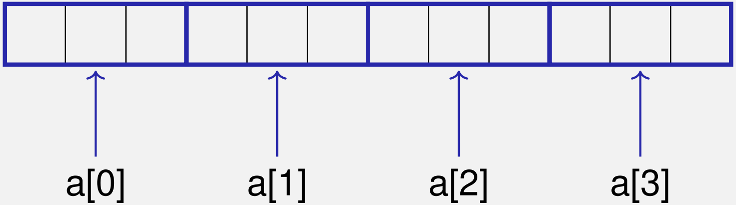
The L-value



value i

a [*expr*]

has type *T* and refers to the i-th element of the vector *a* (counting from 0!)



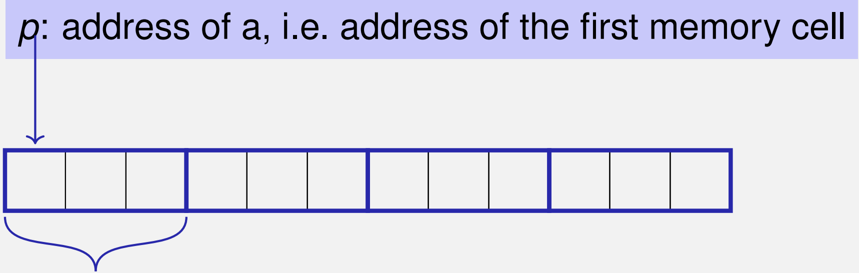
Random Access

$$a\,[\,expr\,]$$

The value i of *expr* is called *index*.
`[]`: subscript operator

Random Access

- Random access is very efficient:

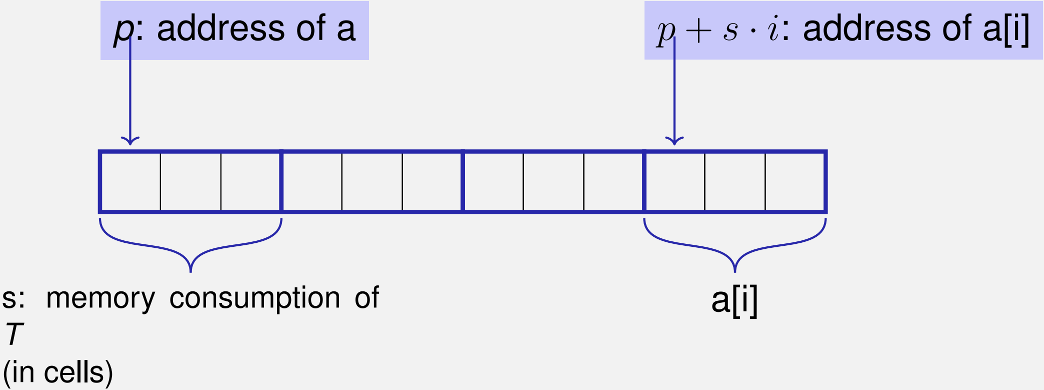


p: address of a, i.e. address of the first memory cell

s: memory consumption of
T
(in cells)

Random Access

- Random access is very efficient:



p: address of a

$p + s \cdot i$: address of a[i]

s: memory consumption of T (in cells)

a[i]

Vector Initialization

- `std::vector<int> a (5);`

  The five elements of `a` are zero intialized)

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- `std::vector<int> a (5);`
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- `std::vector<int> a (5, 2);`
  the 5 elements of `a` are initialized with 2.
- `std::vector<int> a {4, 3, 5, 2, 1};`
  the vector is initialized with an *initialization list*.
- `std::vector<int> a;`
  An initially empty vector is created.

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- `std::vector<int> a;`
  An initially empty vector is created.

Attention

- Accessing elements outside the valid bounds of a vector leads to undefined behavior.

```
std::vector arr (10);
for (int i=0; i<=10; ++i)
  arr[i] = 30;
```

Attention

- Accessing elements outside the valid bounds of a vector leads to undefined behavior.

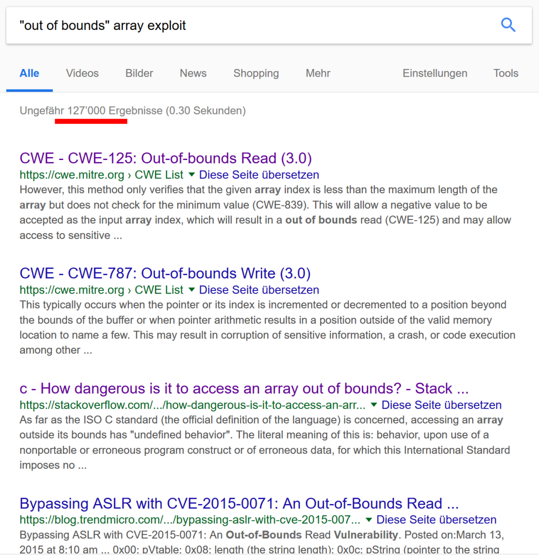
```
std::vector arr (10);
for (int i=0; i<=10; ++i)
  arr[i] = 30; // runtime error: access to arr[10]!
```

Attention

Bound Checks

When using a subscript operator on a vector, it is the sole *responsibility of the programmer* to check the validity of element accesses.

Consequences of illegal index accesses



Ungefähr 127'000 Ergebnisse (0.30 Sekunden)

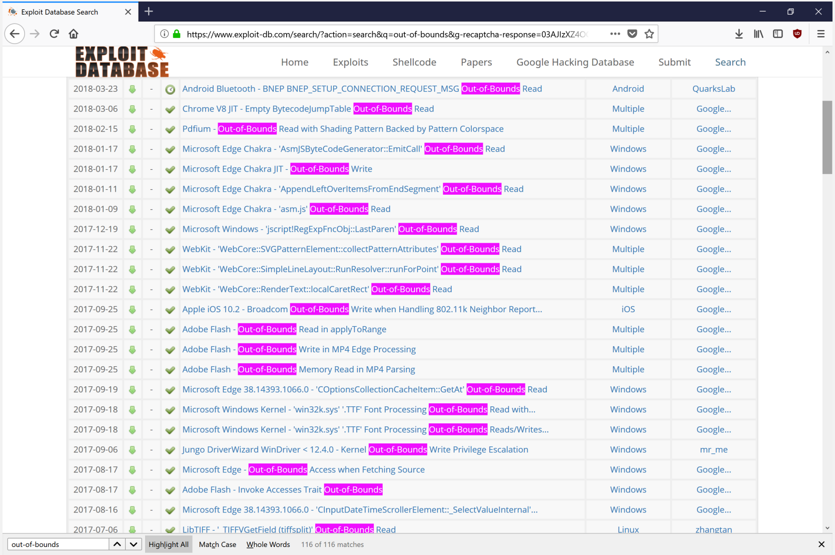
CWE - CWE-125: Out-of-bounds Read (3.0)
https://cwe.mitre.org › CWE List ▼ Diese Seite übersetzen
However, this method only verifies that the given **array** index is less than the maximum length of the **array** but does not check for the minimum value (CWE-839). This will allow a negative value to be accepted as the input **array** index, which will result in a **out of bounds** read (CWE-125) and may allow access to sensitive ...

CWE - CWE-787: Out-of-bounds Write (3.0)
https://cwe.mitre.org › CWE List ▼ Diese Seite übersetzen
This typically occurs when the pointer or its index is incremented or decremented to a position beyond the bounds of the buffer or when pointer arithmetic results in a position outside of the valid memory location to name a few. This may result in corruption of sensitive information, a crash, or code execution among other ...

c - How dangerous is it to access an array out of bounds? - Stack ...
https://stackoverflow.com/.../how-dangerous-is-it-to-access-an-arr... ▼ Diese Seite übersetzen
As far as the ISO C standard (the official definition of the language) is concerned, accessing an **array** outside its bounds has "undefined behavior". The literal meaning of this is: behavior, upon use of a nonportable or erroneous program construct or of erroneous data, for which this International Standard imposes no ...

Bypassing ASLR with CVE-2015-0071: An Out-of-Bounds Read ...
https://blog.trendmicro.com/.../bypassing-aslr-with-cve-2015-007... ▼ Diese Seite übersetzen
Bypassing ASLR with CVE-2015-0071: An **Out-of-Bounds** Read **Vulnerability**. Posted on:March 13, 2015 at 8:10 am ... 0x00: pVtable; 0x08: length (the string length); 0x0c: pString (pointer to the string

Consequences of illegal index accesses



2018-03-23	⬇	-	◷	Android Bluetooth - BNEP BNEP_SETUP_CONNECTION_REQUEST_MSG Out-of-Bounds Read	Android	QuarksLab
2018-03-06	⬇	-	✓	Chrome V8 JIT - Empty BytecodeJumpTable Out-of-Bounds Read	Multiple	Google...
2018-02-15	⬇	-	✓	Pdfium - Out-of-Bounds Read with Shading Pattern Backed by Pattern Colorspace	Multiple	Google...
2018-01-17	⬇	-	✓	Microsoft Edge Chakra - 'AsmJSByteCodeGenerator::EmitCall' Out-of-Bounds Read	Windows	Google...
2018-01-17	⬇	-	✓	Microsoft Edge Chakra JIT - Out-of-Bounds Write	Windows	Google...
2018-01-11	⬇	-	✓	Microsoft Edge Chakra - 'AppendLeftOverItemsFromEndSegment' Out-of-Bounds Read	Windows	Google...
2018-01-09	⬇	-	✓	Microsoft Edge Chakra - 'asm.js' Out-of-Bounds Read	Windows	Google...
2017-12-19	⬇	-	✓	Microsoft Windows - 'jscript!RegExpFncObj::LastParen' Out-of-Bounds Read	Windows	Google...
2017-11-22	⬇	-	✓	WebKit - 'WebCore::SVGPatternElement::collectPatternAttributes' Out-of-Bounds Read	Multiple	Google...
2017-11-22	⬇	-	✓	WebKit - 'WebCore::SimpleLineLayout::RunResolver::runForPoint' Out-of-Bounds Read	Multiple	Google...
2017-11-22	⬇	-	✓	WebKit - 'WebCore::RenderText::localCaretRect' Out-of-Bounds Read	Multiple	Google...
2017-09-25	⬇	-	✓	Apple iOS 10.2 - Broadcom Out-of-Bounds Write when Handling 802.11k Neighbor Report...	iOS	Google...
2017-09-25	⬇	-	✓	Adobe Flash - Out-of-Bounds Read in applyToRange	Multiple	Google...
2017-09-25	⬇	-	✓	Adobe Flash - Out-of-Bounds Write in MP4 Edge Processing	Multiple	Google...
2017-09-25	⬇	-	✓	Adobe Flash - Out-of-Bounds Memory Read in MP4 Parsing	Multiple	Google...
2017-09-19	⬇	-	✓	Microsoft Edge 38.14393.1066.0 - 'COptionsCollectionCacheItem::GetAt' Out-of-Bounds Read	Windows	Google...
2017-09-18	⬇	-	✓	Microsoft Windows Kernel - 'win32k.sys' '.TTF' Font Processing Out-of-Bounds Read with...	Windows	Google...
2017-09-18	⬇	-	✓	Microsoft Windows Kernel - 'win32k.sys' '.TTF' Font Processing Out-of-Bounds Reads/Writes...	Windows	Google...
2017-09-06	⬇	-	✓	Jungo DriverWizard WinDriver < 12.4.0 - Kernel Out-of-Bounds Write Privilege Escalation	Windows	mr_me
2017-08-17	⬇	-	✓	Microsoft Edge - Out-of-Bounds Access when Fetching Source	Windows	Google...
2017-08-17	⬇	-	✓	Adobe Flash - Invoke Accesses Trait Out-of-Bounds	Windows	Google...
2017-08-16	⬇	-	✓	Microsoft Edge 38.14393.1066.0 - 'CInputDateTimeScrollerElement::_SelectValueInternal'...	Windows	Google...
2017-07-06	⬇	-	✓	LibTIFF - '_TIFFVGetField (tiffsplit)' Out-of-Bounds Read	Linux	zhangtan

Vectors are Comfortable

```
std::vector<int> v (10);
v.at(5) = 3; // with bound check
v.push_back(8); // 8 is appended
std::vector<int> w = v; // w is initialized with v
int sz = v.size(); // sz = 11
```

Characters and Texts

- We have seen texts before:

```
std::cout << "Prime numbers in {2,...,999}:\n";
```

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$$\texttt{std::cout << }\underbrace{\texttt{"Prime numbers in \{2,...,999\}:\textbackslash n"}}_{\text{String-Literal}}\texttt{;}$$

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Characters and Texts

- We have seen texts before:

$$\texttt{std::cout << }\underbrace{\texttt{"Prime numbers in \{2,...,999\}:\textbackslash n"}}_{\text{String-Literal}}\texttt{;}$$

- can we really work with texts? Yes:

Character:	Value of the fundamental type **`char`**
Text:	**`std::string`** ≈ vector of **`char`** elements

The type `char` (“character”)

- represents printable characters (e.g. `'a'`) and *control characters* (e.g. `'\n'`)

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char c = 'a'
```

↑

defines variable c of type `char` with value `'a'`

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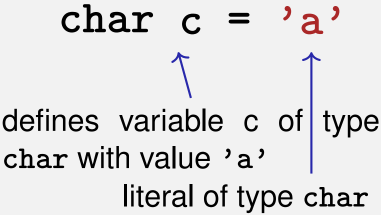

```
char c = 'a'
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defines variable c of type `char` with value `'a'`

literal of type `char`

The type `char` (“character”)

is formally an integer type

- values convertible to `int` / `unsigned int`

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domain:
$\{-128, \ldots, 127\}$ or $\{0, \ldots, 255\}$

The ASCII-Code

- defines concrete conversion rules
  `char` ⟶ `int / unsigned int`

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- defines concrete conversion rules
  `char` ⟶ `int` / `unsigned int`
- is supported on nearly all platforms

  Zeichen ⟶ $\{0, \ldots, 127\}$
  `'A', 'B', ... , 'Z'` ⟶ $65, 66, ..., 90$
  `'a', 'b', ... , 'z'` ⟶ $97, 98, ..., 122$
  `'0', '1', ... , '9'` ⟶ $48, 49, ..., 57$

-
```
for (char c = 'a'; c <= 'z'; ++c)
    std::cout << c;
```

```
abcdefghijklmnopqrstuvwxyz
```

The ASCII-Code

- defines concrete conversion rules
  `char` ⟶ `int` / `unsigned int`
- is supported on nearly all platforms

  Zeichen $\longrightarrow \{0, \ldots, 127\}$
  `'A', 'B', ... , 'Z'` $\longrightarrow 65, 66, ..., 90$
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-
```
for (char c = 'a'; c <= 'z'; ++c)
    std::cout << c;
```

```
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```

Extension of ASCII: UTF-8

- Internationalization of Software ⇒ large character sets required. Common today: unicode, 100 symbol sets, 110000 characters.

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- ASCII can be encoded with 7 bits. An eighth bit can be used to encode further 128 characters – this is history

Extension of ASCII: UTF-8

- Internationalization of Software ⇒ large character sets required. Common today: unicode, 100 symbol sets, 110000 characters.
- ASCII can be encoded with 7 bits. An eighth bit can be used to indicate the appearance of further bits.

Bits	Encoding
7	0xxxxxxx
11	110xxxxx 10xxxxxx
16	1110xxxx 10xxxxxx 10xxxxxx
21	11110xxx 10xxxxxx 10xxxxxx 10xxxxxx
26	111110xx 10xxxxxx 10xxxxxx 10xxxxxx 10xxxxxx
31	1111110x 10xxxxxx 10xxxxxx 10xxxxxx 10xxxxxx 10xxxxxx

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- Internationalization of Software ⇒ large character sets required. Common today: unicode, 100 symbol sets, 110000 characters.
- ASCII can be encoded with 7 bits. An eighth bit can be used to indicate the appearance of further bits.

Bits	Encoding
7	`0xxxxxxx`
11	`110xxxxx 10xxxxxx`
16	`1110xxxx 10xxxxxx 10xxxxxx`
21	`11110xxx 10xxxxxx 10xxxxxx 10xxxxxx`
26	`111110xx 10xxxxxx 10xxxxxx 10xxxxxx 10xxxxxx`
31	`1111110x 10xxxxxx 10xxxxxx 10xxxxxx 10xxxxxx 10xxxxxx`

Einige Zeichen in UTF-8

Symbol	Codierung (jeweils 16 Bit)
ﯹ	`11101111 10101111 10111001`

Einige Zeichen in UTF-8

Symbol	Codierung (jeweils 16 Bit)
ﯹ	11101111 10101111 10111001
☠	11100010 10011000 10100000
☃	11100010 10011000 10000011
☙	11100010 10011000 10011001

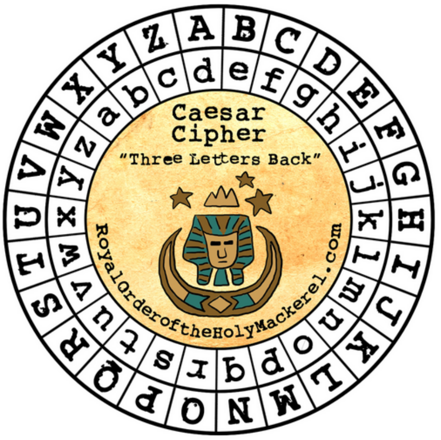
Einige Zeichen in UTF-8

Symbol	Codierung (jeweils 16 Bit)
ﯹ	11101111 10101111 10111001
☠	11100010 10011000 10100000
☃	11100010 10011000 10000011
☙	11100010 10011000 10011001
A	01000001

Caesar-Code

Replace every printable character in a text by its pre-pre-predecessor.

’ ’	(32)	→	’|’	(124)
’!’	(33)	→	’}’	(125)
		...		
’D’	(68)	→	’A’	(65)
’E’	(69)	→	’B’	(66)
		...		
∼	(126)	→	’{’	(123)


Caesar
Cipher
"Three Letters Back"
RoyalOrderoftheHolyMackerel.com
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
a b c d e f g h i j k l m n o p q r s t u v w x y z

Caesar-Code: shift-Function

```
// pre: divisor > 0
// post: return the remainder of dividend / divisor
//      with 0 <= result < divisor
int mod(int dividend, int divisor);

// POST: if c is one of the 95 printable ASCII characters, c is
//      cyclically shifted s printable characters to the right
char shift(char c, int s) {
    if (c >= 32 && c <= 126) { // c printable
      c = 32 + mod(c - 32 + s,95)};
    }
    return c;
}
```

Caesar-Code: `shift`-Function

```
// pre: divisor > 0
// post: return the remainder of dividend / divisor
//       with 0 <= result < divisor
int mod(int dividend, int divisor);

// POST: if c is one of the 95 printable ASCII characters, c is
//       cyclically shifted s printable characters to the right
char shift(char c, int s) {
    if (c >= 32 && c <= 126) { // c printable
      c = 32 + mod(c - 32 + s,95)};
    }
    return c;
}
```

"- 32" transforms interval $[32, 126]$ to $[0, 94]$
"32 +" transforms interval $[0, 94]$ back to $[32, 126]$
mod(x,95) is the representative of $x(\bmod 95)$ in interval $[0, 94]$

Caesar-Code: caesar-Function

```
// POST: Each character read from std::cin was shifted cyclically
//       by s characters and afterwards written to std::cout
void caesar(int s) {
  std::cin >> std::noskipws; // #include <ios>

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

spaces and newline characters shall *not* be ignored

Caesar-Code: `caesar`-Function

```
// POST: Each character read from std::cin was shifted cyclically
//       by s characters and afterwards written to std::cout
void caesar(int s) {
  std::cin >> std::noskipws; // #include <ios>

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

Conversion to `bool`: returns *false* if and only if the input is empty.

Caesar-Code: caesar-Function

```
// POST: Each character read from std::cin was shifted cyclically
//       by s characters and afterwards written to std::cout
void caesar(int s) {
  std::cin >> std::noskipws; // #include <ios>

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

shifts only printable characters.

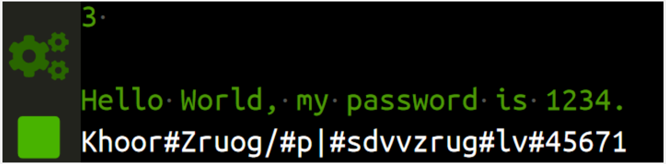
Caesar-Code: Main Program

```
int main() {
  int s;
  std::cin >> s;

  // Shift input by s
  caesar(s);

  return 0;
}
```

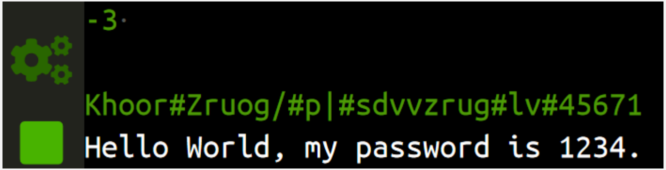
Encode: shift by n (here: 3)



```
3·

Hello·World,·my·password·is·1234.
Khoor#Zruog/#p|#sdvvzrug#lv#45671
```

Encode: shift by $-n$ (here: -3)



```
-3·

Khoor#Zruog/#p|#sdvvzrug#lv#45671
Hello World, my password is 1234.
```

Caesar-Code: Generalisation

```
void caesar(int s) {
  std::cin >> std::noskipws;

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

- Currently only from `std::cin` to `std::cout`

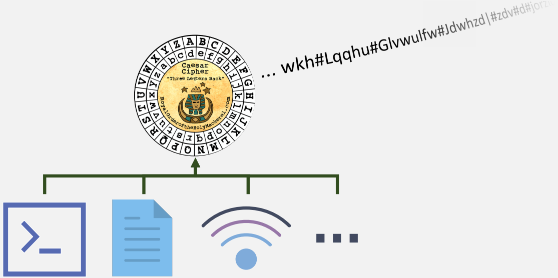
Caesar-Code: Generalisation

```
void caesar(int s) {
  std::cin >> std::noskipws;

  char next;
  while (std::cin >> next) {
    std::cout << shift(next, s);
  }
}
```

- Currently only from `std::cin` to `std::cout`

- Better: from arbitrary character source (console, file, ...) to arbitrary character sink (console, ...)


... wkh#Lqqhu#Glvwulfw#Jdwhzd|#zdv#d#jorz
Caesar Cipher
"Three Letters Back"
RoyalOrderoftheHolyMackerel.com
...

Icons: flaticon.com; authors Smashicons, Kirill Kazachek; CC 3.0 BY

Caesar-Code: Generalisation

```
void caesar(std::istream& in,
            std::ostream& out,
            int s) {

  in >> std::noskipws;

  char next;
  while (in >> next) {
    out << shift(next, s);
  }
}
```

- `std::istream`/`std::ostream` is an *generic input/output stream* of `char`s

Caesar-Code: Generalisation

```
void caesar(std::istream& in,
            std::ostream& out,
            int s) {

  in >> std::noskipws;

  char next;
  while (in >> next) {
    out << shift(next, s);
  }
}
```

- `std::istream`/`std::ostream` is an *generic input/output stream* of `char`s
- Function is called with *specific streams*, e.g.: Console (`std::cin`/`cout`), Files (`std::i`/`ofstream`), Strings (`std::i`/`ostringstream`)

Caesar-Code: Generalisation, Example 1

```
#include <iostream>
...

// in void main():
caesar(std::cin, std::cout, s);
```

Calling the generalised `caesar` function: from `std::cin` to `std::cout`

Caesar-Code: Generalisation, Example 2

```
#include <iostream>
#include <fstream>
...

// in void main():
std::string from_file_name = ...; // Name of file to read from
std::string to_file_name = ...; // Name of file to write to
std::ifstream from(from_file_name); // Input file stream
std::ofstream to(to_file_name); // Output file stream

caesar(from, to, s);
```

Calling the generalised `caesar` function: from file to file

Caesar-Code: Generalisation, Example 3

```
#include <iostream>
#include <sstream>
...

// in void main():
std::string plaintext = "My password is 1234";
std::istringstream from(plaintext);

caesar(from, std::cout, s);
```

Calling the generalised `caesar` function: from a string to `std::cout`

13. Vectors and Strings II

Strings, Multidimensional Vector/Vectors of Vectors, Shortest Paths, Vectors as Function Arguments

Texts

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- Text “`to be or not to be`” could be represented as `vector<char>`
- Texts are ubiquitous, however, and thus have their own typ in the standard library: `std::string`
- Requires `#include <string>`

Using std::string

- Declaration, and initialisation with a literal:

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```

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std::string text(n, ’a’)
```

- Comparing texts:

```
if (text1 == text2) ...
```

Using `std::string`

- Querying size:

```
for (unsigned int i = 0; i < text.size(); ++i) ...
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Using std::string

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```

Using `std::string`

- Querying size:

```
for (unsigned int i = 0; i < text.size(); ++i) ...
```

- Reading single characters:

```
if (text[0] == ’a’) ... // or text.at(0)
```

- Writing single characters:

```
text[0] = ’b’; // or text.at(0)
```

Using `std::string`

- Concatenate strings:

```
text = ":-";
text += ")";
assert(text == ":-)");
```

- Many more operations; if interested, see `https://en.cppreference.com/w/cpp/string`

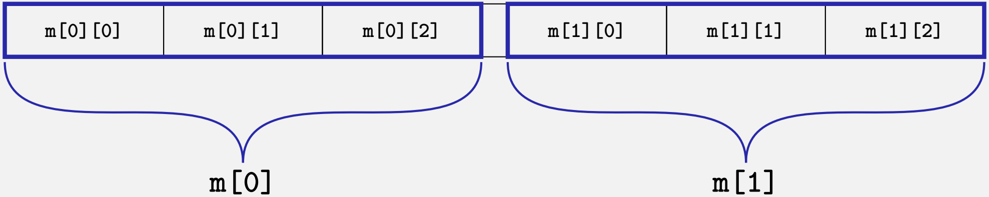
Multidimensional Vectors

- For storing multidimensional structures such as tables, matrices, ...
- ... *vectors of vectors* can be used:

```
std::vector<std::vector<int>> m; // An empty matrix
```

Multidimensional Vectors

In memory: flat



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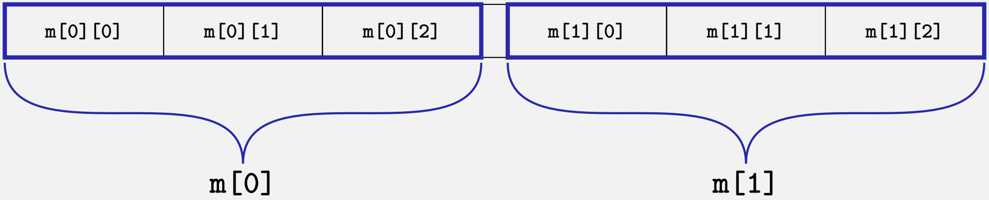

in our head: matrix

		columns		
		0	1	2
rows	0	m[0][0]	m[0][1]	m[0][2]
	1	m[1][0]	m[1][1]	m[1][2]

Multidimensional Vectors: Initialisation Examples

Using literals:

```
// A 3—by—5 matrix
std::vector<std::vector<std::string>> m = {
  {"ZH", "BE", "LU", "BS", "GE"},
  {"FR", "VD", "VS", "NE", "JU"},
  {"AR", "AI", "OW", "IW", "ZG"}
};

assert(m[1][2] == "VS");
```

Multidimensional Vectors: Initialisation Examples

Fill to specific size:

```
unsigned int a = ...;
unsigned int b = ...;

// An a-by-b matrix with all ones
std::vector<std::vector<int>>
  m(a, std::vector<int>(b, 1));
```

Multidimensional Vectors: Initialisation Examples

Fill to specific size:

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// An a-by-b matrix with all ones
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  m(a, std::vector<int>(b, 1));
```

(Many further ways of initialising a vector exist)

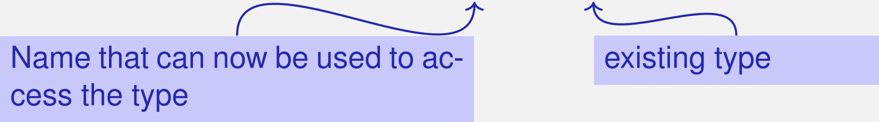
Multidimensional Vectors and Type Aliases

- Also possible: vectors of vectors of vectors of ...:
  `std::vector<std::vector<std::vector<...>>>`
- Type names can obviously become looooooong

Multidimensional Vectors and Type Aliases

- Also possible: vectors of vectors of vectors of ...:
  `std::vector<std::vector<std::vector<...>>>`
- Type names can obviously become looooooong
- The declaration of a *type alias* helps here:

`using` *Name* = *Typ*;



Name that can now be used to access the type

existing type

Type Aliases: Example

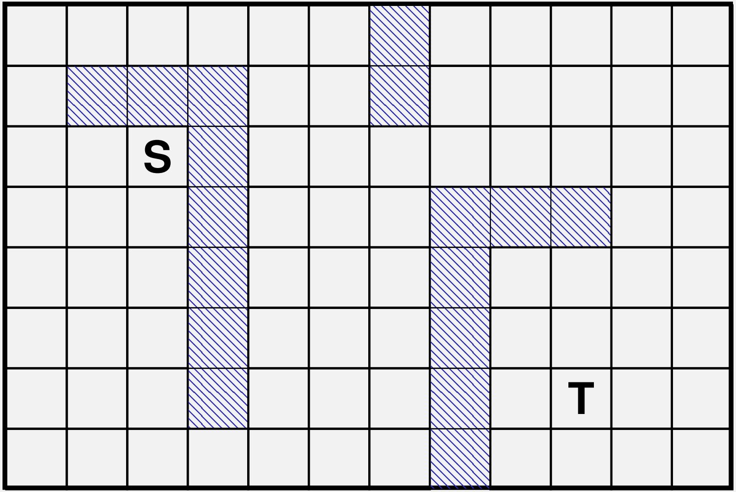
```
#include <iostream>
#include <vector>
using imatrix = std::vector<std::vector<int>>;

// POST: Matrix ’m’ was printed to stream ’to’
void print(imatrix m, std::ostream to);

int main() {
  imatrix m = ...;
  print(m, std::cout);
}
```

Application: Shortest Paths

Factory hall ($n \times m$ square cells)



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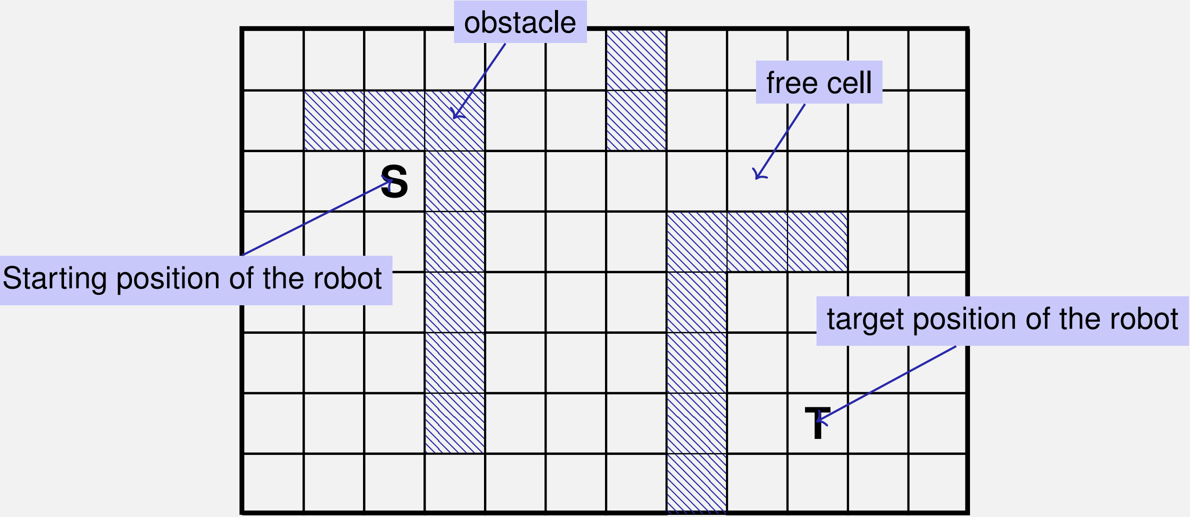

obstacle

free cell

Starting position of the robot

target position of the robot

Application: Shortest Paths

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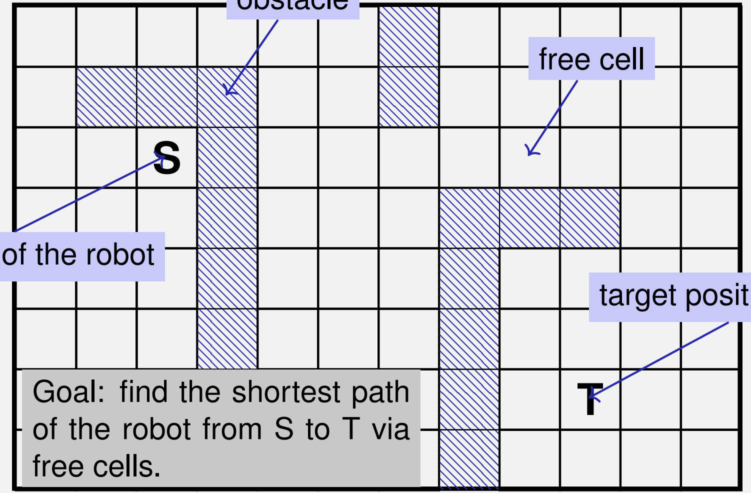

obstacle

free cell

Starting position of the robot

target position of the robot

Goal: find the shortest path of the robot from S to T via free cells.

This problem appears to be different

Find the *lengths* of the shortest paths to *all* possible targets.

4	5	6	7	8	9	▨	15	16	17	18	19
3	▨	▨	▨	9	10	▨	14	15	16	17	18
2	1	0	▨	10	11	12	13	14	15	16	17
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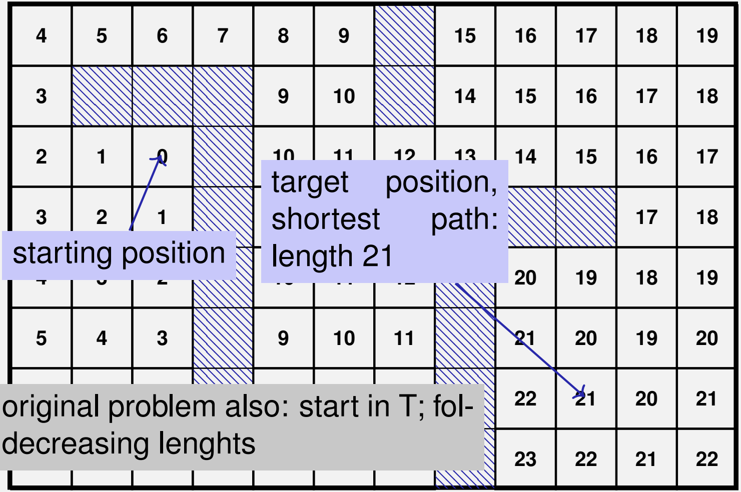

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target position, shortest path: length 21

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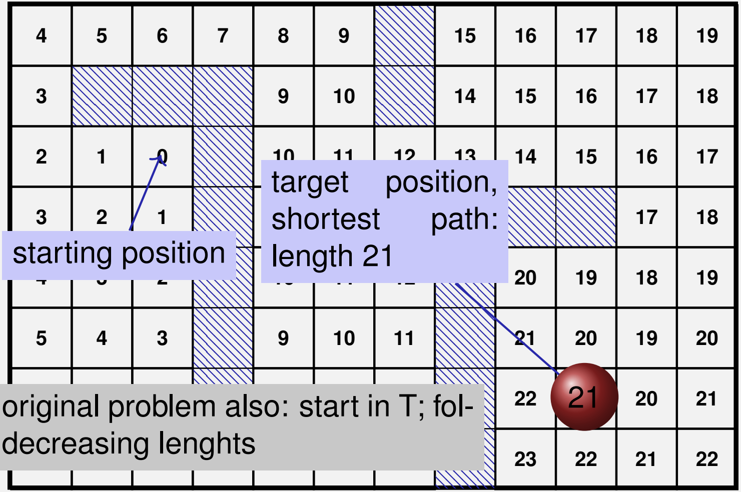

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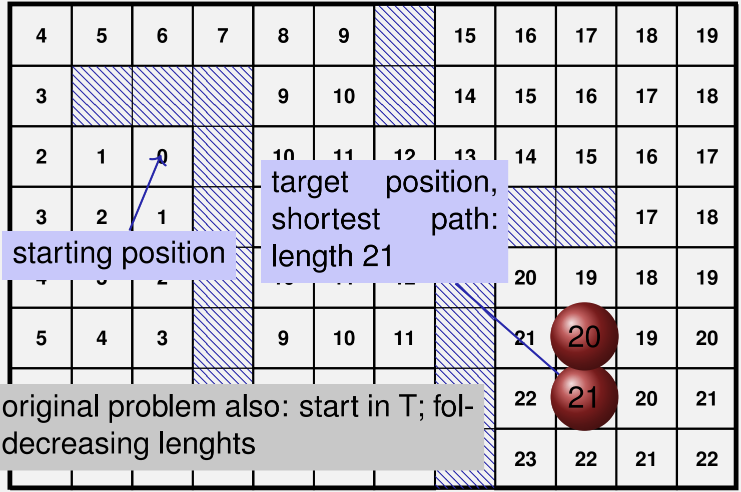

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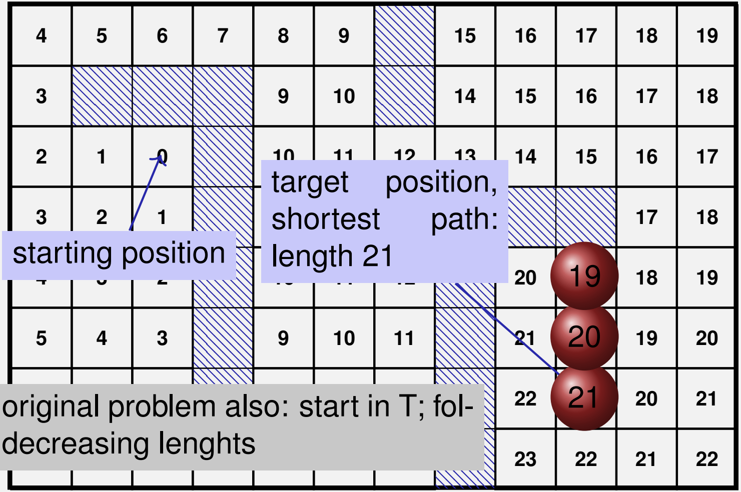

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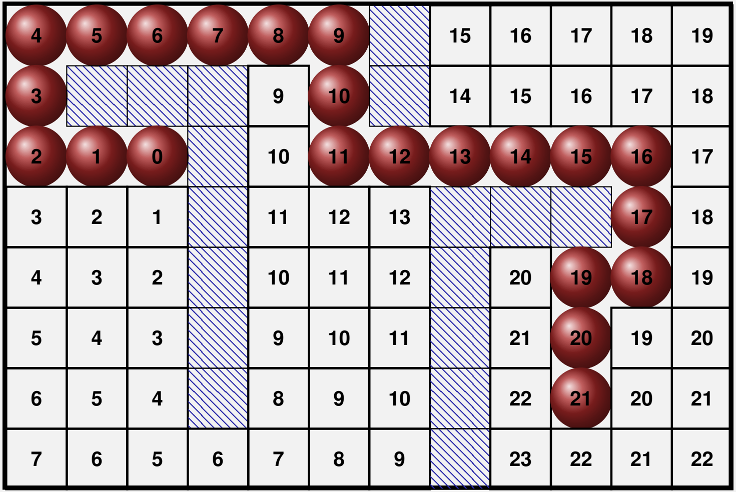

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14. Recursion 1

Mathematical Recursion, Termination, Call Stack, Examples, Recursion vs. Iteration, n-Queen Problem, Lindenmayer Systems

Mathematical Recursion

- Many mathematical functions can be naturally defined recursively.

