8. Fundamental Data Structures

Abstract data types stack, queue, implementation variants for linked lists, [Ottman/Widmayer, Kap. 1.5.1-1.5.2, Cormen et al, Kap. 10.1.-10.2]

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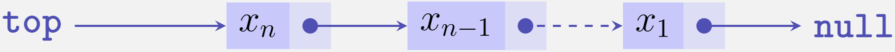
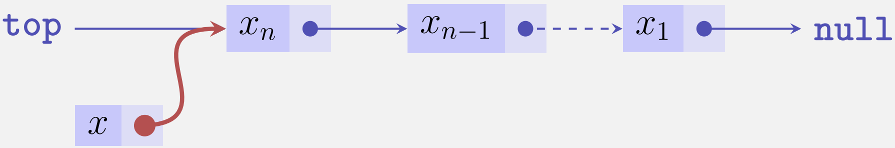
- **emptyStack**$()$: Returns an empty stack.

Implementation Push



push(x, S):

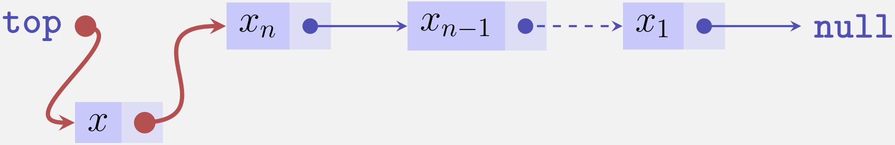
Implementation Push



`push`(x, S):

1. Create new list element with x and pointer to the value of `top`.

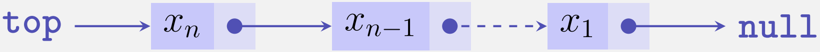
Implementation Push



push(x, S):

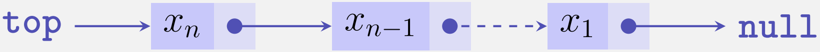
1. Create new list element with x and pointer to the value of **top**.
2. Assign the node with x to **top**.

Implementation Pop



$\texttt{pop}(S)$:

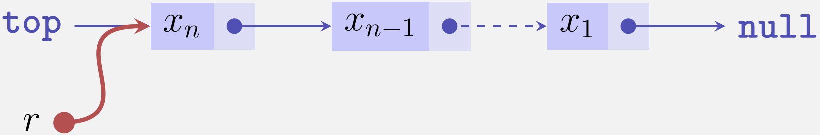
Implementation Pop



`pop`(S):

1. If `top`=`null`, then return `null`

Implementation Pop



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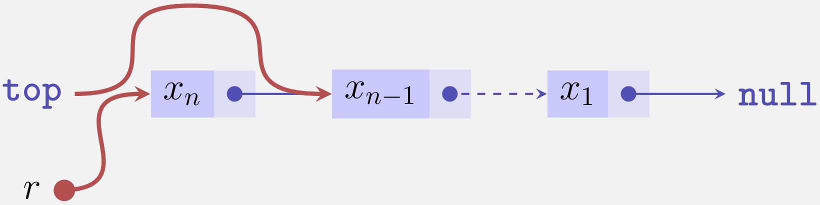

pop(S):

1. If top=null, then return null
2. otherwise memorize pointer p of top in r.
3. Set top to $p.next$ and return r

Analysis

Each of the operations `push`, `pop`, `top` and `isEmpty` on a stack can be executed in $\mathcal{O}(1)$ steps.

