

Datenstrukturen und Algorithmen

Exercise 12

FS 2021

Program of today

- 1 Feedback of last exercise
- 2 MaxFlow
- 3 C++ Threads
- 4 Two Quizzes

1. Feedback of last exercise

Exercise Union-Find

```
class UnionFind{
    std::vector<size_t> parents_;
public:
    UnionFind(size_t size) : parents_(size, size) { };

    size_t find(size_t index){
        while(parents_[index] != parents_.size())
            index = parents_[index];
        return index;
    }

    void unite(size_t a, size_t b){
        parents_[find(a)] = b;
    }
};
```

Union-Find with Map (Exercise Kruskal)

```
class UnionFind {
private:
    std::unordered_map<NodeP,NodeP> parent;
    std::unordered_map<NodeP,unsigned> depth;
public:
    void MakeSet(NodeP n){
        parent[n] = n; depth[n] = 0;
    }
    NodeP Find(NodeP n){
        while (parent[n] != n){
            n = parent[n];
        }
        return n;
    }
}
```

Optimizing Union

```
bool Union(NodeP l, NodeP r){
    l = Find(l);
    r = Find(r);
    if (l == r){
        return false;
    } else {
        if (depth[l] < depth[r])
            std::swap(l,r);
        parent[r] = l;
        if (depth[l] == depth[r])
            depth[l]++;
        return true;
    }
}
```

Alternative: optimizing Find

```
NodeP Find(NodeP n){
    NodeP root = n;
    while (parent[root] != root){
        root = parent[root];
    }
    while (parent[n] != root){
        auto next = parent[n];
        parent[n] = root;
        n = next;
    }
    return root;
}
```

no depth required

Kruskal

```
std::vector<Edge> result;
std::sort(edges.begin(), edges.end(),
    [](const Edge& l, const Edge& r) {return l.length < r.length; }
);

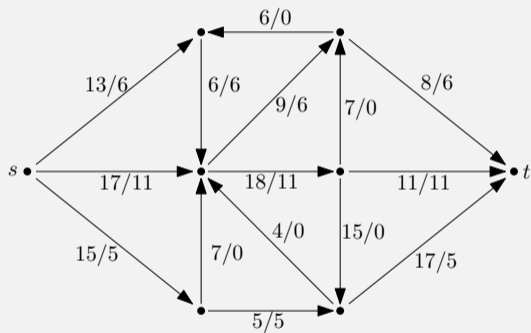
UnionFind uf;
for (auto n: nodes)
    uf.MakeSet(n);

for (const auto& e: edges){
    if (uf.Union(e.source, e.target)){
        result.push_back(e);
    }
}
return result;
```

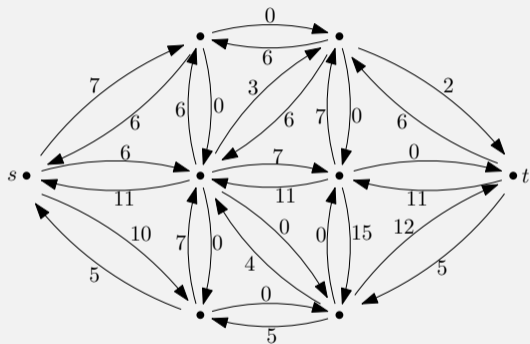
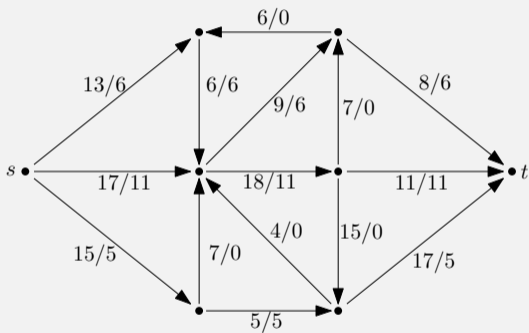

TSP

```
std::vector<Edge> Graph::TSP(){
    std::vector<Edge> mst = Kruskal();
    std::unordered_map<NodeP, std::vector<Edge>> adj;
    for (const auto& e: mst){
        adj[e.source].push_back(e);
        adj[e.target].emplace_back(e.target, e.source);
    }
    return DFS(adj, mst[0].source);
}
```

Exercise Manual Max-Flow



Exercise Manual Max-Flow



2. MaxFlow

Flow

A *Flow* $f : V \times V \rightarrow \mathbb{R}$ fulfills the following conditions:

- *Bounded Capacity:*

For all $u, v \in V$: $f(u, v) \leq c(u, v)$.