Mathematical Recursion

- Many mathematical functions can be naturally defined recursively.
- This means, the function appears in its own definition

$$n! = \begin{cases} 1, & \text{if } n \leq 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

Recursion in C++: In the same Way!

$$n! = \begin{cases} 1, & \text{if } n \leq 1 \\ n \cdot (n-1)!, & \text{otherwise} \end{cases}$$

```
// POST: return value is n!
unsigned int fac (unsigned int n)
{
  if (n <= 1)
    return 1;
  else
    return n * fac (n-1);
}
```

Infinite Recursion

- is as bad as an infinite loop. . .

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```
void f()
{
  f(); // f() -> f() -> ... stack overflow
}
```

Infinite Recursion

- is as bad as an infinite loop…
- …but even worse: it burns time and memory

```
void f()
{
  f(); // f() -> f() -> ... stack overflow
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```

Ein Euro ist ein Euro.

Wim Duisenberg, erster Präsident der EZB

Recursive Functions: Termination

As with loops we need

- progress towards termination

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`fac(n)`:
terminates immediately for $n \leq 1$, otherwise the function is called recusively with `< n` .

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`fac(n)`:
terminates immediately for $n \leq 1$, otherwise the function is called recusively with < n .

“n is getting smaller for each call”

Recursive Functions: Evaluation

Example: `fac(4)`

```
// POST: return value is n!
unsigned int fac (unsigned int n)
{
  if (n <= 1) return 1;
  return n * fac(n-1); // n > 1
}
```

Call of `fac(4)`

Recursive Functions: Evaluation

Example: `fac(4)`

```
// POST: return value is n!
unsigned int fac (unsigned int n)
{ // n = 4
  if (n <= 1) return 1;
  return n * fac(n-1); // n > 1
}
```

Initialization of the formal argument

Recursive Functions: Evaluation

Example: `fac(4)`

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// POST: return value is n!
unsigned int fac (unsigned int n)
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Evaluation of the return expression

Recursive Functions: Evaluation

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recursive call with argument $n - 1 == 3$

Recursive Functions: Evaluation

Example: `fac(4)`

```
// POST: return value is n!
unsigned int fac (unsigned int n)
{ // n = 3
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Recursive Functions: Evaluation

Example: `fac(4)`

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// POST: return value is n!
unsigned int fac (unsigned int n)
{ // n = 3
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}
```

Now there are two n. That of `fac(4)` and that of `fac(3)`

Initialization of the formal argument

Recursive Functions: Evaluation

Example: `fac(4)`

```
// POST: return value is n!
unsigned int fac (unsigned int n)
{
  if (n <= 1) return 1;
  return n * fac(n-1); // n > 1
}
```

The n of the current call is used: $n = 3$

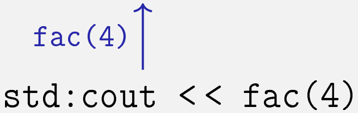
Initialization of the formal argument

The Call Stack

```
std:cout << fac(4)
```

The Call Stack

For each function call:



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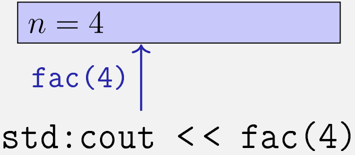
For each function call:

- push value of the call argument onto the stack



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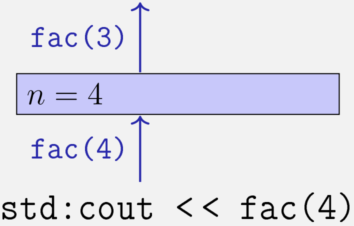

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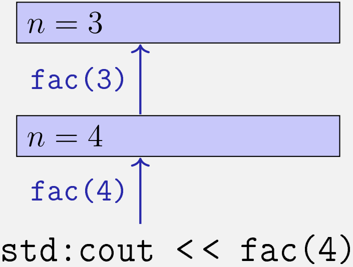

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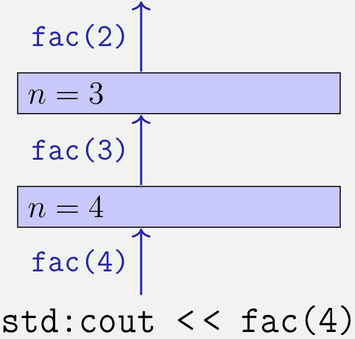

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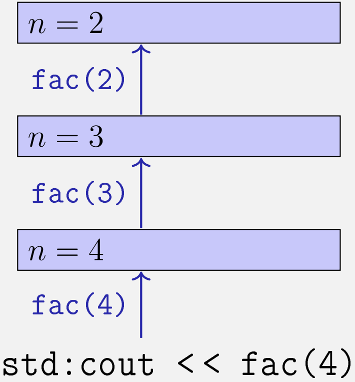

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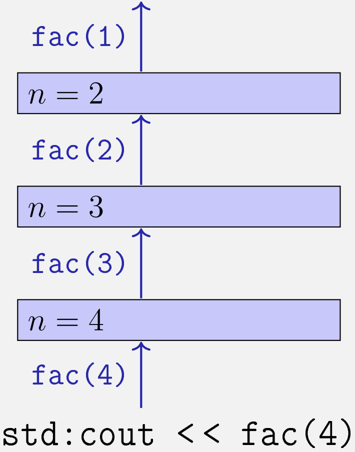

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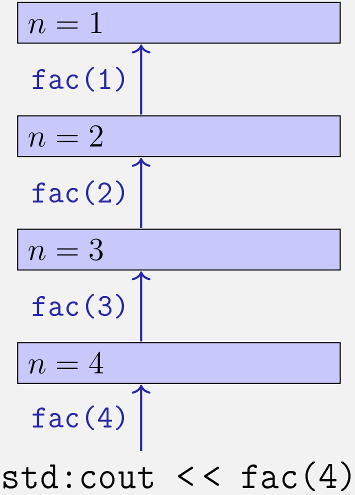


$n = 1$

`fac(1)`

$n = 2$

`fac(2)`

$n = 3$

`fac(3)`

$n = 4$

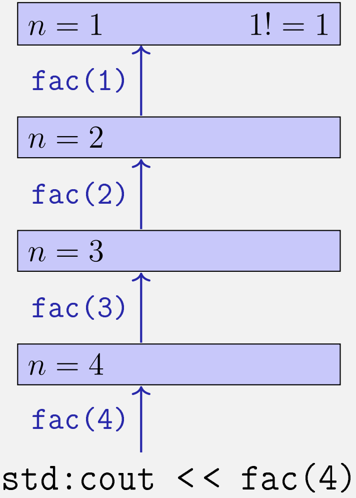
`fac(4)`

`std:cout << fac(4)`

The Call Stack

For each function call:

- push value of the call argument onto the stack
- always work with the top value



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fac(4)

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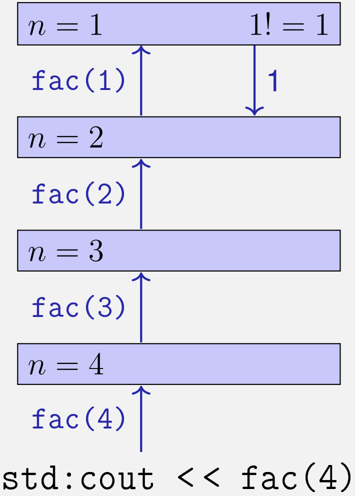
The Call Stack

For each function call:

- push value of the call argument onto the stack
- always work with the top value
- at the end of the call the top value is removed from the stack



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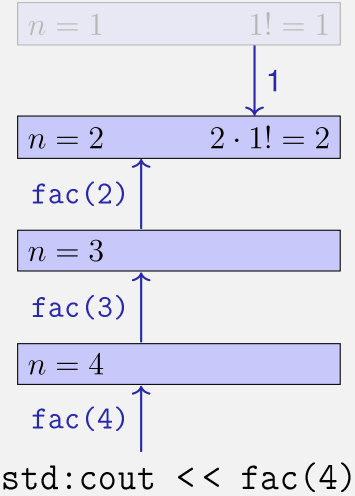

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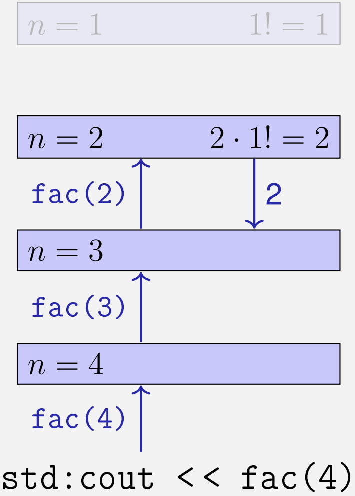

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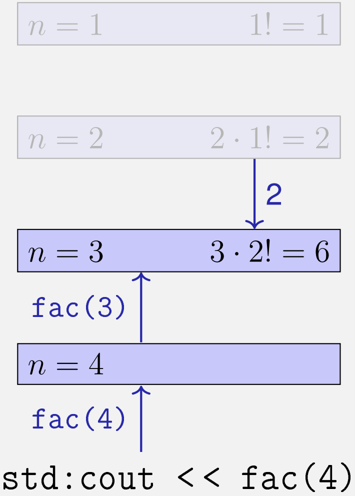

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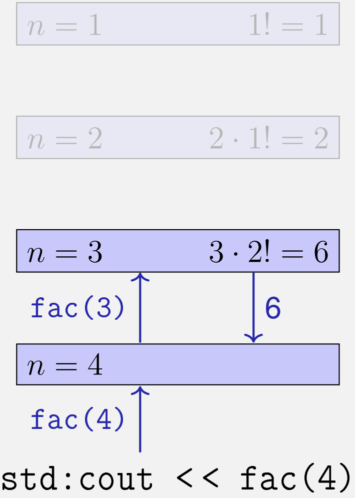

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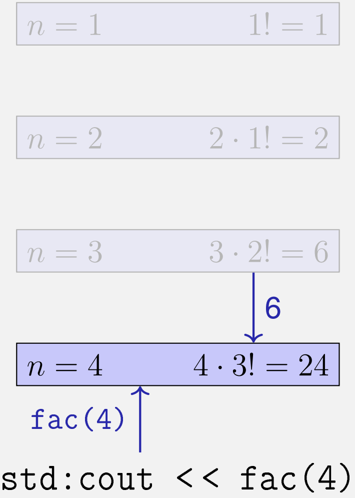
- always work with the top value
- at the end of the call the top value is removed from the stack



The Call Stack

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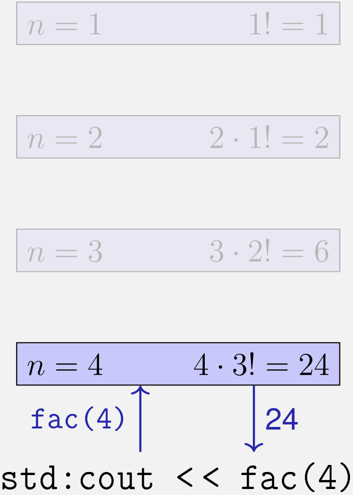

```
std:cout << fac(4)
```

The Call Stack

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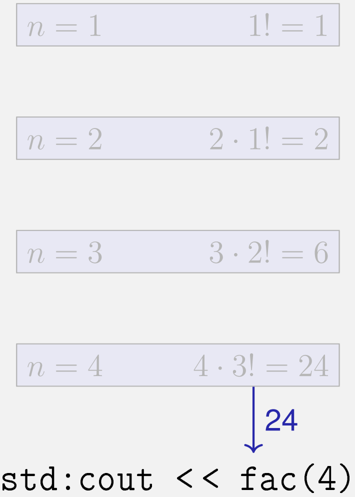

$n = 1$ $1! = 1$

$n = 2$ $2 \cdot 1! = 2$

$n = 3$ $3 \cdot 2! = 6$

$n = 4$ $4 \cdot 3! = 24$

24

```
std:cout << fac(4)
```

Euclidean Algorithm

- finds the greatest common divisor $\gcd(a, b)$ of two natural numbers a and b

Euclidean Algorithm

- finds the greatest common divisor $\gcd(a, b)$ of two natural numbers a and b
- is based on the following mathematical recursion (proof in the lecture notes):

$$\gcd(a, b) = \begin{cases} a, & \text{if } b = 0 \\ \gcd(b, a \bmod b), & \text{otherwise} \end{cases}$$

Euclidean Algorithm in C++

$$\gcd(a,b) = \begin{cases} a, & \text{if } b = 0 \\ \gcd(b, a \bmod b), & \text{otherwise} \end{cases}$$

```
unsigned int gcd (unsigned int a, unsigned int b)
{
  if (b == 0)
    return a;
  else
    return gcd (b, a % b);
}
```

Euclidean Algorithm in C++

$$\gcd(a,b) = \begin{cases} a, & \text{if } b = 0 \\ \gcd(b, a \bmod b), & \text{otherwise} \end{cases}$$

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unsigned int gcd (unsigned int a, unsigned int b)
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  if (b == 0)
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    return gcd (b, a % b);
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```

Termination: $a \bmod b < b$, thus b gets smaller in each recursive call.

Fibonacci Numbers

$$F_n := \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{if } n > 1 \end{cases}$$

Fibonacci Numbers

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$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots$$

Fibonacci Numbers in Zurich

Fibonacci Numbers in $\mathrm{C}{+}{+}$

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```
unsigned int fib (unsigned int n)
{
  if (n == 0) return 0;
  if (n == 1) return 1;
  return fib (n-1) + fib (n-2); // n > 1
}
```

Fibonacci Numbers in C++

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}
```

Correctness and termination are clear.

Fibonacci Numbers in $\mathrm{C}{+}{+}$

Laufzeit

`fib(50)` takes “forever” because it computes F_{48} two times, F_{47} 3 times, F_{46} 5 times, F_{45} 8 times, F_{44} 13 times, F_{43} 21 times ... F_1 ca. 10^9 times (!)

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Fast Fibonacci Numbers

Idea:

- Compute each Fibonacci number only once, in the order $F_0, F_1, F_2, \ldots, F_n$!

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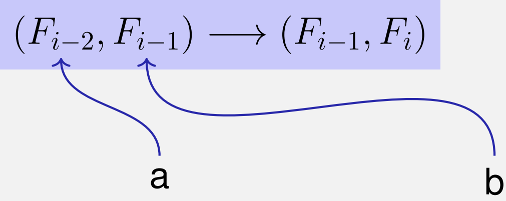
Fast Fibonacci Numbers

Idea:

- Compute each Fibonacci number only once, in the order $F_0, F_1, F_2, \ldots, F_n$!
- Memorize the most recent two numbers (variables `a` and `b`)!
- Compute the next number as a sum of `a` and `b`!

Fast Fibonacci Numbers in $\mathrm{C}{+}{+}$

```
unsigned int fib (unsigned int n){
  if (n == 0) return 0;
  if (n <= 2) return 1;
  unsigned int a = 1;  // F_1
  unsigned int b = 1;  // F_2
  for (unsigned int i = 3; i <= n; ++i){
    unsigned int a_old = a;  // F_i-2
    a = b;                   // F_i-1
    b += a_old;              // F_i-1 += F_i-2 -> F_i
  }
  return b;
}
```



$$(F_{i-2}, F_{i-1}) \longrightarrow (F_{i-1}, F_i)$$

Fast Fibonacci Numbers in $C{+}{+}$

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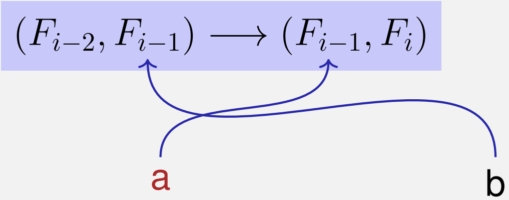

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Fast Fibonacci Numbers in $\mathrm{C}{+}{+}$

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  }
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```

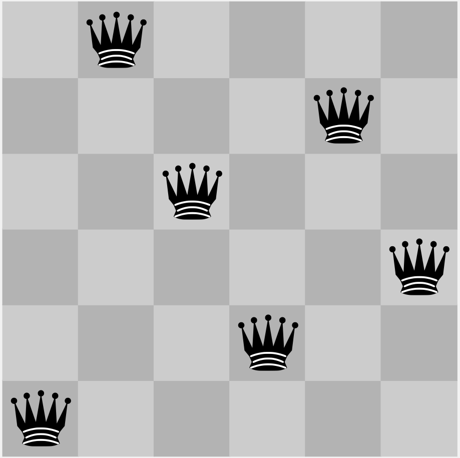
very fast, also for `fib(50)`

$$(F_{i-2}, F_{i-1}) \longrightarrow (\underset{\overset{\uparrow}{a}}{F_{i-1}}, \underset{\overset{\uparrow}{b}}{F_i})$$

The Power of Recursion

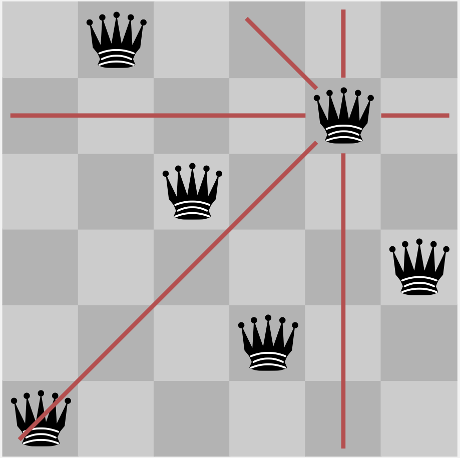
- Some problems appear to be hard to solve without recursion. With recursion they become significantly simpler.
- Examples: *The n-Queens-Problem*, The towers of Hanoi, *Sudoku-Solver*, Expression Parsers, Reversing In- or Output, Searching in Trees, Divide-And-Conquer (e.g. sorting)

The n-Queens Problem



- Provided is a $n\ timesn$ chessboard
- For example $n = 6$
- Question: is it possiblt to position n queens such that no two queens threaten each other?

The n-Queens Problem



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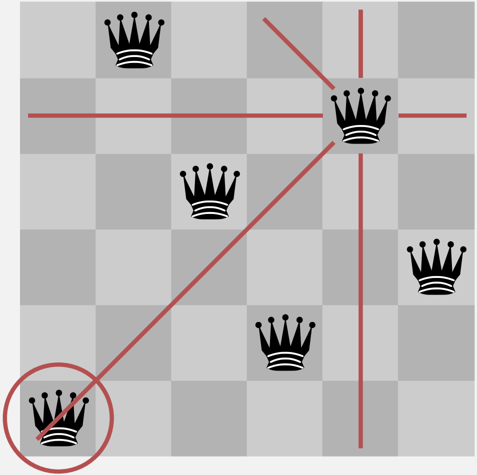

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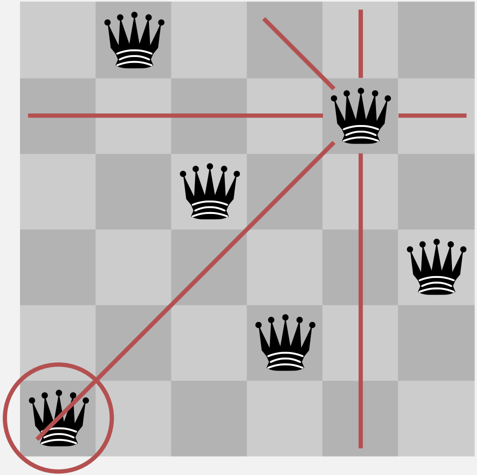

- Provided is a $n timesn$ chessboard
- For example $n = 6$
- Question: is it possiblt to position n queens such that no two queens threaten each other?
- If yes, how many solutions are there?

Solution?

- Try all possible placements?

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- $\binom{n^2}{n}$ possibilities. Too many!

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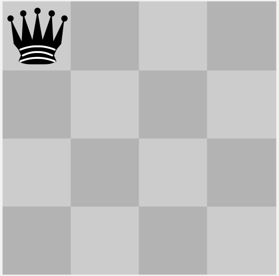
Solution?

- Try all possible placements?
- $\binom{n^2}{n}$ possibilities. Too many!
- n^n possibilities. Better – but still too many.
- Idea: Do not follow paths that obviously fail. (Backtracking)

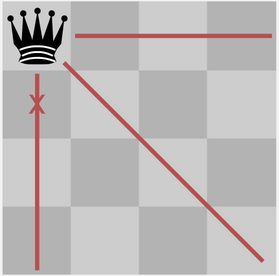
Solution with Backtracking



First Queen

queens
0
0
0
0

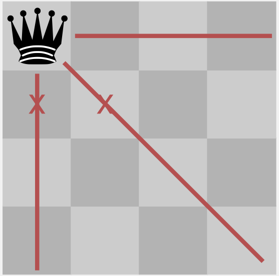
Solution with Backtracking



Forbidden Squares: no other queens may be here.

queens
0
0
0
0

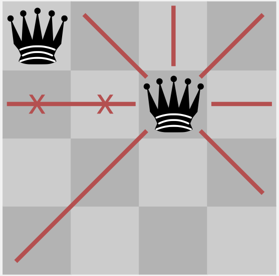
Solution with Backtracking



Forbidden Squares: no other queens may be here.

queens
0
1
0
0

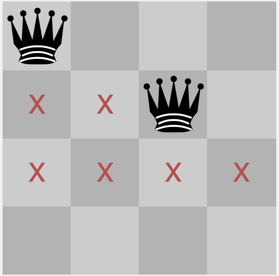
Solution with Backtracking



Second Queen in next row (no collision)

queens
0
2
0
0

Solution with Backtracking

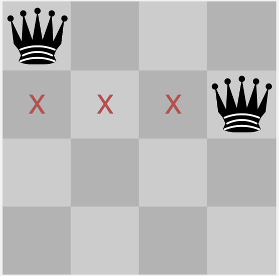


All squares in next row forbiden. Track back !

queens

0
2
4
0

Solution with Backtracking

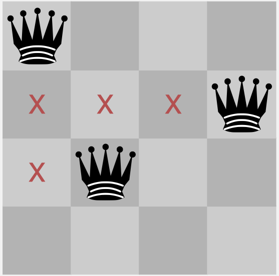


Move queen one step further and try again

queens

0
3
0
0

Solution with Backtracking

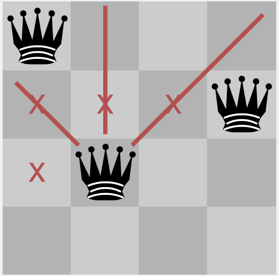


next row

queens

queens
0
3
1
0

Solution with Backtracking

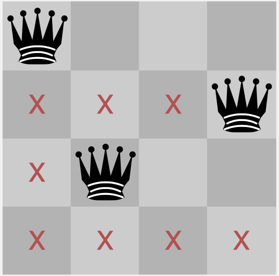


Ok (only previous queens have to be tested)

queens

0
3
1
0

Solution with Backtracking

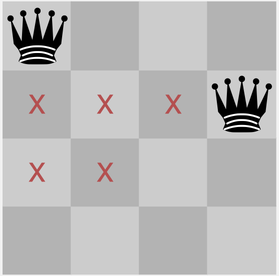


All squares of the next row forbidden. Track back.

queens

0
3
1
4

Solution with Backtracking

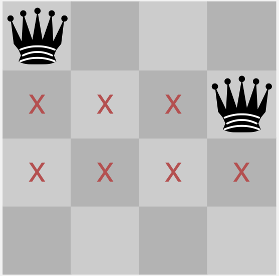


Continue in previous row.

queens

0
3
1
0

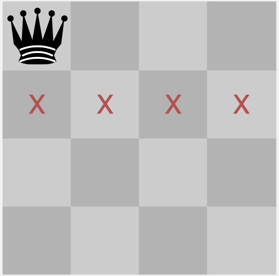
Solution with Backtracking



Remaining squares also forbidden. Track back!

queens
0
3
4
0

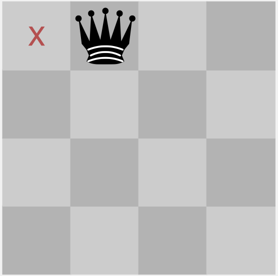
Solution with Backtracking



All squares of this row did not yield a solution. Track back!

queens
0
4
0
0

Solution with Backtracking

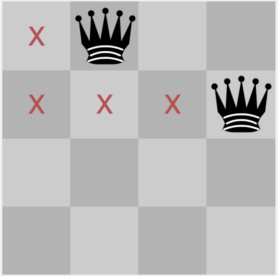


again advance queen by one square

queens

queens
1
0
0
0

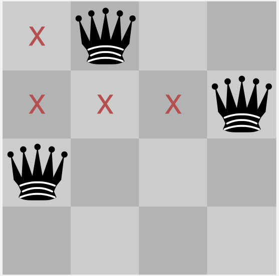
Solution with Backtracking



next row

queens
1
3
0
0

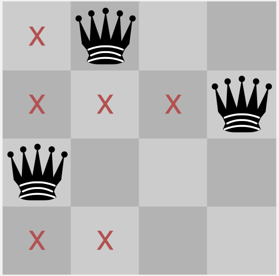
Solution with Backtracking



next row

queens
1
3
0
0

Solution with Backtracking

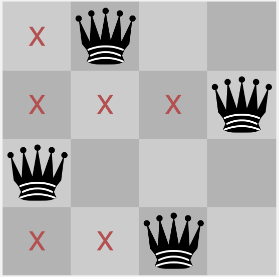


next row

queens

queens
1
3
0
1

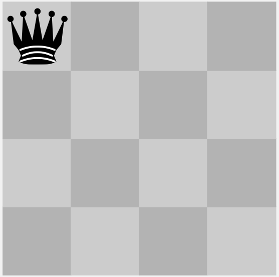
Solution with Backtracking

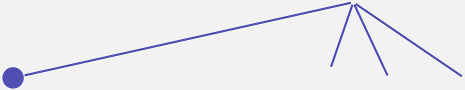


Found a solution

queens
1
3
0
2

Search Strategy Visualized as a Tree





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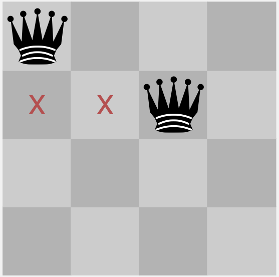

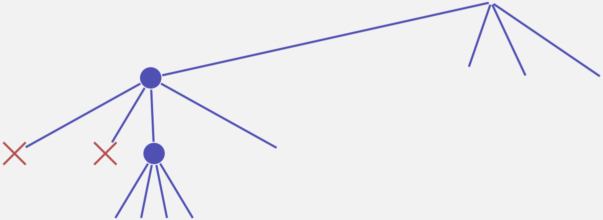

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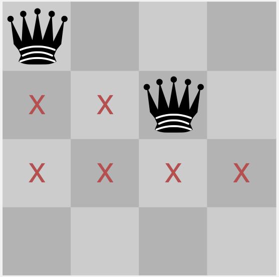

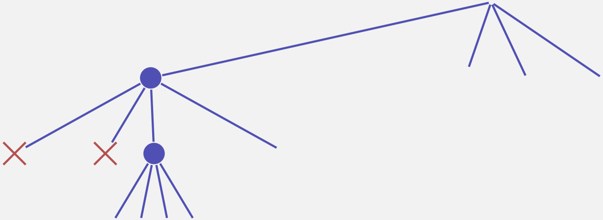

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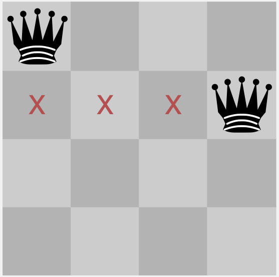

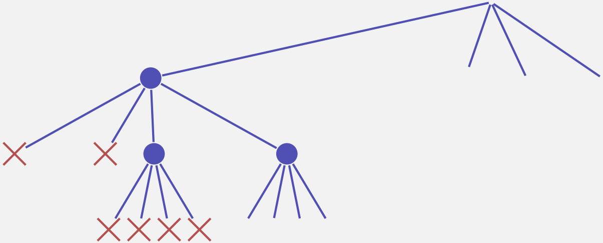

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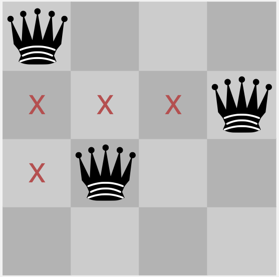

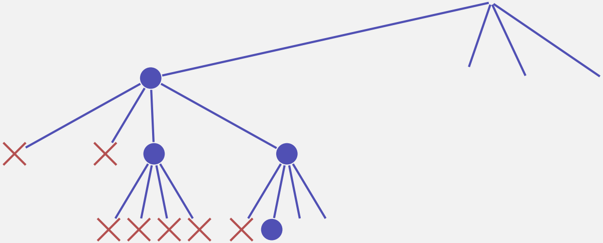

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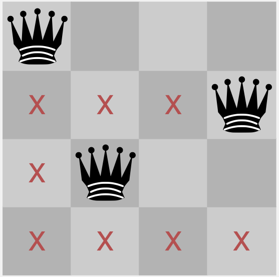

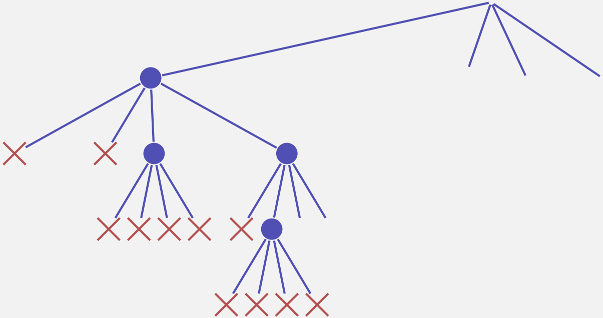

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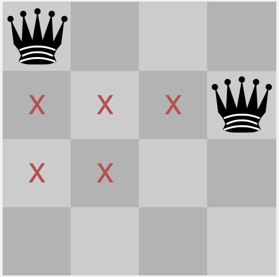

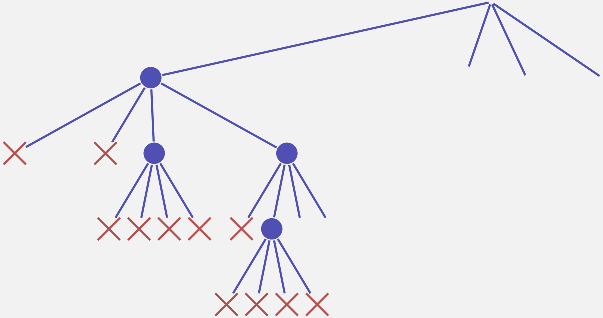

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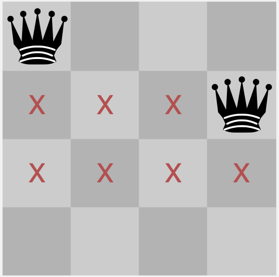

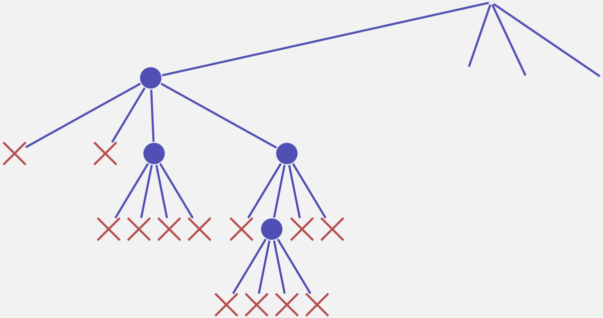

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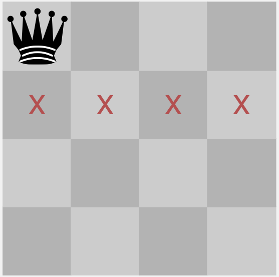
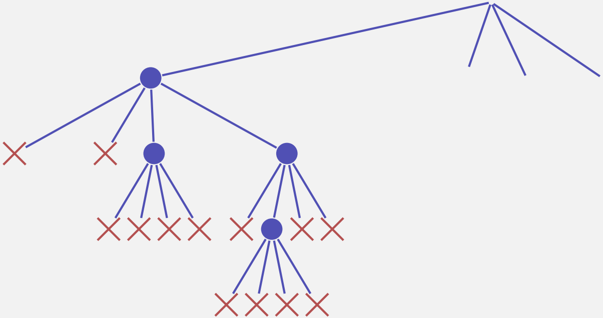
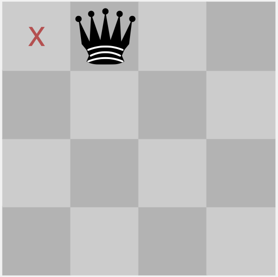
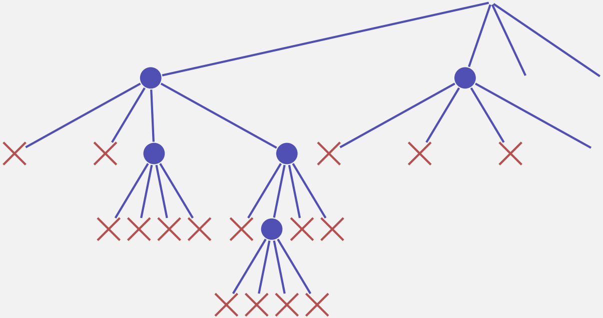


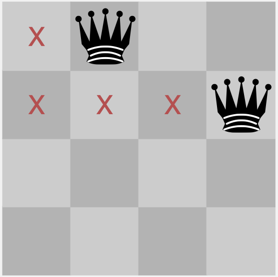


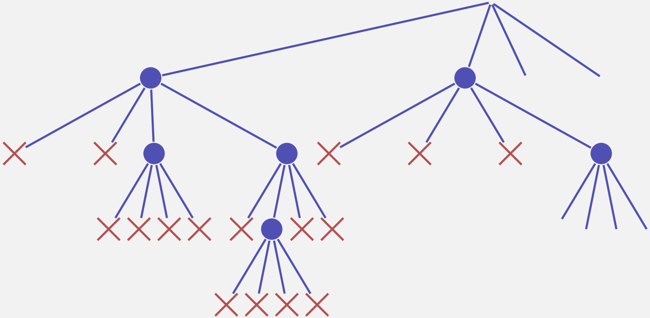
Search Strategy Visualized as a Tree



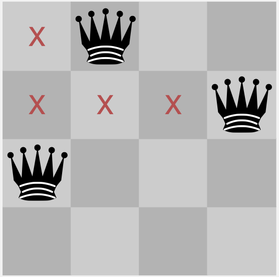


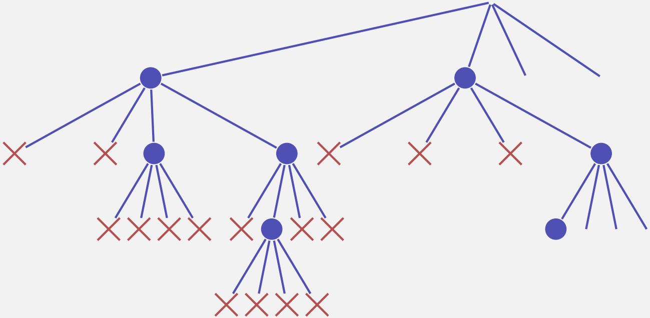
Search Strategy Visualized as a Tree



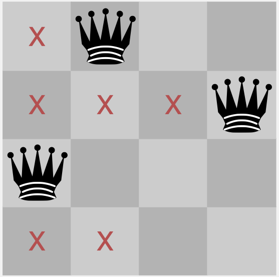


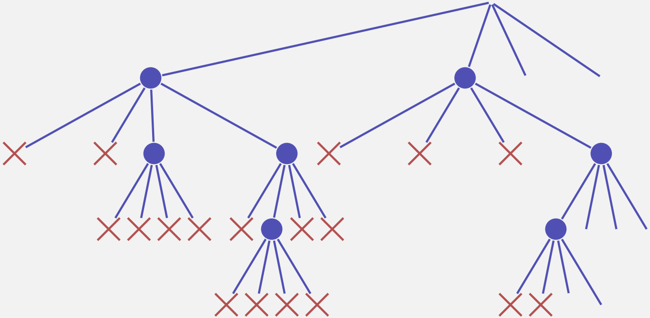
Search Strategy Visualized as a Tree



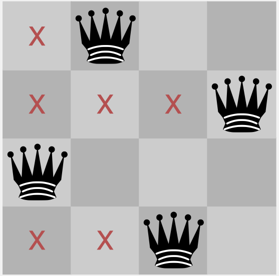


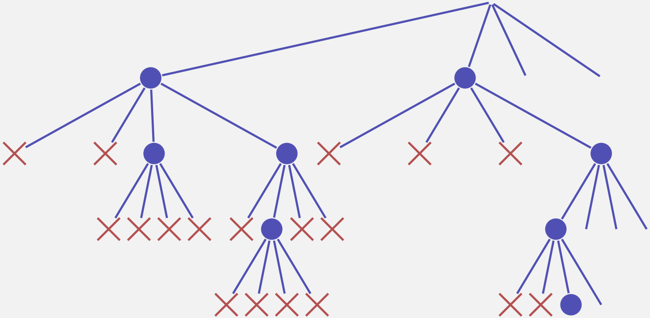
Search Strategy Visualized as a Tree





Search Strategy Visualized as a Tree





Check Queen

```
using Queens = std::vector<unsigned int>;

// post: returns if queen in the given row is valid, i.e.
//      does not share a common row, column or diagonal
//      with any of the queens on rows 0 to row-1
bool valid(const Queens& queens, unsigned int row){
  unsigned int col = queens[row];
  for (unsigned int r = 0; r != row; ++r){
    unsigned int c = queens[r];
    if (col == c || col - row == c0 - r || col + row == c + r)
      return false; // same column or diagonal
  }
  return true; // no shared column or diagonal
}
```

Recursion: Find a Solution

```
// pre: all queens from row 0 to row−1 are valid,
//      i.e. do not share any common row, column or diagonal
// post: returns if there is a valid position for queens on
//      row .. queens.size(). if true is returned then the
//      queens vector contains a valid configuration.
bool solve(Queens& queens, unsigned int row){
  if (row == queens.size())
    return true;
  for (unsigned int col = 0; col != queens.size(); ++col){
    queens[row] = col;
    if (valid(queens, row) && solve(queens,row+1))
       return true; // (else check next position)
  }
  return false; // no valid configuration found
}
```

Recursion: Count all Solutions

```
// pre: all queens from row 0 to row-1 are valid,
//   i.e. do not share any common row, column or diagonal
// post: returns the number of valid configurations of the
//   remaining queens on rows row ... queens.size()
int nSolutions(Queens& queens, unsigned int row){
  if (row == queens.size())
    return 1;
  int count = 0;
  for (unsigned int col = 0; col != queens.size(); ++col){
    queens[row] = col;
    if (valid(queens, row))
      count += nSolutions(queens,row+1);
  }
  return count;
}
```

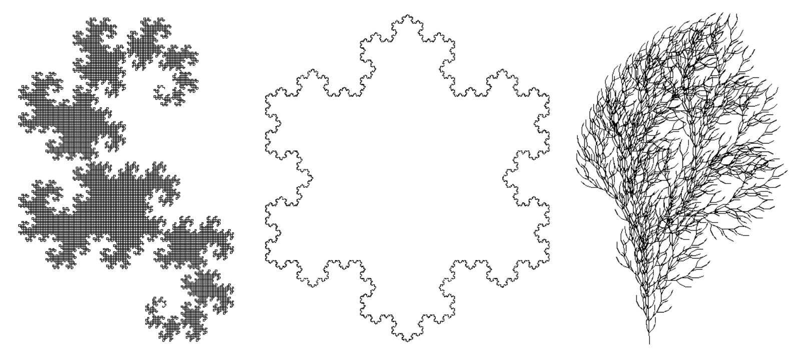
Main Program

```
// pre: positions of the queens in vector queens
// post: output of the positions of the queens in a graphical way
void print(const Queens& queens);

int main(){
  int n;
  std::cin >> n;
  Queens queens(n);
  if (solve(queens,0)){
    print(queens);
    std::cout << "# solutions:" << nSolutions(queens,0) << std::endl;
  } else
    std::cout << "no solution" << std::endl;
  return 0;
}
```

Lindenmayer-Systems (L-Systems)

Fractals from Strings and Turtles



Definition and Example

- alphabet Σ

- $\{\mathrm{F}, +, -\}$

Definition and Example

- alphabet Σ
- Σ^*: finite words over Σ

- $\{ \mathrm{F} , + , - \}$

Definition and Example

- alphabet Σ
- Σ^*: finite words over Σ
- production $P : \Sigma \to \Sigma^*$

- $\{\mathrm{F}, +, -\}$
-

c	$P(c)$
F	F + F +
$+$	$+$
$-$	$-$

Definition and Example

- alphabet Σ
- Σ^*: finite words over Σ
- production $P : \Sigma \rightarrow \Sigma^*$
- initial word $s_0 \in \Sigma^*$

- $\{\,\mathrm{F}\,,\,+\,,\,-\,\}$
-

c	$P(c)$
F	$\mathrm{F} + \mathrm{F} +$
$+$	$+$
$-$	$-$

- F

Definition and Example

- alphabet Σ
- Σ^*: finite words over Σ
- production $P : \Sigma \to \Sigma^*$
- initial word $s_0 \in \Sigma^*$

- $\{\mathrm{F}, +, -\}$
-

c	$P(c)$
F	F + F +
$+$	$+$
$-$	$-$

- F

Definition

The triple $\mathcal{L} = (\Sigma, P, s_0)$ is an L-System.