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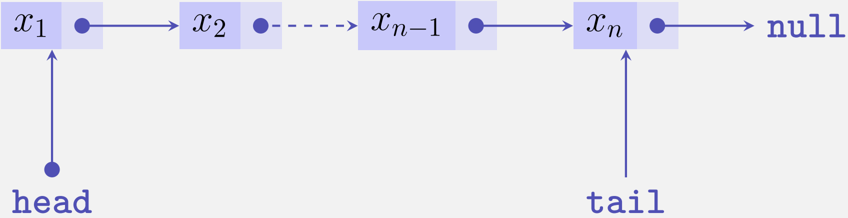
Implementation Queue



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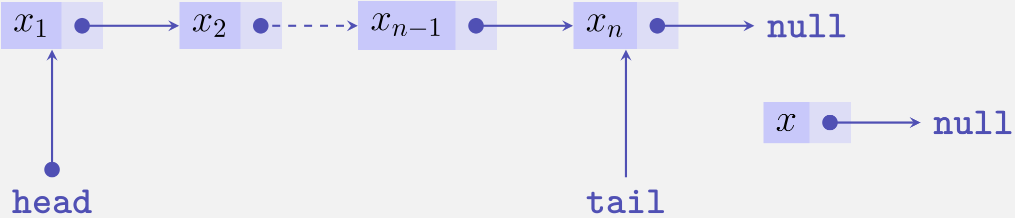

`enqueue`(x, S):

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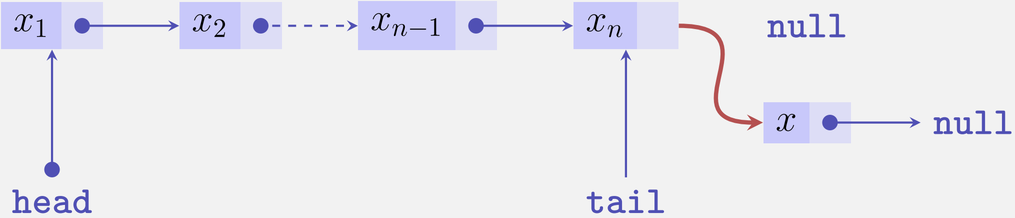
Implementation Queue



$\texttt{enqueue}(x, S)$:

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2. If `tail` $\neq$ `null`, then set `tail.next` to the node with x.

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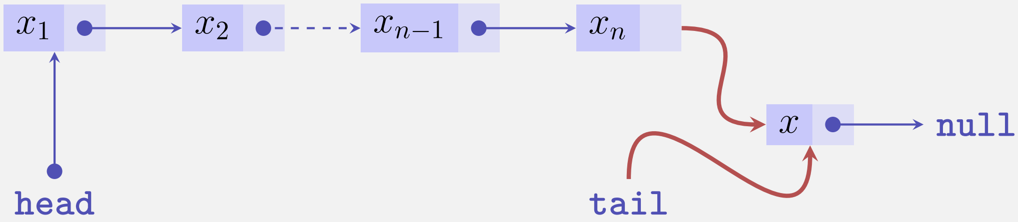

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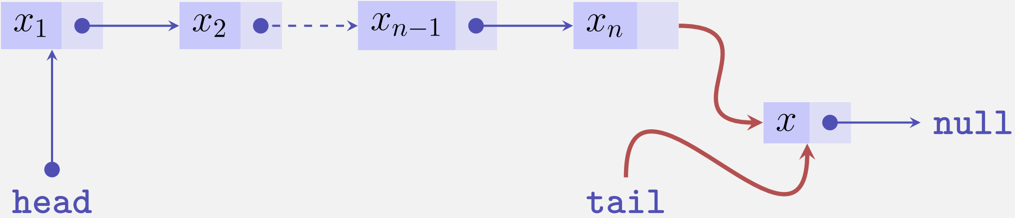

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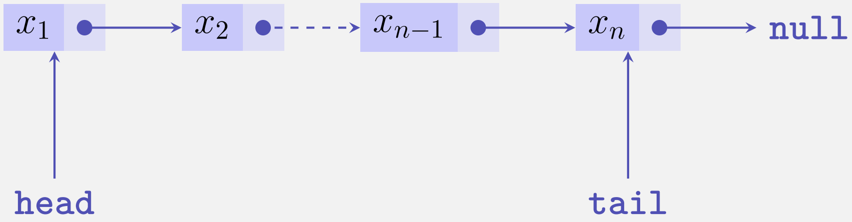
4. If **head** $=$ **null**, then set **head** to **tail**.

