Datenstrukturen und Algorithmen

Exercise 9

FS 2021

Program of today

1. Feedback of last exercises
2. Recap Theory

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2. Recap Theory

Quiz: Runtimes of simple Operations

Operation	Matrix	List
Find neighbours/successors of $v \in V$		
find $v \in V$ without neighbour/successor		
$(u, v) \in E$?		
Insert edge		
Delete edge		

Quiz: Runtimes of simple Operations

Operation	Matrix	List
Find neighbours/successors of $v \in V$	$\Theta(n)$	
find $v \in V$ without neighbour/successor		
$(u, v) \in E$?		
Insert edge		
Delete edge		

Quiz: Runtimes of simple Operations

Operation	Matrix	List
Find neighbours/successors of $v \in V$	$\Theta(n)$	$\Theta(\deg^+ v)$
find $v \in V$ without neighbour/successor		
$(u, v) \in E$?		
Insert edge		
Delete edge		

Quiz: Runtimes of simple Operations

Operation	Matrix	List
Find neighbours/successors of $v \in V$	$\Theta(n)$	$\Theta(\deg^+ v)$
find $v \in V$ without neighbour/successor	$\Theta(n^2)$	
$(u, v) \in E$?		
Insert edge		
Delete edge		

Quiz: Runtimes of simple Operations

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Find neighbours/successors of $v \in V$	$\Theta(n)$	$\Theta(\deg^+ v)$
find $v \in V$ without neighbour/successor	$\Theta(n^2)$	$\Theta(n)$
$(u, v) \in E$?		
Insert edge		
Delete edge		

Quiz: Runtimes of simple Operations

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Find neighbours/successors of $v \in V$	$\Theta(n)$	$\Theta(\deg^+ v)$
find $v \in V$ without neighbour/successor	$\Theta(n^2)$	$\Theta(n)$
$(u, v) \in E$?	$\Theta(1)$	
Insert edge		
Delete edge		

Quiz: Runtimes of simple Operations

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find $v \in V$ without neighbour/successor	$\Theta(n^2)$	$\Theta(n)$
$(u, v) \in E$?	$\Theta(1)$	$\Theta(\deg^+ v)$
Insert edge		
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Insert edge	$\Theta(1)$	
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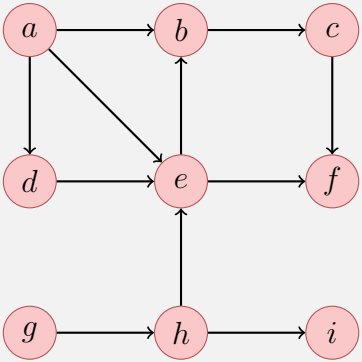
Insert edge	$\Theta(1)$	$\Theta(1)$
Delete edge	$\Theta(1)$	$\Theta(\deg^+ v)$

Breadth-First-Search BFS

BFS starting from a:

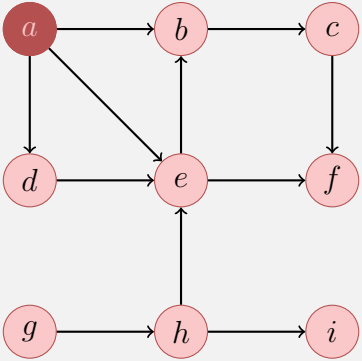


BFS-Tree: Distances and Parents

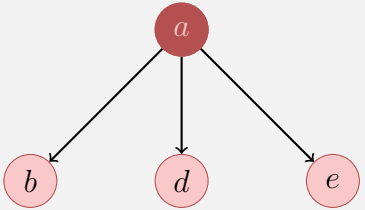
distance 0

Breadth-First-Search BFS

BFS starting from a:



BFS-Tree: Distances and Parents

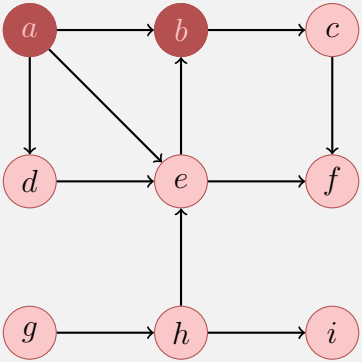


distance 0

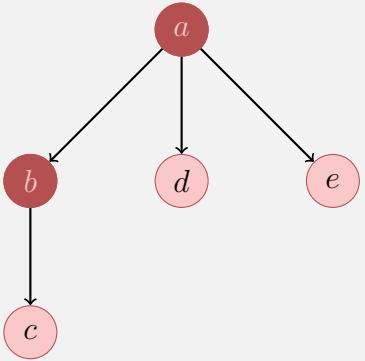
distance 1

Breadth-First-Search BFS

BFS starting from a:



BFS-Tree: Distances and Parents



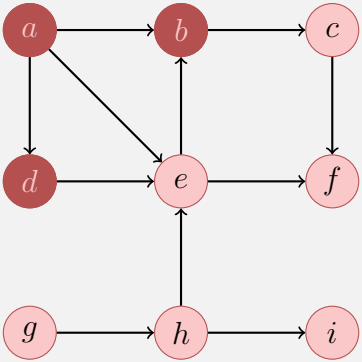
distance 0

distance 1

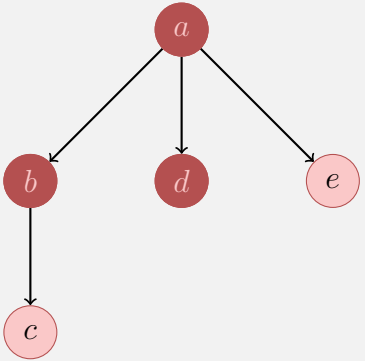
distance 2

Breadth-First-Search BFS

BFS starting from a:



BFS-Tree: Distances and Parents



distance 0

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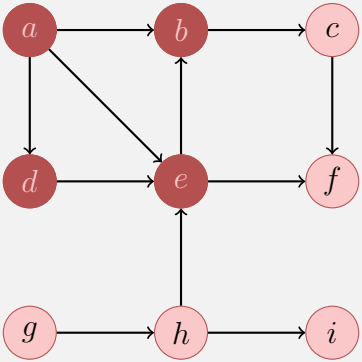
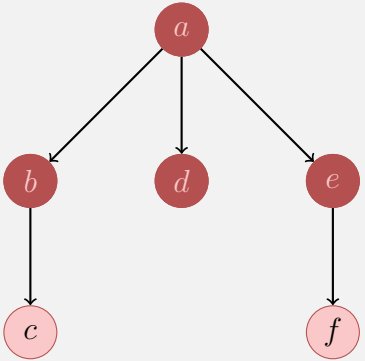
Breadth-First-Search BFS

BFS starting from a:



BFS-Tree: Distances and Parents



distance 0

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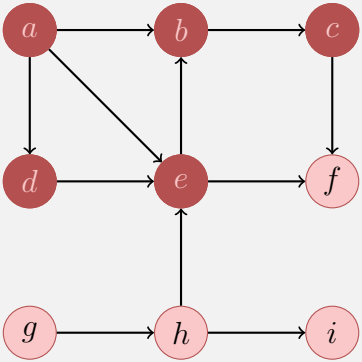
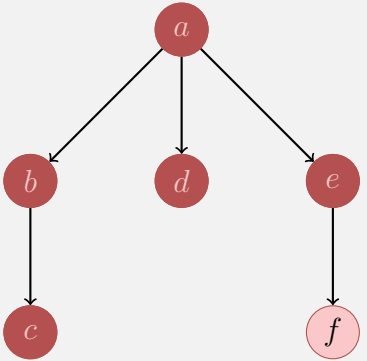
distance 2

Breadth-First-Search BFS

BFS starting from a:



BFS-Tree: Distances and Parents



distance 0

distance 1

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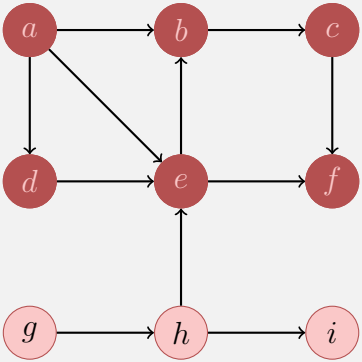
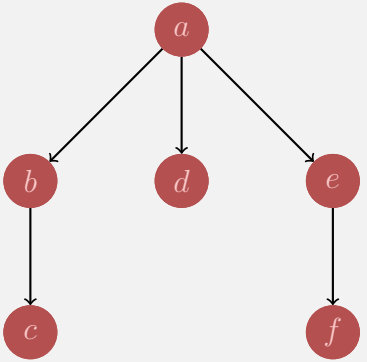
Breadth-First-Search BFS