The Language Described

Wörter $w_0, w_1, w_2, \ldots \in \Sigma^*$:

$$P(\mathrm{F}) = \mathrm{F} + \mathrm{F} +$$

$$w_0 \;:=\; s_0 \qquad\qquad w_0 \;:=\; \mathrm{F}$$

The Language Described

Wörter $w_0, w_1, w_2, \ldots \in \Sigma^*$:

$P(\mathrm{F}) = \mathrm{F} + \mathrm{F} +$

$$w_0 := s_0 \qquad w_0 := \mathrm{F}$$

$$w_1 := P(w_0) \qquad w_1 := \mathrm{F} + \mathrm{F} +$$

The Language Described

Wörter $w_0, w_1, w_2, \ldots \in \Sigma^*$:

$$P(\mathrm{F}) = \mathrm{F} + \mathrm{F} +$$

$$\begin{aligned} w_0 &:= s_0 & w_0 &:= \mathrm{F} \\ w_1 &:= P(w_0) & w_1 &:= \mathrm{F} + \mathrm{F} + \\ w_2 &:= P(w_1) & w_2 &:= \mathrm{F} + \mathrm{F} + + \mathrm{F} + \mathrm{F} + + \end{aligned}$$

Definition

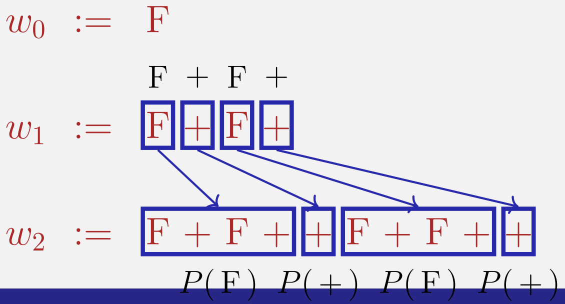
$$P(c_1 c_2 \ldots c_n) := P(c_1) P(c_2) \ldots P(c_n)$$

The Language Described

Wörter $w_0, w_1, w_2, \ldots \in \Sigma^*$:

$P(\mathrm{F}) = \mathrm{F} + \mathrm{F} +$

$$\begin{aligned} w_0 &:= s_0 \\ w_1 &:= P(w_0) \\ w_2 &:= P(w_1) \end{aligned}$$



$w_0 := \mathrm{F}$

$\mathrm{F} + \mathrm{F} +$

$w_1 := \boxed{\mathrm{F}}\,\boxed{+}\,\boxed{\mathrm{F}}\,\boxed{+}$

$w_2 := \boxed{\mathrm{F} + \mathrm{F} +}\,\boxed{+}\,\boxed{\mathrm{F} + \mathrm{F} +}\,\boxed{+}$

$P(\mathrm{F}) \quad P(+) \quad P(\mathrm{F}) \quad P(+)$

Definition

$$P(c_1 c_2 \ldots c_n) := P(c_1) P(c_2) \ldots P(c_n)$$

The Language Described

Wörter $w_0, w_1, w_2, \ldots \in \Sigma^*$:

$P(\mathrm{F}) = \mathrm{F} + \mathrm{F} +$

$$w_0 := s_0 \qquad w_0 := \mathrm{F}$$

$$w_1 := P(w_0) \qquad w_1 := \mathrm{F} + \mathrm{F} +$$

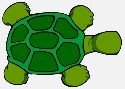
$$w_2 := P(w_1) \qquad w_2 := \mathrm{F} + \mathrm{F} + + \mathrm{F} + \mathrm{F} + +$$

Definition

$$P(c_1 c_2 \ldots c_n) := P(c_1) P(c_2) \ldots P(c_n)$$

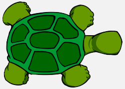
Turtle Graphics

Turtle with position and direction



Turtle Graphics

Turtle with position and direction

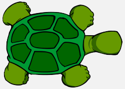


Turtle understands 3 commands:

F : move one step forwards	$+$: rotate by 90 degrees	$-$: rotate by -90 degrees

Turtle Graphics

Turtle with position and direction



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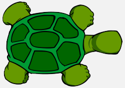
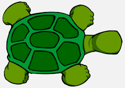
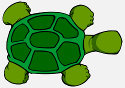


$+$: rotate by 90 degrees

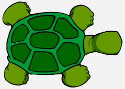


$-$: rotate by -90 degrees



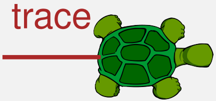
Turtle Graphics

Turtle with position and direction

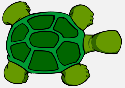


Turtle understands 3 commands:

F : move one step forwards ✓



$+$: rotate by 90 degrees



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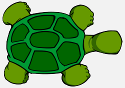
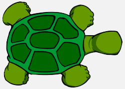


Turtle Graphics

Turtle with position and direction



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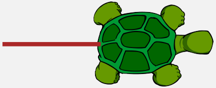
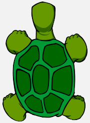
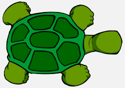


$+$: rotate by 90 degrees ✓

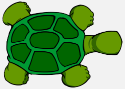


$-$: rotate by -90 degrees



Turtle Graphics

Turtle with position and direction



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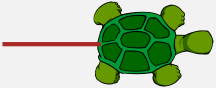
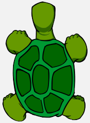


$+$: rotate by 90 degrees ✓



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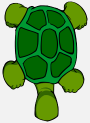
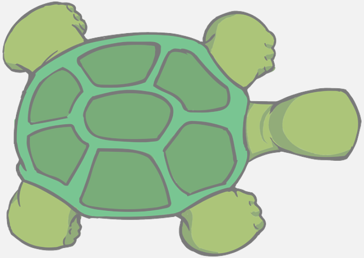


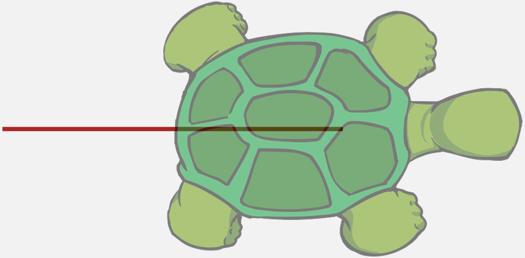
Draw Words!

$$w_1 = \mathrm{F} + \mathrm{F} +$$



Draw Words!

$$w_1 = \textcolor{red}{\mathrm{F}} + \mathrm{F} +$$



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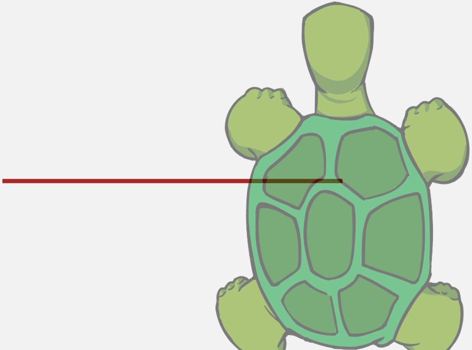

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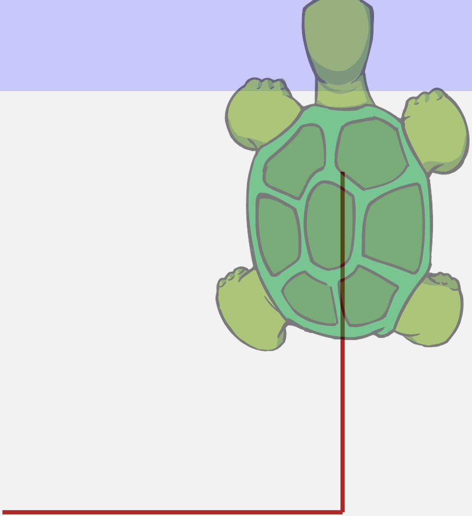
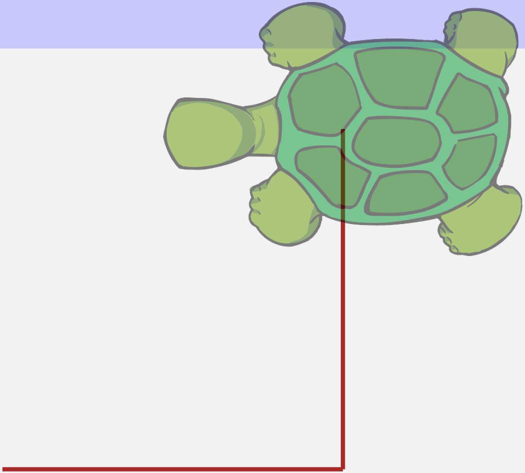


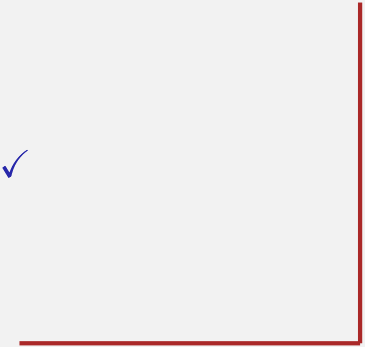
Draw Words!



$$w_1 = \mathrm{F} + \mathrm{F} \color{red}{+}$$

Draw Words!

$w_1 = \mathrm{F} + \mathrm{F} + \checkmark$



lindenmayer: Main Program

word $w_0 \in \Sigma^*$:

```
int main () {
  std::cout << "Maximal Recursion Depth =? ";
  unsigned int n;
  std::cin >> n;

  std::string w = "F"; // w_0
  produce(w,n);

  return 0;
}
```

lindenmayer: Main Program

word $w_0 \in \Sigma^*$:

```
int main () {
  std::cout << "Maximal Recursion Depth =? ";
  unsigned int n;
  std::cin >> n;

  std::string w = "F"; // w_0
  produce(w,n);

  return 0;
}
```

$w = w_0 = \mathrm{F}$

lindenmayer: production

```
// POST: recursively iterate over the production of the characters
//       of a word.
//       When recursion limit is reached, the word is "drawn"
void produce(std::string word, int depth){
  if (depth > 0){
    for (unsigned int k = 0; k < word.length(); ++k)
      produce(replace(word[k]), depth-1);
  } else {
    draw_word(word);
  }
}
```

lindenmayer: production

```
// POST: recursively iterate over the production of the characters
//       of a word.
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void produce(std::string word, int depth){
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      produce(replace(word[k]), depth-1);
  } else {
    draw_word(word);
  }
}
```

$w = w_i \to w = w_{i+1}$

lindenmayer: production

```
// POST: recursively iterate over the production of the characters
//       of a word.
//       When recursion limit is reached, the word is "drawn"
void produce(std::string word, int depth){
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lindenmayer: production

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  } else {
    draw_word(word);
  }
}
```

draw $w = w_n$!

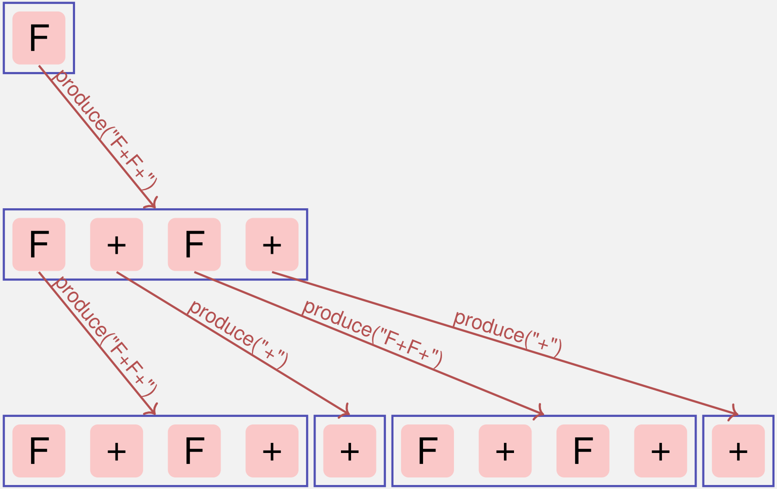
lindenmayer: replace

```
// POST: returns the production of c
std::string replace (const char c)
{
  switch (c) {
  case ’F’:
    return "F+F+";
  default:
    return std::string (1, c); // trivial production c —> c
  }
}
```

lindenmayer: draw

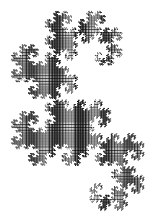
```
// POST: draws the turtle graphic interpretation of word
void draw_word (const std::string& word)
{
  for (unsigned int k = 0; k < word.length(); ++k)
    switch (word[k]) {
    case ’F’:
      turtle::forward(); // move one step forward
      break;
    case ’+’:
      turtle::left(90); // turn counterclockwise by 90 degrees
      break;
    case ’–’:
      turtle::right(90); // turn clockwise by 90 degrees
    }
}
```

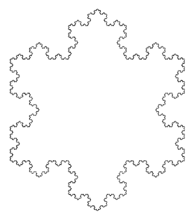
The Recursion

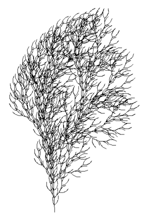


L-Systeme: Erweiterungen

- arbitrary symbols without graphical interpetation
- arbitrary angles (snowflake)
- saving and restoring the state of the turtle → plants (bush)







15. Recursion 2

Building a Calculator, Formal Grammars, Extended Backus Naur Form (EBNF), Parsing Expressions

Motivation: Calculator

Example

Input: `3 + 5`
Output: `8`

- binary Operators `+`, `-`, `*`, `/` and numbers

Motivation: Calculator

Example

Input: `3 / 5`
Output: `0.6`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic

Motivation: Calculator

Example

Input: `3 + 5 * 20`
Output: `103`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++

Motivation: Calculator

Example

Input: `(3 + 5) * 20`
Output: `160`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses

Motivation: Calculator

Example

Input: `-(3 + 5) + 20`

Output: `12`

- binary Operators `+`, `-`, `*`, `/` and numbers
- floating point arithmetic
- precedences and associativities like in C++
- parentheses
- unary operator `-`

Naive Attempt (without Parentheses)

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != '=') {
    double rval;
    std::cin >> rval;

    if (op == '+')
        lval += rval;
    else if (op == '*')
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

Seems to work...

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != ’=’) {
    double rval;
    std::cin >> rval;

    if (op == ’+’)
        lval += rval;
    else if (op == ’*’)
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 1 * 2 * 3 * 4 =
Result 24
```

Oops, Multiplication first. . .

```
double lval;
std::cin >> lval;

char op;
while (std::cin >> op && op != ’=’) {
    double rval;
    std::cin >> rval;

    if (op == ’+’)
        lval += rval;
    else if (op == ’*’)
        lval *= rval;
    else ...
}
std::cout << "Ergebnis " << lval << "\n";
```

```
Input 2 + 3 * 3 =
Result 15
```

Analyzing the Problem

Example

Input:

$$13 + \ldots$$

Analyzing the Problem

Example

Input:

$$13 + 4 * \ldots$$

Analyzing the Problem

Example

Input:

$$13 + 4 * (15 - \ldots$$

Analyzing the Problem

Example

Input:

$$13 + 4 * (15 - 7 * \ldots$$

Analyzing the Problem

Example

Input:

$$\underbrace{13 + 4 * (15 - 7 *}_{\text{Needs to be stored such that evaluation can be performed}} 3) =$$

Analyzing the Problem

Example

Result:

$$13 + 4*(15 - 21)$$

Analyzing the Problem

Example

Result:

$$13+4*(-6)$$

Analyzing the Problem

Example

Result:

$$13 + (-24)$$

Analyzing the Problem

Example

Result:

$$-11$$

Analyzing the Problem

Example

Expression:

$$13 + 4 * (15 - 7 * 3)$$

Example

This

Analyzing the Problem

Example

Expression:

$$13 + 4 * (15 - 7 * 3)$$

Example

This lecture

Analyzing the Problem

Example

Expression:

$$13 + 4 * (15 - 7 * 3)$$

Example

This lecture is

Analyzing the Problem

Example

Expression:

$$13 + 4 * (15 - 7 * 3)$$

Example

This lecture is pretty

Analyzing the Problem

Example

Expression:

$$13 + 4 * (15 - 7 * 3)$$

Example

This lecture is pretty much

Analyzing the Problem

Example

Expression:

$$13 + 4 * (15 - 7 * 3)$$

Example

This lecture is pretty much recursive.

Analyzing the Problem

$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

We will store symbols elegantly using recursion.

We need a new formal tool (that is independent of C++).

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$$13 + 4 * (15 - 7 * 3)$$

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$$13 + 4 * (15 - 7 * 3)$$

“Understanding an expression requires lookahead to upcoming symbols!

We will store symbols elegantly using recursion.

We need a new formal tool (that is independent of $\mathrm{C}++$).

Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

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- Strings: finite sequences of symbols

A formal grammar defines which strings are valid.

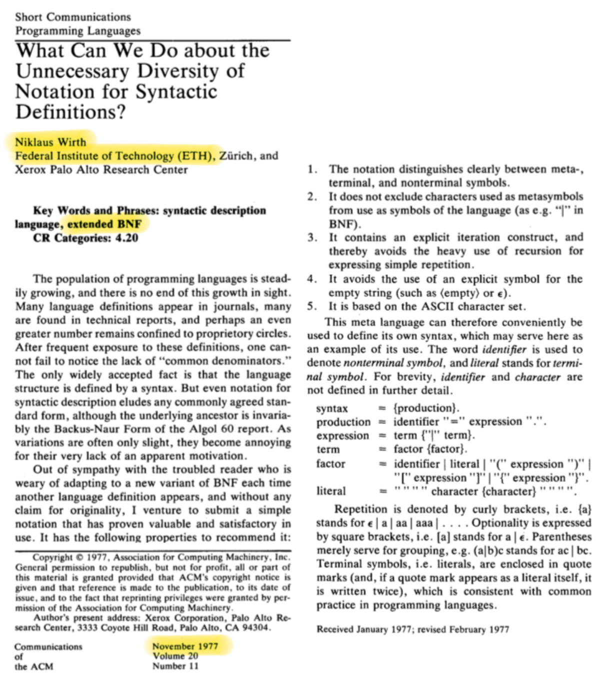
Formal Grammars

- Alphabet: finite set of symbols
- Strings: finite sequences of symbols

A formal grammar defines which strings are valid.

To describe the formal grammar, we use:

Extended Backus Naur Form (EBNF)



What Can We Do about the Unnecessary Diversity of Notation for Syntactic Definitions?

Niklaus Wirth
Federal Institute of Technology (ETH), Zürich, and
Xerox Palo Alto Research Center

Key Words and Phrases: syntactic description language, extended BNF
CR Categories: 4.20

The population of programming languages is steadily growing, and there is no end of this growth in sight. Many language definitions appear in journals, many are found in technical reports, and perhaps an even greater number remains confined to proprietory circles. After frequent exposure to these definitions, one cannot fail to notice the lack of "common denominators." The only widely accepted fact is that the language structure is defined by a syntax. But even notation for syntactic description eludes any commonly agreed standard form, although the underlying ancestor is invariably the Backus-Naur Form of the Algol 60 report. As variations are often only slight, they become annoying for their very lack of an apparent motivation.

Out of sympathy with the troubled reader who is weary of adapting to a new variant of BNF each time another language definition appears, and without any claim for originality, I venture to submit a simple notation that has proven valuable and satisfactory in use. It has the following properties to recommend it:

1. The notation distinguishes clearly between meta-, terminal, and nonterminal symbols.
2. It does not exclude characters used as metasymbols from use as symbols of the language (as e.g. "|" in BNF).
3. It contains an explicit iteration construct, and thereby avoids the heavy use of recursion for expressing simple repetition.
4. It avoids the use of an explicit symbol for the empty string (such as ⟨empty⟩ or ϵ).
5. It is based on the ASCII character set.

This meta language can therefore conveniently be used to define its own syntax, which may serve here as an example of its use. The word *identifier* is used to denote *nonterminal symbol*, and *literal* stands for *terminal symbol*. For brevity, *identifier* and *character* are not defined in further detail.

```
syntax      =  {production}.
production  =  identifier "=" expression ".".
expression  =  term {"|" term}.
term        =  factor {factor}.
factor      =  identifier | literal | "(" expression ")" |
               "[" expression "]" | "{" expression "}".
literal     =  """" character {character} """".
```

Repetition is denoted by curly brackets, i.e. {a} stands for ϵ | a | aa | aaa | . . . . Optionality is expressed by square brackets, i.e. [a] stands for a | ϵ. Parentheses merely serve for grouping, e.g. (a|b)c stands for ac | bc. Terminal symbols, i.e. literals, are enclosed in quote marks (and, if a quote mark appears as a literal itself, it is written twice), which is consistent with common practice in programming languages.

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Expressions

```
-(3-(4-5))*(3+4*5)/6
```

What do we need in a grammar?

Expressions

$$-(\underline{3}-(\underline{4}-\underline{5}))*(\underline{3}+\underline{4}*\underline{5})/\underline{6}$$

What do we need in a grammar?

- Number

Expressions

$$-\underline{(}3-\underline{(}4-5\underline{))}*\underline{(}3+4*5\underline{)}/6$$

What do we need in a grammar?

- Number , (?)

Expressions

$$\underline{-}(3-(4-5))*(3+4*5)/6$$

What do we need in a grammar?

- Number , (?)
  -Number, -(?)

Expressions

$$-(3-(4-5))\underline{*}(3+4\underline{*}5)\underline{/}6$$

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- ? * ?,   ? / ?,   ...

Expressions

$$-(3\underline{-}(4\underline{-}5))*(3\underline{+}4*5)/6$$

What do we need in a grammar?

- Number , (?)
  −Number, −(?)
- ? ∗ ?,   ? / ?,   ...
- ? − ?,   ? + ?,   ...

Expressions Multiplication/Division

`-(3-(4-5))*(3+4*5)/6`

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- ? * ?, ? / ?, ...
- ? - ?, ? + ?, ...

Factor

Expressions Multiplication/Division

$$-(3-(4-5))*(3+4*5)/6$$

What do we need in a grammar?

- Number , (?)
  −Number, −(?)
- Factor ∗ Factor,
  Factor / Factor , ...
- ? − ?, ? + ?, ...

Factor

Expressions Addition/Subtraction

$$-(3-(4-5))*(3+4*5)/6$$

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor ∗ Factor,
  Factor / Factor , ...
- ? − ?, ? + ?, ...

Factor

Term

Addition/Subtraction

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- ? - ?, ? + ?, ...

Factor

Term

Expressions

-(3-(4-5))*(3+4*5)/6

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term,
  Term - Term, ...

Factor

Term

Expressions

```
-(3-(4-5))*(3+4*5)/6
```

What do we need in a grammar?

- Number , (?)
  -Number, -(?)
- Factor * Factor, Factor
  Factor / Factor , ...
- Term + Term,
  Term - Term, ...

Factor

Term

Expression

Expressions

```
-(3-(4-5))*(3+4*5)/6
```

What do we need in a grammar?

- Number , (?)
  −Number, −(?)
- Factor ∗ Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term − Term, ...

Factor

Term

Expression

Expressions

$$-(3-(4-5))*(3+4*5)/6$$

What do we need in a grammar?

- Number , (Expression)
  −Number, −(Expression)
- Factor ∗ Factor, Factor
  Factor / Factor , ...
- Term + Term, Term
  Term − Term, ...

Factor

Term

Expression

The EBNF for Expressions

A factor is

- a number,
- an expression in parentheses or
- a negated factor.

```
factor    = unsigned_number
          | "(" expression ")"
          | "−" factor .
```

The EBNF for Expressions

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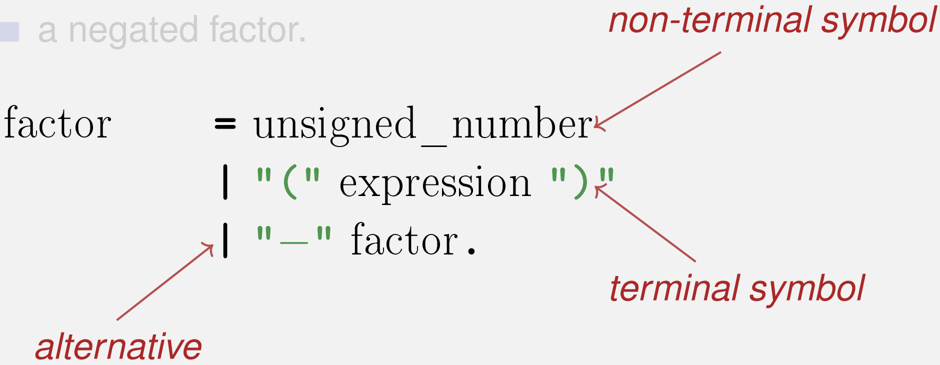

```
factor        = unsigned_number
              | "(" expression ")"
              | "−" factor.
```

non-terminal symbol

terminal symbol

alternative

The EBNF for Expressions

A term is

- factor,
- factor $*$ factor, factor / factor,
- factor $*$ factor $*$ factor, factor / factor $*$ factor, ...
- ...

```
term = factor { "*" factor | "/" factor }.
```

The EBNF for Expressions

A term is

- factor,
- factor * factor, factor / factor,
- factor * factor * factor, factor / factor * factor, ...
- ...

term = factor { "*" factor | "/" factor }.

The EBNF for Expressions

A term is

- factor,
- factor $*$ factor, factor / factor,
- factor $*$ factor $*$ factor, factor / factor $*$ factor, ...
- ...

term = factor { "$*$" factor | "/" factor }.

The EBNF for Expressions

A term is

- factor,
- factor ∗ factor, factor / factor,
- factor ∗ factor ∗ factor, factor / factor ∗ factor, ...
- ...

term = factor { "∗" factor | "/" factor }.

The EBNF for Expressions

A term is

- factor,
- factor ∗ factor, factor / factor,
- factor ∗ factor ∗ factor, factor / factor ∗ factor, ...
- ...

term = factor { "∗" factor | "/" factor }.

optional repetition

The EBNF for Expressions

```
factor     = unsigned_number
             | "(" expression ")"
             | "−" factor.

term       = factor { "∗" factor | "/" factor }.

expression = term { "+" term |"−" term }.
```

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.

Parsing

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- **Parser:** A program for parsing.

Parsing

- **Parsing:** Check if a string is valid according to the EBNF.
- **Parser:** A program for parsing.
- **Useful:** From the EBNF we can (nearly) automatically generate a parser:
  - Rules become functions
  - Alternatives and options become `if`–statements.
  - Nonterminial symbols on the right hand side become function calls
  - Optional repetitions become `while`–statements

Rules

```
factor     = unsigned_number
             | "(" expression ")"
             | "−" factor.

term       = factor { "∗" factor | "/" factor }.

expression = term { "+" term | "−" term }.
```

Functions (Parser)

Expression is read from an input stream.

```
// POST: returns true if and only if is = factor ...
//       and in this case extracts factor from is
bool factor (std::istream& is);

// POST: returns true if and only if is = term ...,
//       and in this case extracts all factors from is
bool term (std::istream& is);

// POST: returns true if and only if is = expression ...,
//       and in this case extracts all terms from is
bool expression (std::istream& is);
```

Functions (Parser with Evaluation)

Expression is read from an input stream.

```
// POST: extracts a factor from is
//       and returns its value
double factor (std::istream& is);

// POST: extracts a term from is
//       and returns its value
double term (std::istream& is);

// POST: extracts an expression from is
//       and returns its value
double expression (std::istream& is);
```

One Character Lookahead. . .

. . . to find the right alternative.

```
// POST: leading whitespace characters are extracted
//       from is, and the first non-whitespace character
//       is returned (0 if there is no such character)
char lookahead (std::istream& is)
{
    if (is.eof())           // eof: end of file (checks if stream is finished)
        return 0;
    is >> std::ws;          // skip all whitespaces
    if (is.eof())
        return 0;           // end of stream
    return is.peek();       // next character in is
}
```

Cherry-Picking

. . . to extract the desired character.

```
// POST: if ch matches the next lookahead then consume it
//       and return true; return false otherwise
bool consume (std::istream& is, char ch)
{
    if (lookahead(is) == ch){
        is >> ch;
        return true;
    }
    return false ;
}
```

Evaluating Factors

```
double factor (std::istream& is)
{
    double v;
    if (consume(is, '(')) {
        v = expression (is);
        consume(is, ')');
    } else if (consume(is, '-')) {
        v = -factor (is);
    } else {
        is >> v;
    }
    return v;
}
```

```
factor = "(" expression ")"
       | "-" factor
       | unsigned_number.
```

Evaluating Terms

```
double term (std::istream& is)
{
    double value = factor (is);
    while(true){
        if (consume(is, '*'))
            value *= factor (is);
        else if (consume(is, '/'))
            value /= factor(is)
        else
            return value;
    }
}
```

```
term = factor { "*" factor | "/" factor }.
```

Evaluating Expressions

```
double expression (std::istream& is)
{
  double value = term(is);
  while(true){
    if (consume(is, '+'))
      value += term (is);
    else if (consume(is, '-'))
      value -= term(is)
    else
      return value;
  }
}
```

```
expression = term { "+" term | "-" term }.
```

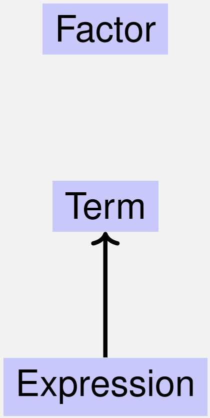
Recursion!

Factor

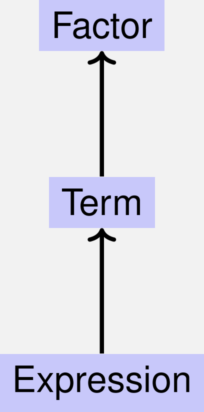
Term

Expression

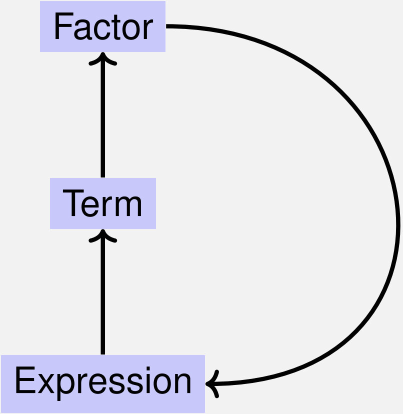
Recursion!



Recursion!



Recursion!



Factor

Term

Expression

EBNF — and it works!

EBNF (`calculator.cpp`, Evaluation from left to right):

```
factor     = unsigned_number
           | "(" expression ")"
           | "-" factor.

term       = factor { "*" factor | "/" factor }.

expression = term { "+" term | "-" term }.
```

```
std::stringstream input ("1-2-3");
std::cout << expression (input) << "\n"; // -4
```

16. Structs

Rational Numbers, Struct Definition

Calculating with Rational Numbers

- Rational numbers ($\mathbb{Q}$) are of the form $\frac{n}{d}$ with n and d in $\mathbb{Z}$
- $\mathrm{C}{+}{+}$does not provide a built-in type for rational numbers

Calculating with Rational Numbers

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- $\mathrm{C{+}{+}}$does not provide a built-in type for rational numbers

Goal

We build a $\mathrm{C{+}{+}}$-type for rational numbers ourselves! ☺

Vision

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;
std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

A First Struct

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

A First Struct

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

member variable (numerator)

member variable (denominator)

A First Struct

```
struct rational {
  int n;
  int d; // INV: d != 0
};
```

member variable

member variable

- `struct` defines a new *type*

A First Struct

```
struct rational {
  int n;
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};
```
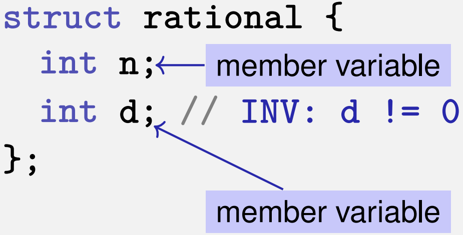

member variable

member variable

- `struct` defines a new *type*
- formal range of values: *cartesian product* of the value ranges of existing types

A First Struct



```
struct rational {
   int n;
   int d; // INV: d != 0
};
```

- `struct` defines a new *type*
- formal range of values: *cartesian product* of the value ranges of existing types
- real range of values: `rational` ⊊ `int` × `int`.

Accessing Member Variables

```
struct rational {
    int n;
    int d; // INV: d != 0
};

rational add (rational a, rational b){
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
```

$$\frac{r_n}{r_d} := \frac{a_n}{a_d} + \frac{b_n}{b_d} = \frac{a_n \cdot b_d + a_d \cdot b_n}{a_d \cdot b_d}$$

Input

```
// Input r
rational r;
std::cout << "Rational number r:\n";
std::cout << " numerator =? ";
std::cin >> r.n;
std::cout << " denominator =? ";
std::cin >> r.d;

// Input s the same way
rational s;
...
```

Vision comes within Reach ...

