1. Integers

Evaluation of Arithmetic Expressions, Associativity and Precedence, Arithmetic Operators, Domain of Types `int`, `unsigned int`

Celsius to Fahrenheit

```
// Program: fahrenheit.cpp
// Convert temperatures from Celsius to Fahrenheit.
#include <iostream>

int main() {
  // Input
  std::cout << "Temperature in degrees Celsius =? ";
  int celsius;
  std::cin >> celsius;

  // Computation and output
  std::cout << celsius << " degrees Celsius are "
            << 9 * celsius / 5 + 32  << " degrees Fahrenheit.\n";
  return 0;
}
```


`9 * celsius / 5 + 32`

- Arithmetic expression,
- contains three literals, a variable, three operator symbols

How to put the expression in parentheses?

Precedence

Multiplication/Division before Addition/Subtraction

`9 * celsius / 5 + 32`

bedeutet

`(9 * celsius / 5) + 32`

Rule 1: precedence

Multiplicative operators (`*`, `/`, `%`) have a higher precedence ("bind more strongly") than additive operators (`+`, `-`)

Associativity

From left to right

`9 * celsius / 5 + 32`

bedeutet

`((9 * celsius) / 5) + 32`

Rule 2: Associativity

Arithmetic operators (*, /, %, +, −) are left associative: operators of same precedence evaluate from left to right

Arity

Rule 3: Arity

Unary operators +, − first, then binary operators +, −.

`-3 - 4`

means

`(-3) - 4`

Parentheses

Any expression can be put in parentheses by means of

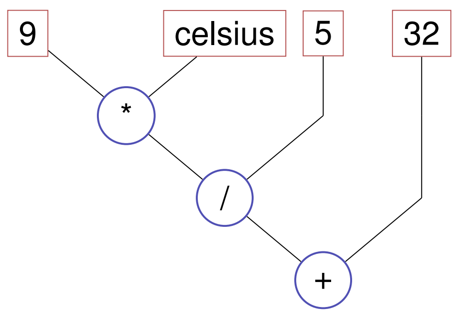
- associativities
- precedences
- arities (number of operands)

of the operands in an unambiguous way (Details in the lecture notes).

Expression Trees

Parentheses yield the expression tree

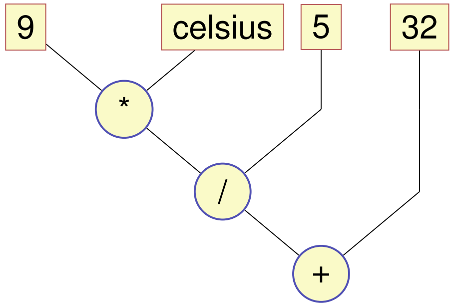
`(((9 * celsius) / 5) + 32)`



Evaluation Order

"From top to bottom" in the expression tree

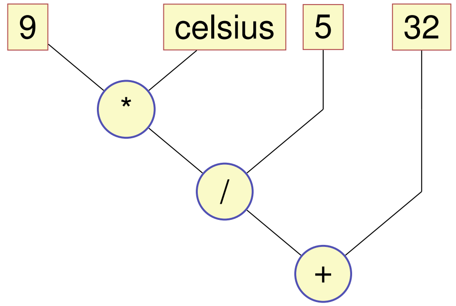
```
9 * celsius / 5 + 32
```



Evaluation Order

Order is not determined uniquely:

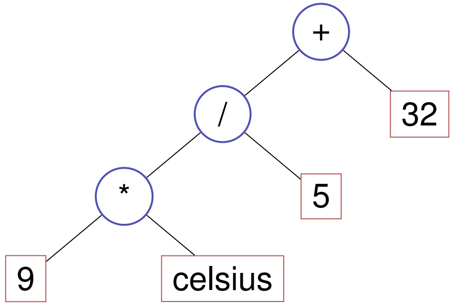
```
9 * celsius / 5 + 32
```



Expression Trees – Notation

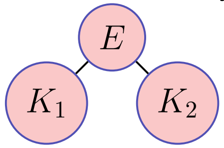
Common notation: root on top

```
9 * celsius / 5 + 32
```



Evaluation Order – more formally

- Valid order: any node is evaluated *after* its children



In $\mathrm{C{+}{+}}$, the valid order to be used is not defined.

- "Good expression": any valid evaluation order leads to the same result.
- Example for a "bad expression": `(a+b)*(a++)`

Evaluation order

Guideline

Avoid modifying variables that are used in the same expression more than once.

Arithmetic operations

	Symbol	Arity	Precedence	Associativity
Unary +	+	1	16	right
Negation	-	1	16	right
Multiplication	*	2	14	left
Division	/	2	14	left
Modulus	%	2	14	links
Addition	+	2	13	left
Subtraction	-	2	13	left

All operators: [R-value $\times$] R-value $\to$ R-value

Assignment expression – in more detail

- Already known: `a = b` means
  Assignment of `b` (R-value) to `a` (L-value).
  Returns: L-value
- What does `a = b = c` mean?
- Answer: assignment is right-associative

`a = b = c` $\Longleftrightarrow$ `a = (b = c)`

Example multiple assignment:
`a = b = 0` $\Longrightarrow$ `b=0; a=0`

Division and Modulus

- Operator / implements integer division

`5 / 2` has value 2

- In `fahrenheit.cpp`

```
9 * celsius / 5 + 32

15 degrees Celsius are 59 degrees Fahrenheit
```

- Mathematically equivalent... but not in C++!

```
9 / 5 * celsius + 32

15 degrees Celsius are 47 degrees Fahrenheit
```

Division and Modulus

- Modulus-operator computes the rest of the integer division

`5 / 2` has value 2, `5 % 2` has value 1.

