Informatik II

Übung 6

FS 2019

Program Today

1. Repetition Lectures
2. String-Hashing and Computing with Modulo
3. In-Class-Exercises: Sliding Window

Hashing well-done

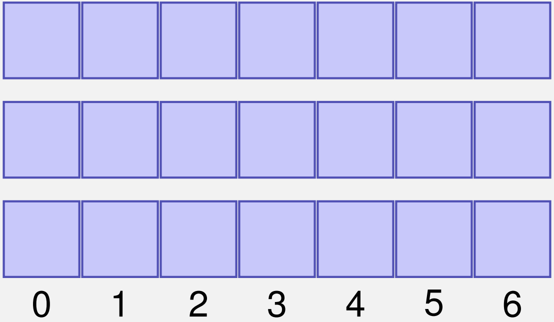
Useful Hashing. . .

- distributes the keys as uniformly as possible in the hash table.
- avoids probing over long areas of used entries (e.g. primary clustering).
- avoids using the same probing sequence for keys with the same hash value (e.g. secondary clustering).

Hashing Examples

Insert the keys $25, 4, 17, 45$ into the hash table, using the function $h(k) = k \bmod 7$ and probing to the right, $h(k) + s(j, k)$:

- linear probing,
  $s(j, k) = j$.
- quadratic probing,
  $s(j, k) = (-1)^{j+1}\lceil j/2 \rceil^2$.
- Double Hashing,
  $s(j, k) = j \cdot (1 + (k \bmod 5))$.



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				25		
0	1	2	3	4	5	6

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Linear				17	25	4	45
Quadratic			45	17	25	4	
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		45	17	25	4	
		4	17	25	45	

Computing with Modulo

$$
\begin{aligned}
(a+b) \bmod m &= ((a \bmod m) + (b \bmod m)) \bmod m \\
(a-b) \bmod m &= ((a \bmod m) - (b \bmod m) + m) \bmod m \\
(a \cdot b) \bmod m &= ((a \bmod m) \cdot (b \bmod m)) \bmod m
\end{aligned}
$$

Exercise: Compute

$$12746357 \bmod 11$$

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\begin{aligned}
&12746357 \bmod 11\\
&= (7 + 5 \cdot 10 + 3 \cdot 10^2 + 6 \cdot 10^3 + 4 \cdot 10^4 + 7 \cdot 10^5 + 2 \cdot 10^6 + 1 \cdot 10^7) \bmod 11
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&= (7 + 50 + 3 + 60 + 4 + 70 + 2 + 10) \bmod 11
\end{aligned}
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For the second equality we used the fact that $10^2 \bmod 11 = 1$.

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&= (7 + 50 + 3 + 60 + 4 + 70 + 2 + 10) \bmod 11 \\
&= (7 + 6 + 3 + 5 + 4 + 4 + 2 + 10) \bmod 11 \\
&= 8 \bmod 11.
\end{aligned}
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Implementation Hash(String) in Java

$$h_{c,m}(s) = \left(\sum_{i=0}^{k-1} s_i \cdot c^{k-1-i} \right) \bmod m$$

```
int ComputeHash(int C, int M, String s) {
  int hash = 0;
  for (int i = 0; i < s.length(); ++i){
    hash = (C * hash % M + s.charAt(i)) % M;
  }
  return hash;
}
```

In-Class-Exercises: Sliding Window

`https://expert.ethz.ch/print/ifbaug2/SS19/e06_examples`

Given a String `text` of length n, we want to find the shortest substring `text`$[l, r]$, which contains each of the characters ‘a’, ‘b’ and ‘c’ at least once.

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- Idea: Consider a fixed substring `text`$[l, r]$:
  - If it is missing some characters $\rightarrow$ increase substring length.
  - If it contains all 3 characters $\rightarrow$ decrease substring length.

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- Sliding Window Approach.

In-Class-Exercises: Sliding Window

Sliding Window Approach:

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Sliding Window Approach:

Time: $\mathcal{O}(n)$.

- In each step we enlarge the sliding window to the right or decrease it on the left. Hence there can be at most $2n$ steps.
- We hash a constant number of characters, hence HashMap operations will take time $\mathcal{O}(1)$.

Comparison to Rabin-Karp Exercise

Rabin-Karp: We are looking for a specific Substring “abc”, and not just its individual Characters ‘a’, ‘b’, ‘c’!

- Easier, since our Sliding Window always has the same length!
- But at the same time more difficult, since the order of the characters matters!

Questions / Suggestions?