```
// computation
const rational t = add (r, s);

// output
std::cout << "Sum is " << t.n << "/" << t.d << ".\n";
```

Struct Defintions: Examples

```
struct rational_vector_3 {
  rational x;
  rational y;
  rational z;
};
```

underlying types can be fundamental or user defined

Struct Definitions: Examples

```
struct extended_int {
  // represents value if is_positive==true
  // and -value otherwise
  unsigned int value;
  bool is_positive;
};
```

the underlying types can be different

Structs: Initialization and Assignment

```
rational s;
```

← member variables are uninitialized

```
rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

member-wise initialization:

`t.n = 1, t.d = 5`

Structs: Initialization and Assignment

`rational s;`

`rational t = {1,5};`

`rational u = t;` ← member-wise copy

`t = u;`

`rational v = add (u,t);`

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;

t = u;  ←  member-wise copy

rational v = add (u,t);
```

Structs: Initialization and Assignment

```
rational s;

rational t = {1,5};

rational u = t;

t = u;

rational v = add (u,t);
```

⟵ member-wise copy

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

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- member-wise comparison does not make sense in general...

Comparing Structs?

For each fundamental type (`int, double,...`) there are comparison operators `==` and `!=` , not so for structs! Why?

- member-wise comparison does not make sense in general...
- ...otherwise we had, for example, $\frac{2}{3} \neq \frac{4}{6}$

User Defined Operators

Instead of

```
rational t = add(r, s);
```

we would rather like to write

```
rational t = r + s;
```

User Defined Operators

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rational t = add(r, s);
```

we would rather like to write

```
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```

This can be done with *Operator Overloading (→ next week)*.

17. Classes

Overloading Functions and Operators, Encapsulation, Classes, Member Functions, Constructors

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }            // f1
int sq (int x) { ... }                  // f2
int pow (int b, int e) { ... }          // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);
std::cout << sq (1.414);
std::cout << pow (2);
std::cout << pow (3,3);
```

Function Overloading

- A function is defined by name, types, number and order of arguments

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double sq (double x) { ... }             // f1
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- A function is defined by name, types, number and order of arguments

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double sq (double x) { ... }          // f1
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int pow (int b, int e) { ... }        // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);     // compiler chooses f2
std::cout << sq (1.414);
std::cout << pow (2);
std::cout << pow (3,3);
```

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }            // f1
int sq (int x) { ... }                  // f2
int pow (int b, int e) { ... }          // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);     // compiler chooses f2
std::cout << sq (1.414); // compiler chooses f1
std::cout << pow (2);
std::cout << pow (3,3);
```

Function Overloading

- A function is defined by name, types, number and order of arguments

```
double sq (double x) { ... }          // f1
int sq (int x) { ... }                // f2
int pow (int b, int e) { ... }        // f3
int pow (int e) { return pow (2,e); } // f4
```

- the compiler automatically chooses the function that fits “best” for a function call

```
std::cout << sq (3);     // compiler chooses f2
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std::cout << pow (2);    // compiler chooses f4
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Function Overloading

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```
double sq (double x) { ... }          // f1
int sq (int x) { ... }                // f2
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std::cout << pow (2);    // compiler chooses f4
std::cout << pow (3,3);  // compiler chooses f3
```

Operator Overloading

- Operators are special functions and can be overloaded
- Name of the operator *op*:

```
operatorop
```

Adding `rational` Numbers – Before

```
// POST: return value is the sum of a and b
rational add (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = add (r, s);
```

Adding `rational` Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = r + s;
```

Adding rational Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = r + s;
```

infix notation

Adding `rational` Numbers – After

```
// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
const rational t = operator+ (r, s);
```

equivalent but less handy: functional notation

Unary Minus

Only one argument:

```
// POST: return value is -a
rational operator- (rational a)
{
    a.n = -a.n;
    return a;
}
```

Comparison Operators

can be defined such that they do the right thing:

Comparison Operators

can be defined such that they do the right thing:

```
// POST: returns true iff a == b
bool operator== (rational a, rational b)
{
    return a.n * b.d == a.d * b.n;
}
```

Comparison Operators

can be defined such that they do the right thing:

```
// POST: returns true iff a == b
bool operator== (rational a, rational b)
{
    return a.n * b.d == a.d * b.n;
}
```

$$\frac{2}{3} = \frac{4}{6} \qquad \checkmark$$

Arithmetic Assignment

We want to write

```
rational r;
r.n = 1; r.d = 2;                    // 1/2

rational s;
s.n = 1; s.d = 3;                    // 1/3

r += s;
std::cout << r.n << "/" << r.d;      // 5/6
```

Operator +=

```
rational& operator+= (rational& a, rational b)
{
    a.n = a.n * b.d + a.d * b.n;
    a.d *= b.d;
    return a;
}
```

Operator +=

```
rational& operator+= (rational& a, rational b)
{
    a.n = a.n * b.d + a.d * b.n;
    a.d *= b.d;
    return a;
}
```

- The L-value `a` is increased by the value of `b` and returned as L-value

In/Output Operators

can also be overloaded.

- Before:

```
std::cout << "Sum is "
          << t.n << "/" << t.d << "\n";
```

- After (desired):

```
std::cout << "Sum is "
          << t << "\n";
```

In/Output Operators

can be overloaded as well:

```
// POST: r has been written to out
std::ostream& operator<< (std::ostream& out,
                          rational r)
{
    return out << r.n << "/" << r.d;
}
```

In/Output Operators

can be overloaded as well:

```
// POST: r has been written to out
std::ostream& operator<< (std::ostream& out,
                          rational r)
{
    return out << r.n << "/" << r.d;
}
```

writes `r` to the output stream
and returns the stream as L-value.

Input

```
// PRE: in starts with a rational number
// of the form "n/d"
// POST: r has been read from in
std::istream& operator>> (std::istream& in,
                          rational& r){
    char c; // separating character ’/’
    return in >> r.n >> c >> r.d;
}
```

reads `r` from the input stream
and returns the stream as L-value.

Goal Attained!

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;

std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

Goal Attained!

```
// input
std::cout << "Rational number r =? ";
rational r;
std::cin >> r;

std::cout << "Rational number s =? ";
rational s;
std::cin >> s;

// computation and output
std::cout << "Sum is " << r + s << ".\n";
```

operator >>

operator +

operator<<

A new Type with Functionality…

```
struct rational {
    int n;
    int d; // INV: d != 0
};

// POST: return value is the sum of a and b
rational operator+ (rational a, rational b)
{
    rational result;
    result.n = a.n * b.d + a.d * b.n;
    result.d = a.d * b.d;
    return result;
}
...
```

. . . should be in a Library!

`rational.h`:

- Definition of a struct `rational`
- Function declarations

`rational.cpp`:

- arithmetic operators (`operator+`, `operator+=`, ...)
- relational operators (`operator==`, `operator>`, ...)
- in/output (`operator >>`, `operator <<`, ...)

Thought Experiment

The three core missions of ETH:

- research

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- education

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We found a startup: RAT PACK®!

Thought Experiment

The three core missions of ETH:

- research
- education
- technology transfer

We found a startup: RAT PACK®!

- Selling the `rational` library to customers
- ongoing development according to customer’s demands

The Customer is Happy

“Buying RAT PACK® has been a game-changing move to put us on the forefront of cutting-edge technology in social media engineering.”

B. Labla, CEO

The Customer is Happy

. . . and programs busily using `rational`.

The Customer is Happy

…and programs busily using `rational`.

- output as `double`-value $(\frac{3}{5} \rightarrow 0.6)$

The Customer is Happy

. . . and programs busily using `rational`.

- output as `double`-value $(\frac{3}{5} \rightarrow 0.6)$

```
// POST: double approximation of r
double to_double (rational r)
{
  double result = r.n;
  return result / r.d;
}
```

The Customer Wants More

“Can we have rational numbers with an extended value range?”

The Customer Wants More

“Can we have rational numbers with an extended value range?”

- Sure, no problem, e.g.:

```
~~struct rational {~~
  ~~int n;~~
  ~~int d;~~
~~};~~
```

$\Rightarrow$

```
struct rational {
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

New Version of RAT PACK®

It sucks, nothing works any more!

New Version of RAT PACK®

It sucks, nothing works any more!

- What is the problem?

New Version of RAT PACK®

It sucks, nothing works any more!

- What is the problem?

$-\frac{3}{5}$ *is sometimes* 0.6, *this cannot be true!*

New Version of RAT PACK®

It sucks, nothing works any more!

- What is the problem?

$-\frac{3}{5}$ *is sometimes* 0.6*, this cannot be true!*

- That is your fault. Your conversion to `double` is the problem, our library is correct.

New Version of RAT PACK®

It sucks, nothing works any more!

- What is the problem?

$-\frac{3}{5}$ is sometimes 0.6, this cannot be true!

- That is your fault. Your conversion to `double` is the problem, our library is correct.

Up to now it worked, therefore the new version is to blame!

Liability Discussion

```
// POST: double approximation of r
double to_double (rational r){
  double result = r.n;
  return result / r.d;
}
```

Liability Discussion

```
// POST: double approximation of r
double to_double (rational r){
  double result = r.n;
  return result / r.d;
}
```

correct using. . .

```
struct rational {
  int n;
  int d;
};
```

Liability Discussion

```
// POST: double approximation of r
double to_double (rational r){
  double result = r.n;
  return result / r.d;
}
```

correct using...

```
struct rational {
  int n;
  int d;
};
```

...not correct using

```
struct rational {
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

Liability Discussion

```
// POST: double approximation of r
double to_double (rational r){
  double result = r.n;
  return result / r.d;
}
```

r.is_positive and result.is_positive do not appear.

correct using…

```
struct rational {
  int n;
  int d;
};
```

…not correct using

```
struct rational {
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

We are to Blame!!

- Customer sees and uses our representation of rational numbers (initially `r.n, r.d`)

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- Customer sees and uses our representation of rational numbers (initially `r.n, r.d`)
- When we change it (`r.n, r.d, r.is_positive`), the customer's programs do not work anymore.

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- No customer is willing to adapt the programs when the version of the library changes.

We are to Blame!!

- Customer sees and uses our representation of rational numbers (initially `r.n, r.d`)
- When we change it (`r.n, r.d, r.is_positive`), the customer's programs do not work anymore.
- No customer is willing to adapt the programs when the version of the library changes.

⇒ RAT PACK® is history…

Idea of Encapsulation (Information Hiding)

- A type is uniquely defined by its *value range* and its *functionality*

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- ⇒ The customer is not provided with representation but with functionality!

Idea of Encapsulation (Information Hiding)

- A type is uniquely defined by its *value range* and its *functionality*
- The representation should not be visible.
- ⇒ The customer is not provided with representation but with functionality!

↑

```
str.length(),
v.push_back(1),...
```

Classes

- provide the concept for encapsulation in $\mathrm{C}{+}{+}$

Classes

- provide the concept for encapsulation in $\mathrm{C{+}{+}}$
- are a variant of structs

Classes

- provide the concept for encapsulation in C++
- are a variant of structs
- are provided in many object oriented programming languages

Encapsulation: `public` / `private`

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

is used instead of `struct` if anything at all shall be “hidden”

Encapsulation: `public` / `private`

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

is used instead of `struct` if anything at all shall be “hidden”

only difference

- `struct`: by default *nothing* is hidden
- `class` : by default *everything* is hidden

Encapsulation: public / private

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

Application Code

```
rational r;
r.n = 1;      // error: n is private
r.d = 2;      // error: d is private
int i = r.n; // error: n is private
```

Encapsulation: public / private

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

Good news: r.d = 0 cannot happen any more by accident.

Application Code

```
rational r;
r.n = 1;      // error: n is private
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Encapsulation: public / private

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

Good news: `r.d = 0` cannot happen any more by accident.

Bad news: the customer cannot do anything any more . . .

Application Code

```
rational r;
r.n = 1;      // error: n is private
r.d = 2;      // error: d is private
int i = r.n; // error: n is private
```

Encapsulation: `public` / `private`

```
class rational {
  int n;
  int d; // INV: d != 0
};
```

Good news: `r.d = 0` cannot happen any more by accident.

Bad news: the customer cannot do anything any more . . .

. . . and we can't, either.
(no `operator+`,. . .)

Application Code

```
rational r;
r.n = 1;      // error: n is private
r.d = 2;      // error: d is private
int i = r.n; // error: n is private
```

Member Functions: Declaration

```
class rational {
public:
    // POST: return value is the numerator of this instance
    int numerator () const {
      return n;
    }
    // POST: return value is the denominator of this instance
    int denominator () const {
      return d;
    }
private:
    int n;
    int d; // INV: d!= 0
};
```

Member Functions: Declaration

```cpp
class rational {
public:
   // POST: return value is the numerator of this instance
   int numerator () const {
     return n;
   }
   // POST: return value is the denominator of this instance
   int denominator () const {
     return d;
   }
private:
   int n;
   int d; // INV: d!= 0
};
```

public area

Member Functions: Declaration

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class rational {
public:
    // POST: return value is the numerator of this instance
    int numerator () const {
      return n;
    }
    // POST: return value is the denominator of this instance
    int denominator () const {
      return d;
    }
private:
    int n;
    int d; // INV: d!= 0
};
```

public area

member function

Member Functions: Declaration

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class rational {
public:
    // POST: return value is the numerator of this instance
    int numerator () const {
      return n;
    }
    // POST: return value is the denominator of this instance
    int denominator () const {
      return d;
    }
private:
    int n;
    int d; // INV: d!= 0
};
```

public area

member function

member functions have access to private data

Member Functions: Call

```
// Definition des Typs
class rational {
    ...
};
...
// Variable des Typs
rational r;

int n = r.numerator();   // Zaehler
int d = r.denominator(); // Nenner
```

member access

Member Functions: Definition

```
// POST: returns numerator of this instance
int numerator () const
{
  return n;
}
```

Member Functions: Definition ???

```
// POST: returns numerator of this instance
int numerator ()  const
{
  return n;
}
```

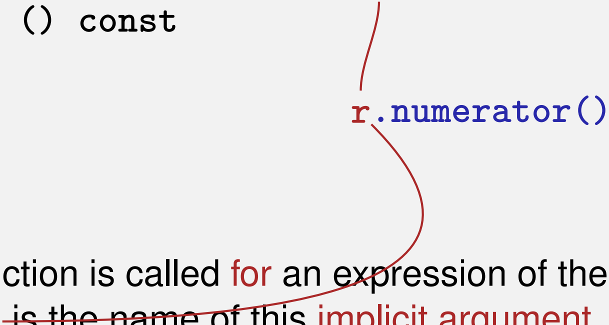
Member Functions: Definition

```
// POST: returns numerator of this instance
int numerator () const
{
  return n;                          r.numerator()
}
```

- A member function is called for an expression of the class.

Member Functions: Definition

```
// POST: returns numerator of this instance
int numerator () const
{
  return n;
}
```



r.numerator()

- A member function is called for an expression of the class. in the function, this is the name of this implicit argument.

Member Functions: Definition

```
// POST: returns numerator of this instance
int numerator ()  const
{
  return n;                   r.numerator()
}
```

- A member function is called for an expression of the class. in the function, `this` is the name of this implicit argument.
- const refers to the instance `this`

Member Functions: Definition

```
// POST: returns  numerator of this instance
int numerator () const
{
  return n;
}
```

r.numerator()

- A member function is called for an expression of the class. in the function, `this` is the name of this implicit argument.
- const refers to the instance `this`
- `n` is the shortcut for `this->n` (precise explanation of “`->`” next week)

`const` and Member Functions

```
class rational {
public:
  int numerator () const
  { return n; }
  void set_numerator (int N)
  { n = N;}
...
}
```

```
rational x;
x.set_numerator(10); // ok;
const rational y = x;
int n = y.numerator(); // ok;
y.set_numerator(10); // error;
```

The `const` at a member function is to promise that an instance cannot be changed via this function.

`const` items can only call `const` member functions.

Comparison

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return n;
    }
};

rational r;
...
std::cout << r.numerator();
```

Comparison

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return this->n;
    }
};

rational r;
...
std::cout << r.numerator();
```

Comparison

Roughly like this it were ...

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return this->n;
    }
};

rational r;
...
std::cout << r.numerator();
```

Comparison

Roughly like this it were ...

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return this->n;
    }
};

rational r;
...
std::cout << r.numerator();
```

... without member functions

```
struct bruch {
    int n;
    ...
};

int numerator (const bruch& dieser)
{
    return dieser.n;
}

bruch r;
..
std::cout << numerator(r);
```

Member-Definition: In-Class

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return n;
    }
    ....
};
```

- No separation between declaration and definition (bad for libraries)

Member-Definition: In-Class vs. Out-of-Class

```
class rational {
    int n;
    ...
public:
    int numerator () const
    {
        return n;
    }
    ....
};
```

- No separation between declaration and definition (bad for libraries)

```
class rational {
    int n;
    ...
public:
    int numerator () const;
    ...
};

int rational::numerator () const
{
  return n;
}
```

- This also works.

Initialisation? Constructors!

```
class rational
{
public:
    rational (int num, int den)
        : n (num), d (den)
    {
        assert (den != 0);
    }
...
};
...
rational r (2,3); // r = 2/3
```

Initialisation? Constructors!

```
class rational
{
public:
    rational (int num, int den)
        : n (num), d (den)
    {
        assert (den != 0);
    }
 ...
};
 ...
rational r (2,3); // r = 2/3
```

Initialization of the member variables

function body.

Initialisation “rational = int”?

```
class rational
{
public:
    rational (int num)
        : n (num), d (1)
    {}
 ...
};
 ...
rational r (2);    // explicit initialization with 2
rational s = 2;    // implicit conversion
```

Initialisation “rational = int”?

```
class rational
{
public:
    rational (int num)
        : n (num), d (1)
    {}   ⟵ empty function body
 ...
};
 ...
rational r (2);    // explicit initialization with 2
rational s = 2;    // implicit conversion
```

The Default Constructor

```
class rational
{
public:
     ...
     rational ()
        : n (0), d (1)
     {}
...
};
...
rational r;    // r = 0
```

empty list of arguments

The Default Constructor

```
class rational
{
public:
      ...
      rational ()
          : n (0), d (1)
      {}
 ...
};
 ...
rational r;     // r = 0
```

empty list of arguments

⇒ There are no uninitiatlized variables of type rational any more!

Alterantively: Deleting a Default Constructor

```
class rational
{
public:
      ...
      rational () = delete;
 ...
};
 ...
rational r;     // error: use of deleted function 'rational::rational()
```

⇒ There are no uninitiatlized variables of type rational any more!

RAT PACK® Reloaded . . .

Customer’s program now looks like this:

```
// POST: double approximation of r
double to_double (const rational r)
{
  double result = r.numerator();
  return result / r.denominator();
}
```

RAT PACK® Reloaded . . .

Customer’s program now looks like this:

```
// POST: double approximation of r
double to_double (const rational r)
{
  double result = r.numerator();
  return result / r.denominator();
}
```

- We can adapt the member functions together with the representation ✓

RAT PACK® Reloaded . . .

before

```
class rational {
...
private:
  int n;
  int d;
};
```

RAT PACK® Reloaded ...

before

```
class rational {
...
private:
  int n;
  int d;
};
```

```
int numerator () const
{
  return n;
}
```

RAT PACK® Reloaded . . .

before

```
class rational {
...
private:
  int n;
  int d;
};
```

```
int numerator () const
{
  return n;
}
```

after

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

RAT PACK® Reloaded ...

before

```
class rational {
...
private:
  int n;
  int d;
};
```

```
int numerator () const
{
  return n;
}
```

after

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

RAT PACK® Reloaded ?

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const
{
  if (is_positive)
    return n;
  else {
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}
```

RAT PACK® Reloaded ?

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const
{
  if (is_positive)
    return n;
  else {
    int result = n;
    return −result;
  }
}
```

- value range of nominator and denominator like before

RAT PACK® Reloaded ?

```
class rational {
...
private:
  unsigned int n;
  unsigned int d;
  bool is_positive;
};
```

```
int numerator () const
{
  if (is_positive)
    return n;
  else {
    int result = n;
    return -result;
  }
}
```

- value range of nominator and denominator like before
- possible overflow in addition

Encapsulation still Incompleete

Customer’s point of view (`rational.h`):

```
class rational {
public:
   // POST: returns numerator of *this
   int numerator () const;
   ...
private:
  // none of my business
};
```

Encapsulation still Incompleete

Customer’s point of view (`rational.h`):

```
class rational {
public:
   // POST: returns numerator of *this
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   ...
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  // none of my business
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```

- We determined denominator and nominator type to be `int`

Encapsulation still Incompleete

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class rational {
public:
   // POST: returns numerator of *this
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   ...
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  // none of my business
};
```

- We determined denominator and nominator type to be `int`
- Solution: encapsulate not only data but alsoe types.

Fix: “our” type `rational::integer`

Customer’s point of view (`rational.h`):

```
public:
   using integer = long int; // might change
   // POST: returns numerator of *this
   integer numerator () const;
```

Fix: “our” type rational::integer

Customer’s point of view (rational.h):

```
public:
   using integer = long int; // might change
   // POST: returns numerator of *this
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```

- We provide an additional type!

Fix: “our” type `rational::integer`

Customer’s point of view (`rational.h`):

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public:
   using integer = long int; // might change
   // POST: returns numerator of *this
   integer numerator () const;
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- We provide an additional type!
- Determine only Functionality, e.g:
  - implicit conversion `int` → `rational::integer`

Fix: “our” type `rational::integer`

Customer’s point of view (`rational.h`):

```
public:
   using integer = long int; // might change
   // POST: returns numerator of *this
   integer numerator () const;
```

- We provide an additional type!
- Determine only Functionality, e.g:
  - implicit conversion `int` → `rational::integer`
  - function `double to_double (rational::integer)`

RAT PACK® Revolutions

Finally, a customer program that remains stable

```
// POST: double approximation of r
double to_double (const rational r)
{
  rational::integer n = r.numerator();
  rational::integer d = r.denominator();
  return to_double (n) / to_double (d);
}
```

18. Dynamic Data Structures I

Dynamic Memory, Addresses and Pointers, Const-Pointer Arrays, Array-based Vectors

Recap: `vector<`T`>`

- Can be initialised with arbitrary size **`n`**

Recap: vector<T>

- Can be initialised with arbitrary size **n**
- Supports various operations:

```
e = v[i];          // Get element
v[i] = e;          // Set element
l = v.size();      // Get size
v.push_front(e);   // Prepend element
v.push_back(e);    // Append element
...
```

Recap: vector<T>

- Can be initialised with arbitrary size `n`
- Supports various operations:

```
e = v[i];          // Get element
v[i] = e;          // Set element
l = v.size();      // Get size
v.push_front(e);   // Prepend element
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...
```

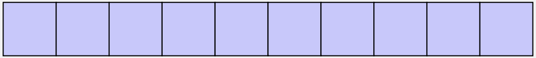
- A vector is a *dynamic data structure*, whose size may change at runtime

Our Own Vector!

- Today, we'll implement our own vector: `vec`
- Step 1: `vec<int>` (today)
- Step 2: `vec<`T`>` (later, only superficially)

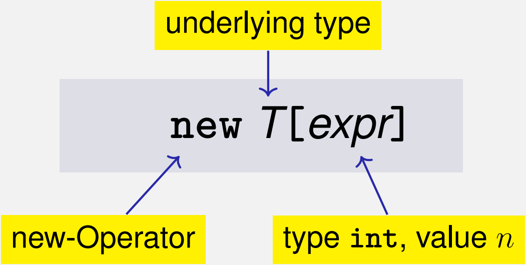
Vectors in Memory

Already known: A vector has a *contiguous* memory layout



Question: How to *allocate* a chunk of memory of *arbitrary* size during runtime, i.e. *dynamically*?

`new` for Arrays



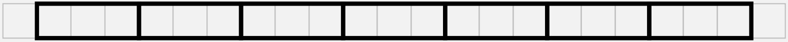
underlying type

`new` *T*[*expr*]

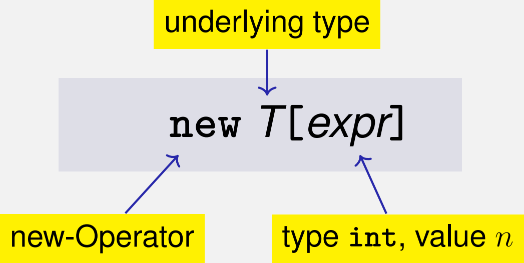
new-Operator

type `int`, value n

- **Effect**: new contiguous chunk of memory n elements of type *T* is allocated



`new` for Arrays



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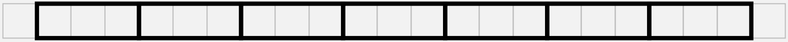
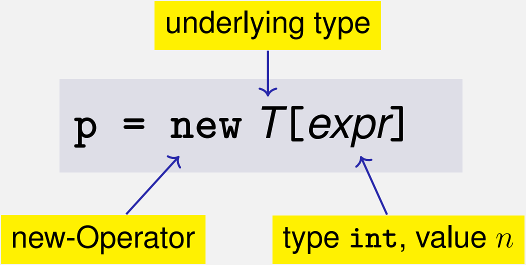


- This chunk of memory is called an *array* (of length n)

`new` for Arrays

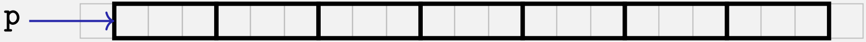


underlying type

`p = new` *T*[*expr*]

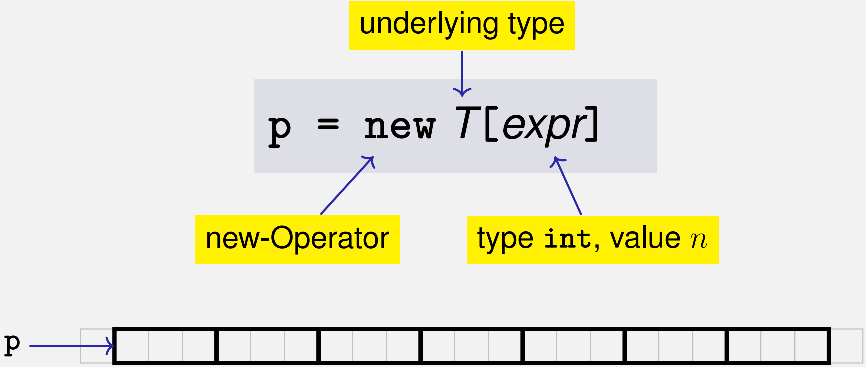
new-Operator

type `int`, value n



- **Type**: A pointer $T*$ (more soon)

`new` for Arrays



underlying type

`p = new` *T*[*expr*]

new-Operator

type `int`, value n

p

- **Type**: A pointer $T*$ (more soon)
- **Value**: the starting address of the memory chunk

Outlook: `new` and `delete`

`new` *T*[*expr*]

- So far: memory (local variables, function arguments) “lives” only inside a function call

Outlook: `new` and `delete`

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Outlook: `new` and `delete`

`new` *T*`[`*expr*`]`

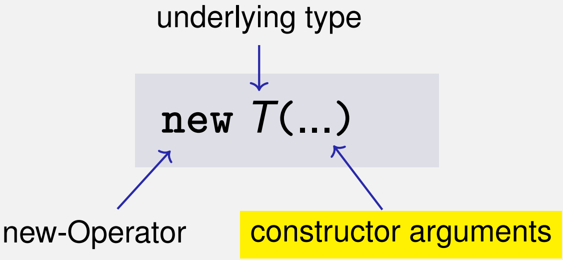
- So far: memory (local variables, function arguments) “lives” only inside a function call
- But now: memory chunk inside vector must not “die” before the vector itself
- Memory allocated with **`new`** is *not* automatically deallocated (= released)

Outlook: `new` and `delete`

`new` T[*expr*]

- So far: memory (local variables, function arguments) “lives” only inside a function call
- But now: memory chunk inside vector must not “die” before the vector itself
- Memory allocated with `new` is *not* automatically deallocated (= released)
- Every `new` must have a matching `delete` that releases the memory explicitly → **in two weeks**

`new` (Without Arrays)



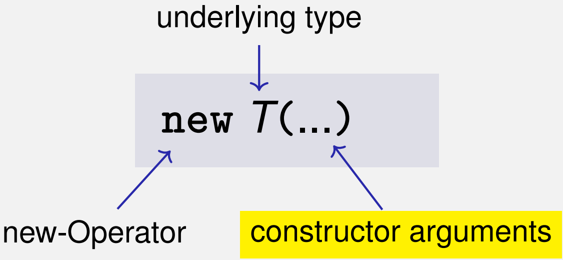
underlying type

`new` *T*(...)

new-Operator

constructor arguments

`new` (Without Arrays)



underlying type

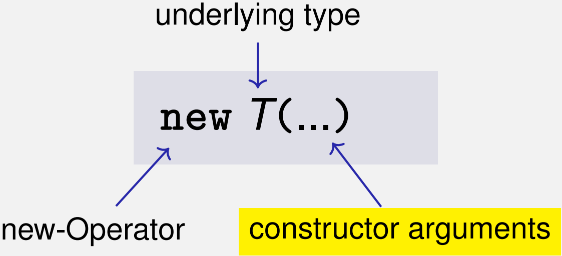
`new` *T*(...)

new-Operator

constructor arguments

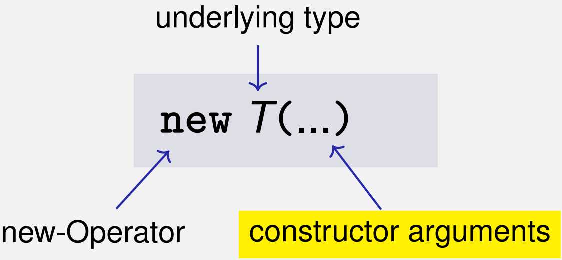
- **Effect**: memory for a new object of type *T* is allocated . . .

`new` (Without Arrays)



- **Effect**: memory for a new object of type *T* is allocated . . .
- . . . and initialized by means of the matching constructor

`new` (Without Arrays)



underlying type

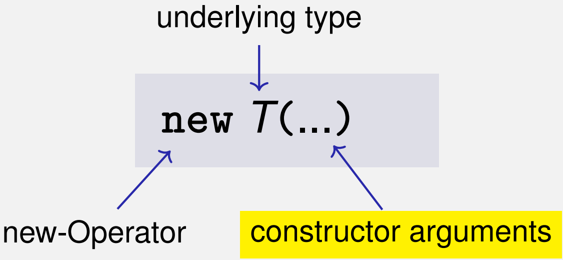
`new` *T*(...)

new-Operator

constructor arguments

- **Effect**: memory for a new object of type *T* is allocated . . .
- . . . and initialized by means of the matching constructor
- **Value**: address of the new T object, **Type**: Pointer $T*$

`new` (Without Arrays)



underlying type

`new` T`(...)`

new-Operator

constructor arguments

- **Effect**: memory for a new object of type *T* is allocated …
- …and initialized by means of the matching constructor
- **Value**: address of the new T object, **Type**: Pointer $T*$
- Also true here: object “lives” until deleted explicitly (usefulness will become clearer later)

Pointer Types

`T*`   Pointer type for base type `T`

An expression of type `T*` is called *pointer (to `T`)*

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`T*` Pointer type for base type `T`

An expression of type `T*` is called *pointer (to `T`)*

```
int* p; // Pointer to an int
std::string* q; // Pointer to a std::string
```

Pointer Types

Value of a pointer to `T` is the *address* of an object of type `T`

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Value of a pointer to `T` is the *address* of an object of type `T`

```
int* p = ...;
std::cout << p; // e.g. 0x7ffd89d5f7cc
```

Pointer Types

Value of a pointer to `T` is the *address* of an object of type `T`

```
int* p = ...;
std::cout << p; // e.g. 0x7ffd89d5f7cc
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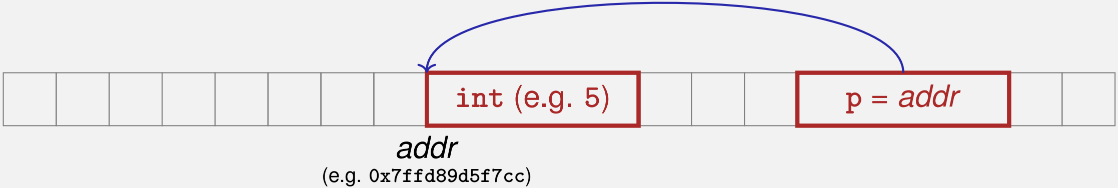

Address Operator

Question: How to obtain an object's address?

1. Directly, when creating a new object via `new`

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2. For existing objects: via the *address operator* &

&*expr*

expr: l-value of type T

Address Operator

Question: How to obtain an object's address?

1. Directly, when creating a new object via `new`
2. For existing objects: via the *address operator* &

$$\&expr \longleftarrow \text{expr: l-value of type } T$$

- **Value** of the expression: the *address* of object (l-value) *expr*

Address Operator

Question: How to obtain an object's address?

1. Directly, when creating a new object via `new`

2. For existing objects: via the *address operator* &

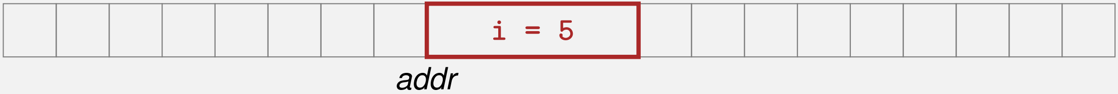
&expr

expr: l-value of type T

- **Value** of the expression: the *address* of object (l-value) *expr*
- **Type** of the expression: A pointer $T*$ (of type T)

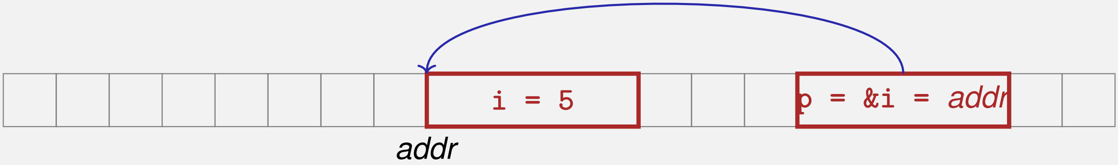
Address Operator

```
int i = 5; // i initialised with 5
```



Address Operator

```
int i = 5; // i initialised with 5
int∗ p = &i; // p initialised with address of i
```



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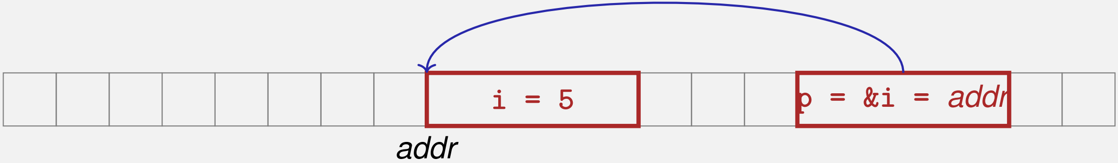


Next question: How to “follow” a pointer?

Dereference Operator

Answer: by using the *dereference operator* *

$*$expr ← expr: r-value of type T*

Dereference Operator

Answer: by using the *dereference operator* ∗

∗*expr* ← expr: r-value of type T*

- **Value** of the expression: the *value* of the object located at the address denoted by *expr*

Dereference Operator

Answer: by using the *dereference operator* $*$

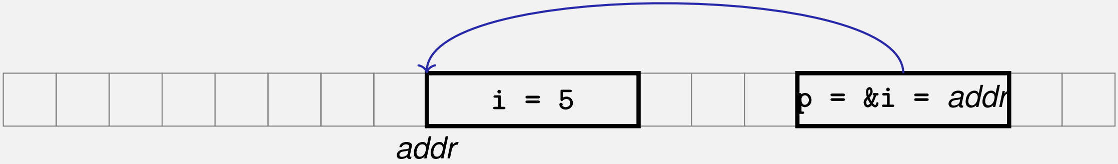
$*$*expr*

expr: r-value of type T*

- **Value** of the expression: the *value* of the object located at the address denoted by *expr*
- **Type** of the expression: T

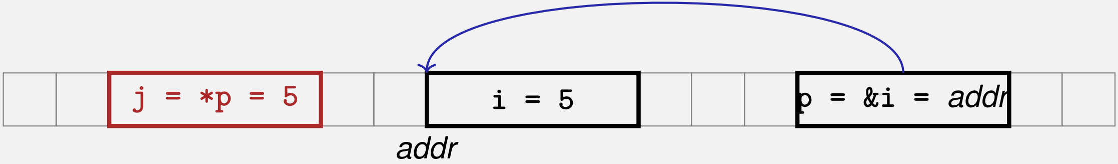
Dereference Operator

```
int i = 5;
int* p = &i; // p = address of i
```

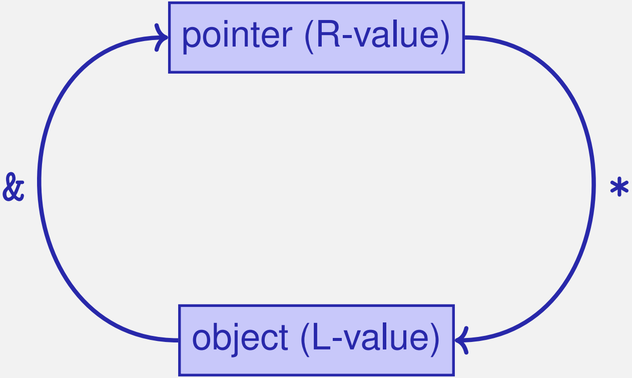


Dereference Operator

```
int i = 5;
int∗ p = &i; // p = address of i
int j = ∗p; // j = 5
```



Address and Dereference Operator



Pointer Types

A T* must actually point to a T

```
int* p = ...;  // p points to an int
double* q = p; // but q to a double → compiler error!
```

Mnenmonic Trick

The declaration

```
T* p;      // p is of the type “pointer to T”
```

Mnenmonic Trick

The declaration

```
T* p;      // p is of the type “pointer to T”
```

can be read as

```
T *p;      // *p is of type T
```

Null-Pointer

- Special pointer value that signals that no object is pointed to
- represented b the literal `nullptr` (convertible to `T*`)

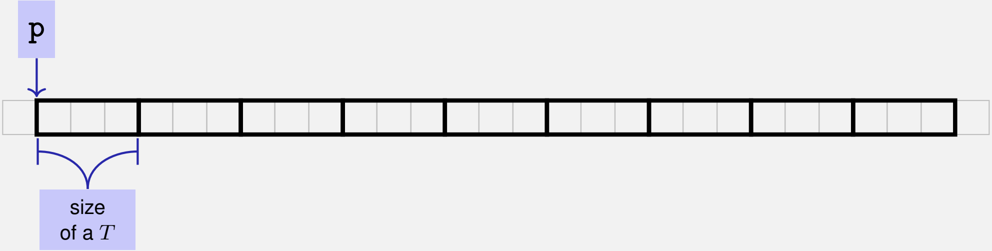
```
int∗ p = nullptr;
```

- Cannot be dereferenced (runtime error)
- Exists to avoid undefined behaviour