- It holds that:

`(a / b) * b + a % b` has the value of `a`.

Increment and decrement

- Increment / Decrement a number by one is a frequent operation
- works like this for an L-value:

```
expr = expr + 1.
```

Disadvantages

- relatively long
- expr is evaluated twice (effects!)

In-/Decrement Operators

Post-Increment

```
expr++
```

Value of expr is increased by one, the *old* value of expr is returned (as R-value)

Pre-increment

```
++expr
```

Value of expr is increased by one, the *new* value of expr is returned (as L-value)

Post-Dekrement

```
expr--
```

Value of expr is decreased by one, the *old* value of expr is returned (as R-value)

Prä-Dekrement

```
--expr
```

Value of expr is increased by one, the *new* value of expr is returned (as L-value)

In-/decrement Operators

	use	arity	prec	assoz	L-/R-value
Post-increment	`expr++`	1	17	left	L-value → R-value
Pre-increment	`++expr`	1	16	right	L-value → L-value
Post-decrement	`expr--`	1	17	left	L-value → R-value
Pre-decrement	`--expr`	1	16	right	L-value → L-value

In-/Decrement Operators

Example

```
int a = 7;
std::cout << ++a << "\n"; // 8
std::cout << a++ << "\n"; // 8
std::cout << a << "\n"; // 9
```


In-/Decrement Operators

Is the expression

`++expr;` ← we favour this

equivalent to

`expr++;`?

Yes, but

- Pre-increment can be more efficient (old value does not need to be saved)
- Post In-/Decrement are the only left-associative unary operators (not very intuitive)

C++ vs. ++C

Strictly speaking our language should be named ++C because

- it is an advancement of the language C
- while C++ returns the old C.

Arithmetic Assignments

`a += b`

⇔

`a = a + b`

analogously for `-`, `*`, `/` and `%`

Arithmetic Assignments

	Gebrauch	Bedeutung
`+=`	`expr1 += expr2`	`expr1 = expr1 + expr2`
`-=`	`expr1 -= expr2`	`expr1 = expr1 - expr2`
`*=`	`expr1 *= expr2`	`expr1 = expr1 * expr2`
`/=`	`expr1 /= expr2`	`expr1 = expr1 / expr2`
`%=`	`expr1 %= expr2`	`expr1 = expr1 % expr2`

Arithmetic expressions evaluate expr1 only once.
Assignments have precedence 4 and are right-associative.

Binary Number Representations

Binary representation ("Bits" from $\{0, 1\}$)

$$b_n b_{n-1} \dots b_1 b_0$$

corresponds to the number $b_n \cdot 2^n + \dots + b_1 \cdot 2 + b_0$

Example: 101011 corresponds to 43.

Least Significant Bit (LSB)

Most Significant Bit (MSB)

Binary Numbers: Numbers of the Computer?

Truth: Computers calculate using binary numbers.


NEUE ZÜRCHER ZEITUNG

TECHNIK

Das programmgesteuerte Rechengerät an der Eidgenössischen Technischen Hochschule in Zürich

Binary Numbers: Numbers of the Computer?

Stereotype: computers are talking 0/1 gibberish

Hexadecimal Numbers

Numbers with base 16

$$h_n h_{n-1} \dots h_1 h_0$$

corresponds to the number

$$h_n \cdot 16^n + \cdots + h_1 \cdot 16 + h_0.$$

notation in C++: prefix `0x`

Example: `0xff` corresponds to 255.

Hex Nibbles

hex	bin	dec
0	0000	0
1	**0001**	**1**
2	**0010**	**2**
3	0011	3
4	**0100**	**4**
5	0101	5
6	0110	6
7	0111	7
8	**1000**	**8**
9	1001	9
a	1010	10
b	1011	11
c	1100	12
d	1101	13
e	1110	14
f	1111	15

Why Hexadecimal Numbers?

