Informatik II

Übung 12

FS 2019

Program Today

1. Feedback of last exercise
2. Repetition of Lecture
3. In-Class-Exercise (practical)

1. Feedback of last exercise

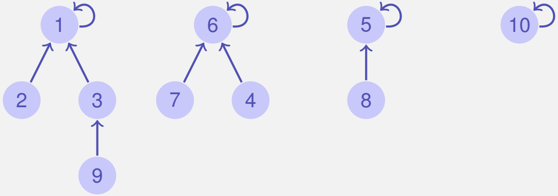
2. Repetition of Lecture

Union-Find Algorithm MST-Kruskal(G)

Input: Weighted Graph $G = (V, E, c)$
Output: Minimum spanning tree with edges A.

```
Sort edges by weight c(e_1) ≤ ... ≤ c(e_m)
A ← ∅
for k = 1 to |V| do
    MakeSet(k)
for k = 1 to m do
    (u, v) ← e_k
    if Find(u) ≠ Find(v) then
        Union(Find(u), Find(v))
        A ← A ∪ e_k
    else                                  // conceptual: R ← R ∪ e_k
return (V, A, c)
```

Implementation Union-Find



Representation as array:

Index	1	2	3	4	5	6	7	8	9	10
Parent	**1**	1	1	6	**5**	**6**	5	5	3	**10**

Implementation Union-Find

Index	1	2	3	4	5	6	7	8	9	10
Parent	**1**	1	1	6	**5**	**6**	5	5	3	**10**

Make-Set(i)	$p[i] \leftarrow i$; **return** i
Find(i)	**while** $(p[i] \neq i)$ **do** $i \leftarrow p[i]$ **return** i
Union(i, j) [1]	$p[j] \leftarrow i$;

[1] i and j need to be names (roots) of the sets. Otherwise use Union(Find(i),Find(j))

Optimisation of the runtime for Find

Tree may degenerate. Example: Union$(8, 7)$, Union$(7, 6)$, Union$(6, 5)$, ...

Index	1	2	3	4	5	6	7	8	..
Parent	1	1	2	3	4	5	6	7	..

Worst-case running time of Find in $\Theta(n)$.

Optimisation of the runtime for Find

Idea: always append smaller tree to larger tree. Requires additional size information (array) g

Make-Set(i)	$p[i] \leftarrow i$; $g[i] \leftarrow 1$; **return** i
Union(i, j)	**if** $g[j] > g[i]$ **then** swap(i, j) $p[j] \leftarrow i$ **if** $g[i] = g[j]$ **then** $g[i] \leftarrow g[i] + 1$

$\Rightarrow$ Tree depth (and worst-case running time for Find) in $\Theta(\log n)$

Further improvement

Link all nodes to the root when Find is called.

Find(i):

$j \leftarrow i$
while $(p[i] \neq i)$ **do** $i \leftarrow p[i]$
while $(j \neq i)$ **do**
    $t \leftarrow j$
    $j \leftarrow p[j]$
    $p[t] \leftarrow i$
return i

Cost: amortised *nearly* constant (inverse of the Ackermann-function).[2]

[2]We do not go into details here.

Running time of Kruskal's Algorithm

- Sorting of the edges: $\Theta(|E| \log |E|) = \Theta(|E| \log |V|)$. [3]
- Initialisation of the Union-Find data structure $\Theta(|V|)$
- $|E| \times$ Union(Find(x),Find(y)): $\mathcal{O}(|E| \log |E|) = \mathcal{O}(|E| \log |V|)$.

Overal $\Theta(|E| \log |V|)$.

[3] because G is connected: $|V| \leq |E| \leq |V|^2$

3. In-Class-Exercise (practical)

Union-Find datastructure and its optimisations

Questions / Suggestions?