```
int∗ p; // p could point to anything
int∗ q = nullptr; // q explicitly points nowhere
```

Pointer Arithmetic: Pointer plus `int`

```
T* p = new T[n]; // p points to first array element
```



How to point to rear elements?

Pointer Arithmetic: Pointer plus `int`

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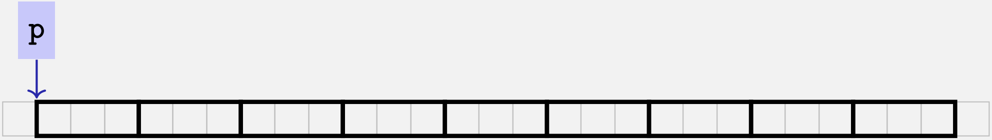

How to point to rear elements? → *Pointer arithmetic*:

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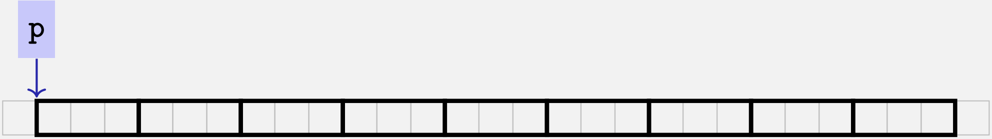

How to point to rear elements? → *Pointer arithmetic*:

- `p` yields the *value* of the *first* array element, `*p` its *value*

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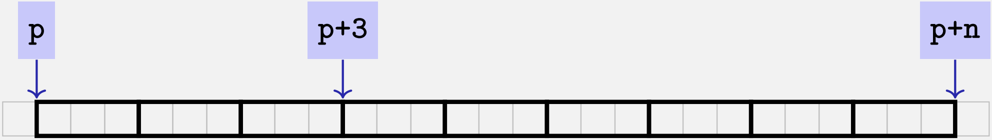

How to point to rear elements? → *Pointer arithmetic*:

- `p` yields the *value* of the *first* array element, `*p` its *value*
- `*(p + i)` yields the value of the `i`*th* array element, for $0 \leq$ `i` $<$ `n`

Pointer Arithmetic: Pointer plus `int`

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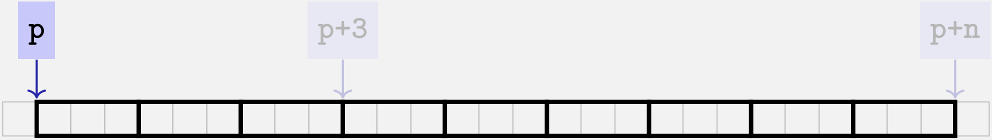
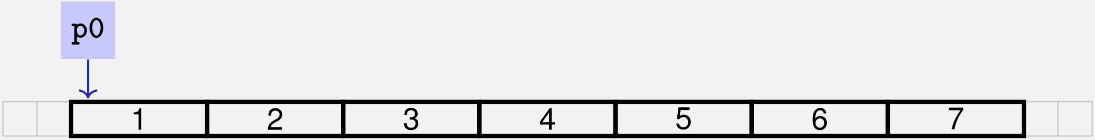


How to point to rear elements? → *Pointer arithmetic*:

- `p` yields the *value* of the *first* array element, `*p` its *value*
- `*(p + i)` yields the value of the `i`*th* array element, for $0 \leq \texttt{i} < \texttt{n}$
- `*p` is equivalent to `*(p + 0)`

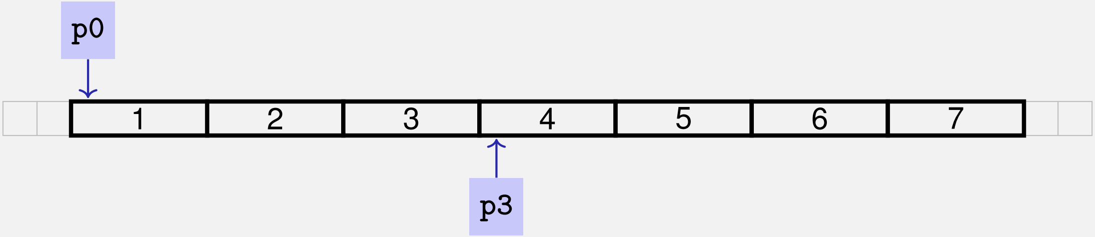
Pointer Arithmetic: Pointer plus `int`

```
int∗ p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
```



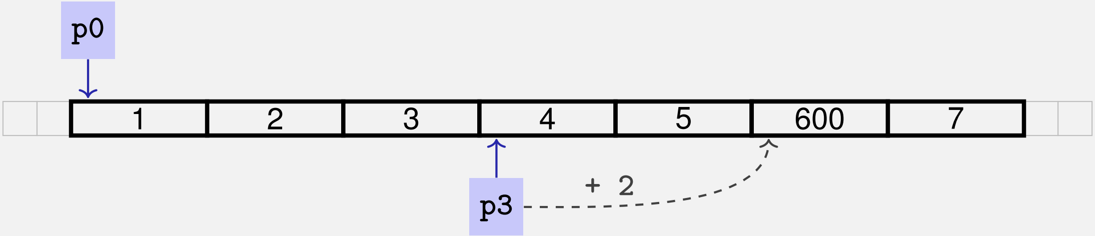
Pointer Arithmetic: Pointer plus `int`

```
int∗ p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
int∗ p3 = p0 + 3; // p3 points to 4th element
```



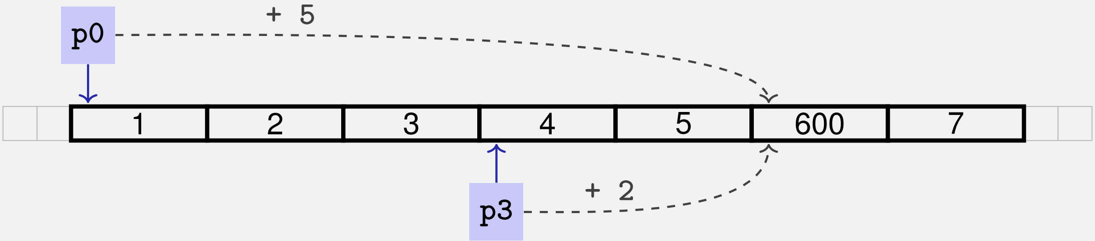
Pointer Arithmetic: Pointer plus `int`

```
int* p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
int* p3 = p0 + 3; // p3 points to 4th element
*(p3 + 2) = 600; // set value of 6th element to 600
std::cout << *(p0 + 5);
```



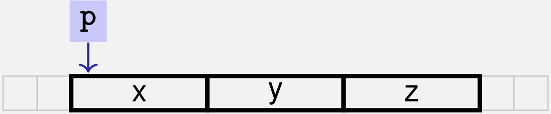
Pointer Arithmetic: Pointer plus `int`

```
int∗ p0 = new int[7]{1,2,3,4,5,6,7}; // p0 points to 1st element
int∗ p3 = p0 + 3; // p3 points to 4th element
∗(p3 + 2) = 600; // set value of 6th element to 600
std::cout << ∗(p0 + 5); // output 6th element’s value (i.e. 600)
```



Sequential Pointer Iteration

```
char* p = new char[3]{'x', 'y', 'z'};
```



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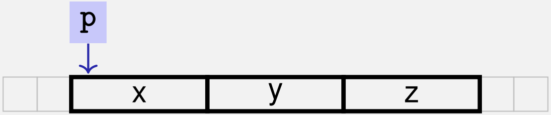

```
  for (char* it = p;
          it != p + 3;
          ++it) {

    std::cout << *it << ' ';
  }
```

Sequential Pointer Iteration

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char* p = new char[3]{'x', 'y', 'z'};
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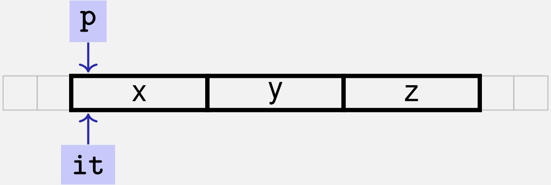

```
  for (char* it = p;
          it != p + 3;
          ++it) {

    std::cout << *it << ' ';
  }
```

it points to first element

Sequential Pointer Iteration

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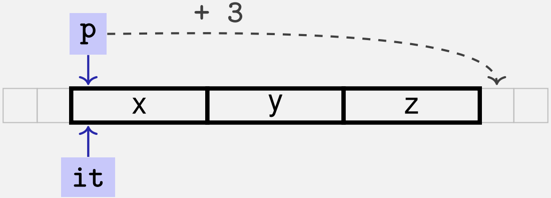
char* p = new char[3]{'x', 'y', 'z'};
```



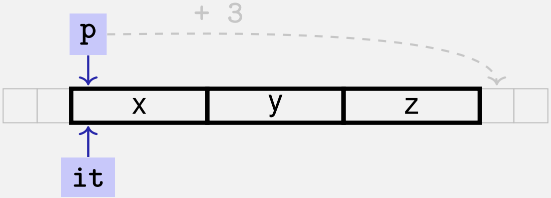
```
  for (char* it = p;
       it != p + 3;
       ++it) {

    std::cout << *it << ' ';
  }
```

Abort if end reached

Sequential Pointer Iteration

```
char* p = new char[3]{'x', 'y', 'z'};
```



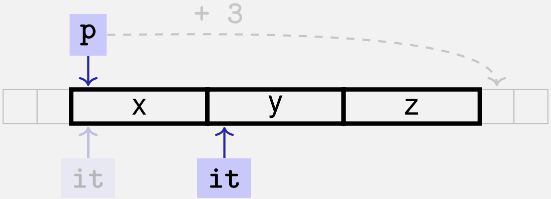
```
for (char* it = p;
      it != p + 3;
      ++it) {

  std::cout << *it << ' ';
}
```

Output current element: 'x'

Sequential Pointer Iteration

```
char* p = new char[3]{'x', 'y', 'z'};
```



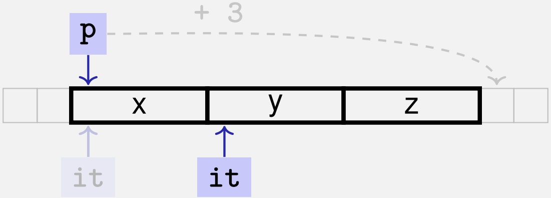
```
  for (char* it = p;
       it != p + 3;
       ++it) {

    std::cout << *it << ' ';  // x
  }
```

Advance pointer element-wise

Sequential Pointer Iteration

```
char* p = new char[3]{'x', 'y', 'z'};
```



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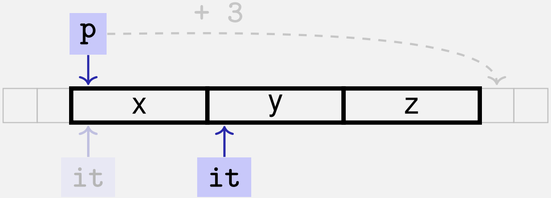

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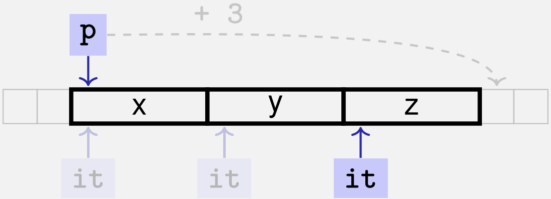

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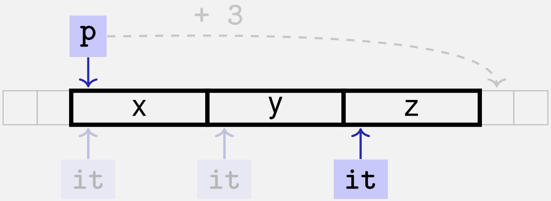

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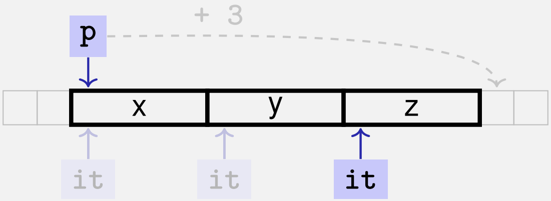

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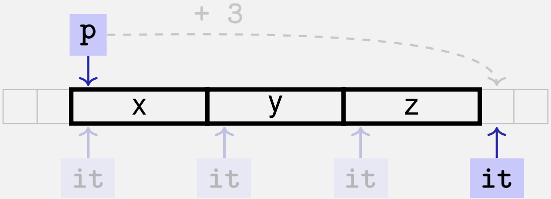

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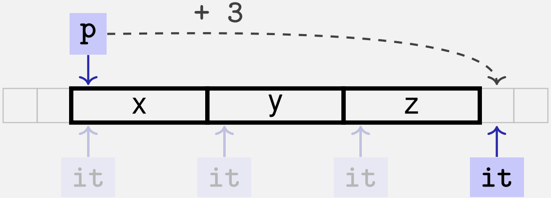

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          it != p + 3;
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  }
```

Random Access to Arrays

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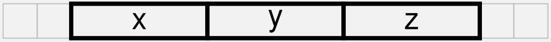

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- can also be written as `p[i]`

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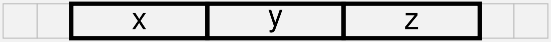


- The expression `*(p + i)`
- can also be written as `p[i]`
- E.g. `p[1] == *(p + 1) == 'y'`

Random Access to Arrays

iteration over an array via indices and *random access*:

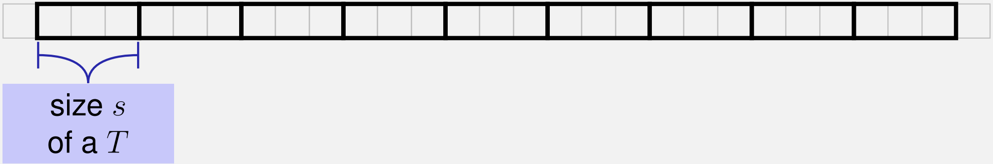
```
char* p = new char[3]{'x', 'y', 'z'};

for (int i = 0; i < 3; ++i)
  std::cout << p[i] << ' ';
```

But: this is less *efficient* than the previously shown *sequential* access via pointer iteration

Random Access to Arrays

```
T* p = new T[n];
```



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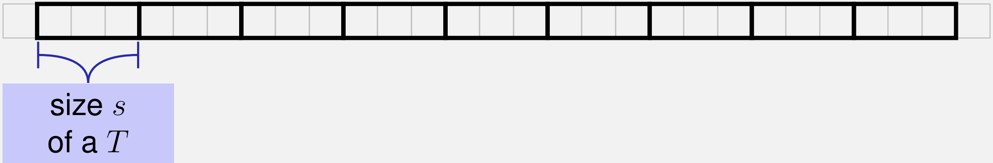

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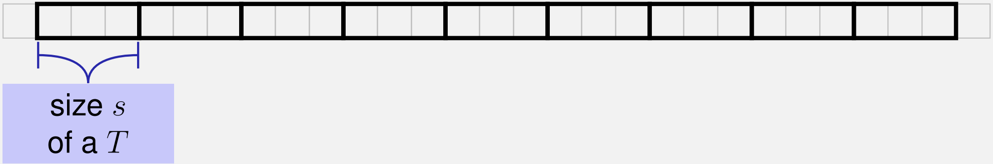

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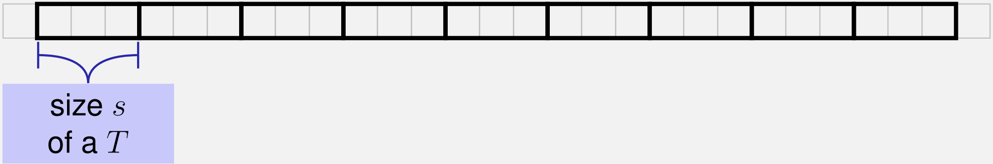

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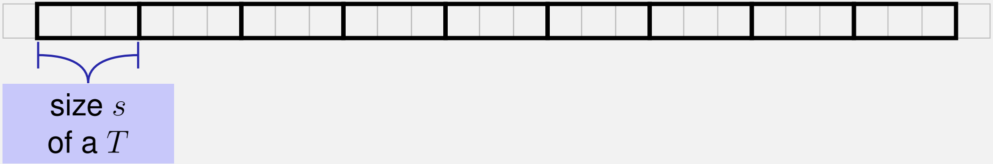

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- Iteration via *random access* (`p[0]`, `p[1]`, `...`) costs one addition and one multiplication per access
- Iteration via *sequentiall access* (`++p`, `++p`, `...`) costs only one addition per access
- Sequential access is thus to be preferred for iterations

Reading a book . . . with random access

Random Access

- open book on page 1
- close book
- open book on pages 2-3
- close book
- open book on pages 4-5
- close book
- ....

Reading a book . . . with sequential access

Random Access

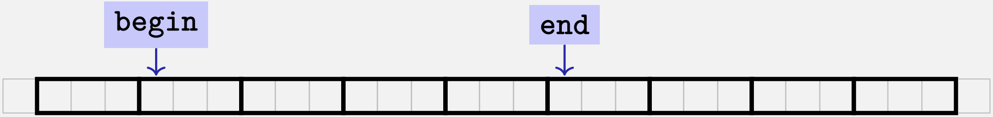
- open book on page 1
- close book
- open book on pages 2-3
- close book
- open book on pages 4-5
- close book
- ....

Sequential Access

- open book on page 1
- turn the page
- turn the page
- turn the page
- turn the page
- turn the page
- ...

Arrays in Functions

C++*covention*: arrays (or a segment of it) are passed using two pointers



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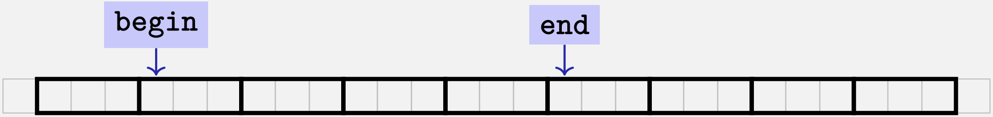

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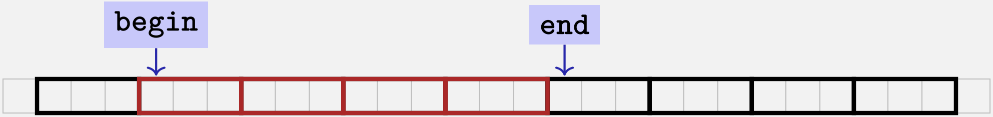

- `begin`: Pointer to the first element
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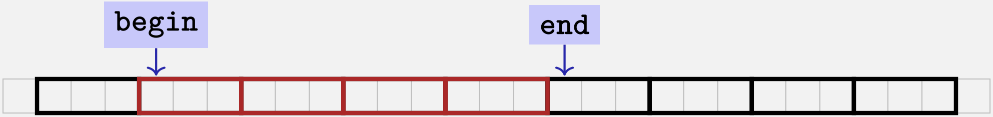

- `begin`: Pointer to the first element
- `end`: Pointer *past* the last element
- `[begin, end)` Designates the elements of the segment of the array
- `[begin, end)` is empty if `begin == end`
- `[begin, end)` must be a *valid range*, i.e. a (pot. empty) array segment

Arrays in (mutating) Functions: `fill`

```
// PRE: [begin, end) is a valid range
// POST: Every element within [begin, end) was set to value
void fill(int* begin, int* end, int value) {
  for (int* p = begin; p != end; ++p)
    *p = value;
}
```

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}


...
int* p = new int[5];
fill(p, p+5, 1); // Array at p becomes {1, 1, 1, 1, 1}
```

Functions with/without Effect

- Pointers can (like references) be used for functions with effect.
  Example: `fill`

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- Pointers can (like references) be used for functions with effect. Example: `fill`
- But many functions don’t have an effect, they only read the data
- ⇒ Use of `const`

Functions with/without Effect

- Pointers can (like references) be used for functions with effect. Example: `fill`
- But many functions don't have an effect, they only read the data
- ⇒ Use of `const`
- So far, for example:

```
const int zero = 0;
const int& nil = zero;
```

Positioning of Const

`const` T is equivalent to T `const` (and can be written like this):

```
const int zero = ...   ⟺   int const zero = ...
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Both keyword orders are used in praxis

Const and Pointers

Read the declaration from right to left

```
int const p;
```

`p` is a constant integer

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Read the declaration from right to left

`int const p;`	`p` is a constant integer
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// PRE: [begin, end) is a valid range
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void print(
    int const* const begin,
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    std::cout << *p << ' ';
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Pointer, *not const*, to const int

Arrays, `new`, Pointer: Conclusion

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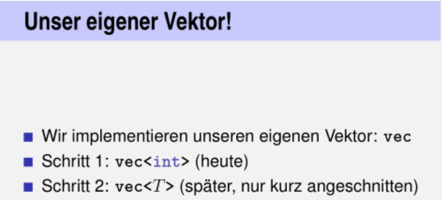
- Memory allocated by `new` is *not* automatically released (more on this soon)
- Pointers and references are related, both “link” to objects in memory. See also additional the slides `pointers.pdf`)

Array-based Vector

- Vectors . . . that somehow rings a bell 🤔



Unser eigener Vektor!

- Wir implementieren unseren eigenen Vektor: `vec`
- Schritt 1: `vec<int>` (heute)
- Schritt 2: `vec<`T`>` (später, nur kurz angeschnitten)

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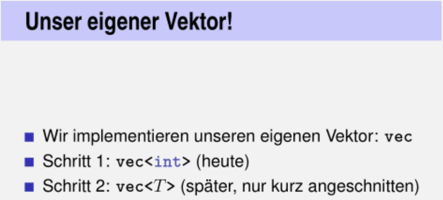

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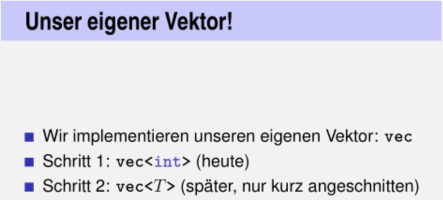

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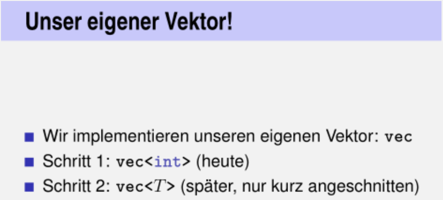
- `avec` – an array-based vector of `int` elements



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Array-based Vector `avec`: Class Signature

```
class avec {
  // Private (internal) state:
  int* elements;
  unsigned int count;









}
```

Pointer to first element

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  // Private (internal) state:
  int* elements; // Pointer to first element
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```

Number of elements

```
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```

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class avec {
  // Private (internal) state:
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public: // Public interface:
  avec(unsigned int size);
  unsigned int size() const;
  int& operator[](int i);
  void print(std::ostream& sink) const;
}
```

Constructor

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Size of vector

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Access an element

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Output elements

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}
```

Constructor avec::avec()

```
avec::avec(unsigned int size)
        : count(size) {

  elements = new int[size];
}
```

Save size

Constructor `avec::avec()`

```
avec::avec(unsigned int size)
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}
```

Allocate memory

Constructor `avec::avec()`

```
avec::avec(unsigned int size)
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}
```

Side remark: vector is not initialised with a default value

Excursion: Accessing Member Variables

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- `elements` is a member variable of our `avec` instance

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avec::avec(unsigned int size): count(size) {
  (*this).elements = new int[size];
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```

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- `elements` is a shorthand for `(*this).elements`

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- `elements` is a shorthand for `(∗this).elements`
- Equivalent, but shorter: `this−>elements`
- Mnemonic trick: “Follow the pointer to the member variable”

Function `avec::size()`

```
int avec::size() const {
  return this->count;
}
```

Doesn’t modify the vector

Function `avec::size()`

```
int avec::size() const  {
  return this->count;
}
```

Return size

Usage example:

```
avec v = avec(7);
assert(v.size() == 7); // ok
```

Function avec::operator[]

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Return ith element

Function avec::operator[]

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Element access with index check:

```
int& avec::at(int i) const {
  assert(0 <= i && i < this->count);

  return this->elements[i];
}
```

Function avec::operator[]

```
int& avec::operator[](int i) {
  return this->elements[i];
}
```

Usage example:

```
avec v = avec(7);
std::cout << v[6]; // Outputs a "random" value
v[6] = 0;
std::cout << v[6]; // Outputs 0
```

Function `avec::print()`

Output elements using sequential access:

```
void avec::print(std::ostream& sink) const {
  for (int* p = this->elements;
       p != this->elements + this->count;
       ++p)
  {
    sink << *p << ' ';
  }
}
```

Pointer to first element

Function `avec::print()`

Output elements using sequential access:

```
void avec::print(std::ostream& sink) const {
  for (int* p = this->elements;
       p != this->elements + this->count;
       ++p)
  {
    sink << *p << ' ';
  }
}
```

Abort iteration if past last element

Function `avec::print()`

Output elements using sequential access:

```
void avec::print(std::ostream& sink) const {
  for (int* p = this->elements;
       p != this->elements + this->count;
       ++p)
  {
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  }
}
```

Advance pointer element-wise

Function avec::print()

Output elements using sequential access:

```
void avec::print(std::ostream& sink) const {
  for (int* p = this->elements;
       p != this->elements + this->count;
       ++p)
  {
    sink << *p << ' ';
  }
}
```

Advance pointer element-wise

Output current element

Function `avec::print()`

Finally: overload output operator:

```
_________________ operator<<(_________________ sink,
                             _________________ vec) {
  vec.print(sink);
  return _________________;
}
```

Function `avec::print()`

Finally: overload output operator:

```
std::ostream& operator<<(std::ostream& sink,
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Function avec::print()

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Observations:

- Constant reference to `vec`, since unchanged

Function `avec::print()`

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std::ostream& operator<<(std::ostream& sink,
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- Constant reference to `vec`, since unchanged
- But not to `sink`: Outputing elements equals change

Function `avec::print()`

Finally: overload output operator:

```
std::ostream& operator<<(std::ostream& sink,
                         const avec& vec) {
  vec.print(sink);
  return sink;
}
```

Observations:

- Constant reference to `vec`, since unchanged
- But not to `sink`: Outputing elements equals change
- `sink` is returned to enable output chaining, e.g.
  `std::cout << v << '\n'`

Further Functions?

```
class avec {
  ...
  void push_front(int e)      // Prepend e to vector
  void push_back(int e)       // Append e to vector
  void remove(unsigned int i) // Cut out ith element
  ...
}
```

Further Functions?

```
class avec {
  ...
  void push_front(int e)       // Prepend e to vector
  void push_back(int e)        // Append e to vector
  void remove(unsigned int i) // Cut out ith element
  ...
}
```

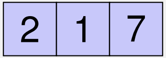
Commonalities: such operations need to change the vector's *size*

Resizing arrays

An allocated block of memory (e.g. `new int[3]`) cannot be resized later on

Resizing arrays

An allocated block of memory (e.g. `new int[3]`) cannot be resized later on



Resizing arrays

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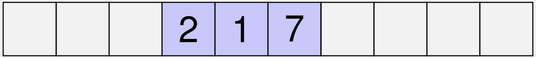


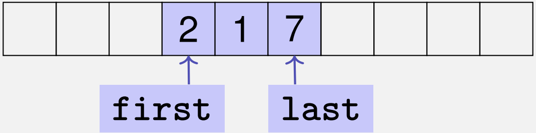
			2	1	7				

Possibility:

- Allocate more memory than initially necessary

Resizing arrays

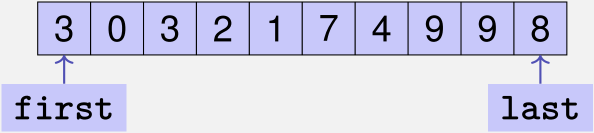
An allocated block of memory (e.g. `new int[3]`) cannot be resized later on



Possibility:

- Allocate more memory than initially necessary
- Fill from inside out, with pointers to first and last element

Resizing arrays



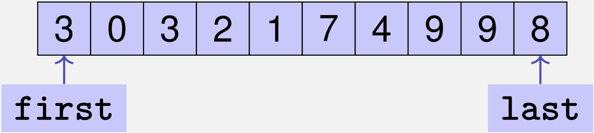
3	0	3	2	1	7	4	9	9	8

`first`

`last`

- But eventually, all slots will be in use

Resizing arrays

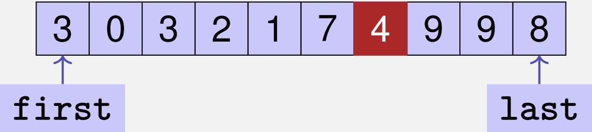


3	0	3	2	1	7	4	9	9	8

`first` `last`

- But eventually, all slots will be in use
- Then unavoidable: Allocate larger memory block and copy data over

Resizing arrays



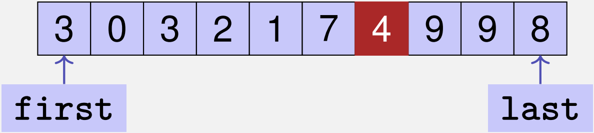
3	0	3	2	1	7	4	9	9	8

`first`

`last`

Deleting elements requires shifting (by copying) all preceding or following elements

Resizing arrays

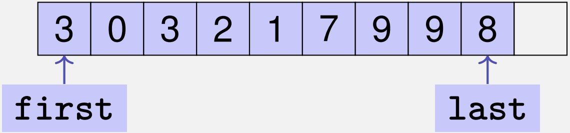


3 0 3 2 1 7 4 9 9 8

first

last

Deleting elements requires shifting (by copying) all preceding or following elements

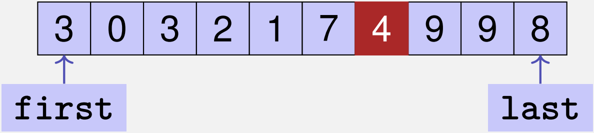


3 0 3 2 1 7 9 9 8

first

last

Resizing arrays

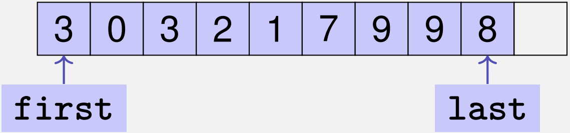


3	0	3	2	1	7	4	9	9	8

first

last

Deleting elements requires shifting (by copying) all preceding or following elements



3	0	3	2	1	7	9	9	8	

first

last

Similar: inserting at arbitrary position

19. Dynamic Data Structures II

Linked Lists, Vectors as Linked Lists

Different Memory Layout: Linked List

- *No* contiguous area of memory and *no* random access

Different Memory Layout: Linked List

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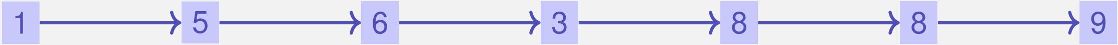
- Each element points to its successor



Different Memory Layout: Linked List

- *No* contiguous area of memory and *no* random access
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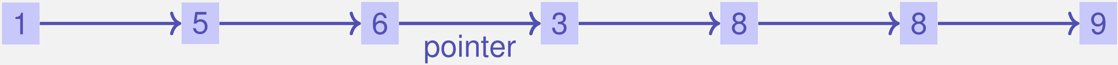
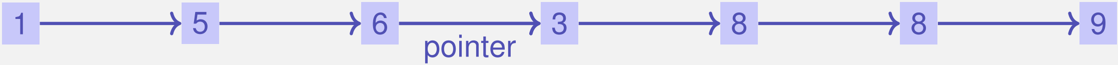


Different Memory Layout: Linked List

- *No* contiguous area of memory and *no* random access
- Each element points to its successor
- Insertion and deletion of *arbitrary* elements is simple



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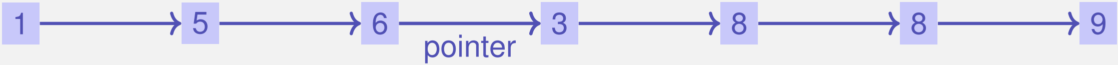
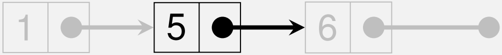


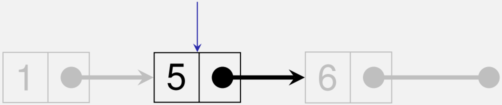
⇒ Our vector can be implemented as a linked list

Linked List: Zoom

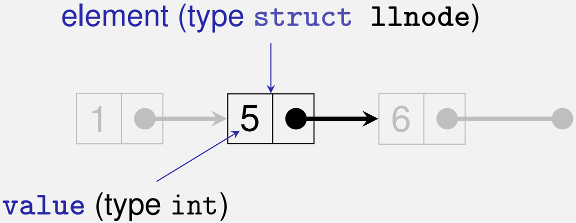


Linked List: Zoom

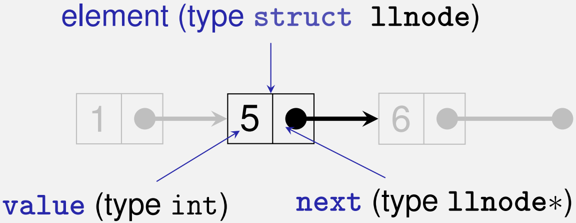
element (type `struct llnode`)



Linked List: Zoom



Linked List: Zoom

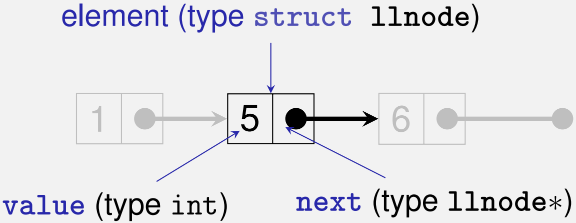


element (type struct llnode)

value (type int)

next (type llnode∗)

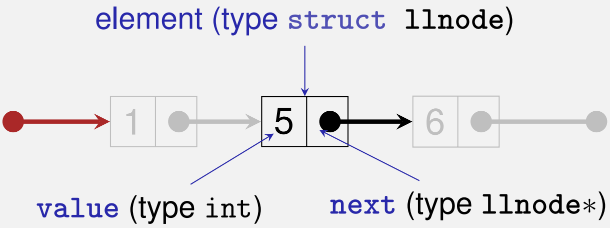
Linked List: Zoom



```
struct llnode {
  int value;
  llnode∗ next;

  llnode(int v, llnode∗ n): value(v), next(n) {} // Constructor
};
```

Vector = Pointer to the First Element



```
class llvec {
  llnode* head;
public:
  // Public interface identical to avec's
  llvec(unsigned int size);
  unsigned int size() const;
  ...
};
```

Function llvec::print()

```
struct llnode {
  int value;
  llnode* next;
  ...
};
```

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';
  }
}
```

Pointer to first element

Function `llvec::print()`

```
struct llnode {
  int value;
  llnode* next;
  ...
};
```

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
         n != nullptr;
         n = n->next)
  {
    sink << n->value << ' ';
  }
}
```

Abort if end reached

Function `llvec::print()`

```
struct llnode {
  int value;
  llnode* next;
  ...
};
```

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';
  }
}
```

Advance pointer element-wise

Function `llvec::print()`

```
struct llnode {
  int value;
  llnode* next;
  ...
};
```

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';
  }
}
```

Output current element

Function `llvec::print()`

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';
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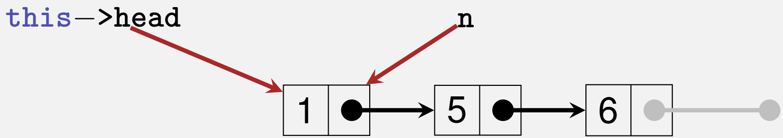
}
```



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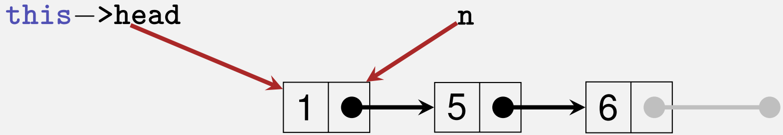
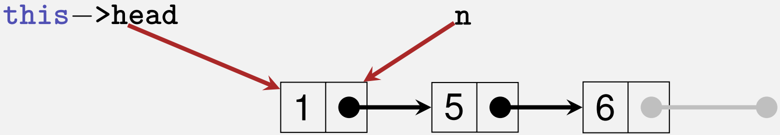


Function llvec::print()

```
void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
  {
    sink << n->value << ' ';  // 1
  }
}
```



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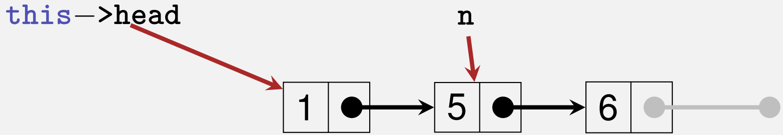

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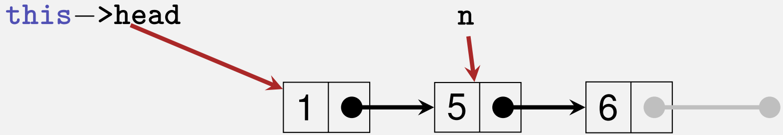

Function `llvec::print()`

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void llvec::print(std::ostream& sink) const {
  for (llnode* n = this->head;
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    sink << n->value << ' ';  // 1 5
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```

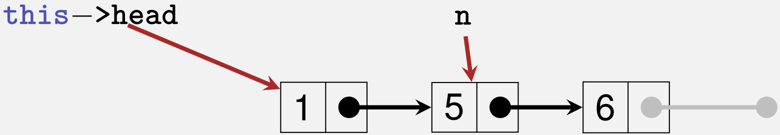

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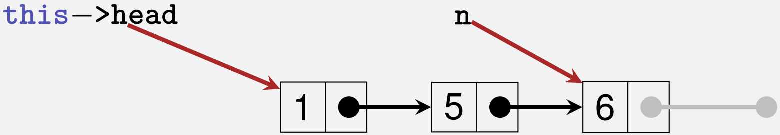

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  }
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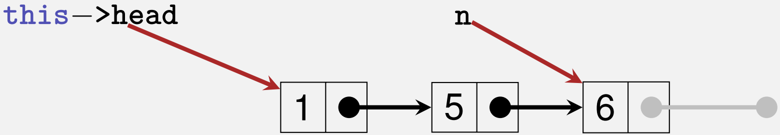
```



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void llvec::print(std::ostream& sink) const {
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       n != nullptr;
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  {
    sink << n->value << ' ';  // 1 5 6
  }
}
```

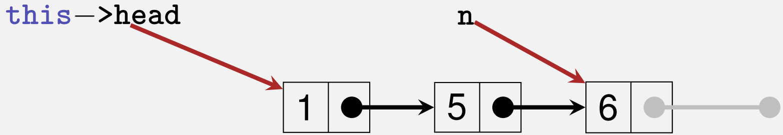

Function `llvec::print()`

```
void llvec::print(std::ostream& sink) const {
  for (llnode∗ n = this−>head;
       n != nullptr;
       n = n−>next)
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  }
}
```
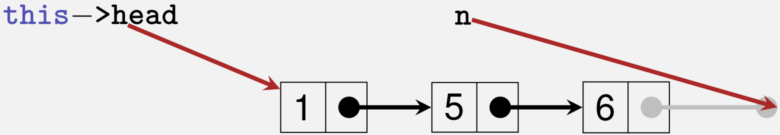



Function llvec::operator[]

Accessing ith Element is implemented similarly to `print()`:

```
int& llvec::operator[](unsigned int i) {
  llnode* n = this->head;

  for (; 0 < i; --i)
    n = n->next;

  return n->value;
}
```

Pointer to first element

Function `llvec::operator[]`

Accessing ith Element is implemented similarly to `print()`:

```
int& llvec::operator[](unsigned int i) {
  llnode* n = this->head;

  for (; 0 < i; --i)
    n = n->next;

  return n->value;
}
```

Step to `i`th element

Function `llvec::operator[]`

Accessing ith Element is implemented similarly to `print()`:

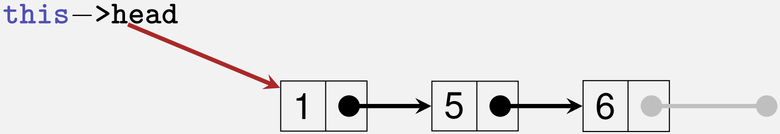
```
int& llvec::operator[](unsigned int i) {
  llnode* n = this->head;

  for (; 0 < i; --i)
    n = n->next;

  return n->value;
}
```

Return `i`th element

Function `llvec::push_front()`

Advantage `llvec`: Prepending elements is very easy:

```
void llvec::push_front(int e) {
  this->head =
    new llnode{e, this->head};
}
```



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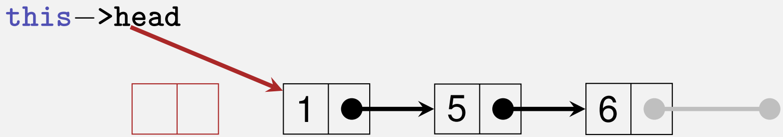

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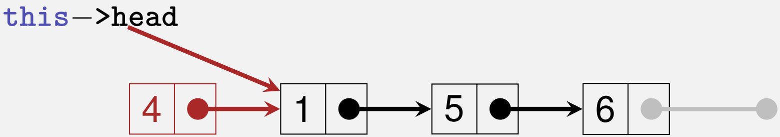

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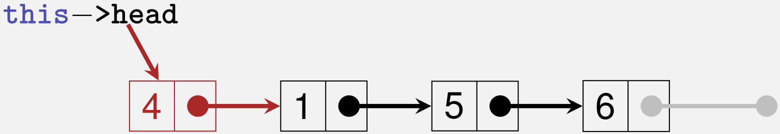

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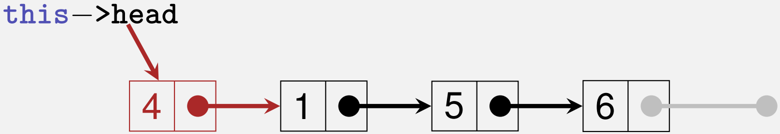

Attention: If the new `llnode` weren't allocated *dynamically*, then it would be deleted (= memory deallocated) as soon as `push_front` terminates

Function `llvec::llvec()`

Constructor can be implemented using `push_front()`:

```
llvec::llvec(unsigned int size) {
  this->head = nullptr;

  for (; 0 < size; --size)
    this->push_front(0);
}
```

`head` initially points to nowhere

Function `llvec::llvec()`

Constructor can be implemented using `push_front()`:

```
llvec::llvec(unsigned int size) {
  this->head = nullptr;

  for (; 0 < size; --size)
    this->push_front(0);
}
```

Prepend 0 `size` times

Function `llvec::llvec()`

Constructor can be implemented using `push_front()`:

```
llvec::llvec(unsigned int size) {
  this->head = nullptr;

  for (; 0 < size; --size)
    this->push_front(0);
}
```

Use case:

```
llvec v = llvec(3);
std::cout << v; // 0 0 0
```

Function `llvec::push_back()`

Simple, but inefficient: traverse linked list to its end and append new element

```
void llvec::push_back(int e) {
  llnode* n = this->head;

  for (; n->next != nullptr; n = n->next);

  n->next =
    new llnode{e, nullptr};
}
```

Start at first element ...

