Informatik II

Übung 7

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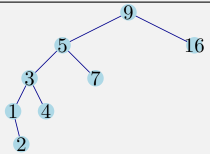
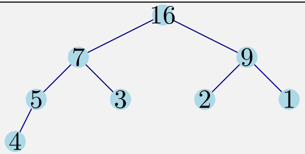
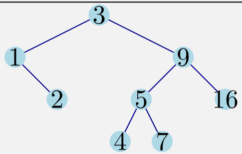
FS 2018

Program Today

1. Recap Binary Trees
2. Repetition Lectures
3. String-Hashing and Computing with Modulo
4. In-Class-Exercises: Sliding Window

Comparison of binary Trees

	Search trees	**Heaps** Min- / Max- Heap	**Balanced trees** AVL, red-black tree
in Java:		PriorityQueue	TreeSet
			
Insertion	$\mathcal{O}(h(T))$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
Search	$\mathcal{O}(h(T))$	$\mathcal{O}(n)$ (!!)	$\mathcal{O}(\log n)$
Deletion	$\mathcal{O}(h(T))$	Search + $\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

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	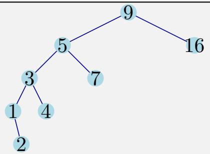	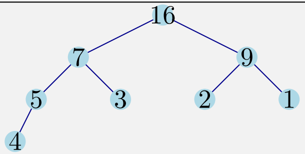	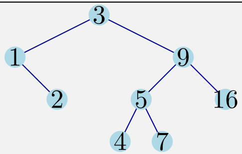
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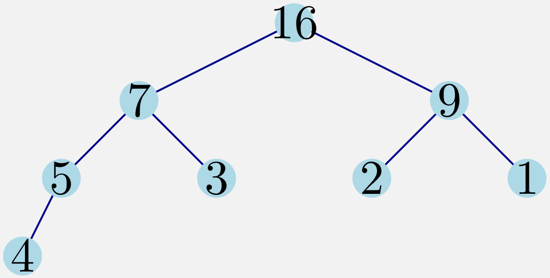
Recall: $\mathcal{O}(\log n) \ll \mathcal{O}(h(T)) \ll \mathcal{O}(n)$

Last week: Pre- / In- / Post- Order



Pre-order:

In-order:

Post-order:

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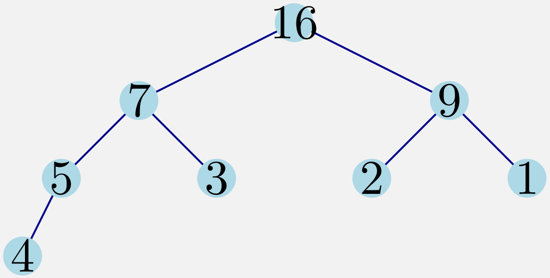

Pre-order: 16 7 5 4 3 9 2 1

In-order:

Post-order:

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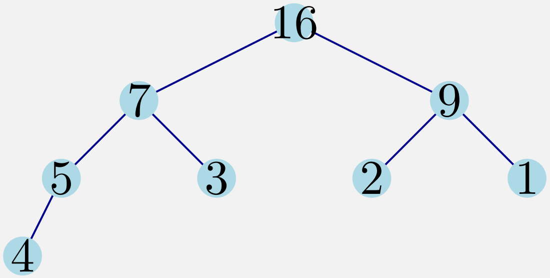

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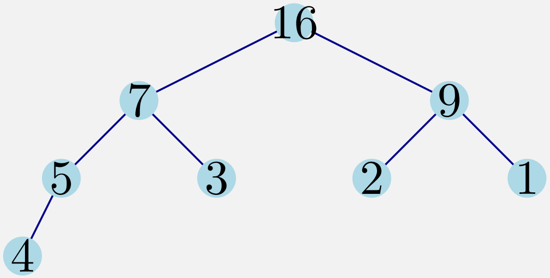

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In-order: 4 5 7 3 16 2 9 1

Post-order: 4 5 3 7 2 1 9 16

Hashing well-done

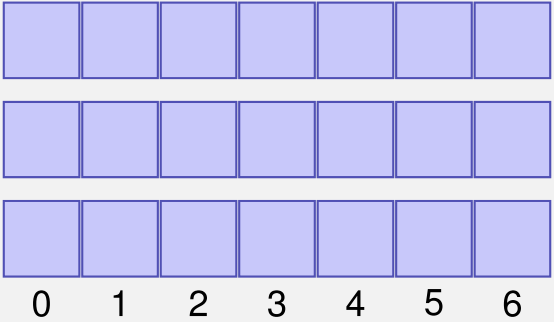
Useful Hashing. . .