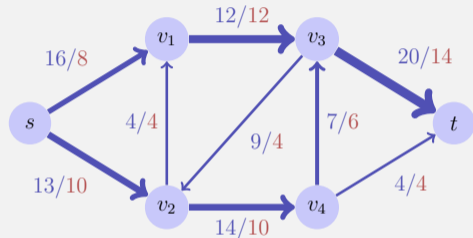
- *Skew Symmetry:*

For all $u, v \in V$: $f(u, v) = -f(v, u)$.

- *Conservation of flow:*

For all $u \in V \setminus \{s, t\}$:

$$\sum_{v \in V} f(u, v) = 0.$$



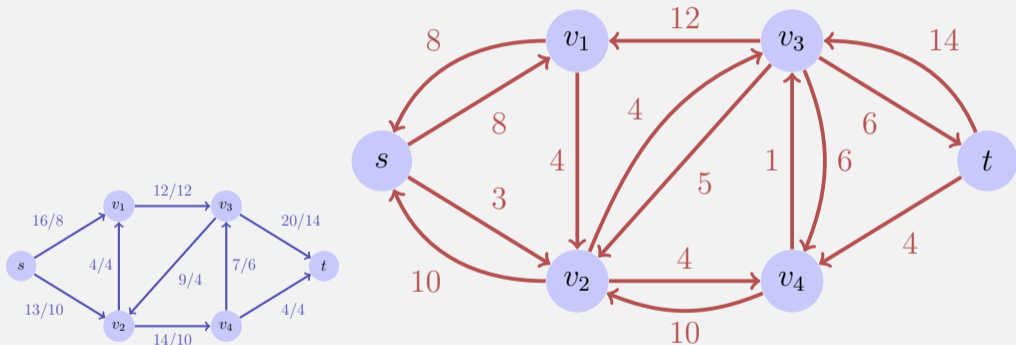
Value of the flow:

$$|f| = \sum_{v \in V} f(s, v).$$

Here $|f| = 18$.

Rest Network

Rest network G_f provided by the edges with positive rest capacity:



Rest networks provide the same kind of properties as flow networks with the exception of permitting antiparallel edges

Augmenting Paths

expansion path p : simple path from s to t in the rest network G_f .

Rest capacity $c_f(p) = \min\{c_f(u, v) : (u, v) \text{ edge in } p\}$

Max-Flow Min-Cut Theorem

Theorem

Let f be a flow in a flow network $G = (V, E, c)$ with source s and sink t . The following statements are equivalent:

- 1 f is a maximal flow in G*
- 2 The rest network G_f does not provide any expansion paths*
- 3 It holds that $|f| = c(S, T)$ for a cut (S, T) of G .*

Algorithm Ford-Fulkerson(G, s, t)

Input: Flow network $G = (V, E, c)$

Output: Maximal flow f .

for $(u, v) \in E$ **do**

$f(u, v) \leftarrow 0$

while Exists path $p : s \rightsquigarrow t$ in rest network G_f **do**

$c_f(p) \leftarrow \min\{c_f(u, v) : (u, v) \in p\}$

foreach $(u, v) \in p$ **do**

if $(u, v) \in E$ **then**

$f(u, v) \leftarrow f(u, v) + c_f(p)$

else

$f(v, u) \leftarrow f(v, u) + c_f(p)$

Edmonds-Karp Algorithm

Choose in the Ford-Fulkerson-Method for finding a path in G_f the expansion path of shortest possible length (e.g. with BFS)

Theorem

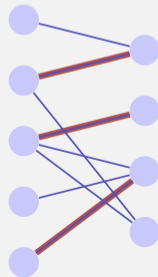
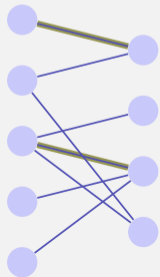
*When the Edmonds-Karp algorithm is applied to some integer valued flow network $G = (V, E)$ with source s and sink t then the number of flow increases applied by the algorithm is in $\mathcal{O}(|V| \cdot |E|)$
 \Rightarrow Overall asymptotic runtime: $\mathcal{O}(|V| \cdot |E|^2)$*

Application: maximal bipartite matching

Given: bipartite undirected graph $G = (V, E)$.

