ETH zürich

Exercise Session 9

Data Structures and Algorithms, D-MATH, ETH Zurich

Program of today

Feedback of last exercise

Repetition Theory

- Activity Selection
- Huffman Coding
- Recursive Problem-Solving Strategies

In-Class-Exercise (practical)

Hints for current tasks

1. Feedback of last exercise

2. Repetition Theory

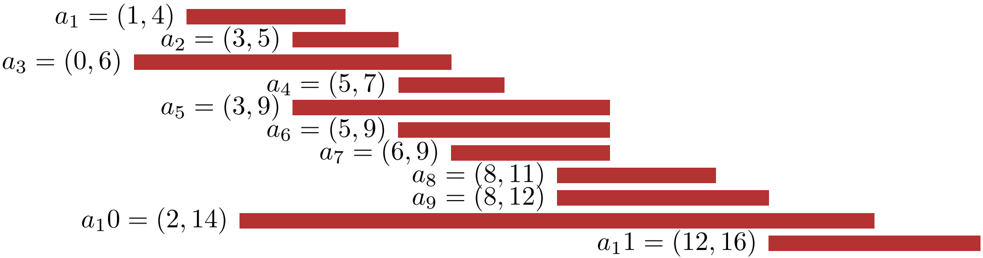
Greedy Choice

A problem with a recursive solution can be solved with a **greedy algorithm** if it has the following properties:

- The problem has **optimal substructure**: the solution of a problem can be constructed with a combination of solutions of sub-problems.
- The problem has the **greedy choice property**: The solution to a problem can be constructed, by using a local property that does not depend on the solution of the sub-problems.

Activity Selection

Coordination of activities that use a common resource exclusively.
Activities $S = \{a_1, a_2, \ldots, a_n\}$ with start- and finishing times $0 \leq s_i \leq f_i < \infty$, increasingly sorted by finishing times.



Activity Selection Problem: Find a maximal subset (maxium number of elements) of compatible (non-intersecting) activities.

Dynamic Programming Approach?

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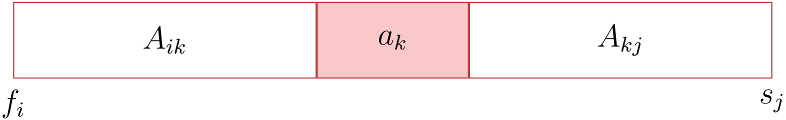
Let $a_k \in A_{ij}$ and $A_{ik} = S_{ik} \cap A_{ij}$, $A_{ki} = S_{kj} \cap A_{ij}$, thus $A_{ij} = A_{ik} + \{a_k\} + A_{kj}$.



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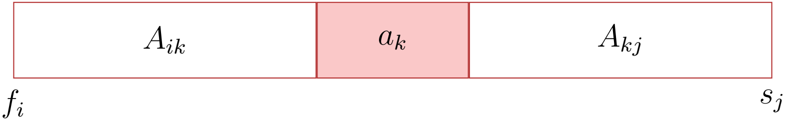

Straightforward: A_{ik} and A_{kj} must be maximal, otherwise
$A_{ij} = A_{ik} + \{a_k\} + A_{kj}$ would not be maximal.

Dynamic Programming Approach?

Let $c_{ij} = |A_{ij}|$.
Then the following recursion holds

$$c_{ij} = \begin{cases} 0 & \text{falls } S_{ij} = \emptyset, \\ \max_{a_k \in S_{ij}} \{c_{ik} + c_{kj} + 1\} & \text{falls } S_{ij} \neq \emptyset. \end{cases}$$

$\Rightarrow$ Dynamic programming.

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But there is a simpler alternative.

Greedy

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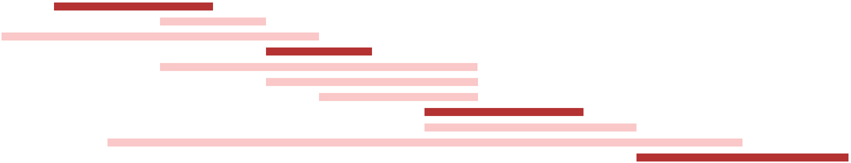


Greedy

Theorem 1

Given: The set of subproblem S_k, and an activity a_m from S_k with the earliest end time. Then a_m is contained in a maximal subset of compatible activities from S_k.