Invariants



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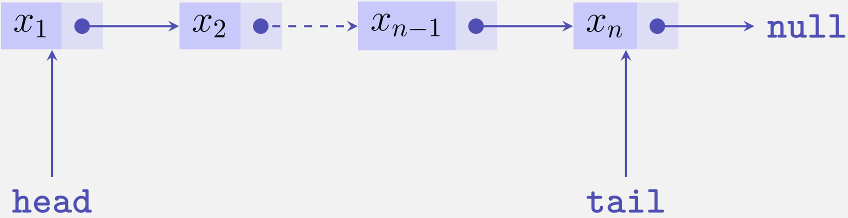

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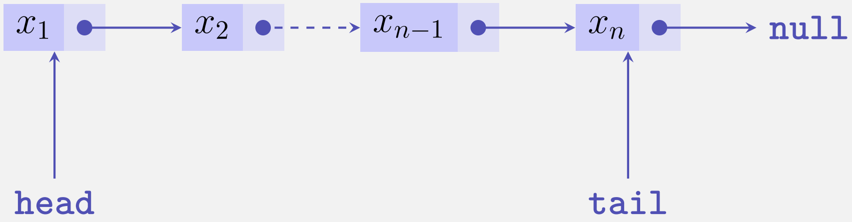

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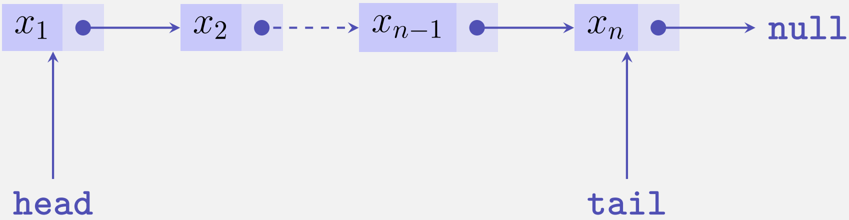

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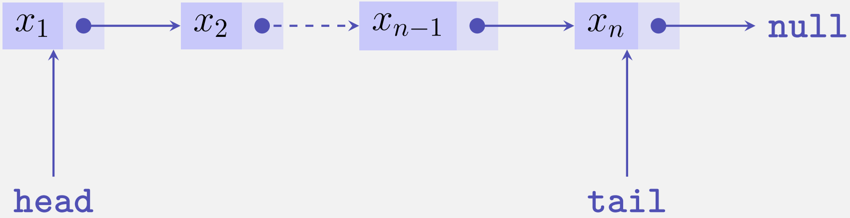

dequeue(S):

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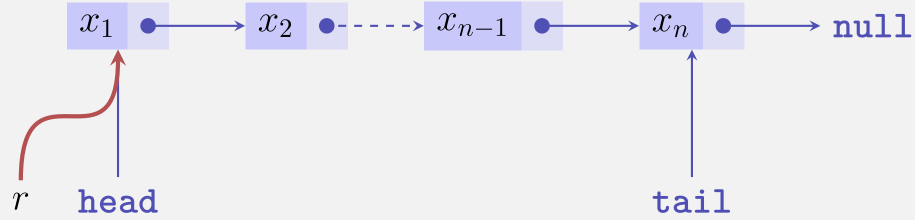

$\texttt{dequeue}(S)$:

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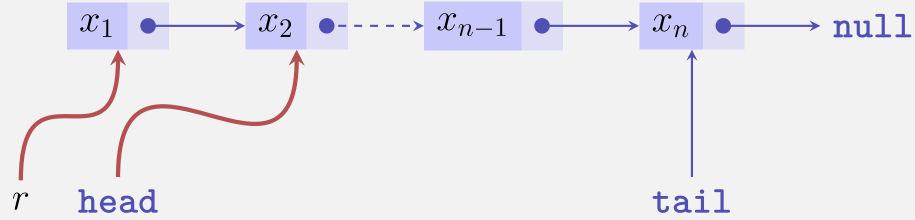