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... and go to the last element

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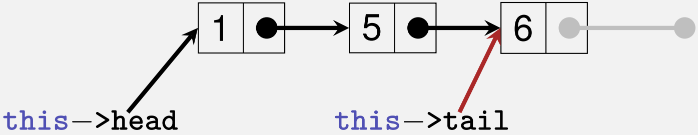
Append new element to currently last

Function `llvec::push_back()`

- More efficient, but also slightly more complex:
  1. Second pointer, pointing to the last element: `this->tail`



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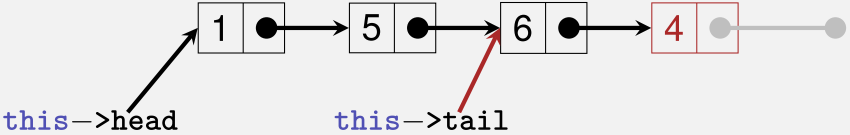

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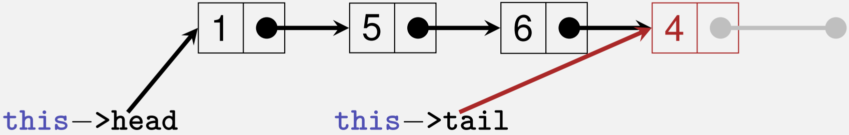

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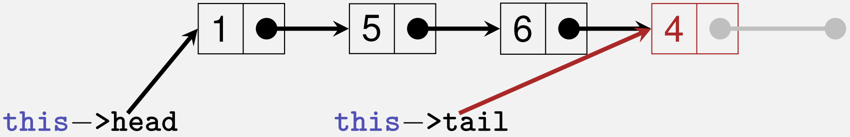

- But: Several corner cases, e.g. vector still empty, must be accounted for

Function `llvec::size()`

Simple, but inefficient: *compute* size by counting

```
unsigned int llvec::size() const {
  unsigned int c = 0;

  for (llnode* n = this->head;
       n != nullptr;
       n = n->next)
    ++c;

  return c;
}
```

Count initially 0

Function `llvec::size()`

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Count linked-list length

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}
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Return count

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More efficient, but also slightly more complex: *maintain* size as member variable

1. Add member variable `unsigned int count` to class `llvec`
2. `this−>count` must now be updated *each* time an operation (such as `push_front`) affects the vector's size

Efficiency: Arrays vs. Linked Lists

- Memory: our `avec` requires roughly n `int`s (vector size n), our `llvec` roughly $3n$ `int`s (a pointer typically requires 8 byte)

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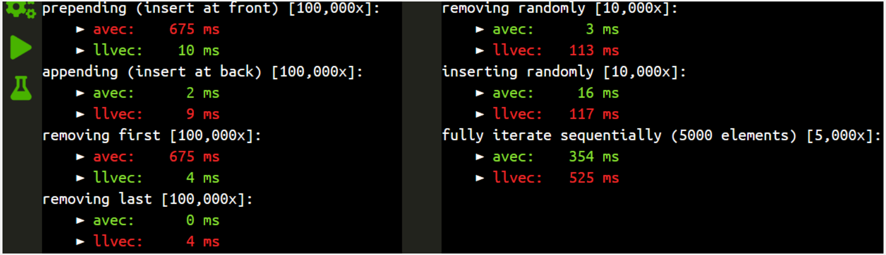
- Runtime (with `avec` = `std::vector`, `llvec` = `std::list`):



```
prepending (insert at front) [100,000x]:
    ► avec:     675 ms
    ► llvec:     10 ms
appending (insert at back) [100,000x]:
    ► avec:       2 ms
    ► llvec:      9 ms
removing first [100,000x]:
    ► avec:     675 ms
    ► llvec:      4 ms
removing last [100,000x]:
    ► avec:       0 ms
    ► llvec:      4 ms
```

```
removing randomly [10,000x]:
    ► avec:       3 ms
    ► llvec:    113 ms
inserting randomly [10,000x]:
    ► avec:      16 ms
    ► llvec:    117 ms
fully iterate sequentially (5000 elements) [5,000x]:
    ► avec:     354 ms
    ► llvec:    525 ms
```

20. Containers, Iterators and Algorithms

Containers, Sets, Iterators, const-Iterators, Algorithms, Templates

Vectors are Containers

- Viewed abstractly, a vector is
  1. A collection of elements
  2. Plus operations on this collection

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- Called *collections* in some other languages, e.g. Java

Container properties

- Each container has certain *characteristic properties*
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  - Inserting at/removing from arbitrary index is potentially inefficient
  - Looking for a specific element is potentially inefficient
  - Can contain the same element more than once
  - Elements are in insertion order (ordered but not sorted)

Containers in $\mathrm{C}{+}{+}$

- Nearly every application requires maintaining and manipulating arbitrarily many data records

Containers in C++

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- But with different requirements (e.g. only append elements, hardly ever remove, often search elements, . . .)

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- That's why C++'s standard library includes several containers with different properties, see `https://en.cppreference.com/w/cpp/container`
- Many more are available from 3rd-party libraries, e.g. `https://www.boost.org/doc/libs/1_68_0/doc/html/container.html`, `https://github.com/abseil/abseil-cpp`

Example Container: `std::unordered_set<`T`>`

- A *mathematical set* is an unordered, duplicate-free collection of elements:

$$\{1,2,1\} = \{1,2\} = \{2,1\}$$

- In $\mathrm{C{+}{+}}$: `std::unordered_set<`T`>`

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  - Efficient “element contained?” check
  - Efficient insertion and removal of elements
- Side remark: implemented as a hash table

Use Case std::unordered_set<T>

Problem:

- given a sequence of pairs *(name, percentage)* of Code Expert submissions . . .

```
// Input: file submissions.txt
Friedrich 90
Schwerhoff 10
Lehner 20
Schwerhoff 11
```

Use Case `std::unordered_set<`T`>`

Problem:

- given a sequence of pairs *(name, percentage)* of Code Expert submissions . . .

```
// Input: file submissions.txt
Friedrich 90
Schwerhoff 10
Lehner 20
Schwerhoff 11
```

- . . . determine the submitters that achieved at least 50%

```
// Output
Friedrich
```

Use Case std::unordered_set<T>

```
std::ifstream in("submissions.txt");
std::unordered_set<std::string> names;

std::string name;
unsigned int score;

while (in >> name >> score) {
  if (50 <= score)
    names.insert(name);
}

std::cout << "Unique submitters: "
          << names << '\n';
```

Open submissions.txt

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Set of names, initially empty

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Pair *(name, score)*

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          << names << ’\n’;
```

Input next pair

Use Case std::unordered_set<T>

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std::ifstream in("submissions.txt");
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std::string name;
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while (in >> name >> score) {
  if (50 <= score)
    names.insert(name);
}

std::cout << "Unique submitters: "
          << names << ’\n’;
```

Record name if score suffices

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}

std::cout << "Unique submitters: "
          << names << ’\n’;
```

Output recorded names

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set instead of unsorted_set . . .

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. . . and the output is in alphabetical order

Printing Containers

- Recall: `avec::print()` and `llvec::print()`

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- What about printing `set`, `unordered_set`, . . .?

Printing Containers

- Recall: `avec::print()` and `llvec::print()`
- What about printing `set`, `unordered_set`, ...?
- Commonality: iterate over container elements and print them

Similar Functions

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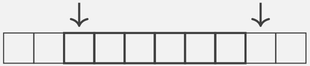
Recall: Iterating With Pointers

- Iteration over an *array*:



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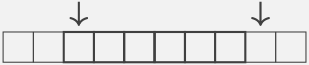

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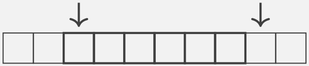

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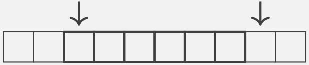

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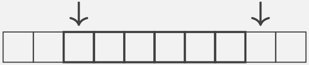

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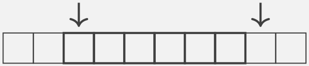

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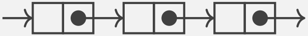

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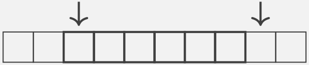

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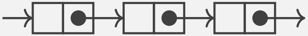

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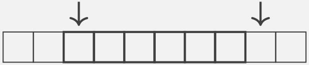

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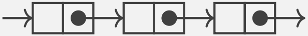

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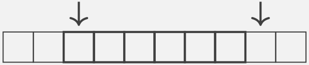

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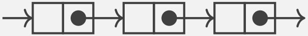

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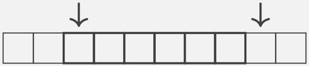

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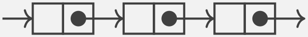

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Iterators

- Iteration requires only the previously shown four operations
- But their implementation depends on the container
- $\Rightarrow$ Each $\mathrm{C{+}{+}}$container implements their own *Iterator*
- Given a container `c`:
  - `it = c.begin()`: Iterator pointing to the first element

Iterators

- Iteration requires only the previously shown four operations
- But their implementation depends on the container
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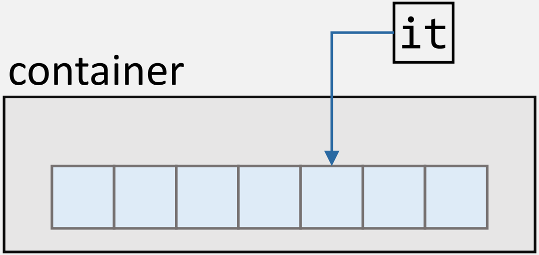
  - `it = c.end()`: Iterator pointing *behind* the last element
  - `*it`: Access current element
  - `++it`: Advance iterator by one element
- Iterators are essentially pimped pointers

Iterators

- Iterators allow accessing different containers in a *uniform* way: `*it`, `++it`, etc.



Iterators

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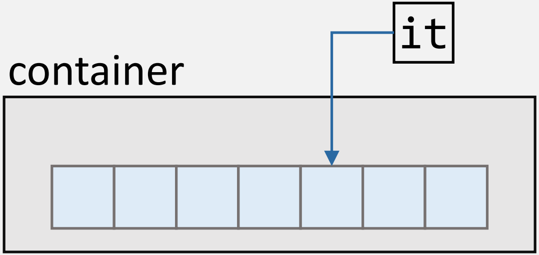
- Users remain independent of the container implementation



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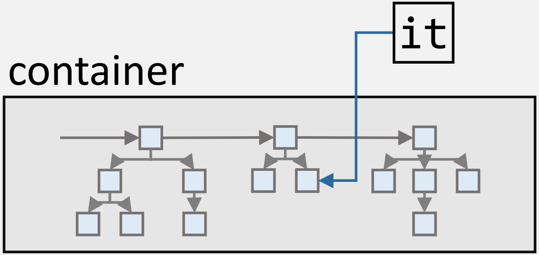

Iterators

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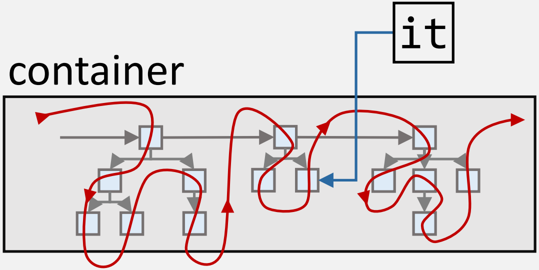

Iterators

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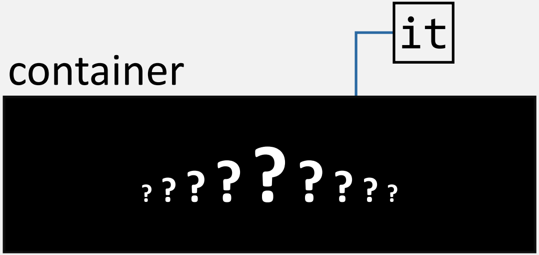

Iterators

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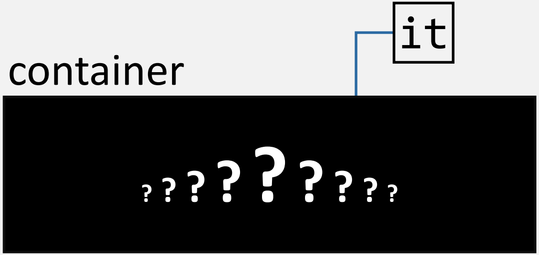

Example: Iterate over `std::vector`

`it` is an iterator specific to `std::vector<int>`

```
std::vector<int> v = {1, 2, 3};

for (std::vector<int>::iterator it = v.begin();
     it != v.end();
     ++it)  {

  *it = -*it;
}

std::cout << v; // -1 -2 -3
```

Example: Iterate over `std::vector`

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```

`it` initially points to the first element

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```cpp
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for (std::vector<int>::iterator it = v.begin();
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}

std::cout << v; // -1 -2 -3
```

Abort if `it` reached the end

Example: Iterate over `std::vector`

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for (std::vector<int>::iterator it = v.begin();
     it != v.end();
     ++it) {

  *it = -*it;
}

std::cout << v; // -1 -2 -3
```

Advance `it` element-wise

Example: Iterate over `std::vector`

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std::vector<int> v = {1, 2, 3};

for (std::vector<int>::iterator it = v.begin();
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     ++it)  {

  *it = -*it;
}

std::cout << v; // -1 -2 -3
```

Negate current element ($e \to -e$)

Example: Iterate over `std::vector`

```
std::vector<int> v = {1, 2, 3};

for (std::vector<int>::iterator it = v.begin();
     it != v.end();
     ++it)  {

  *it = -*it;
}

std::cout << v; // -1 -2 -3
```

Example: Iterate over `std::vector`

Recall: type aliases can be used to shorten often-used type names

```
using ivit = std::vector<int>::iterator; // int-vector iterator

for (ivit it = v.begin();
     ...
```

Negate as a Function

```
void neg(std::vector<int>& v) {
  for (std::vector<int>::iterator it = v.begin();
      it != v.end();
      ++it) {

    *it = -*it;
  }
}

// in main():
std::vector<int> v = {1, 2, 3};
neg(v); // v = {-1, -2, -3}
```

Negate as a Function

```
void neg(std::vector<int>& v) {
  for (std::vector<int>::iterator it = v.begin();
       it != v.end();
       ++it) {

    *it = -*it;
  }
}

// in main():
std::vector<int> v = {1, 2, 3};
neg(v); // v = {-1, -2, -3}
```

Disadvantage: Always negates the complete vector

Negate as a Function

Better: negate inside a specific *range (interval)*

```
void neg(std::vector<int>::iterator begin;
         std::vector<int>::iterator end) {

  for (std::vector<int>::iterator it = begin;
      it != end;
      ++it) {

    *it = -*it;
  }
}
```

Negate elements in interval [`begin`, `end`)

Negate as a Function

Better: negate inside a specific *range (interval)*

```
void neg(std::vector<int>::iterator start;
         std::vector<int>::iterator end);

// in main():
std::vector<int> v = {1, 2, 3};
neg(v.begin(), v.begin() + (v.size() / 2));
```

← Negate first half

Algorithms Library in $\mathrm{C}{+}{+}$

- The $\mathrm{C}{+}{+}$standard library includes lots of useful algorithms (functions) that work on iterator-defined intervals [*begin, end*)

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Algorithms Library in C++

- The C++standard library includes lots of useful algorithms (functions) that work on iterator-defined intervals [*begin, end*)
- For example **`find`**, **`fill`** and **`sort`**
- See also `https://en.cppreference.com/w/cpp/algorithm`

An iterator for `llvec`

We need:

1. An `llvec`-specific iterator with at least the following functionality:
   - Access current element: `operator*`
   - Advance iterator: `operator++`
   - End-reached check: `operator!=` (or `operator==`)

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2. Member functions `begin()` and `end()` for `llvec` to get an iterator to the beginning and past the end, respectively

Iterator `avec::iterator` (Step 1/2)

```
class llvec {
  ...
public:
  class iterator {
    ...
  };

  ...
}
```

- The iterator belongs to our vector, that’s why `iterator` is a public *inner class* of `llvec`

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class llvec {
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  class iterator {
    ...
  };


  ...
}
```

- The iterator belongs to our vector, that’s why `iterator` is a public *inner class* of `llvec`
- Instances of our iterator are of type `llvec::iterator`

Iterator `llvec::iterator` (Step 1/2)

```
class iterator {
  llnode∗ node;

public:
  iterator(llnode∗ n);
  iterator& operator++();
  int& operator∗() const;
  bool operator!=(const iterator& other) const;
};
```

Pointer to current vector element

Iterator `llvec::iterator` (Step 1/2)

```
class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
  bool operator!=(const iterator& other) const;
};
```

Create iterator to specific element

Iterator `llvec::iterator` (Step 1/2)

```
class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
  bool operator!=(const iterator& other) const;
};
```

Advance iterator by one element

Iterator `llvec::iterator` (Step 1/2)

```
class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
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};
```

Access current element

Iterator llvec::iterator (Step 1/2)

```
class iterator {
  llnode* node;

public:
  iterator(llnode* n);
  iterator& operator++();
  int& operator*() const;
  bool operator!=(const iterator& other) const;
};
```

Compare with other iterator

Iterator `llvec::iterator` (Step 1/2)

```cpp
// Constructor
llvec::iterator::iterator(llnode* n): node(n)  {}


// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
  assert(this->node != nullptr);

  this->node = this->node->next;

  return *this;
}
```

Iterator `llvec::iterator` (Step 1/2)

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// Constructor
llvec::iterator::iterator(llnode* n): node(n) {}


// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
  assert(this->node != nullptr);

  this->node = this->node->next;

  return *this;
}
```

Let iterator point to `n` initially

Iterator `llvec::iterator` (Step 1/2)

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// Constructor
llvec::iterator::iterator(llnode* n): node(n) {}



// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
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}
```

Iterator `llvec::iterator` (Step 1/2)

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llvec::iterator::iterator(llnode* n): node(n) {}



// Pre-increment
llvec::iterator& llvec::iterator::operator++() {
  assert(this->node != nullptr);

  this->node = this->node->next;

  return *this;
}
```

Return reference to advanced iterator

Iterator `llvec::iterator` (Step 1/2)

```
// Element access
int& llvec::iterator::operator*() const {
  return this->node->value;
}


// Comparison
bool llvec::iterator::operator!=(const llvec::iterator& other)
    const {
  return this->node != other.node;
}
```

Iterator llvec::iterator (Step 1/2)

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bool llvec::iterator::operator!=(const llvec::iterator& other)
    const {
  return this->node != other.node;
}
```

`this` iterator different from `other` if they point to different element

An iterator for `llvec` (Repetition)

We need:

1. An `llvec`-specific iterator with at least the following functionality:
   - Access current element: `operator*`
   - Advance iterator: `operator++`
   - End-reached check: `operator!=` (or `operator==`)

   ✓

2. Member functions **`begin()`** and **`end()`** for **`llvec`** to get an iterator to the beginning and past the end, respectively

Iterator `avec::iterator` (Step 2/2)

```
class llvec {
  ...
public:
  class iterator {...};

  iterator begin();
  iterator end();


  ...
}
```

`llvec` needs member functions to issue iterators pointing *to the beginning* and *past the end*, respectively, of the vector

Iterator `llvec::iterator` (Step 2/2)

```
llvec::iterator llvec::begin() {
  return llvec::iterator(this->head);
}

llvec::iterator llvec::end() {
  return llvec::iterator(nullptr);
}
```

Iterator to first vector element

Iterator `llvec::iterator` (Step 2/2)

```
llvec::iterator llvec::begin() {
  return llvec::iterator(this->head);
}

llvec::iterator llvec::end() {
  return llvec::iterator(nullptr);
}
```

Iterator past last vector element

Const-Iterators

- In addition to `iterator`, every container should also provide a *const-iterator* `const_iterator`
- Const-iterators grant only read access to the underlying Container
- For example for `llvec`:

```
llvec::const_iterator llvec::cbegin() const;
llvec::const_iterator llvec::cend() const;

const int& llvec::const_iterator::operator*() const;
...
```

Const-Iterators

- In addition to `iterator`, every container should also provide a *const-iterator* `const_iterator`
- Const-iterators grant only read access to the underlying Container
- For example for `llvec`:

```
llvec::const_iterator llvec::cbegin() const;
llvec::const_iterator llvec::cend() const;

const int& llvec::const_iterator::operator*() const;
...
```

- Therefore not possible (compiler error): `*(v.cbegin()) = 0`

Const-Iterators

Const-Iterator *can* be used to allow only reading:

```
llvec v = ...;
for (llvec::const_iterator it = v.cbegin(); ...)
  std::cout << *it;
```

It would also possible to use the non-const `iterator` here

Const-Iterators

Const-Iterator *must* be used if the vector is const:

```
const llvec v = ...;
for (llvec::const_iterator it = v.cbegin(); ...)
  std::cout << *it;
```

It is not possible to use `iterator` here (compiler error)

Excursion: Templates

- Goal: A *generic* output operator << for *iterable Containers*: `llvec`, `avec`, `std::vector`, `std::set`, . . .

Excursion: Templates

- Goal: A *generic* output operator << for *iterable Containers*: `llvec`, `avec`, `std::vector`, `std::set`, ...
- I.e. `std::cout << c << 'n'` should work for any such container `c`

Excursion: Templates

Templates enable *type-generic* functions and classes:

- Templates enable the use of *types as arguments*

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container);
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- Templates enable the use of *types as arguments*

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container);
```

We already know the pointy brackets from vectors. Vectors are also implemented as templates.

Excursion: Templates

Templates enable *type-generic* functions and classes:

- Templates enable the use of *types as arguments*

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container);
```

Intuition: operator works for every output stream `sink` of type `S` and every container `container` of type `C`

Excursion: Templates

Templates enable *type-generic* functions and classes:

- Templates enable the use of *types as arguments*

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container);
```

- The compiler *infers* suitable types from the call arguments

```
std::set<int> s = ...;
std::cout << s << '\n';
```

← S = std::ostream, C = std::set<int>

Excursion: Templates

Implementation of << *constrains* `S` and `C` (Compiler errors if not satisfied):

```
template <typename S, typename C>
S& operator<<(S& sink, const C& container) {
  for (typename C::const_iterator it = container.begin();
       it != container.end();
       ++it) {

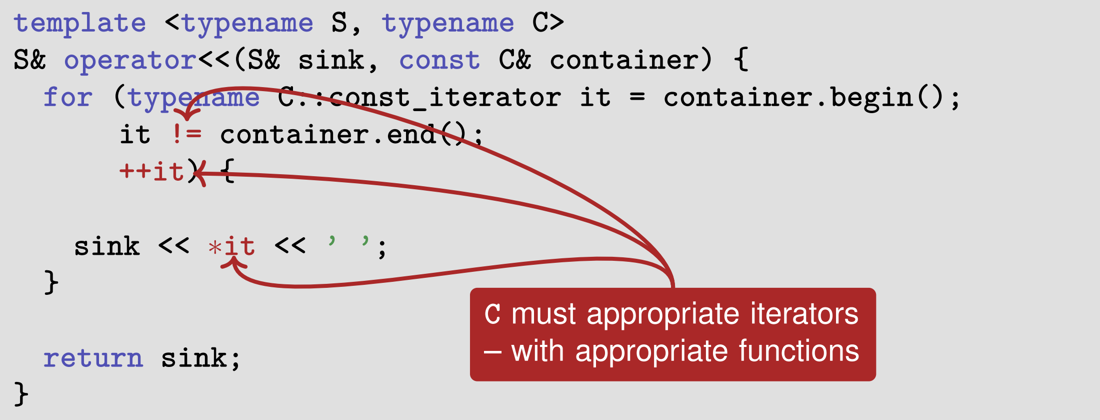
    sink << *it << ' ';
  }

  return sink;
}
```

`C` must appropriate iterators

Excursion: Templates

Implementation of << *constrains* `S` and `C` (Compiler errors if not satisfied):



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  return sink;
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```

`C` must appropriate iterators
– with appropriate functions

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```
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       ++it) {

    sink << *it << ' ';
  }

  return sink;
}
```

`S` must support outputting elements (`*it`) and characters (`' '`)

21. Dynamic Datatypes and Memory Management

Problem

Last week: dynamic data type

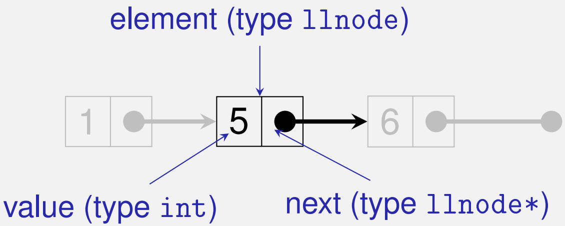
Have allocated dynamic memory, but not released it again. In particular: no functions to remove elements from `llvec`.

Today: correct memory management!

Goal: class stack with memory management

```
class stack{
public:
  // post: Push an element onto the stack
  void push(int value);
  // pre: non-empty stack
  // post: Delete top most element from the stack
  void pop();
  // pre: non-empty stack
  // post: return value of top most element
  int top() const;
  // post: return if stack is empty
  bool empty() const;
  // post: print out the stack
  void print(std::ostream& out) const;
...
```

Recall the Linked List



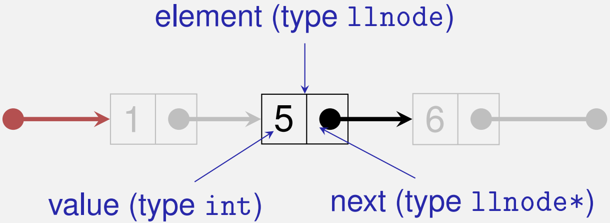
element (type llnode)

value (type int)

next (type llnode*)

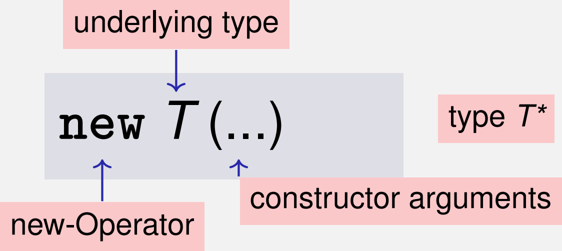
```
struct llnode {
  int value;
  llnode* next;
  // constructor
  llnode (int v, llnode* n) : value (v), next (n) {}
};
```

Stack = Pointer to the Top Element



```
class stack {
public:
  void push (int value);
  ...
private:
  llnode* topn;
};
```

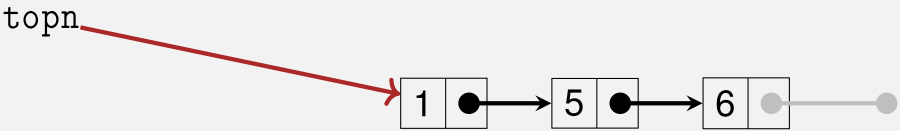
Recall the `new` Expression



- **Effect:** new object of type *T* is allocated in memory . . .
- . . . and initialized by means of the matching constructor.
- **Value:** address of the new object

The `new` Expression push(4)

```
void stack::push(int value){
  topn = new llnode (value, topn);
}
```



The `new` Expression `push(4)`

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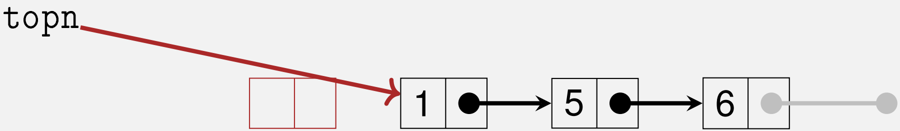

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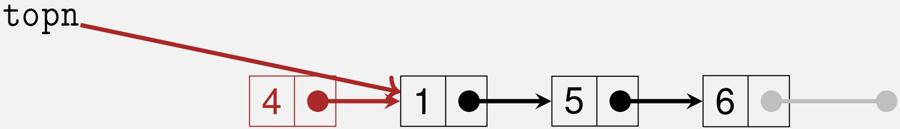

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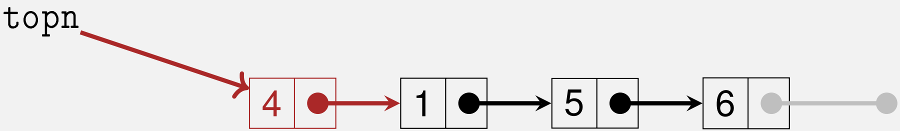


The `delete` Expression

Objects generated with `new` have *dynamic storage duration:* they "live" until they are explicitly *deleted*

The `delete` Expression

Objects generated with **`new`** have *dynamic storage duration:* they "live" until they are explicitly *deleted*

```
delete expr
```

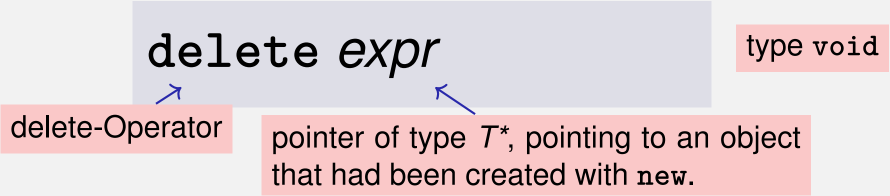
type **`void`**

delete-Operator

pointer of type *T**, pointing to an object that had been created with **`new`**.

The `delete` Expression

Objects generated with **`new`** have *dynamic storage duration:* they “live” until they are explicitly *deleted*



`delete` *expr*

type **`void`**

delete-Operator

pointer of type *T**, pointing to an object that had been created with **`new`**.

- **Effect:** object is *deconstructed* (explanation below)
  ... and *memory is released*.

Who is born must die…

Guideline “Dynamic Memory”

For each `new` there is a matching `delete`!

Who is born must die. . .

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For each `new` there is a matching `delete`!

Non-compliance leads to memory leaks

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- old objects that occupy memory…

Who is born must die. . .

Guideline “Dynamic Memory”

For each `new` there is a matching `delete`!

Non-compliance leads to memory leaks

- old objects that occupy memory. . .
- . . . until it is full (heap overflow)

Careful with `new` and `delete`!

```
rational∗ t = new rational;
rational∗ s = t;
delete s;
int nominator = (*t).denominator();
```

Careful with `new` and `delete`!

```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

Careful with new and delete!

```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

Careful with new and delete!

```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

... and used for releaseing the object

Careful with new and delete!

```
rational* t = new rational;
rational* s = t;
delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

... and used for releaseing the object

error: memory released!

Dereferencing of „dangling pointers”

- Pointer to released objects: *dangling pointers*

Careful with `new` and `delete`!