Let A_k be a maximal subset with compatible activities from S_k, and a_j be an activity from A_k with the earliest end time. If $a_j = a_m \Rightarrow$ done. If $a_j \neq a_m$, then consider $A'_k = A_k - \{a_j\} \cup \{a_m\}$. A'_k consists of compatible activities and is also maximal because $|A'_k| = |A_k|$.

$\blacksquare$

Algorithm RecursiveActivitySelect(s, f, k, n)

Input: Sequence of start and end points (s_i, f_i), $1 \leq i \leq n$, $s_i < f_i$, $f_i \leq f_{i+1}$ for all i. $1 \leq k \leq n$

Output: Set of all compatible activitivies.

$m \leftarrow k + 1$

while $m \leq n$ and $s_m \leq f_k$ **do**

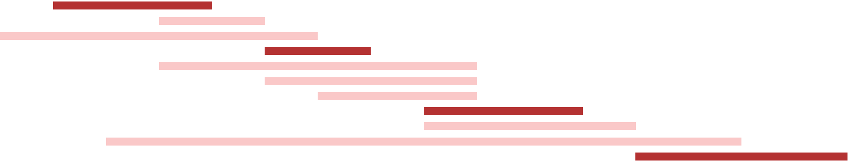
  $m \leftarrow m + 1$

if $m \leq n$ **then**

  return $\{a_m\} \cup$ RecursiveActivitySelect(s, f, m, n)

else

  return $\emptyset$



Algorithm IterativeActivitySelect(s, f, n)

Input: Sequence of start and end points (s_i, f_i), $1 \leq i \leq n$, $s_i < f_i$, $f_i \leq f_{i+1}$ for all i.

Output: Maximal set of compatible activities.

$A \leftarrow \{a_1\}$
$k \leftarrow 1$
for $m \leftarrow 2$ **to** n **do**
  if $s_m \geq f_k$ **then**
    $A \leftarrow A \cup \{a_m\}$
    $k \leftarrow m$
return A

Runtime of both algorithms:

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Runtime of both algorithms: $\Theta(n)$

Huffman's Idea

Tree construction bottom up

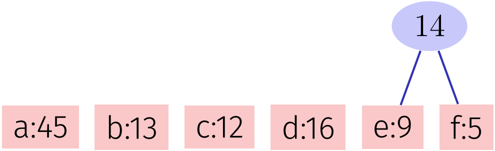
- Start with the set C of code words

a:45 b:13 c:12 d:16 e:9 f:5

Huffman's Idea

Tree construction bottom up

- Start with the set C of code words
- Replace iteriatively the two nodes with smallest frequency by a new parent node.



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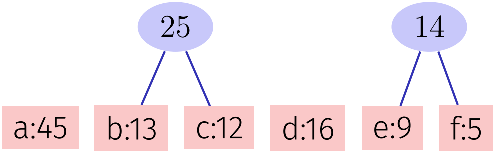

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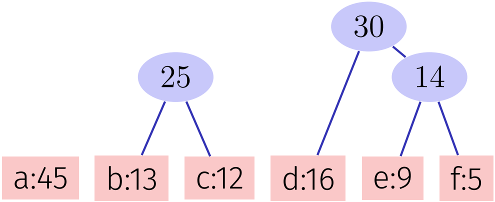

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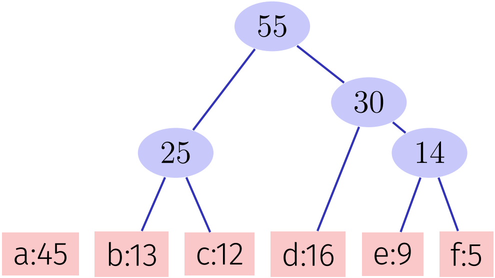

