Datenstrukturen und Algorithmen

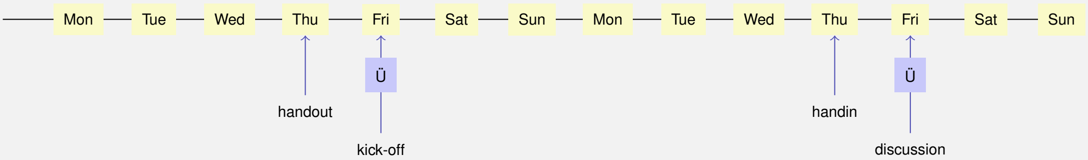
Exercise 1

FS 2021

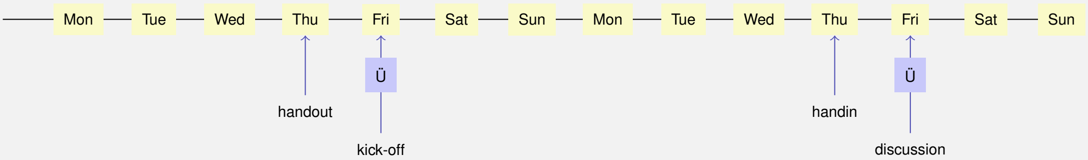
Schedule for today

1. Exercise Process
2. Preparation Theory
   - Aysmptotic Running Time
3. Programming Exercise

Process for the exercises



Process for the exercises



- Thursday:
    - Handout of new exercise sheet (online per Code Expert).
    - Submission of old exercise sheet (online per Code Expert).
- Friday during exercise class:
    - Kick-off presentation of new exercise sheet.
    - Discussion of old exercise sheet.
    - Opportunity to ask questions about lecture and exercises.

2. Preparation Theory

Required for next weeks lectures

Sums

$$\sum_{i=0}^{n} i = ?$$

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

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Why?

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

Why?

Intuition

$$1 + ... + 100 = (1 + 100) + (2 + 99) + (3 + 98) + ... + (50 + 51)$$

Sums

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}$$

Why?

Intuition

$$1 + ... + 100 = (1 + 100) + (2 + 99) + (3 + 98) + ... + (50 + 51)$$

More formally?

Sums

$$\sum_{i=0}^{n}(n-i) = ?$$

Sums

$$\sum_{i=0}^{n}(n-i) = \sum_{i=0}^{n} i$$

Sums

$$\sum_{i=0}^{n}(n-i) = \sum_{i=0}^{n} i$$

$$\Rightarrow 2 \cdot \sum_{i=0}^{n} i = \sum_{i=0}^{n} i + \sum_{i=0}^{n}(n-i)$$

$$= \sum_{i=0}^{n} (i + (n-i)) = \sum_{i=0}^{n} n = (n+1) \cdot n$$

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Sums

$$\sum_{i=0}^{n} i^2 = ?$$

Sums

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Sums

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This you do not need to know by heart. But you should know that it is a polynome of third degree.

[Sums]

How do you derive something like this?

[Sums]

How do you derive something like this? Interesting Trick: On the one hand

$$\sum_{i=0}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 = \sum_{i=0}^{n} i^3 - \sum_{i=0}^{n-1} i^3 = n^3,$$

[Sums]

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on the other hand

$$\begin{aligned}\sum_{i=0}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 &= \sum_{i=1}^{n} i^3 - \sum_{i=1}^{n} (i-1)^3 \\ &= \sum_{i=1}^{n} i^3 - (i-1)^3 = \sum_{i=1}^{n} 3 \cdot i^2 - 3 \cdot i + 1\end{aligned}$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

$$a^x \cdot a^y = ?$$

$$\log_a (x \cdot y) = ?$$

$$\frac{a^x}{a^y} = ?$$

$$\log_a \frac{x}{y} = ?$$

$$a^{x \cdot y} = ?$$

$$\log_a x^y = ?$$

$$\log_a n! = ?$$

$$\log_b x = ?$$

$$a^{\log_b x} = ?$$

Exponents and Logarithms

$$\log_a y = x \Leftrightarrow a^x = y \quad (a > 0, y > 0)$$

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$$a^{x \cdot y} = (a^x)^y \qquad \log_a x^y = y \log_a x$$