```
rational∗ t = new rational;
rational∗ s = t;
delete s;
int nominator = (*t).denominator();
```

memory for t is allocated

other pointers may point to the same object

... and used for releaseing the object

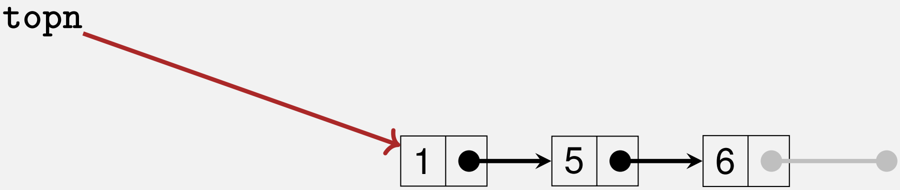
error: memory released!

Dereferencing of „dangling pointers”

- Pointer to released objects: *dangling pointers*
- Releasing an object more than once using `delete` is a similar severe error

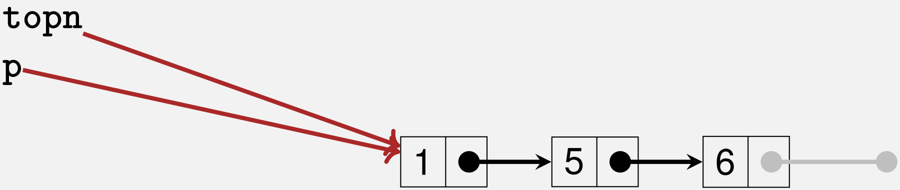
Stack Continued: pop()

```
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```



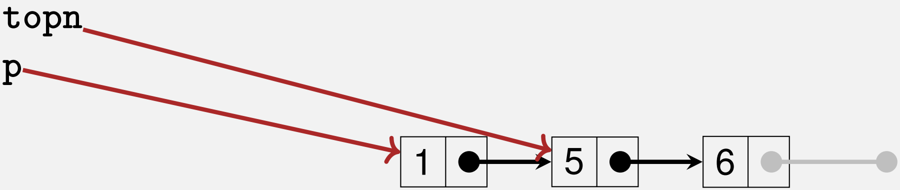
Stack Continued: pop()

```
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```



Stack Continued: pop()

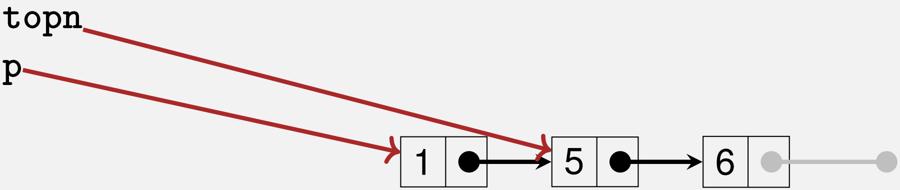
```
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```



Stack Continued: pop()

```
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```

reminder: shortcut for `(*topn).next`



Stack Continued: pop()

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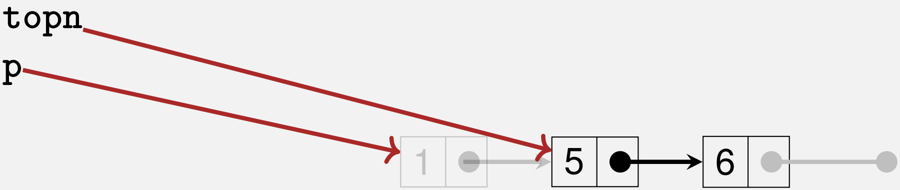
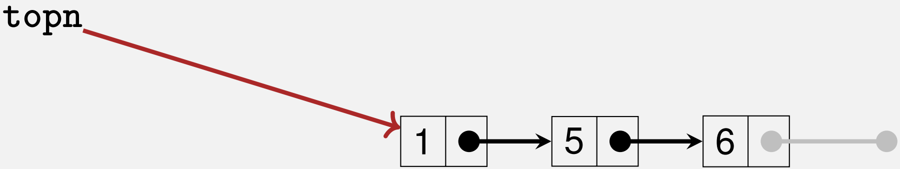
void stack::pop(){
    assert (!empty());
    llnode* p = topn;
    topn = topn->next;
    delete p;
}
```



Print the Stack print()

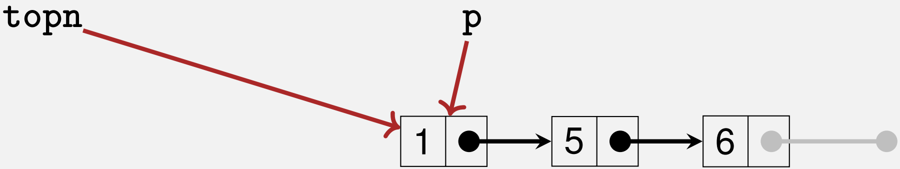
```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " ";
}
```



Print the Stack

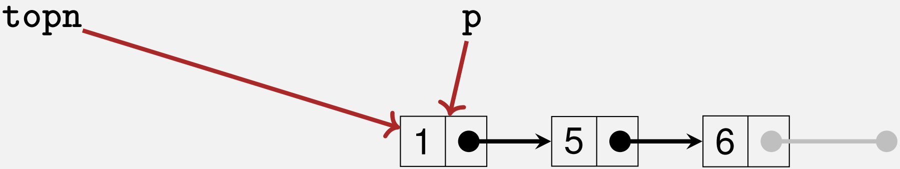
print()

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " ";
}
```



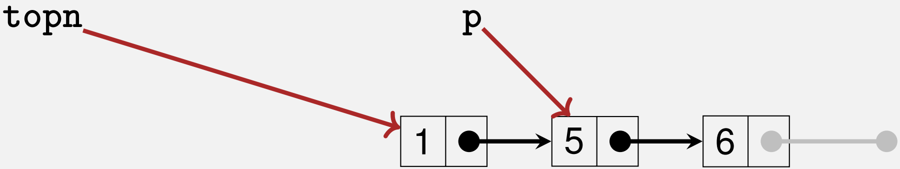
Print the Stack print()

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1
}
```



Print the Stack print()

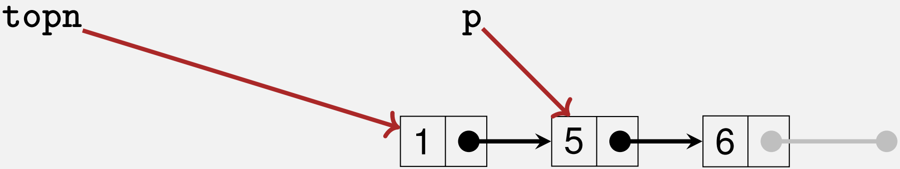
```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1
}
```



Print the Stack

print()

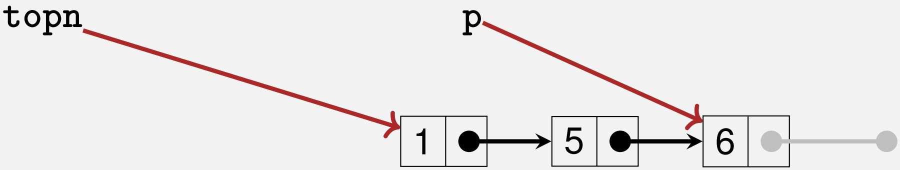
```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1 5
}
```



Print the Stack

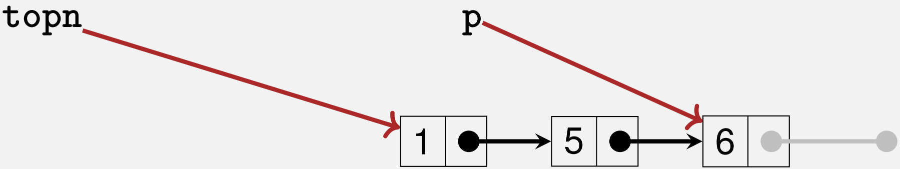
print()

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1 5
}
```



Print the Stack print()

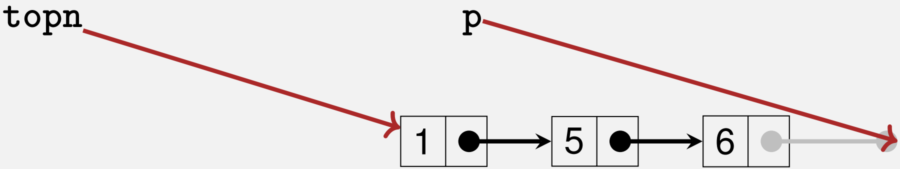
```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1 5 6
}
```



Print the Stack

print()

```
void stack::print (std::ostream& out) const {
  for(const llnode* p = topn; p != nullptr; p = p->next)
    out << p->value << " "; // 1 5 6
}
```



Output Stack: operator<<

```
class stack {
public:
   void push (int value);
   void pop();
   void print (std::ostream& o) const;
   ...
private:
   llnode* topn;
};

// POST: s is written to o
std::ostream& operator<< (std::ostream& o, const stack& s){
   s.print (o);
   return o;
}
```

empty(),top()

```
bool stack::empty() const {
  return top == nullptr;
}

int stack::top() const {
  assert(!empty());
  return topn->value;
}
```

Empty Stack

```
class stack{
public:
  stack() : topn (nullptr) {} // default constructor

  void push(int value);
  void pop();
  void print(std::ostream& out) const;
  int top() const;
  bool empty() const;
private:
  llnode* topn;
}
```

Zombie Elements

```
{
  stack s1; // local variable
  s1.push (1);
  s1.push (3);
  s1.push (2);
  std::cout << s1 << "\n"; // 2 3 1
}
// s1 has died (become invalid)...
```

Zombie Elements

```
{
  stack s1; // local variable
  s1.push (1);
  s1.push (3);
  s1.push (2);
  std::cout << s1 << "\n"; // 2 3 1
}
// s1 has died (become invalid)...
```

- . . . but the three elements of the stack `s1` continue to live (memory leak)!

Zombie Elements

```
{
  stack s1; // local variable
  s1.push (1);
  s1.push (3);
  s1.push (2);
  std::cout << s1 << "\n"; // 2 3 1
}
// s1 has died (become invalid)...
```

- ...but the three elements of the stack `s1` continue to live (memory leak)!
- They should be released together with `s1`.

The Destructor

- The Destructor of class *T* is the unique member function with declaration

  $$\sim T(\,);$$

- is automatically called when the memory duration of a class object ends – i.e. when `delete` is called on an object of type `T*` or when the enclosing scope of an object of type `T` ends.
- If no destructor is declared, it is automatically generated and calls the destructors for the member variables (pointers `topn`, no effect – reason for zombie elements

Using a Destructor, it Works

```
// POST: the dynamic memory of *this is deleted
stack::~stack(){
  while (topn != nullptr){
    llnode* t = topn;
    topn = t->next;
    delete t;
  }
}
```

Using a Destructor, it Works

```
// POST: the dynamic memory of *this is deleted
stack::~stack(){
  while (topn != nullptr){
    llnode* t = topn;
    topn = t->next;
    delete t;
  }
}
```

- automatically deletes all stack elements when the stack is being released

Using a Destructor, it Works

```
// POST: the dynamic memory of *this is deleted
stack::~stack(){
  while (topn != nullptr){
    llnode* t = topn;
    topn = t->next;
    delete t;
  }
}
```

- automatically deletes all stack elements when the stack is being released
- Now our stack class seems to follow the guideline “dynamic memory” (?)

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n";

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n";

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n";

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n";

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

Stack Done?

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop (); // Oops, crash!
```

Stack Done?

Obviously not…

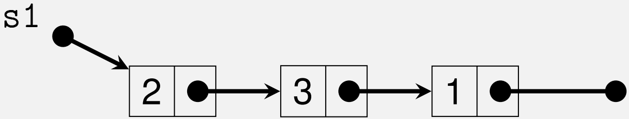
```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop (); // Oops, crash!
```

What has gone wrong?

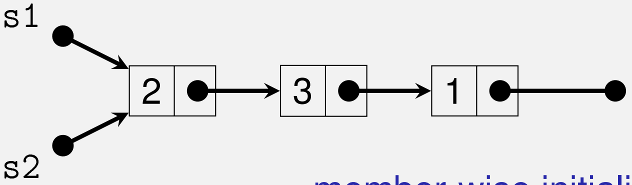


```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?



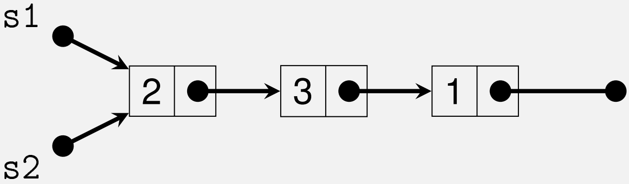
member-wise initialization: copies the `topn` pointer only.

```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?

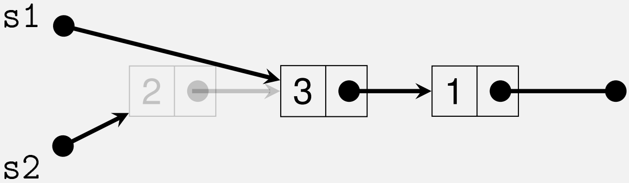


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?

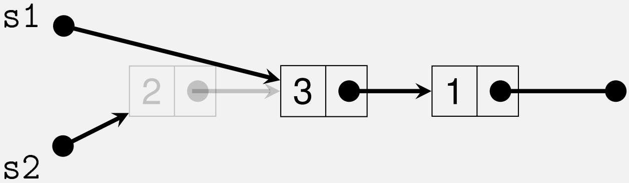


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

What has gone wrong?

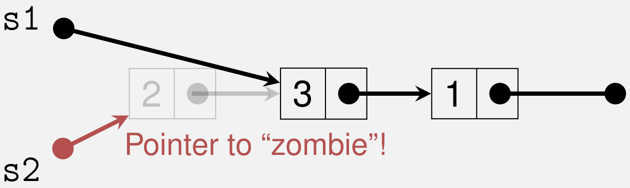


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

What has gone wrong?



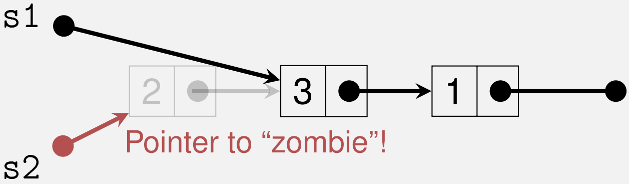
Pointer to “zombie”!

```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

What has gone wrong?



```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();  // Oops, crash!
```

The actual problem

Already this goes wrong:

```
{
  stack s1;
  s1.push(1);
  stack s2 = s1;
}
```

When leaving the scope, both stacks are deconstructed. But both stacks try to delete the same data, because both stacks have *access to the same pointer*.

Possible solutions

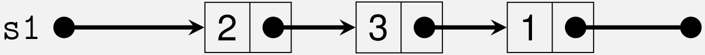
Smart-Pointers (we will not go into details here):

- Count the number of pointers referring to the same objects and delete only when that number goes down to 0 `std::shared_pointer`
- Make sure that not more than one pointer can point to an object: `std::unique_pointer`.

or:

- We make a real copy of all data – as discussed below.

We make a real copy

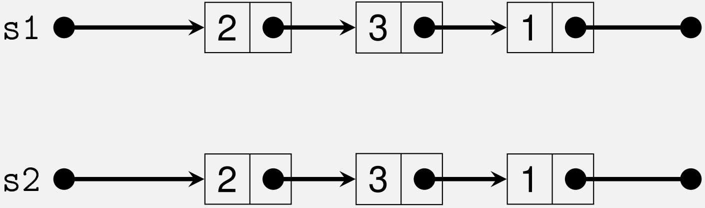


```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

We make a real copy

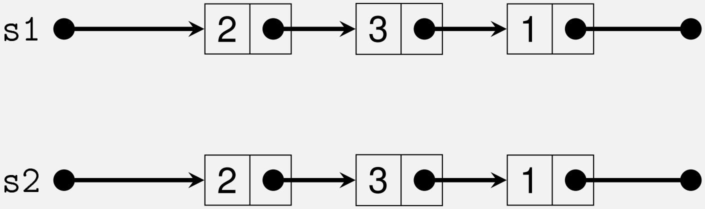


```
...
stack s2 = s1;
std::cout << s2 << "\n";

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

We make a real copy

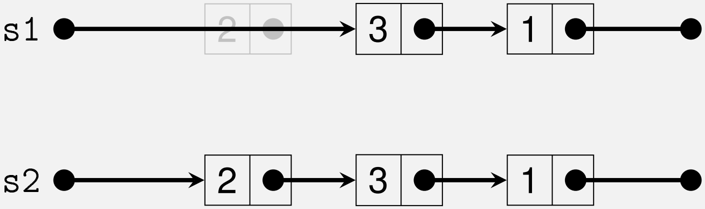


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

We make a real copy

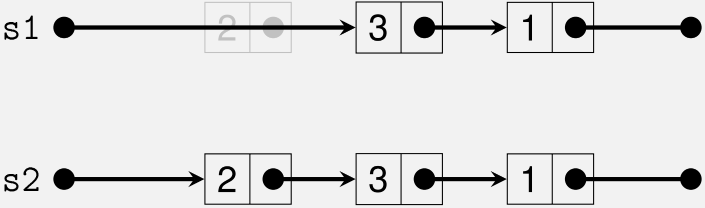


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n";

s2.pop ();
```

We make a real copy

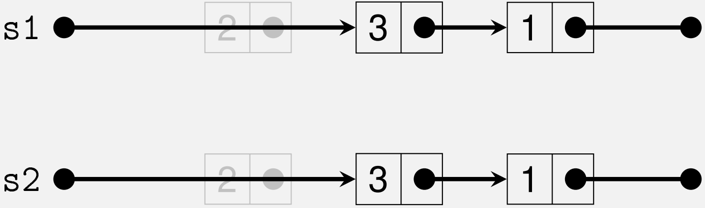


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

We make a real copy

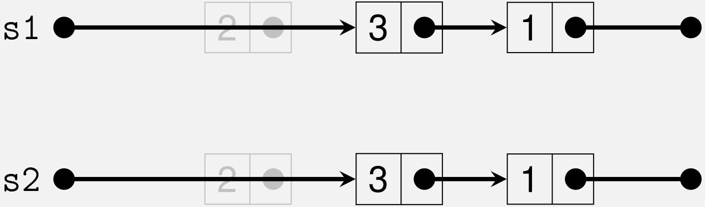


```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();
```

We make a real copy



```
...
stack s2 = s1;
std::cout << s2 << "\n"; // 2 3 1

s1.pop ();
std::cout << s1 << "\n"; // 3 1

s2.pop ();  // ok
```

The Copy Constructor

- The copy constructor of a class *T* is the unique constructor with declaration

  T (`const` *T*& x);

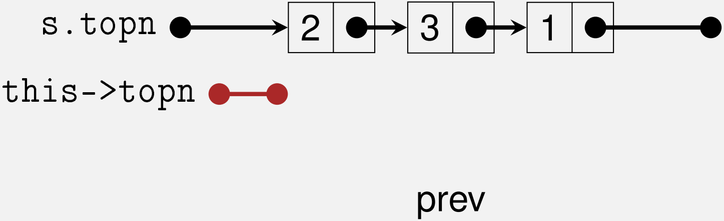
- is automatically called when values of type *T* are initialized with values of type `T`

  ```
  T x = t;      (t of type T)
  T x (t);
  ```

- If there is no copy-constructor declared then it is generated automatically (and initializes member-wise – reason for the problem above

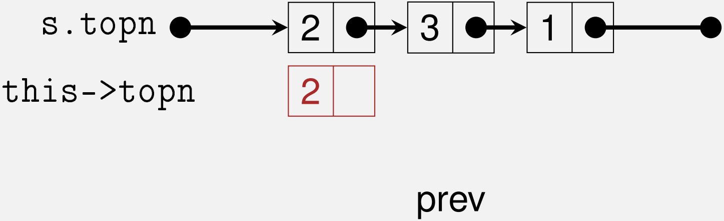
It works with a Copy Constructor

```
// POST: *this is initialized with a copy of s
stack::stack (const stack& s) : topn (nullptr) {
  if (s.topn == nullptr) return;
  topn = new llnode(s.topn->value, nullptr);
  llnode* prev = topn;
  for(llnode* n = s.topn->next; n != nullptr; n = n->next){
    llnode* copy = new llnode(n->value, nullptr);
    prev->next = copy;
    prev = copy;
  }
}
```



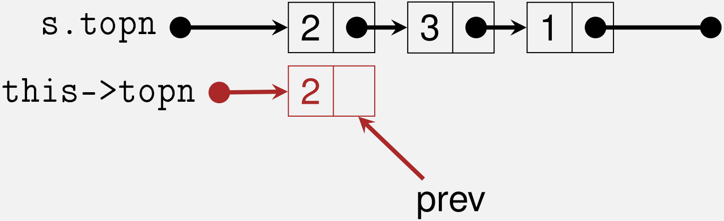
It works with a Copy Constructor

```
// POST: *this is initialized with a copy of s
stack::stack (const stack& s) : topn (nullptr) {
  if (s.topn == nullptr) return;
  topn = new llnode(s.topn->value, nullptr);
  llnode* prev = topn;
  for(llnode* n = s.topn->next; n != nullptr; n = n->next){
    llnode* copy = new llnode(n->value, nullptr);
    prev->next = copy;
    prev = copy;
  }
}
```



It works with a Copy Constructor

```
// POST: *this is initialized with a copy of s
stack::stack (const stack& s) : topn (nullptr) {
  if (s.topn == nullptr) return;
  topn = new llnode(s.topn—>value, nullptr);
  llnode* prev = topn;
  for(llnode* n = s.topn—>next; n != nullptr; n = n—>next){
    llnode* copy = new llnode(n—>value, nullptr);
    prev—>next = copy;
    prev = copy;
  }
}
```



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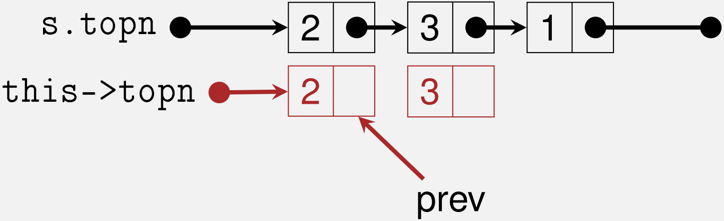

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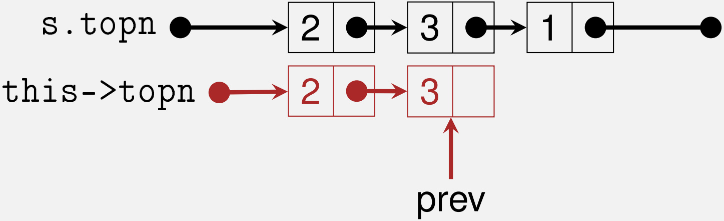

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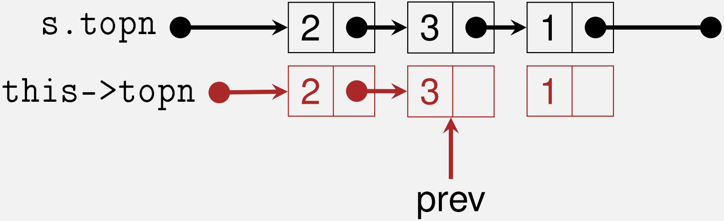

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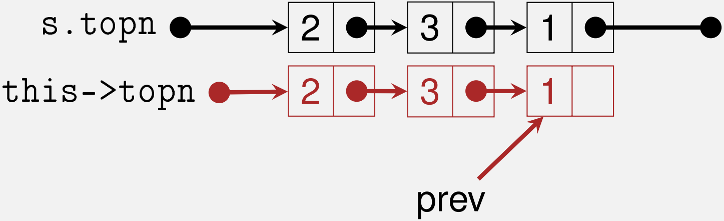

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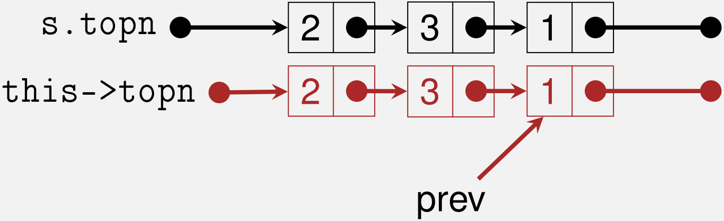

Aside: copy recursively

```
llnode* copy (node* that){
  if (that == nullptr) return nullptr;
  return new llnode(that->value, copy(that->next));
}
```

Elegant, isn't it?

[6] not an overflow of the stack that we are implementing but the call stack

Aside: copy recursively

```
llnode* copy (node* that){
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Elegant, isn’t it? Why did we not do it like this?

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Aside: copy recursively

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Elegant, isn't it? Why did we not do it like this?

Reason: linked lists can become very long. `copy` could then lead to stack overflow[6]. Stack memory is usually smaller than heap memory.

[6] not an overflow of the stack that we are implementing but the call stack

Initialization $\neq$ Assignment!

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2 = s1; // Initialisierung


s1.pop ();
std::cout << s1 << "\n"; // 3 1
s2.pop (); // ok:  Copy Constructor!
```

Initialization $\neq$ Assignment!

```
stack s1;
s1.push (1);
s1.push (3);
s1.push (2);
std::cout << s1 << "\n"; // 2 3 1

stack s2;
s2 = s1; // Zuweisung

s1.pop ();
std::cout << s1 << "\n"; // 3 1
s2.pop (); // Oops, Crash!
```

The Assignment Operator

- Overloading `operator=` as a member function
- Like the copy-constructor without initializer, but additionally
  - Releasing memory for the “old” value
  - Check for self-assignment (`s1=s1`) that should not have an effect
- If there is no assignment operator declared it is automatically generated (and assigns member-wise – reason for the problem above

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//            copy of s (right operand)
stack& stack::operator= (const stack& s)
```

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//              copy of s (right operand)
stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
```

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//             copy of s (right operand)
stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
    stack copy = s; // Copy Construction
```

It works with an Assignment Operator!

```
// POST: *this (left operand) becomes a
//             copy of s (right operand)
stack& stack::operator= (const stack& s){
  if (topn != s.topn){ // no self-assignment
    stack copy = s; // Copy Construction
    std::swap(topn, copy.topn); // now copy has the garbag
```

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  } // copy is cleaned up -> deconstruction
  return *this; // return as L-Value (convention)
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  } // copy is cleaned up -> deconstruction
  return *this; // return as L-Value (convention)
}
```

Cooool trick! ☺

Done

```
class stack{
public:
  stack(); // constructor
  ~stack(); // destructor
  stack(const stack& s); // copy constructor
  stack& operator=(const stack& s); // assignment operator

  void push(int value);
  void pop();
  int top() const;
  bool empty() const;
  void print(std::ostream& out) const;
private:
  llnode* topn;
}
```

Dynamic Datatype

- Type that manages dynamic memory (e.g. our class for a stack)
- Minimal Functionality:
    - Constructors
    - Destructor
    - Copy Constructor
    - Assignment Operator

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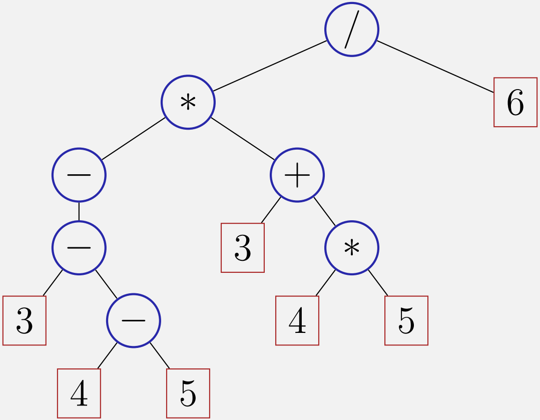
Rule of Three: if a class defines at least one of them, it must define all three

(Expression) Trees

```
-(3-(4-5))*(3+4*5)/6
```



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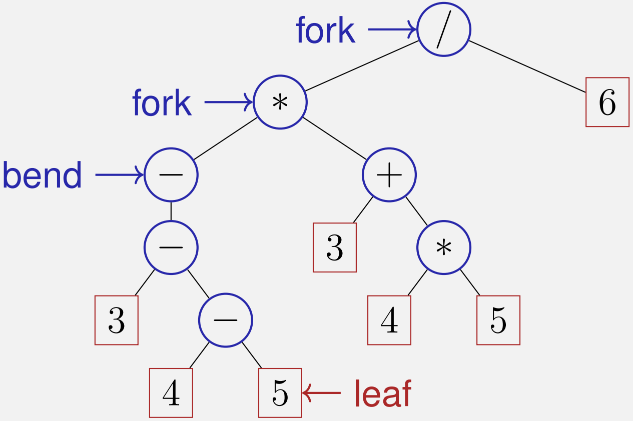

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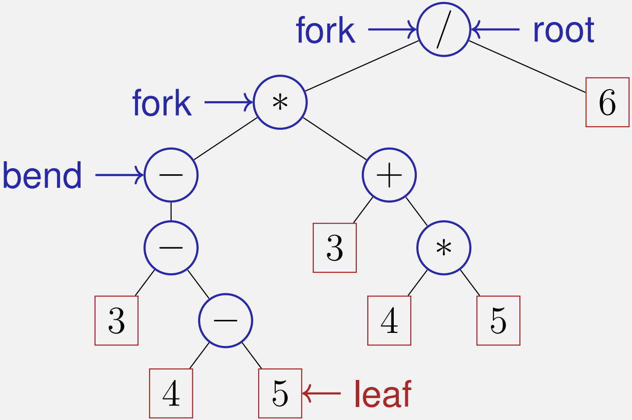

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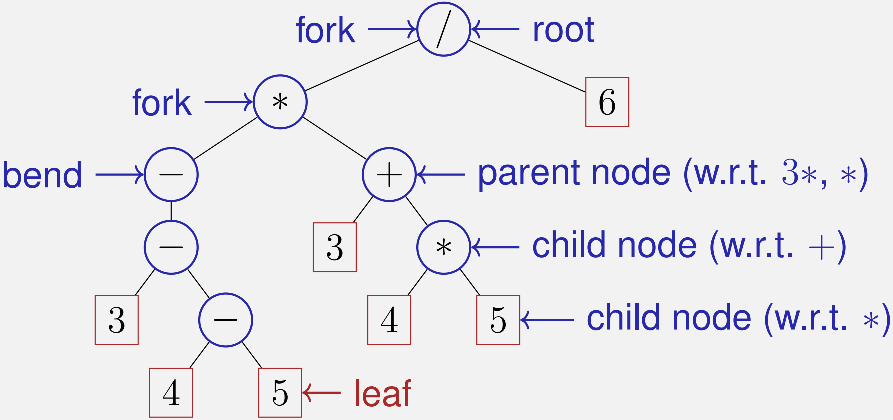
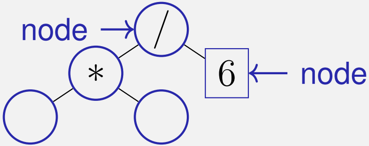


Nodes: Forks, Bends or Leaves



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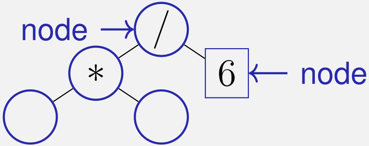

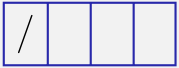

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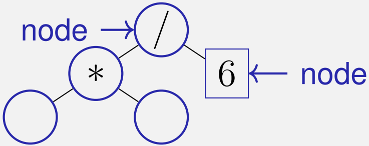

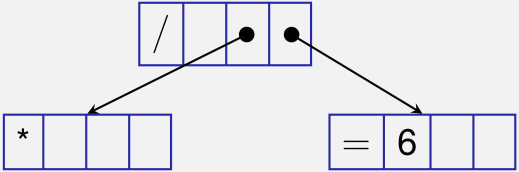

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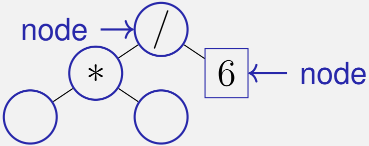

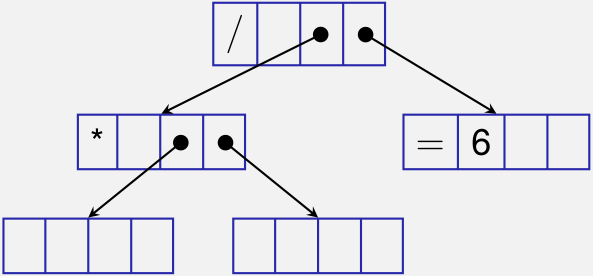

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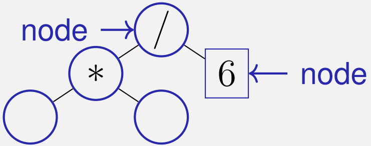

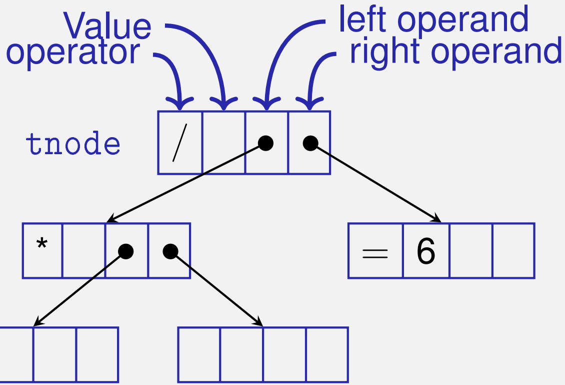

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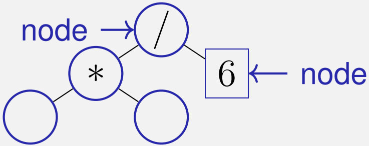
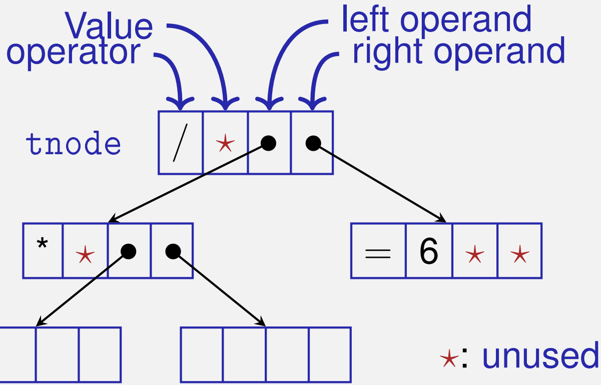
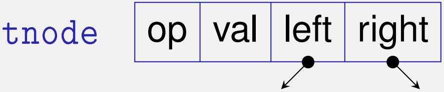




Nodes (`struct tnode`)



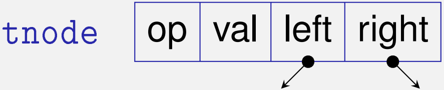
tnode

op | val | left | right

```
struct tnode {
  char op; // leaf node: op is '='
           // internal node: op is '+', '-', '*' or '/'
  double val;
  tnode* left;
  tnode* right;

  tnode(char o, double v, tnode* l, tnode* r)
    : op(o), val(v), left(l), right(r) {}
};
```

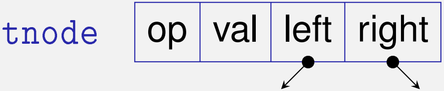
Nodes (`struct tnode`)



```
struct tnode {
  char op; // leaf node: op is '='
           // internal node: op is '+', '−', '∗' or '/'
  double val;
  tnode∗ left; // == nullptr for unary minus
  tnode∗ right;

  tnode(char o, double v, tnode∗ l, tnode∗ r)
    : op(o), val(v), left(l), right(r) {}
};
```

Nodes (struct tnode)



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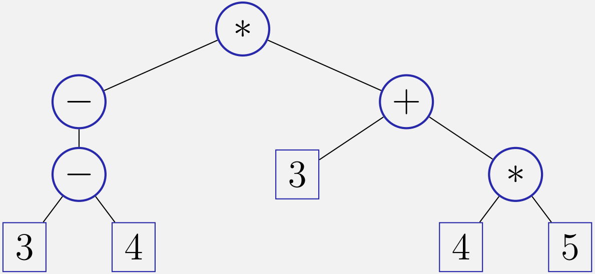
```

Size = Count Nodes in Subtrees



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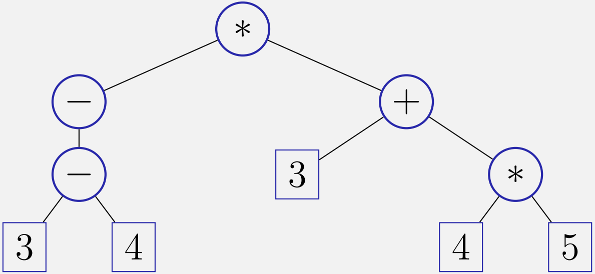
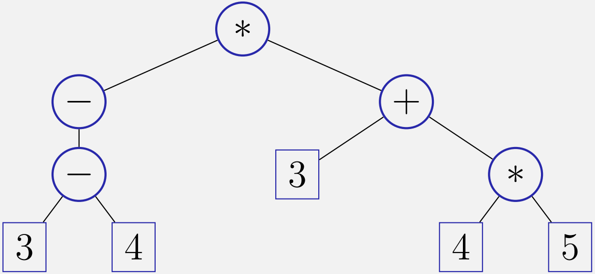


- Size of a leave: 1
- Size of other nodes: 1 + sum of child nodes' size

Size = Count Nodes in Subtrees



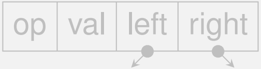
- Size of a leave: 1
- Size of other nodes: 1 + sum of child nodes' size
- E.g. size of the "+"-node is 5

Count Nodes in Subtrees

```
// POST: returns the size (number of nodes) of
//       the subtree with root n
int size (const tnode* n) {
  if (n){ // shortcut for n != nullptr
    return size(n->left) + size(n->right) + 1;
  }
  return 0;
}
```

op	val	left	right

Evaluate Subtrees



op | val | left | right

```
// POST: evaluates the subtree with root n
double eval(const tnode* n){
  assert(n);
  if (n->op == '=') return n->val; ← leaf...
  double l = 0;                         ...or fork:
  if (n->left) l = eval(n->left); ⟵ op unary, or left branch
  double r = eval(n->right);⟵———— right branch
  switch(n->op){
    case '+': return l+r;
    case '-': return l-r;
    case '*': return l*r;
    case '/': return l/r;
    default: return 0;
  }
}
```

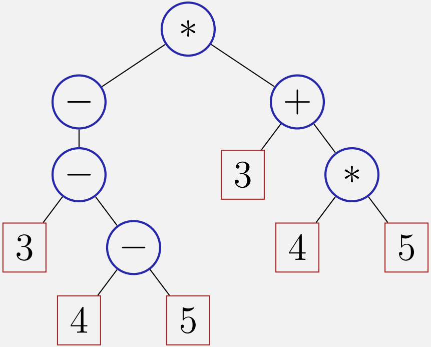
Cloning Subtrees

```
// POST: a copy of the subtree with root n is made
//       and a pointer to its root node is returned
tnode* copy (const tnode* n) {
  if (n == nullptr)
    return nullptr;
  return new tnode (n->op, n->val, copy(n->left), copy(n->right));
}
```

op
val
left
right

Felling Subtrees

```
// POST: all nodes in the subtree with root n are deleted
void clear(tnode* n) {
  if(n){
    clear(n->left);
    clear(n->right);
    delete n;
  }
}
```



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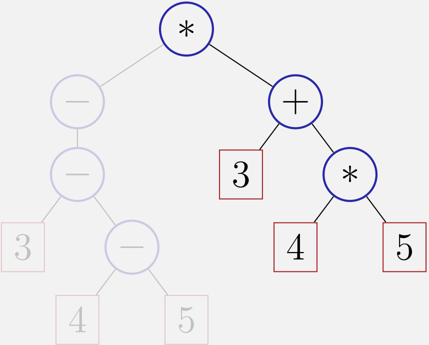

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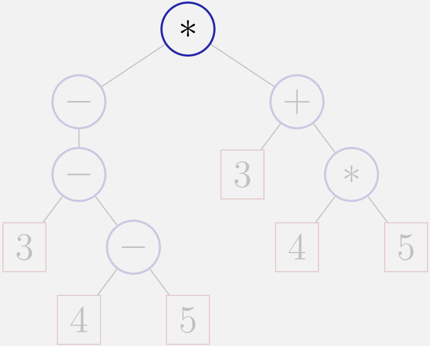

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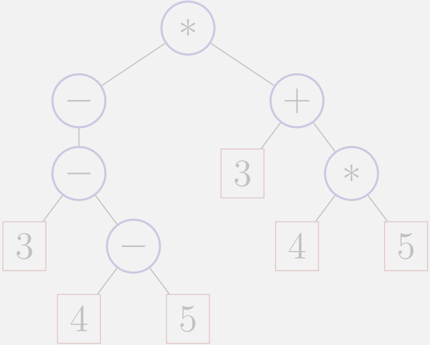


Using Expression Subtrees

