Lindenmayer Systems

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  1. Alphabet Σ - the allowed symbols
  2. Production P - how to replace each symbol
  3. Initial word s_0 - the word to start with

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- Example:
  1. $\Sigma := \{F, +, -\}$
  2. $P := \begin{cases} F \mapsto F + F + \\ + \mapsto + \\ - \mapsto - \end{cases}$
  3. $s_0 := F$

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- How does it look after 3 rounds?

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w_1:

w_2:

w_3:

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Exercise

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Compute the words w_0, w_1, w_2, w_3 for

1. $\Sigma := \{a, b, c\}$
2. $P := \begin{cases} a \mapsto a \\ b \mapsto ac \\ c \mapsto ab \end{cases}$
3. $s_0 := b$

Exercise

Solution:

w_0: b

w_1: ac

w_2: aab

w_3: aaac

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Draw Lindenmayer Systems

Two Step Procedure

- Goal: Draw n-th step of Lindenmayer system

- Done in 2 steps
  1. Obtain n-th step
  2. Draw it

Step 1 – Obtain n-th Word

- Write and use the following two functions
  - **`std::string production (const char c)`**
    - In: symbol e.g. `F`
    - Out: its production e.g. `F+F+`

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  - **`std::string production (const char c)`**
    - In: symbol e.g. `F`
    - Out: its production e.g. `F+F+`
  - **`std::string next_word (const std::string word)`**
    - In: w_n (Word of step `n`) e.g. `FF`
    - Out: w_{n+1} (Word of step `n+1`) e.g. `F+F+F+F+`
    - Applies `production` to each character in w_n and concatenates the results.

Step 2 – Draw It

- Idea: view alphabet as turtle commands

- Example:

Alphabet: $\Sigma := \{F, +, -\}$

F	`turtle::forward()`
$+$	`turtle::left(90)`
$-$	`turtle::right(90)`