$$\log_a n! = \sum_{i=1}^{n} \log i$$

$$\log_b x =? \qquad a^{\log_b x} =?$$

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$$\log_b x = \log_b a \cdot \log_a x$$

$$a^{\log_b x} = x^{\log_b a}$$

To see the last line, replace $x \to a^{\log_a x}$

Comparisons

$$\frac{n^2}{2^n} \underset{n\to\infty}{\longrightarrow} ?$$

Comparisons

$$\frac{n^2}{2^n} \xrightarrow[n \to \infty]{} 0$$

Comparisons

$$\frac{n^{10000}}{2^n} \underset{n\to\infty}{\longrightarrow} ?$$

Comparisons

$$\frac{n^{10000}}{2^n} \xrightarrow[n \to \infty]{} 0$$

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \xrightarrow[n \to \infty]{} ?$$

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \xrightarrow[n\to\infty]{} 0$$

Comparisons

$d > 1, c > 0$

$$\frac{n^c}{d^n} \underset{n\to\infty}{\longrightarrow} 0$$

because

$$\frac{n^c}{d^n} = \frac{2^{\log_2 n^c}}{2^{\log_2 d^n}} = 2^{c\cdot\log_2 n - n\log_2 d}$$

Comparisons

$$\frac{n}{\log n} \underset{n \to \infty}{\longrightarrow} ?$$

Comparisons

$$\frac{n}{\log n} \xrightarrow[n \to \infty]{} \infty$$

Comparisons

$$\frac{n \log n}{\sqrt{n}} \underset{n \to \infty}{\longrightarrow} ?$$

Comparisons

$$\frac{n \log n}{\sqrt{n}} \xrightarrow[n \to \infty]{} \infty$$

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} ?$$

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$$

Comparisons

$$\frac{\log_2 n^2}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$$

$$\begin{aligned}
\log_2 n^2 &= 2\log_2 n \\
\sqrt{n} &= n^{1/2} = 2^{\log_2 n^{1/2}} = \left(\sqrt{2}\right)^{\log_2 n} \\
\frac{\log n^2}{\sqrt{n}} &= 2\frac{\log_2 n}{\left(\sqrt{2}\right)^{\log_2 n}}
\end{aligned}$$

which behaves because of $\log_2 n \to \infty$ for $n \to \infty$ like $2\frac{n}{\left(\sqrt{2}\right)^n}$

Warm-up

- What is a problem?

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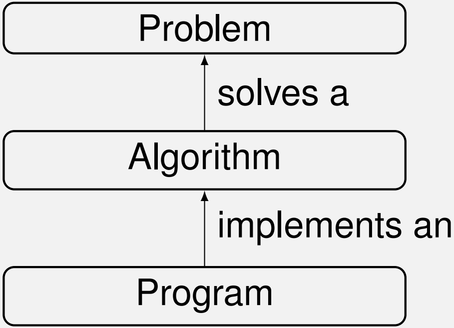
- What is a program?

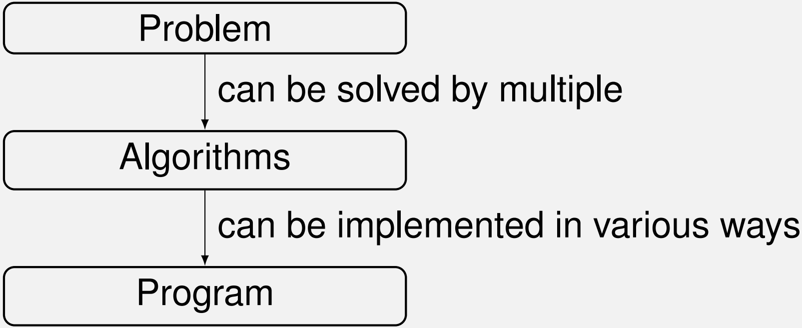
Warm-up

- What is a problem?
- What is an algorithm?
  - ➜ well-defined computing procedure to compute output data from input data.
- What is a program?
  - ➜ Concrete implementation of an algorithm