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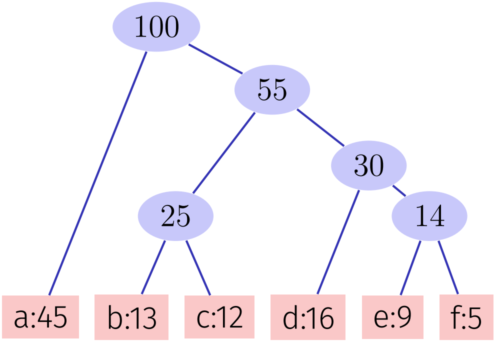

Algorithm Huffman(C)

Input: code words $c \in C$
Output: Root of an optimal code tree

$n \leftarrow |C|$
$Q \leftarrow C$
for $i = 1$ **to** $n - 1$ **do**
  allocate a new node z
  z.left $\leftarrow$ ExtractMin(Q) // extract word with minimal frequency.
  z.right $\leftarrow$ ExtractMin(Q)
  z.freq $\leftarrow$ z.left.freq $+$ z.right.freq
  Insert(Q, z)
return ExtractMin(Q)

Recursive Problem-Solving Strategies

Brute Force Enumeration	Backtracking	Divide and Conquer	Dynamic Programming	Greedy

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Recursive Enumerability	Constraint Satisfaction, Partial Validation	Optimal Substructure	Optimal Substructure, Overlapping Subproblems	Optimal Substructure, Greedy Choice Property

Recursive Problem-Solving Strategies

Brute Force Enumeration	**Backtracking**	**Divide and Conquer**	**Dynamic Programming**	**Greedy**
Recursive Enumerability	Constraint Satisfaction, Partial Validation	Optimal Substructure	Optimal Substructure, Overlapping Subproblems	Optimal Substructure, Greedy Choice Property
DFS, BFS, all Permutations, Tree Traversal	n-Queen, Sudoku, m-Coloring, SAT-Solving, naive TSP	Binary Search, Mergesort, Quicksort, Hanoi Towers, FFT	Bellman Ford, Warshall, Rod-Cutting, LAS, Editing Distance, Knapsack Problem DP	Dijkstra, Kruskal, Huffmann Coding

3. In-Class-Exercise (practical)

Complement the DP implementation to compute an optimal search tree. $\longrightarrow$ CodeExpert

4. Hints for current tasks

Huffman Coding

Huffman Code:

Use `std::map` (`#include <map>`)

```
std::map<std::string,int> observations;
// simple access to elements
++observations["cat"];
++observations["mouse"];
++observations["mouse"];

// a map is a collection of std::pair
// show all entries
for (auto x:observations){
  std::cout << "observations of " << x.first << ":" << x.second << std::enc
}
```

Huffman Code:

Use `std::priority_queue` **(**`#include <queue>`**)**

```
struct MyClass {
    int x;
    MyClass(int X): x{X} {};
};

struct compare{
    bool operator() (const MyClass& a, const MyClass& b){
        return a.x < b.x;
    }
};
//...
std::priority_queue<MyClass, std::vector<MyClass>, compare> q;
q.push(MyClass(10));
```

Huffman Code:

Use Smart Pointers `std::shared_ptr` **(**`#include <memory>`**)**

```
struct List {
    int value;
    std::shared_ptr<List> next;
    List(std::shared_ptr<List> n, int v): value{v}, next{n} {};
};
...
// automatic memory management, we do not need to care
std::shared_ptr<List> l = std::make_shared<List>(nullptr, 10);
l = std::make_shared<List>(l, 20);
while (l != nullptr){ // output: 20 10
  std::cout << l->value << std::endl;
  l = l->next;
}
```

Huffman Node

```
using SharedNode=std::shared_ptr<Node>;
struct Node{
    char value;
    int frequency;
    SharedNode left;
    SharedNode right;

    // constructor for leafs
    Node(char v, int f): value{v}, frequency{f},
      left{nullptr}, right{nullptr} {}
    // constructor for inner nodes
    Node(SharedNode l, SharedNode r): value{0},
      frequency{l->frequency + r->frequency}, left{l}, right{r} {};
};
```