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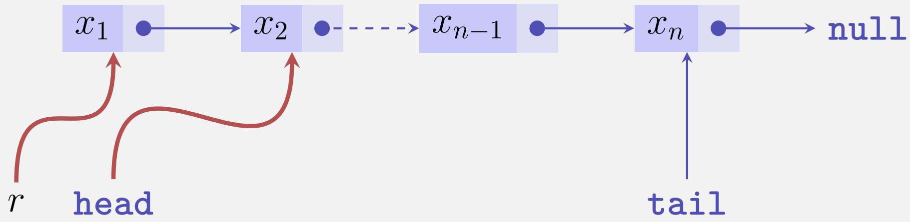

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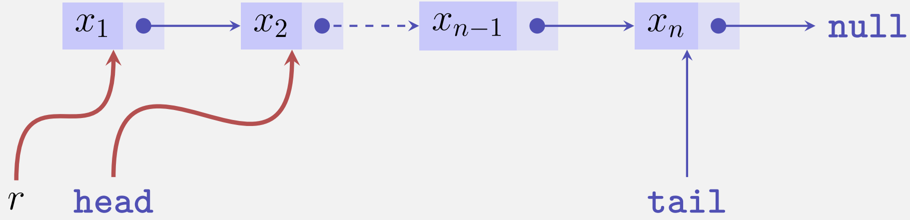

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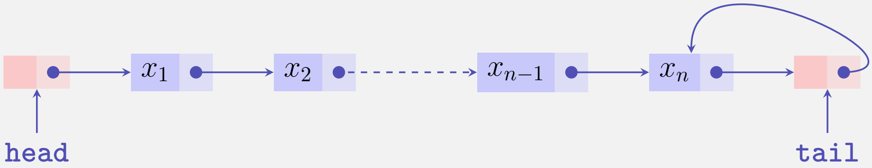
2. Set the pointer of head to head.next.
3. Is now $\texttt{head} = \texttt{null}$ then set tail to null.
4. Return the value of r.

Analysis

Each of the operations `enqueue`, `dequeue`, `head` and `isEmpty` on the queue can be executed in $\mathcal{O}(1)$ steps.

Implementation Variants of Linked Lists

List with dummy elements (sentinels).



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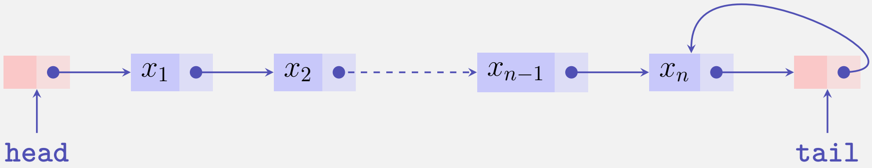

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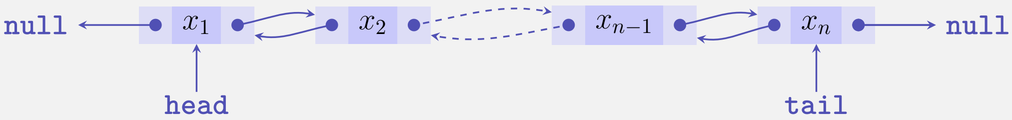
Variant: like this with pointer of an element stored singly indirect. (Example: pointer to x_3 points to x_2.)

Implementation Variants of Linked Lists

Doubly linked list



Overview

	enqueue	delete	search	concat
(A)	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
(B)	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$
(C)	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$	$\Theta(1)$
(D)	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$	$\Theta(1)$

(A) = singly linked
(B) = Singly linked with dummy element at the beginning and the end
(C) = Singly linked with indirect element addressing
(D) = doubly linked