Warm-up



Warm-up


Problem
can be solved by multiple
Algorithms
can be implemented in various ways
Program

Efficiency

Problem	Complexity	Minimal (asymptotic) cost over all algorithms that solve the problem.
Algorithm	Cost	Number of elementary operations
Program	Computing time	Measurable value on an actual machine.

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➜ Estimation of cost or computing time depending on the input size, denoted by n.

Asymptotic behavior

- What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

Asymptotic behavior

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➜ Sets of functions!

Asymptotic behavior

- What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

→ Sets of functions!

Repetition, sets A, B:

subset	$A \subseteq B$
proper subset	$A \subsetneq B$
intersection	$A \cap B$

Asymptotic behavior

Given: function $f : \mathbb{N} \to \mathbb{R}$.

Definition:

$$\mathcal{O}(g) = \{f : \mathbb{N} \to \mathbb{R} | \exists c > 0, n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$
$$\Omega(g) = \{f : \mathbb{N} \to \mathbb{R} | \exists c > 0, n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq c \cdot g(n) \leq f(n)\}$$
$$\Theta(g) = \mathcal{O}(g) \cap \Omega(g)$$

Used slightly more seldom

Given: function $f : \mathbb{N} \to \mathbb{R}$.

Definition:

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$$o(g) = \{f : \mathbb{N} \to \mathbb{R} | \forall c > 0 \; \exists n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$$\Omega(g) = \{f : \mathbb{N} \to \mathbb{R} | \exists c > 0, n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq c \cdot g(n) \leq f(n)\}$$

$$\omega(g) = \{f : \mathbb{N} \to \mathbb{R} | \forall c > 0 \; \exists n_0 \in \mathbb{N} | \forall n \geq n_0 : 0 \leq c \cdot g(n) \leq f(n)\}$$

$f \in o(g)$: f grows much slower than g

$f \in \omega(g)$: f grows much faster than g

Useful information for the exercise

Theorem

1. $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f \in \mathcal{O}(g), \mathcal{O}(f) \subsetneq \mathcal{O}(g)$.
2. $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C > 0$ (C constant) $\Rightarrow f \in \Theta(g)$.
3. $\frac{f(n)}{g(n)} \underset{n\to\infty}{\longrightarrow} \infty \Rightarrow g \in \mathcal{O}(f), \mathcal{O}(g) \subsetneq \mathcal{O}(f)$.

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3. $\frac{f(n)}{g(n)} \underset{n\to\infty}{\to} \infty \Rightarrow g \in \mathcal{O}(f)$, $\mathcal{O}(g) \subsetneq \mathcal{O}(f)$.

Example

1. $\lim_{n\to\infty} \frac{n}{n^2} = 0 \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2)$.
2. $\lim_{n\to\infty} \frac{2n}{n} = 2 > 0 \Rightarrow 2n \in \Theta(n)$.
3. $\frac{n^2}{n} \underset{n\to\infty}{\to} \infty \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2)$.

Property

$$f_1 \in \mathcal{O}(g), f_2 \in \mathcal{O}(g) \Rightarrow f_1 + f_2 \in \mathcal{O}(g)$$

Examples

$$\mathcal{O}(g) = \{f : \mathbb{N} \to \mathbb{R} |\ \exists c > 0, \exists n_0 \in \mathbb{N} : \forall n \geq n_0 : 0 \leq f(n) \leq c \cdot g(n)\}$$

$f(n)$	$f \in \mathcal{O}(?)$	Example
$3n + 4$		
$2n$		
$n^2 + 100n$		
$n + \sqrt{n}$		