- distributes the keys as uniformly as possible in the hash table.
- avoids probing over long areas of used entries (e.g. primary clustering).
- avoids using the same probing sequence for keys with the same hash value (e.g. secondary clustering).

Hashing Examples

Insert the keys $25, 4, 17, 45$ into the hash table, using the function $h(k) = k \bmod 7$ and probing to the right, $h(k) + s(j, k)$:

- linear probing,
  $s(j, k) = j$.
- quadratic probing,
  $s(j, k) = (-1)^{j+1} \lceil j/2 \rceil^2$.
- Double Hashing,
  $s(j, k) = j \cdot (1 + (k \bmod 5))$.



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				25		
0	1	2	3	4	5	6

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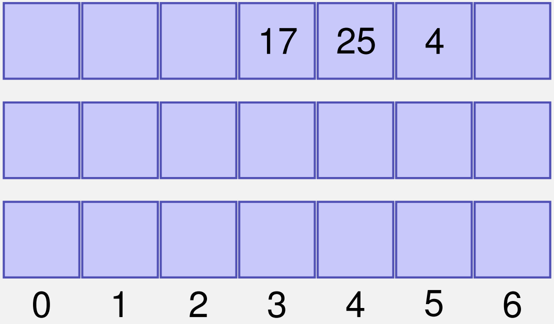

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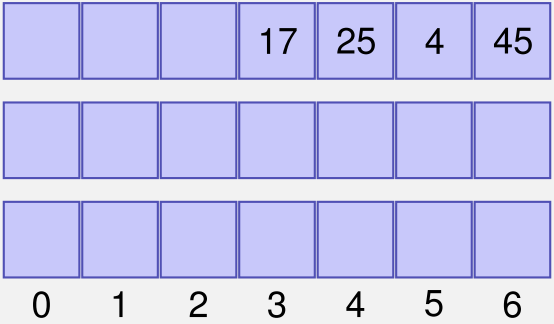

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Computing with Modulo

$$(a + b) \bmod m = ((a \bmod m) + (b \bmod m)) \bmod m$$
$$(a - b) \bmod m = ((a \bmod m) - (b \bmod m) + m) \bmod m$$
$$(a \cdot b) \bmod m = ((a \bmod m) \cdot (b \bmod m)) \bmod m$$

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$$12746357 \bmod 11$$

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\begin{aligned}
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&= (7 + 5 \cdot 10 + 3 \cdot 10^2 + 6 \cdot 10^3 + 4 \cdot 10^4 + 7 \cdot 10^5 + 2 \cdot 10^6 + 1 \cdot 10^7) \bmod 11 \\
&= (7 + 50 + 3 + 60 + 4 + 70 + 2 + 10) \bmod 11
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For the second equality we used the fact that $10^2 \bmod 11 = 1$.

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&= (7 + 50 + 3 + 60 + 4 + 70 + 2 + 10) \bmod 11 \\
&= (7 + 6 + 3 + 5 + 4 + 4 + 2 + 10) \bmod 11 \\
&= 8 \bmod 11.
\end{aligned}
$$

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Implementation Hash(String) in Java

$$h_{c,m}(s) = \left(\sum_{i=0}^{k-1} s_i \cdot c^{k-1-i} \right) \bmod m$$

```
int ComputeHash(int C, int M, String s) {
  int hash = 0;
  for (int i = 0; i < s.length(); ++i){
    hash = (C * hash % M + s.charAt(i)) % M;
  }
  return hash;
}
```

In-Class-Exercises: Sliding Window

Given a String `text` of length n, we want to find the shortest substring `text`$[l, r]$, which contains each of the characters ‘a’, ‘b’ and ‘c’ at least once.

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- Sliding Window Approach.

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Time: $\mathcal{O}(n)$.

- In each step we enlarge the sliding window to the right or decrease it on the left. Hence there can be at most $2n$ steps.
- We hash a constant number of characters, hence HashMap operations will take time $\mathcal{O}(1)$.

Comparison to Exercise 7.3

Exercise 7.3: We are looking for a specific Substring “abc”, and not just its individual Characters ‘a’, ‘b’, ‘c’!

- Easier, since our Sliding Window always has the same length!
- But at the same time more difficult, since the order of the characters matters!

Questions / Suggestions?