- A Hex-Nibble requires exactly 4 bits. Numbers 1, 2, 4 and 8 represent bits 0, 1, 2 and 3.
- „compact representation of binary numbers"

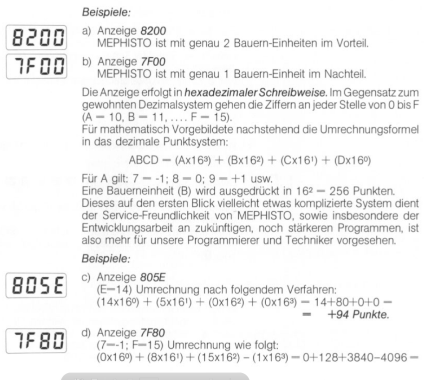
32-bit numbers consist of eight hex-nibbles: `0x00000000 -- 0xffffffff` .
`0x400` $= 1Ki = 1'024.$
`0x100000` $= 1Mi = 1'048'576.$
`0x40000000` $= 1Gi = 1'073.741,824.$
`0x80000000`: highest bit of a 32-bit number is set
`0xffffffff`: all bits of a 32-bit number are set
„`0x8a20aaf0` is an address in the upper 2G of the 32-bit address space"

Example: Hex-Colors

$$\#\underbrace{00}_{r}\underbrace{FF}_{g}\underbrace{00}_{b}$$

Why Hexadecimal Numbers?

"For programmers and technicians" (Excerpt of a user manual of the chess computers *Mephisto II*, 1981)



Beispiele:

8200 a) Anzeige *8200*
MEPHISTO ist mit genau 2 Bauern-Einheiten im Vorteil.

7F00 b) Anzeige *7F00*
MEPHISTO ist mit genau 1 Bauern-Einheit im Nachteil.

Die Anzeige erfolgt in ***hexadezimaler Schreibweise.*** Im Gegensatz zum gewohnten Dezimalsystem gehen die Ziffern an jeder Stelle von 0 bis F (A = 10, B = 11, ... F = 15).
Für mathematisch Vorgebildete nachstehend die Umrechnungsformel in das dezimale Punktsystem:

ABCD = (Ax16^3) + (Bx16^2) + (Cx16^1) + (Dx16^0)

Für A gilt: 7 = -1; 8 = 0; 9 = +1 usw.
Eine Bauerneinheit (B) wird ausgedrückt in 16^2 = 256 Punkten.
Dieses auf den ersten Blick vielleicht etwas komplizierte System dient der Service-Freundlichkeit von MEPHISTO, sowie insbesondere der Entwicklungsarbeit an zukünftigen, noch stärkeren Programmen, ist also mehr für unsere Programmierer und Techniker vorgesehen.

Beispiele:

805E c) Anzeige *805E*
(E=14) Umrechnung nach folgendem Verfahren:
(14x16^0) + (5x16^1) + (0x16^2) + (0x16^3) = 14+80+0+0 =
= *+94 Punkte.*

7F80 d) Anzeige *7F80*
(7=-1; F=15) Umrechnung wie folgt:
(0x16^0) + (8x16^1) + (15x16^2) – (1x16^3) = 0+128+3840–4096 =

http://www.zanchetta.net/default.aspx?Categorie=ECHIQUIERS&Page=documentations

Why Hexadecimal Numbers?

The NZZ could have saved a lot of space ...

Domain of the Type `int`

- Representation with B bits. Domain comprises the 2^B integers:

$$\{-2^{B-1}, -2^{B-1}+1, \ldots, -1, 0, 1, \ldots, 2^{B-1}-2, 2^{B-1}-1\}$$

- On most platforms $B = 32$
- For the type **`int`** C++ guarantees $B \geq 16$
- Background: Section 2.2.8 (Binary Representation) in the lecture notes.

Domain of Type `int`

```
// Program: limits.cpp
// Output the smallest and the largest value of type int.

#include <iostream>
#include <limits>

int main()
{
  std::cout << "Minimum int value is "
            << std::numeric_limits<int>::min() << ".\n"
            << "Maximum int value is "
            << std::numeric_limits<int>::max() << ".\n";
  return 0;
}
```

For example

```
Minimum int value is -2147483648.
Maximum int value is 2147483647.
```


Over- and Underflow

- Arithmetic operations (`+`,`-`,`*`) can lead to numbers outside the valid domain.
- Results can be incorrect!

`power8.cpp`: $15^8 = -1732076671$

`power20.cpp`: $3^{20} = -808182895$

- There is *no error message!*

The Type `unsigned int`

- Domain
$$\{0, 1, \ldots, 2^B - 1\}$$
- All arithmetic operations exist also for `unsigned int`.
- Literals: `1u, 17u` ...

Mixed Expressions

- Operators can have operands of different type (e.g. `int` and `unsigned int`).

```
17 + 17u
```

- Such mixed expressions are of the "more general" type `unsigned int`.
- `int`-operands are *converted* to `unsigned int`.

Conversion

`int` Value	Sign	`unsigned int` Value
x	≥ 0	x
x	< 0	$x + 2^B$

Using two complements representation, nothing happens internally

Conversion "reversed"

The declaration

```
int a = 3u;
```

converts `3u` to `int`.

The value is preserved because it is in the domain of `int`; otherwise the result depends on the implementation.

Signed Number Representation

- (Hopefully) clear by now: binary number representation without sign, e.g.

$$[b_{31}b_{30}\dots b_0]_u \quad \widehat{=} \quad b_{31}\cdot 2^{31} + b_{30}\cdot 2^{30} + \dots + b_0$$

- Obviously required: use a bit for the sign.
- Looking for a consistent solution

> The representation with sign should coincide with the unsigned solution as much as possible. Positive numbers should arithmetically be treated equal in both systems.