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$n^2 + 100n$		
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$2n$	$\mathcal{O}(n)$	$c = 2, n_0 = 0$
$n^2 + 100n$	$\mathcal{O}(n^2)$	$c = 2, n_0 = 100$
$n + \sqrt{n}$	$\mathcal{O}(n)$	$c = 2, n_0 = 1$

Examples

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- $n \in \mathcal{O}(n^2)$

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- $3n^2 \in \mathcal{O}(2n^2)$

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- $3n^2 \in \mathcal{O}(2n^2)$ correct but uncommon:

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- $2n^2 \in \mathcal{O}(n)$

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  Omit constants: $3n^2 \in \mathcal{O}(n^2)$.
- $2n^2 \in \mathcal{O}(n)$ is wrong:

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  Omit constants: $3n^2 \in \mathcal{O}(n^2)$.
- $2n^2 \in \mathcal{O}(n)$ is wrong: $\frac{2n^2}{n} = 2n \underset{n\to\infty}{\to} \infty$!

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- $2n^2 \in \mathcal{O}(n)$ is wrong: $\frac{2n^2}{n} = 2n \underset{n\to\infty}{\to} \infty$!
- $\mathcal{O}(n) \subseteq \mathcal{O}(n^2)$

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- $2n^2 \in \mathcal{O}(n)$ is wrong: $\frac{2n^2}{n} = 2n \underset{n\to\infty}{\to} \infty$!
- $\mathcal{O}(n) \subseteq \mathcal{O}(n^2)$ is correct

Examples

- $n \in \mathcal{O}(n^2)$ correct, but too imprecise:
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Quiz

$1 \in \mathcal{O}(15)$?

Quiz

$1 \in \mathcal{O}(15)$? ✓ better $1 \in \mathcal{O}(1)$

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Quiz: A good strategy?

... Then I simply buy a new machine

1

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... Then I simply buy a new machine If today I can solve a problem of size n, then with a 10 or 100 times faster machine I can solve ...[1]

Komplexität	(speed $\times 10$)	(speed $\times 100$)
$\log_2 n$		
n		
n^2		
2^n		

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n^2	$n \to 3.16 \cdot n$	$n \to 10 \cdot n$
2^n	$n \to n + 3.32$	$n \to n + 6.64$

[1] To see this, you set $f(n') = c \cdot f(n)$ ($c = 10$ or $c = 100$) and solve for n'

Asymptotic Running Times with Θ

```
void run(int n){
  for (int i = 1; i<n; ++i)
    for (int j = 1; j<n; ++j)
      op();
}
```

How often is `op()` called?

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Asymptotic Running Times with Θ

```
void run(int n){
  for(int i = 1; i <= n; ++i)
    for(int j = 1; j*j <= n; ++j)
      for(int k = n; k >= 2; --k)
        op();
}
```

How often is `op()` called?

3. Programming Exercise

Sum in sub-interval (naive algorithm)

Input: A sequence of n numbers $(a_0, a_1, \dots, a_{n-1})$ and a sub-interval $I = [x_0, x_1]$

Output: $\sum_{i=x_0}^{x_1} a_i$.

$\mathcal{S} \leftarrow 0$
for $i \in \{x_0, \dots, x_1\}$ **do**
  $\mathcal{S} \leftarrow \mathcal{S} + a_i$
return $\mathcal{S}$

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Idea

- Use the prefix sum to compute the sum of arbitrary sub-intervals with constant complexity
- Generalization to two dimensions.

Multidimensional vectors

Definition

```
std::vector< std::vector<int> > my_vec( n_rows,
std::vector<int>(n_cols,init_value) );
```

Indexing

```
my_vec[row][col]
```

Classes

```
class Insurance { // Definition
public: // public section
        Insurance(double rate) {rate_ = rate;} // Construktor
        double get_rate() {return rate_;} // member function
private: // private section
        double rate_; // data member
};

int main() {
        Insurance insurance(2.);
        std::cout << insurance.get_rate();
        return 0;
}
```

Questions or Suggestions?