

Informatik II

Übung 3

FS 2020

Program of Today

- 1 Repetition Theory
 - Problems, Algorithms, Programs
 - Asymptotic Running Time
- 2 In-class Exercise (Code-Expert)

1. Repetition Theory

Warm-up

- What is a problem?

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- What is an algorithm?

Warm-up

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- What is an algorithm?
 - well-defined computing procedure to compute output data from input data.

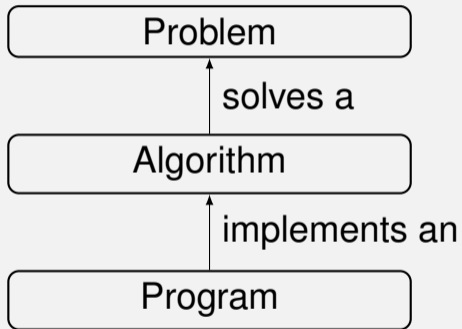
Warm-up

- What is a problem?
- What is an algorithm?
 - well-defined computing procedure to compute output data from input data.
- What is a program?

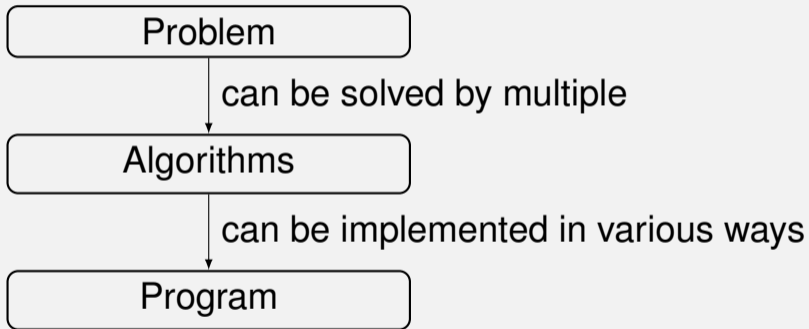
Warm-up

- What is a problem?
- What is an algorithm?
 - well-defined computing procedure to compute output data from input data.
- What is a program?
 - Concrete implementation of an algorithm

Warm-up



Warm-up



Efficiency

Problem	Complexity	Minimal (asymptotic) cost over all algorithms that solve the problem.
Algorithm	Cost	Number of elementary operations
Program	Computing time	Measurable value on an actual machine.

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- Estimation of cost or computing time depending on the input size, denoted by n .

Asymptotic behavior

- What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

Asymptotic behavior

- What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?
- Sets of functions!

Asymptotic behavior

■ What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

→ Sets of functions!

Repetition, sets A, B :

subset $A \subseteq B$

proper subset $A \subsetneq B$

intersection $A \cap B$

Asymptotic behavior

Given: function $f : \mathbb{N} \rightarrow \mathbb{R}$.

Definition:

$$\mathcal{O}(g) = \{f : \mathbb{N} \rightarrow \mathbb{R}^+ \mid \exists c > 0, n_0 \in \mathbb{N} \mid \forall n \geq n_0 : f(n) \leq c \cdot g(n)\}$$

$$\Omega(g) = \{f : \mathbb{N} \rightarrow \mathbb{R}^+ \mid \exists c > 0, n_0 \in \mathbb{N} \mid \forall n \geq n_0 : c \cdot g(n) \leq f(n)\}$$

$$\Theta(g) = \mathcal{O}(g) \cap \Omega(g)$$

Useful information for the exercise

Theorem

- 1 $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f \in \mathcal{O}(g), \mathcal{O}(f) \subsetneq \mathcal{O}(g).$
- 2 $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = C > 0$ (C constant) $\Rightarrow f \in \Theta(g).$
- 3 $\frac{f(n)}{g(n)} \xrightarrow[n \rightarrow \infty]{} \infty \Rightarrow g \in \mathcal{O}(f), \mathcal{O}(g) \subsetneq \mathcal{O}(f).$

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Beispiel

- 1 $\lim_{n \rightarrow \infty} \frac{n}{n^2} = 0 \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2).$
- 2 $\lim_{n \rightarrow \infty} \frac{2n}{n} = 2 > 0 \Rightarrow 2n \in \Theta(n).$
- 3 $\frac{n^2}{n} \xrightarrow[n \rightarrow \infty]{} \infty \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2).$

Property

$$f_1 \in \mathcal{O}(g), f_2 \in \mathcal{O}(g) \Rightarrow f_1 + f_2 \in \mathcal{O}(g)$$

Examples

$$\mathcal{O}(g) = \{f : \mathbb{N} \rightarrow \mathbb{R} \mid \exists c > 0, \exists n_0 \in \mathbb{N} : \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$f(n)$	$f \in \mathcal{O}(?)$	Example
$3n + 4$		
$2n$		
$n^2 + 100n$		
$n + \sqrt{n}$		

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$n^2 + 100n$	$\mathcal{O}(n^2)$	$c = 2, n_0 = 100$
$n + \sqrt{n}$	$\mathcal{O}(n)$	$c = 2, n_0 = 1$

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- $\mathcal{O}(n) \subseteq \mathcal{O}(n^2)$ is correct
- $\Theta(n) \subseteq \Theta(n^2)$ is wrong $n \notin \Omega(n^2) \supset \Theta(n^2)$

Quiz

$1 \in \mathcal{O}(15)$?

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$1 \in \mathcal{O}(15)$? ✓ better $1 \in \mathcal{O}(1)$

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Quiz: A good strategy?

... Then I simply buy a new machine

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... Then I simply buy a new machine If today I can solve a problem of size n , then with a 10 or 100 times faster machine I can solve ...¹

Komplexität	(speed $\times 10$)	(speed $\times 100$)
-------------	----------------------	-----------------------

$\log_2 n$		
------------	--	--

n		
-----	--	--

n^2		
-------	--	--

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n^2	$n \rightarrow 3.16 \cdot n$	$n \rightarrow 10 \cdot n$
2^n	$n \rightarrow n + 3.32$	$n \rightarrow n + 6.64$

¹To see this, you set $f(n') = c \cdot f(n)$ ($c = 10$ or $c = 100$) and solve for n'

Asymptotic Running Times with Θ

```
void run(int n){  
    for (int i = 1; i<n; ++i)  
        for (int j = 1; j<n; ++j)  
            op();  
}
```

How often is `op()` called?

Asymptotic Running Times with Θ

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Asymptotic Running Times with Θ

```
void run(int n){  
    for(int i = 1; i <= n; ++i)  
        for(int j = 1; j*j <= n; ++j)  
            for(int k = n; k >= 2; --k)  
                op();  
}
```

How often is `op()` called?

2. In-class Exercise (Code-Expert)

Crucial preparation for the next large topic