BFS starting from a:



BFS-Tree: Distances and Parents



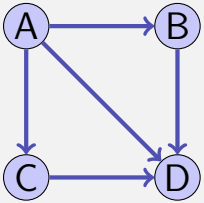
distance 0

distance 1

distance 2

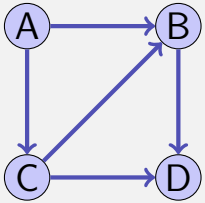
Quiz: Topological Sorting

In how many ways can the following directed graphs be topologically sorted each?



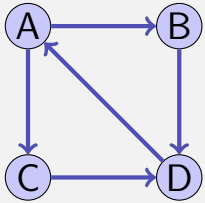
number sortings

?



number sortings

?



number sortings

?

Quiz: Topological Sorting

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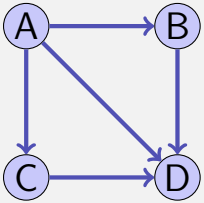
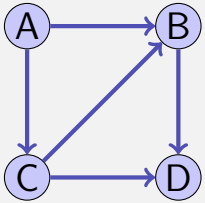


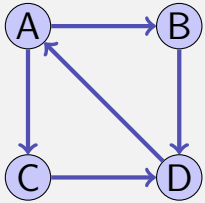
number sortings

2



number sortings

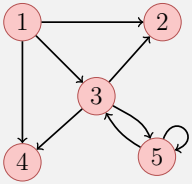
1



number sortings

0

Adjacency Matrix Product



$$B := A_G^2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}^2 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 \end{pmatrix}$$

Algorithm $A \cdot A$

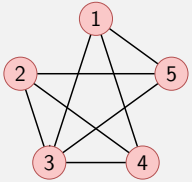
Input: (Adjacency-)Matrix $A = (a_{ij})_{i,j=1\ldots n}$
Output: Matrix Product $B = (b_{ij})_{i,j=1\ldots n} = A \cdot A$

$B \leftarrow 0$
for $r \leftarrow 1$ **to** n **do**
  for $c \leftarrow 1$ **to** n **do**
    for $k \leftarrow 1$ **to** n **do**
      $b_{rc} \leftarrow b_{rc} + a_{rk} \cdot a_{kc}$ // Number of Paths
return B

Counts number of paths of length 2

Quiz: Number triangles

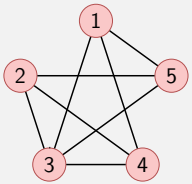
Question: How many triangular path does an undirected graph contain?



Quiz: Number triangles

Question: How many triangular path does an undirected graph contain?

Answer: Remove all cycles (diagonal entries). Compute A_G^3. $a_{ii}^{(3)}$ determines the number of paths of length 3 that contain i.

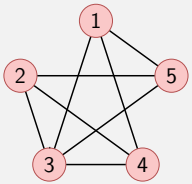


$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}^3 = \begin{pmatrix} 4 & 4 & 8 & 8 & 8 \\ 4 & 4 & 8 & 8 & 8 \\ 8 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 4 & 4 \\ 8 & 8 & 8 & 4 & 4 \end{pmatrix}$$

Quiz: Number triangles

Question: How many triangular path does an undirected graph contain?

Answer: Remove all cycles (diagonal entries). Compute A_G^3. $a_{ii}^{(3)}$ determines the number of paths of length 3 that contain i. There are 6 different permutations of a triangular path. Thus for the number of triangles: $\sum_{i=1}^{n} a_{ii}^{(3)}/6$.



$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}^3 = \begin{pmatrix} 4 & 4 & 8 & 8 & 8 \\ 4 & 4 & 8 & 8 & 8 \\ 8 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 4 & 4 \\ 8 & 8 & 8 & 4 & 4 \end{pmatrix}$$

$\Rightarrow 24/6 = 4$ Dreiecke.

Quiz: Shortest Path

Question: is there a path from i to j? How long is the shortest path (number edges)?

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Answer: exponentiate A_G until for some $k < n$ it holds that $a_{i,j}^{(k)} > 0$. k provides the path length of the shortest path. If $a_{i,j}^{(k)} = 0$ for all $1 \leq k < n$, then there is no path from i to j.

Questions?