```
// Construct a tree for 1 - (-(3 + 7))
tnode* n1 = new tnode('=', 3, nullptr, nullptr);
tnode* n2 = new tnode('=', 7, nullptr, nullptr);
tnode* n3 = new tnode('+', 0, n1, n2);
tnode* n4 = new tnode('-', 0, nullptr, n3);
tnode* n5 = new tnode('=', 1, nullptr, nullptr);
tnode* root = new tnode('-', 0, n5, n4);

// Evaluate the overall tree
std::cout << "1 - (-(3 + 7)) = " << eval(root) << '\n';

// Evaluate a subtree
std::cout << "3 + 7 = " << eval(n3) << '\n';

clear(root); // free memory
```

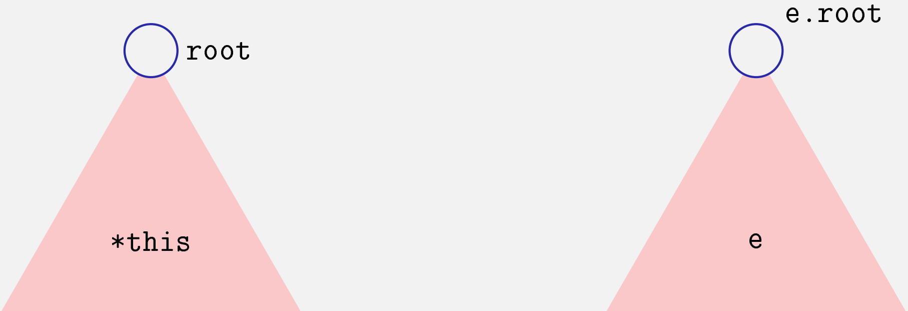
Planting Trees

```
class texpression {
public:
    texpression (double d)
        : root (new tnode (’=’, d, 0, 0)) {}
    ...
private:
    tnode* root;
};
```

creates a tree with one leaf

Letting Trees Grow

```
texpression& texpression::operator-= (const texpression& e)
{
  assert (e.root);
  root = new tnode ('-', 0, root, copy(e.root));
  return *this;
}
```



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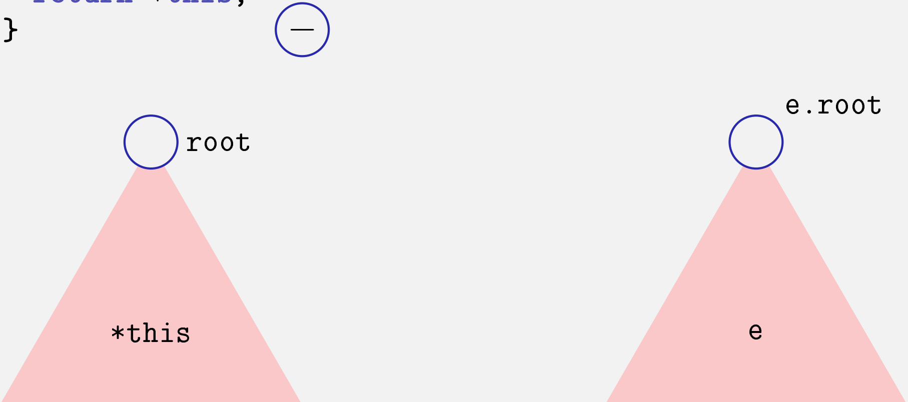

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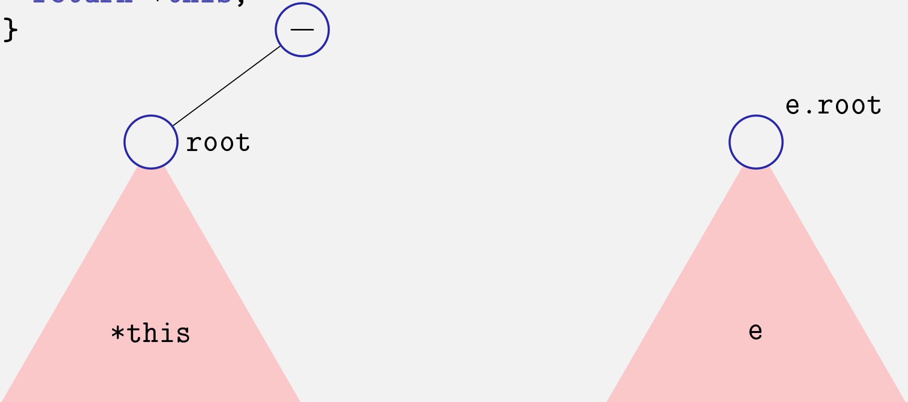

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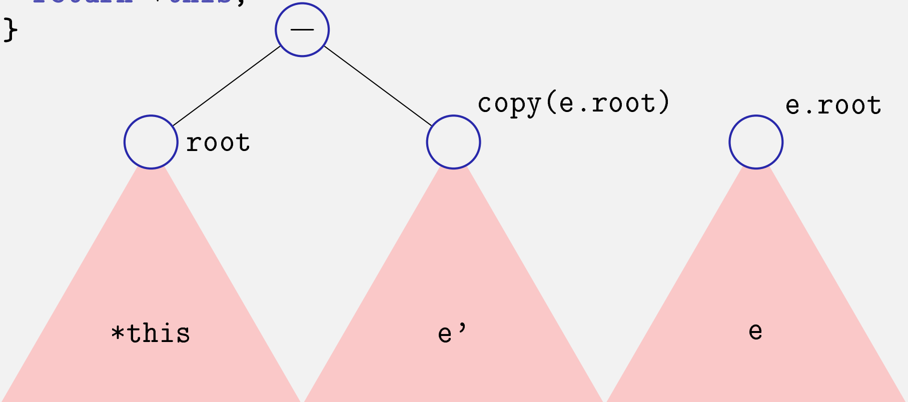
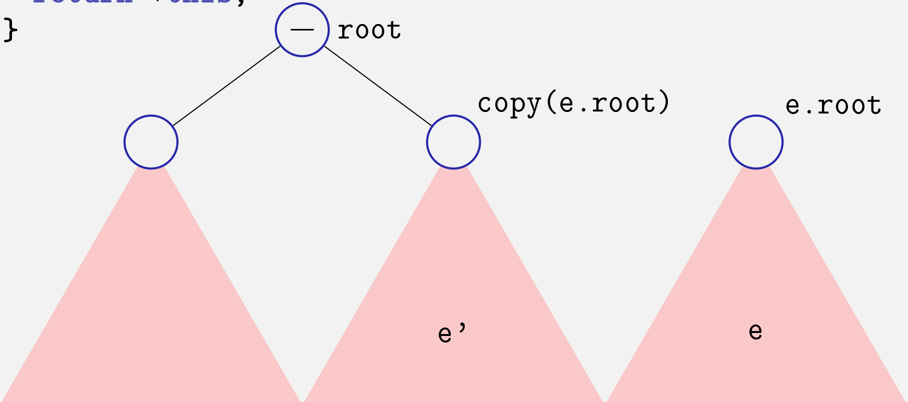
  root = new tnode ('-', 0, root, copy(e.root));
  return *this;
}
```



Letting Trees Grow

```
texpression& texpression::operator-= (const texpression& e)
{
  assert (e.root);
  root = new tnode ('-', 0, root, copy(e.root));
  return *this;
}
```



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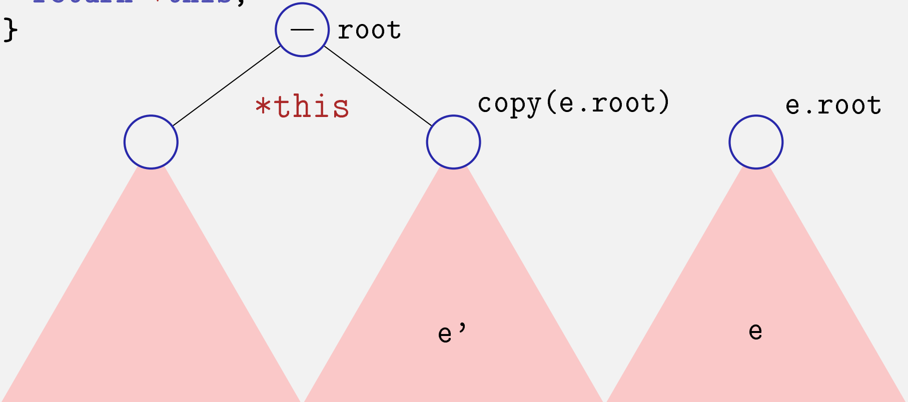

Raising Trees

```
texpression operator− (const texpression& l,
                       const texpression& r){
    texpression result = l;
    return result −= r;
}

texpression a = 3;
texpression b = 4;
texpression c = 5;
texpression d = a−b−c;
```

Raising Trees

```
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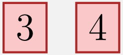
```

$\boxed{3}$

Raising Trees

```
texpression operator- (const texpression& l,
                       const texpression& r){
    texpression result = l;
    return result -= r;
}

texpression a = 3;
texpression b = 4;
texpression c = 5;
texpression d = a-b-c;
```



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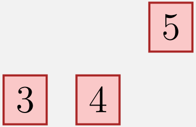

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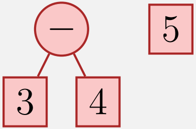

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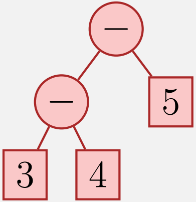

Rule of three: Clone, reproduce and cut trees

```
texpression::~texpression(){
  clear(root);
}

texpresssion::texpression (const texpression& e)
     : root(copy(e.root)) { }

texpression::texpression& operator=(const texpression& e){
  if (root != e.root){
    texpression cp = e;
    std::swap(cp.root, root);
  }
  return *this;
}
```

Concluded

```
class texpression{
public:
  texpression (double d); // constructor
  ~texpression(); // destructor
  texpression (const texpression& e); // copy constructor
  texpression& operator=(const texpression& e); // assignment op
  texpression operator-();
  texpression& operator-=(const texpression& e);
  texpression& operator+=(const texpression& e);
  texpression& operator*=(const texpression& e);
  texpression& operator/=(const texpression& e);
  double evaluate();
private:
 tnode* root;
};
```

From values to trees!

```
// term = factor { "*" factor | "/" factor }
double term (std::istream& is){
  double value = factor (is);
  while (true) {
    if (consume (is, '*'))
      value *= factor (is);
    else if (consume (is, '/'))
      value /= factor (is);
   else
      return value;
  }
}
```

calculator.cpp
(expression value)

From values to trees!

```
using number_type = double;
```

```
// term = factor { "*" factor | "/" factor }
number_type term (std::istream& is){
  number_type value = factor (is);
  while (true) {
    if (consume (is, '*'))
      value *= factor (is);
    else if (consume (is, '/'))
      value /= factor (is);
  else
      return value;
  }
}
```

`double_calculator.cpp`
(expression value)

From values to trees!

```
using number_type = texpression ;

// term = factor { "*" factor | "/" factor }
number_type term (std::istream& is){
  number_type value = factor (is);
  while (true) {
    if (consume (is, '*'))
      value *= factor (is);
    else if (consume (is, '/'))
      value /= factor (is);
   else
      return value;
  }
}
```

`double_calculator.cpp`
(expression value)
→
`texpression_calculator.cpp`
(expression tree)

Concluding Remark

- In this lecture, we have intentionally refrained from implementing member functions in the node classes of the list or tree.[7]
- When there is inheritace and polymorphism used, the implementation of the functionality such as evaluate, print, clear (etc:.) is better implemented in member functions.
- In any case it is not a good idea to implement the memory management of the composite data strcuture list or tree within the nodes.

[7] Parts of the implementations are even simpler (because the case `n==nullptr` can be caught more easily

22. Subtyping, Inheritance and Polymorphism

Expression Trees, Separation of Concerns and Modularisation, Type Hierarchies, Virtual Functions, Dynamic Binding, Code Reuse, Concepts of Object-Oriented Programming

Last Week: Expression Trees

- Goal: Represent arithmetic expressions, e.g.

```
2 + 3 * 2
```

Last Week: Expression Trees

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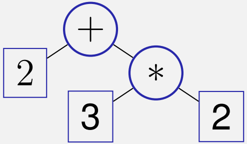
  `2 + 3 * 2`

- Arithmetic expressions form a *tree structure*



Last Week: Expression Trees

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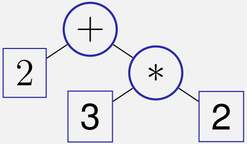

- Expression trees comprise *different* nodes:

Last Week: Expression Trees

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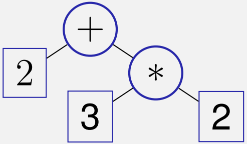
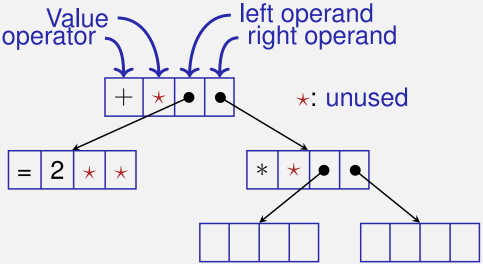


- Expression trees comprise *different* nodes: literals (e.g. 2), binary operators (e.g. $+$), unary operators (e.g. $\sqrt{\ }$), function applications (e.g. $\cos$), etc.

Disadvantages

Implemented via *a single* node type:

```
struct tnode {
  char op; // Operator ('=' for literals)
  double val; // Literal's value
  tnode* left; // Left child (or nullptr)
  tnode* right; // ...
  ...
};
```



Observation: `tnode` is the "sum" of all required nodes (constants, addition, . . .) ⇒ memory wastage, inelegant

Disadvantages

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```
double eval(const tnode* n) {
  if (n->op == '=') return n->val; // n is a constant
  double l = 0;
  if (n->left) l = eval(n->left); // n is not a unary operator
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l+r; // n is an addition node
    case '*': return l*r; // ...
    ...
```

Disadvantages

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  switch(n->op) {
    case '+': return l+r; // n is an addition node
    case '*': return l*r; // ...
    ...
```

⇒ Complex, and therefore error-prone

Disadvantages

```
struct tnode {
  char op;
  double val;
  tnode* left;
  tnode* right;
  ...
};
```

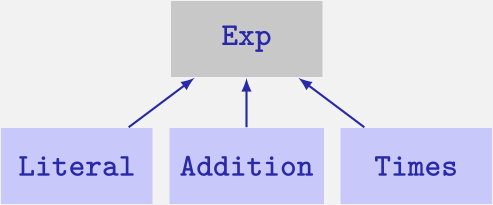
```
double eval(const tnode* n) {
  if (n->op == '=') return n->val;
  double l = 0;
  if (n->left) l = eval(n->left);
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l+r;
    case '*': return l*r;
    ...
```

This code isn't *modular* – we'll change that today!

New Concepts Today

1. Subtyping

- Type hierarchy: **`Exp`** represents general expressions, **`Literal`** etc. are concrete expression



New Concepts Today

1. Subtyping

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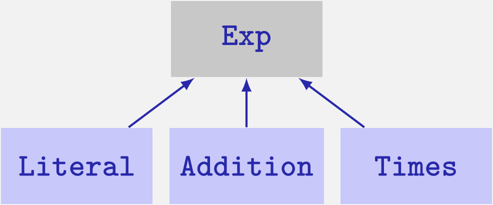
- Every **`Literal`** etc. also is an **`Exp`** (subtype relation)



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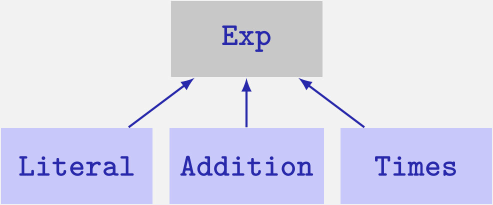


- That's why a `Literal` etc. can be used everywhere, where an `Exp` is expected:

```
Exp∗ e = new Literal(132);
```

New Concepts Today

2. Polymorphism and Dynamic Dispatch

- A variable of *static* type **`Exp`** can “host” expressions of different *dynamic* types:

```
Exp* e = new Literal(2); // e is the literal 2
e = new Addition(e, e); // e is the addition 2 + 2
```

New Concepts Today

2. Polymorphism and Dynamic Dispatch

- A variable of *static* type `Exp` can “host” expressions of different *dynamic* types:

```
Exp* e = new Literal(2); // e is the literal 2
e = new Addition(e, e); // e is the addition 2 + 2
```

- Executed are the member functions of the *dynamic* type:

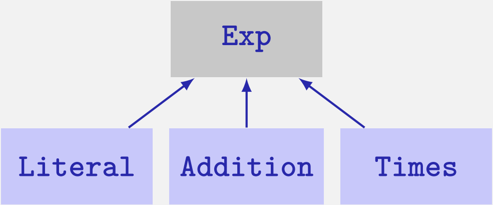
```
Exp* e = new Literal(2);
std::cout << e->eval(); // 2

e = new Addition(e, e);
std::cout << e->eval(); // 4
```

New Concepts Today

3. Inheritance

- Certain functionality is shared among type hierarchy members

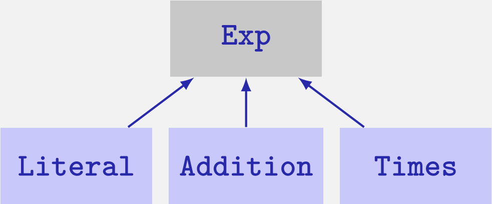


New Concepts Today

3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (**`Addition`**, **`Times`**):

  $1 + size(left\ operand) + size(right\ operand)$



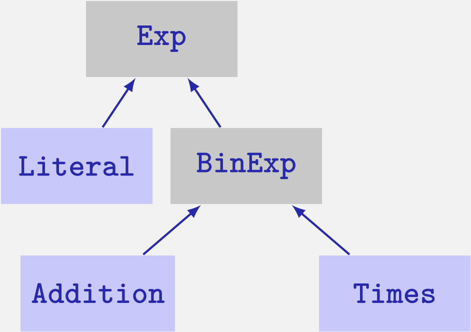
New Concepts Today

3. Inheritance

- Certain functionality is shared among type hierarchy members
- E.g. computing the size (nesting depth) of binary expressions (**`Addition`**, **`Times`**):

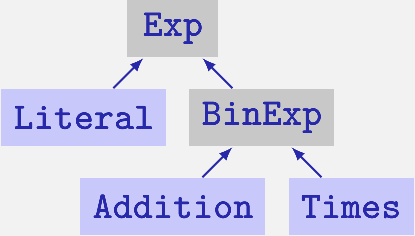
  $1 + size(left\ operand) + size(right\ operand)$

⇒ Implement functionality once, and let subtypes *inherit* it



Advantages

- Subtyping, inheritance and dynamic binding enable *modularisation through spezialisation*

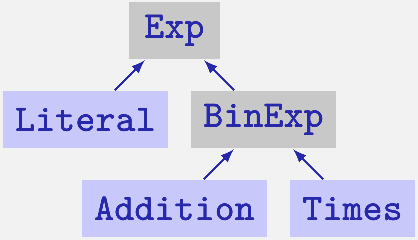


```
Exp* e = new Literal(2);
std::cout << e->eval();

e = new Addition(e, e);
std::cout << e->eval();
```

Advantages

- Subtyping, inheritance and dynamic binding enable *modularisation through spezialisation*
- Inheritance enables sharing common code across modules $\Rightarrow$ *avoid code duplication*



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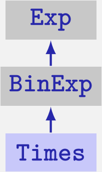
Syntax and Terminology

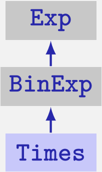


```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```

Syntax and Terminology



```
struct Exp {
  ...
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  ...
}

struct Times : public BinExp {
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}
```

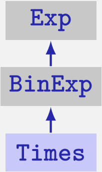
Note: Today, we focus on the new concepts (subtyping, ...) and ignore the orthogonal aspect of encapsulation (`class`, `private` vs. `public` member variables)

Syntax and Terminology

```
struct Exp {
  ...
}

struct BinExp : public Exp {
  ...
}

struct Times : public BinExp {
  ...
}
```



Exp

BinExp

Times

- `BinExp` is a *subclass*[1] of `Exp`

[1] derived class, child class

[2] base class, parent class

Syntax and Terminology

```
struct Exp {
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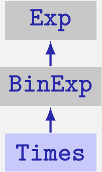

Exp

BinExp

Times

- BinExp is a *subclass*[1] of Exp
- Exp is the *superclass*[2] of BinExp

[1] derived class, child class

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Syntax and Terminology

```
struct Exp {
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struct BinExp : public Exp {
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struct Times : public BinExp {
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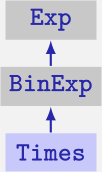

- `BinExp` is a *subclass*[1] of `Exp`
- `Exp` is the *superclass*[2] of `BinExp`
- `BinExp` *inherits* from `Exp`

[1] derived class, child class

[2] base class, parent class

Syntax and Terminology

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struct Exp {
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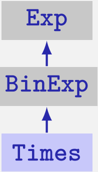

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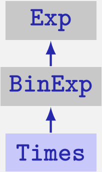
- `BinExp` *inherits* from `Exp`
- `BinExp` *publicly* inherits from `Exp` (`public`), that's why `BinExp` is a *subtype* of `Exp`

[1] derived class, child class

[2] base class, parent class

Syntax and Terminology



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- Analogously: `Times` and `BinExp`

[1] derived class, child class

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Syntax and Terminology

```
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struct BinExp : public Exp {
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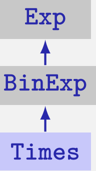

Exp

BinExp

Times

- BinExp is a *subclass*[1] of Exp
- Exp is the *superclass*[2] of BinExp
- BinExp *inherits* from Exp
- BinExp *publicly* inherits from Exp (public), that's why BinExp is a *subtype* of Exp
- Analogously: Times and BinExp
- Subtype relation is transitive: Times is also a subtype of Exp

[1] derived class, child class

[2] base class, parent class

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

Abstract Class Exp and Concrete Class Literal

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

Activates dynamic dispatch

Abstract Class `Exp` and Concrete Class `Literal`

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

Enforces implementation by derived classes . . .

Abstract Class `Exp` and Concrete Class `Literal`

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

. . . that makes `Exp` an *abstract* class

Enforces implementation by derived classes . . .

Abstract Class `Exp` and Concrete Class `Literal`

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {
  double val;

  Literal(double v);
  int size() const;
  double eval() const;
};
```

Abstract Class `Exp` and Concrete Class `Literal`

```
struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {
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```

`Literal` inherits from `Exp` . . .

Abstract Class `Exp` and Concrete Class `Literal`

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struct Exp {
  virtual int size() const = 0;
  virtual double eval() const = 0;
};
```

```
struct Literal : public Exp {
  double val;

  Literal(double v);
  int size() const;
  double eval() const;
};
```

`Literal` inherits from `Exp` . . .

. . . but is otherwise just a regular class

Literal: Implementation

```
Literal::Literal(double v): val(v) {}
```

`Literal`: Implementation

```
Literal::Literal(double v): val(v) {}
```

```
int Literal::size() const {
  return 1;
}
```

Literal: Implementation

```
Literal::Literal(double v): val(v) {}
```

```
int Literal::size() const {
  return 1;
}
```

```
double Literal::eval() const {
  return this->val;
}
```

Subtyping: A Literal is an Expression . . .

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);
```

Subtyping: A Literal is an Expression ...

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);
Exp* e = lit; // OK: Literal is a subtype of Exp
```

Subtyping: A Literal is an Expression . . .

A pointer to a subtype can be used everywhere, where a pointer to a supertype is required:

```
Literal* lit = new Literal(5);
Exp* e = lit; // OK: Literal is a subtype of Exp
```

But not vice versa:

```
Exp* e = ...
Literal* lit = e; // ERROR: Exp is not a subtype of Literal
```

Polymorphie: …a Literal Behaves Like a Literal

```
struct Exp {
  ...
  virtual double eval();
};

double Literal::eval() {
  return this->val;
}
```

```
Exp* e = new Literal(3);
std::cout << e->eval(); // 3
```

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- *virtual* member function: the *dynamic* (here: `Literal`) type determines the member function to be executed
  ⇒ *dynamic binding*

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- Without `Virtual` the *static type* (hier: `Exp`) determines which function is executed

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  virtual double eval();
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double Literal::eval() {
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- *virtual* member function: the *dynamic* (here: `Literal`) type determines the member function to be executed
  ⇒ *dynamic binding*
- Without `Virtual` the *static type* (hier: `Exp`) determines which function is executed
- We won't go into further details

Further Expressions: `Addition` and `Times`

```
struct Addition : public Exp {
  Exp∗ left; // left operand
  Exp∗ right; // right operand
  ...
};
```

Further Expressions: Addition and Times

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

```
struct Times : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

Further Expressions: Addition and Times

```
struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

```
int Addition::size() const {
  return 1 + left->size()
           + right->size();
}
```

```
struct Times : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
  ...
};
```

Further Expressions: Addition and Times

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struct Addition : public Exp {
  Exp* left; // left operand
  Exp* right; // right operand
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😃 Separation of concerns

Further Expressions: Addition and Times

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int Times::size() const {
  return 1 + left->size()
           + right->size();
}
```

😃 Separation of concerns

🤬 Code duplication

Extracting Commonalities . . . : BinExp

```
struct BinExp : public Exp {
  Exp* left;
  Exp* right;

  BinExp(Exp* l, Exp* r);
  int size() const;
};
```

```
BinExp::BinExp(Exp* l, Exp* r): left(l), right(r) {}
```

Extracting Commonalities . . . : BinExp

```
struct BinExp : public Exp {
  Exp* left;
  Exp* right;

  BinExp(Exp* l, Exp* r);
  int size() const;
};
```

```
BinExp::BinExp(Exp* l, Exp* r): left(l), right(r) {}
```

```
int BinExp::size() const {
  return 1 + this->left->size() + this->right->size();
}
```

Note: `BinExp` does not implement `eval` and is therefore also an abstract class, just like `Exp`

...Inheriting Commonalities: Addition

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

. . . Inheriting Commonalities: Addition

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

Addition inherits member variables (left, right) and functions (size) from BinExp

. . . Inheriting Commonalities: Addition

```
struct Addition : public BinExp {
  Addition(Exp∗ l, Exp∗ r);
  double eval() const;
};
```

```
Addition::Addition(Exp∗ l, Exp∗ r): BinExp(l, r) {}
```

Calling the *super constructor* (constructor of `BinExp`) initialises the member variables `left` and `right`

. . . Inheriting Commonalities: Addition

```
struct Addition : public BinExp {
  Addition(Exp* l, Exp* r);
  double eval() const;
};
```

```
Addition::Addition(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Addition::eval() const {
  return
    this->left->eval() +
    this->right->eval();
}
```

...Inheriting Commonalities: `Times`

```
struct Times : public BinExp {
  Times(Exp* l, Exp* r);
  double eval() const;
};
```

```
Times::Times(Exp* l, Exp* r): BinExp(l, r) {}
```

```
double Times::eval() const {
  return
    this->left->eval() *
    this->right->eval();
}
```

Observation: `Additon::eval()` and `Times::eval()` are very similar and could also be unified. However, this would require the concept of *functional programming*, which is outside the scope of this course.

Further Expressions and Operations

- Further expressions, as classes derived from **`Exp`**, are possible, e.g. $-$, $/$, $\sqrt{\ }$, $\cos$, $\log$

Further Expressions and Operations

- Further expressions, as classes derived from `Exp`, are possible, e.g. $-$, $/$, $\sqrt{\ }$, $\cos$, $\log$
- A former bonus exercise (included in today's lecture examples on Code Expert) illustrates possibilities: variables, trigonometric functions, parsing, pretty-printing, numeric simplifications, symbolic derivations, . . .

Mission: Monolithic → Modular ✓

```
struct tnode {
  char op;
  double val;
  tnode* left;
  tnode* right;
  ...
}
```

```
double eval(const tnode* n) {
  if (n->op == '=') return n->val;
  double l = 0;
  if (n->left != 0) l = eval(n->left);
  double r = eval(n->right);
  switch(n->op) {
    case '+': return l + r;
    case '*': return l - r;
    case '-': return l - r;
    case '/': return l / r;
    default:
      // unknown operator
      assert (false);
  }
}
```

```
int size (const tnode* n) const { ... }

...
```

```
struct Literal : public Exp {
  double val;
  ...
  double eval() const {
    return val;
  }
};
```

```
struct Addition : public Exp {
  ...
  double eval() const {
    return left->eval() + right->eval();
  }
};
```

```
struct Times : public Exp {
  ...
  double eval() const {
    return left->eval() * right->eval();
  }
}
```

+

```
struct Cos : public Exp {
  ...
  double eval() const {
    return std::cos(argument->eval());
  }
}
```

And there is so much more . . .

Not shown/discussed:

- Private inheritance (`class B : ~~public~~ A`)
- Subtyping and polymorphism without pointers
- Non-virtuell member functions and static dispatch (`~~virtual~~ double eval()`)
- Overriding inherited member functions and invoking overridden implementations
- Multiple inheritance
- . . .

Object-Oriented Programming

In the last 3rd of the course, several concepts of *object-oriented programming* were introduced, that are briefly summarised on the upcoming slides.

Encapsulation (weeks 10-13):

- Hide the implementation details of types (private section) from users
- Definition of an interface (public area) for accessing values and functionality in a controlled way
- Enables ensuring invariants, and the modification of implementations without affecting user code

Object-Oriented Programming

Subtyping (week 14):

- Type hierarchies, with super- and subtypes, can be created to model relationships between more abstract and more specialised entities
- A subtype supports at least the functionality that its supertype supports – typically more, though, i.e. a subtype extends the interface (public section) of its supertype
- That's why supertypes can be used anywhere, where subtypes are required ...
- ...and functions that can operate on more abstract type (supertypes) can also operate on more specialised types (subtypes)
- The streams introduced in week 7 form such a type hierarchy: `ostream` is the abstract supertyp, `ofstream` etc. are specialised subtypes

Object-Oriented Programming

Polymorphism and *dynamic binding* (week 14):

- A pointer of static typ T_1 can, at runtime, point to objects of (dynamic) type T_2, if T_2 is a subtype of T_1
- When a virtual member function is invoked from such a pointer, the dynamic type determines which function is invoked
- I.e.: despite having the same static type, a different behaviour can be observed when accessing the common interface (member functions) of such pointers
- In combination with subtyping, this enables adding further concrete types (streams, expressions, . . .) to an existing system, without having to modify the latter

Object-Oriented Programming

Inheritance (week 14):

- Derived classes inherit the functionality, i.e. the implementation of member functions, of their parent classes
- This enables sharing common code and thereby avoids code duplication
- An inherited implementation can be overridden, which allows derived classes to behave differently than their parent classes (not shown in this course)

23. Conclusion

Purpose and Format

Name the most important key words to each chapter. Checklist: “does every notion make some sense for me?”

- Ⓜ motivating example for each chapter
- Ⓒ concepts that do not depend from the implementation (language)
- Ⓛ language (C++): all that depends on the chosen language
- Ⓔ examples from the lectures

Kapitelüberblick

- 1. Introduction
- 2. Integers
- 3. Booleans
- 4. Defensive Programming
- 5./6. Control Statements
- 7./8. Floating Point Numbers
- 9./10. Functions
- 11. Reference Types
- 12./13. Vectors and Strings
- 14./15. Recursion
- 16. Structs and Overloading
- 17. Classes
- 18./19. Dynamic Datastructures
- 20. Containers, Iterators and Algorithms
- 21. Dynamic Datatypes and Memory Management
- 22. Subtyping, Polymorphism and Inheritance

1. Introduction

- Ⓜ Euclidean algorithm
- Ⓒ algorithm, Turing machine, programming languages, compilation, syntax and semantics
- values and effects, fundamental types, literals, variables
- Ⓛ include directive `#include <iostream>`
- main function `int main(){...}`
- comments, layout `// Kommentar`
- types, variables, L-value `a` , R-value `a+b`
- expression statement `b=b*b;` , declaration statement `int a;`, return statement `return 0;`

2. Integers

Ⓜ
- Celsius to Fahrenheit

Ⓒ
- associativity and precedence, arity
- expression trees, evaluation order
- arithmetic operators
- binary representation, hexadecimal numbers
- signed numbers, twos complement

Ⓛ
- arithmetic operators `9 * celsius / 5 + 32`
- increment / decrement `expr++`
- arithmetic assignment `expr1 += expr2`
- conversion `int` ↔ `unsigned int`

Ⓔ
- Celsius to Fahrenheit, equivalent resistance

3. Booleans

Ⓒ

- Boolean functions, completeness
- DeMorgan rules

Ⓛ

- the type `bool`
- logical operators `a && !b`
- relational operators `x < y`
- precedences `7 + x < y && y != 3 * z`
- short circuit evaluation `x != 0 && z / x > y`
- the **`assert`**-statement, **`#include <cassert>`**

Ⓔ

- Div-Mod identity.

4. Definsive Programming

Ⓒ

- Assertions and Constants

Ⓛ

- The `assert`-statement, `#include <cassert>`
- `const int speed_of_light=2999792458`

Ⓔ

- Assertions for the GCD

5./6. Control Statements

Ⓜ
- linear control flow vs. interesting programs

Ⓒ
- selection statements, iteration statements
- (avoiding) endless loops, halting problem
- Visibility and scopes, automatic memory
- equivalence of iteration statement

Ⓛ
- if statements `if (a % 2 == 0) {..}`
- for statements `for (unsigned int i = 1; i <= n; ++i) ...`
- while and do-statements `while (n > 1) {...}`
- blocks and branches `if (a < 0) continue;`
- Switch statement `switch(grade) {case 6: }`

Ⓔ
- sum computation (Gauss), prime number tests, Collatz sequence, Fibonacci numbers, calculator, output grades

7./8. Floating Point Numbers

Ⓜ

- correct computation: Celsius / Fahrenheit

Ⓒ

- fixpoint vs. floating point
- holes in the value range
- compute using floating point numbers
- floating point number systems, normalisation, IEEE standard 754
- *guidelines for computing with floating point numbers*

Ⓛ

- types `float`, `double`
- floating point literals `1.23e-7f`

Ⓔ

- Celsius/Fahrenheit, Euler, Harmonic Numbers

9./10. Functions

Ⓜ
- Computation of Powers

Ⓒ
- Encapsulation of Functionality
- functions, formal arguments, arguments
- scope, forward declarations
- procedural programming, modularization, separate compilation
- *Stepwise Refinement*

Ⓛ
- declaration and definition of functions `double pow(double b, int e){ ... }`
- function call `pow (2.0, -2)`
- the type `void`

Ⓔ
- powers, perfect numbers, minimum, calendar

11. Reference Types

Ⓜ

- Swap

Ⓒ

- value- / reference- semantics, pass by value, pass by reference, return by reference
- lifetime of objects / temporary objects
- constants

Ⓛ

- reference type `int& a`
- call by reference, return by reference `int& increment (int& i)`
- const guideline, const references, reference guideline

Ⓔ

- swap, increment

12./13. Vectors and Strings

Ⓜ

- Iterate over data: sieve of erathosthenes

Ⓒ

- vectors, memory layout, random access
- (missing) bound checks
- vectors
- characters: ASCII, UTF8, texts, strings

Ⓛ

- vector types `std::vector<int> a {4,3,5,2,1};`
- characters and texts, the type char `char c = ’a’;`, Konversion nach **`int`**
- vectors of vectors
- Streams `std::istream, std::ostream`

Ⓔ

- sieve of Erathosthenes, Caesar-code, shortest paths

14./15. Recursion

Ⓜ

- recursive math. functions, the n-Queen problem, Lindenmayer systems, a command line calculator

Ⓒ

- recursion
- call stack, memory of recursion
- correctness, termination,
- recursion vs. iteration
- Backtracking, EBNF, formal grammars, parsing

Ⓔ

- factorial, GCD, sudoku-solver, command line calcoulator

16. Structs and Overloading

Ⓜ

- build your own rational number

Ⓒ

- heterogeneous data types
- function and operator overloading
- encapsulation of data

Ⓛ

- struct definition `struct rational {int n; int d;};`
- member access `result.n = a.n * b.d + a.d * b.n;`
- initialization and assignment,
- function overloading `pow(2)` vs. `pow(3,3);`, operator overloading

Ⓔ

- rational numbers, complex numbers

17. Classes

Ⓜ
- rational numbers with encapsulation

Ⓒ
- Encapsulation, Construction, Member Functions

Ⓛ
- classes `class rational { ... };`
- access control `public:` / `private:`
- member functions `int rational::denominator () const`
- The implicit argument of the member functions

Ⓔ
- finite rings, complex numbers

18./19. Dynamic Datastructures

Ⓜ
- Our own vector

Ⓒ
- linked list, allocation, deallocation, dynamic data type

Ⓛ
- The `new` statement
- pointer `int* x;`, Null-pointer `nullptr`.
- address and derference operator `int *ip = &i; int j = *ip;`
- pointer and const `const int *a;`

Ⓔ
- linked list, stack

20. Containers, Iterators and Algorithms

Ⓜ

- vectors are containers

Ⓒ

- iteration with pointers
- containers and iterators
- algorithms

Ⓛ

- Iterators `std::vector<int>::iterator`
- Algorithms of the standard library `std::fill (a, a+5, 1);`
- implement an iterator
- iterators and const

Ⓔ

- output a vector, a set

21. Dynamic Datatypes and Memory Management

Ⓜ

- Stack
- Expression Tree

Ⓒ

- Guideline ”dynamic memory“
- Pointer sharing
- Dynamic Datatype
- Tree-Structure

Ⓛ

- `new` and `delete`
- Destructor `stack::~stack()`
- Copy-Constructor `stack::stack(const stack& s)`
- Assignment operator `stack& stack::operator=(const stack& s)`
- Rule of Three

Ⓔ

- Binary Search Tree

22. Subtyping, Polymorphism and Inheritance

Ⓜ

- extend and generalize expression trees

Ⓒ

- Subtyping
- polymorphism and dynamic binding
- Inheritance

Ⓛ

- base class `struct Exp{}`
- derived class `struct BinExp: public Exp{}`
- abstract class `struct Exp{virtual int size() const = 0...}`
- polymorphie `virtual double eval()`

Ⓔ

- expression node and extensions

The End

End of the Course