Informatik II

Übung 2

FS 2020

Program Today

1. Feedback of last exercise
2. Python
3. Preparation Theory

1. Feedback of last exercise

2. Python

In-Class Exercises

3. Preparation Theory

Required for next weeks lectures

Sums

$$\sum_{i=0}^{n} i = ?$$

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

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Why?

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

Why?

Intuition

$$1 + ... + 100 = (1 + 100) + (2 + 99) + (3 + 98) + ... + (50 + 51)$$

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

Why?

Intuition

$$1 + ... + 100 = (1 + 100) + (2 + 99) + (3 + 98) + ... + (50 + 51)$$

More formally?

Sums

$$\sum_{i=0}^{n}(n-i) = ?$$

Sums

$$\sum_{i=0}^{n}(n-i)=\sum_{i=0}^{n} i$$

Sums

$$\sum_{i=0}^{n}(n-i) = \sum_{i=0}^{n} i$$

$$\Rightarrow 2 \cdot \sum_{i=0}^{n} i = \sum_{i=0}^{n} i + \sum_{i=0}^{n}(n-i)$$

$$= \sum_{i=0}^{n}(i + (n-i)) = \sum_{i=0}^{n} n = (n+1) \cdot n$$

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Sums

$$\sum_{i=0}^{n} i^2 = ?$$

Sums

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Sums

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+2)}{2}$$

This you do not need to know by heart. But you should know that it is a polynome of third degree.

[Sums]

How do you derive something like this?

[Sums]

How do you derive something like this? Interesting Trick: On the one hand

$$\sum_{i=0}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 = \sum_{i=0}^{n} i^3 - \sum_{i=0}^{n-1} i^3 = n^3,$$

[Sums]

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on the other hand

$$\begin{aligned}\sum_{i=0}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 &= \sum_{i=1}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 \\ &= \sum_{i=1}^{n} i^3 - (i-1)^3 = \sum_{i=1}^{n} 3 \cdot i^2 - 3 \cdot i + 1\end{aligned}$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = ?$$

$$\frac{a^x}{a^y} = ?$$

$$a^{x \cdot y} = ?$$

$$\log_b x = ?$$

$$\log_a (x \cdot y) = ?$$

$$\log_a \frac{x}{y} = ?$$

$$\log_a x^y = ?$$

$$\log_a n! = ?$$

$$a^{\log_b x} = ?$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} =?$$

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Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = a^{x+y} \qquad \log_a (x \cdot y) = \log_a x + \log_a y$$

$$\frac{a^x}{a^y} = a^{x-y} \qquad \log_a \frac{x}{y} = \log_a x - \log_a y$$

$$a^{x \cdot y} = (a^x)^y \qquad \log_a x^y = ?$$

$$\log_a n! = ?$$

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$$\log_a n! = \sum_{i=1}^{n} \log i$$

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$$\log_b x = \log_b a \cdot \log_a x$$

$$a^{\log_b x} = x^{\log_b a}$$

To see the last line, replace $x \to a^{\log_a x}$

Comparisons

$$\frac{n^2}{2^n} \underset{n \to \infty}{\longrightarrow} ?$$

Comparisons

$$\frac{n^2}{2^n} \underset{n\to\infty}{\longrightarrow} 0$$

Comparisons

$$\frac{n^{10000}}{2^n} \underset{n\to\infty}{\longrightarrow} ?$$

Comparisons

$$\frac{n^{10000}}{2^n} \xrightarrow[n\to\infty]{} 0$$

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \xrightarrow[n \to \infty]{} ?$$

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \underset{n\to\infty}{\longrightarrow} 0$$

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \xrightarrow[n \to \infty]{} 0$$

because

$$\frac{n^c}{d^n} = \frac{2^{\log_2 n^c}}{2^{\log_2 d^n}} = 2^{c \cdot \log_2 n - n \log_2 d}$$

Comparisons

$$\frac{n}{\log n} \xrightarrow[n \to \infty]{} ?$$

Comparisons

$$\frac{n}{\log n} \xrightarrow[n\to\infty]{} \infty$$

Comparisons

$$\frac{n \log n}{\sqrt{n}} \underset{n \to \infty}{\longrightarrow} ?$$

Comparisons

$$\frac{n \log n}{\sqrt{n}} \xrightarrow[n \to \infty]{} \infty$$

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} ?$$

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$$

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$$

$$\begin{aligned}
\log_2 n^2 &= 2\log_2 n \\
\sqrt{n} &= n^{1/2} = 2^{\log_2 n^{1/2}} = \left(\sqrt{2}\right)^{\log_2 n} \\
\frac{\log n^2}{\sqrt{n}} &= 2\frac{\log_2 n}{\left(\sqrt{2}\right)^{\log_2 n}}
\end{aligned}$$

which behaves because of $\log_2 n \to \infty$ for $n \to \infty$ like $2\frac{n}{\left(\sqrt{2}\right)^n}$

Questions / Suggestions?