Matching M : $M \subseteq E$ such that $|\{m \in M : v \in m\}| \leq 1$ for all $v \in V$.

Maximal Matching M : Matching M , such that $|M| \geq |M'|$ for each matching M' .

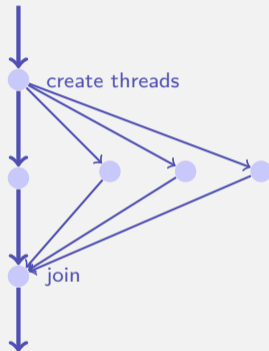


3. C++ Threads

C++11 Threads

```
void hello(int id){  
    std::cout << "hello from " << id << "\n";  
}
```

```
int main(){  
    std::vector<std::thread> tv(3);  
    int id = 0;  
    for (auto & t:tv)  
        t = std::thread(hello, ++id);  
    std::cout << "hello from main \n";  
    for (auto & t:tv)  
        t.join();  
    return 0;  
}
```



Nondeterministic Execution!

One execution:

```
hello from main  
hello from 2  
hello from 1  
hello from 0
```

Other execution:

```
hello from 1  
hello from main  
hello from 0  
hello from 2
```

Other execution:

```
hello from main  
hello from 0  
hello from hello from 1  
2
```

Technical Details I

- With allocating a thread, reference parameters are copied, except explicitly `std::ref` is provided at the construction.

Technical Details I

- With allocating a thread, reference parameters are copied, except explicitly `std::ref` is provided at the construction.

```
void calc( std::vector<int>& very_long_vector ){
    // doing funky stuff with very_long_vector
}

int main(){
    std::vector<int> v( 1000000000 );
    std::thread t1( calc, v );           // bad idea, v is copied
    // here v is unchanged
    std::thread t2( calc, std::ref(v) ); // good idea, v is not copied
    // here v is modified
    std::thread t2( [&v]{calc(v)}; } ); // also good idea
    // here v is modified
    // ...
}
```


Technical Details II

- Threads cannot be copied.

Technical Details II

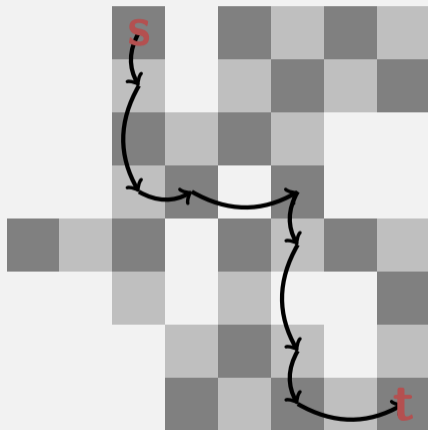
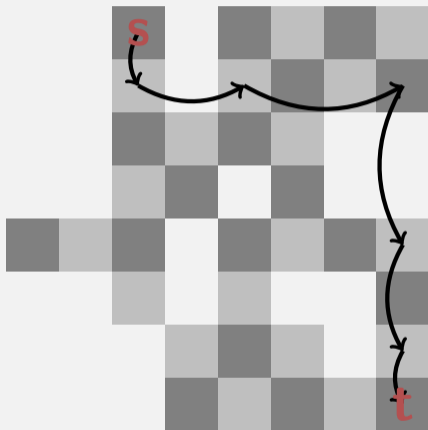
- Threads cannot be copied.

```
{
    std::thread t1(hello);
    std::thread t2;
    t2 = t1; // compiler error
    t1.join();
}
{
    std::thread t1(hello);
    std::thread t2;
    t2 = std::move(t1); // ok
    t2.join();
}
```

4. Two Quizzes

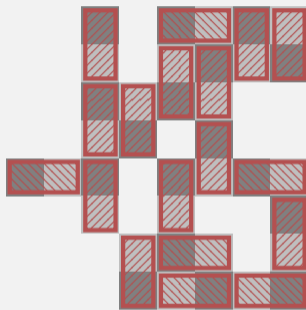
[Exam 2018.01], Tasks 4 and 5

Shortest Path Question



Most important question: What is the corresponding state space?

Max Flow Question



Most important question: How to map this to a max-flow (matching) setup?

Questions?