Datenstrukturen und Algorithmen

Exercise 7

FS 2020

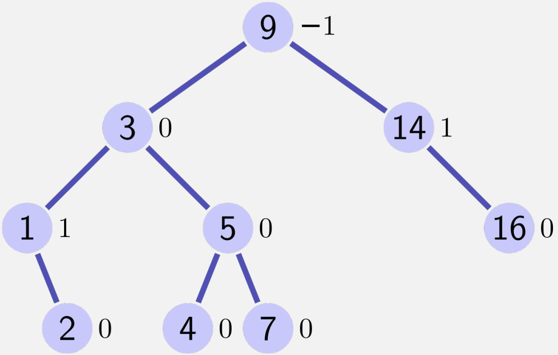
Program of today

1. Feedback of last exercise(s)

2. Repetition theory

AVL insertion

- Given an AVL tree, is there an order that produces the same tree and does not cause any rotations



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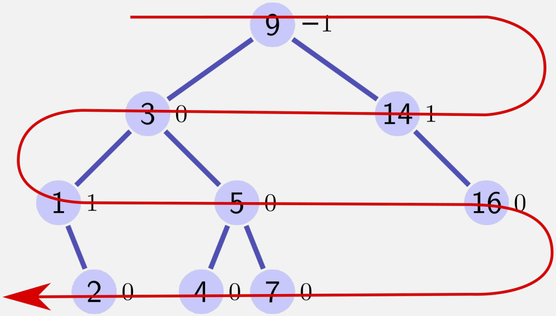

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- By induction over the height of the tree.
- Hypothesis: Keys at height h and lower are placed in the same place and do not cause rotation.
- Step: Show that the traversal is the same as in the original tree, yields same position. Use AVL property of T to show that there will not be a height difference bigger than 1, and therefore no rotation.

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Longest ascending Sequence in matrix

Given $n \times m$ matrix A:

9	27	42	41	48
35	39	8	3	5
12	49	2	38	4
15	47	29	28	6
19	1	25	33	10

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Wanted longest ascending sequence:

$$4, 6, 28, 29, 47, 49$$

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- What is the meaning of each entry?
  - In $T[x][y]$ is the length of the longest ascending sequence that ends in $A[x][y]$
  - In $S[x][y]$ are the coordinates of the predecessor in ascending sequence (if exists)

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- Arbitrary order, if entry is already computed skip it otherwise compute for smaller neighbor recursively.

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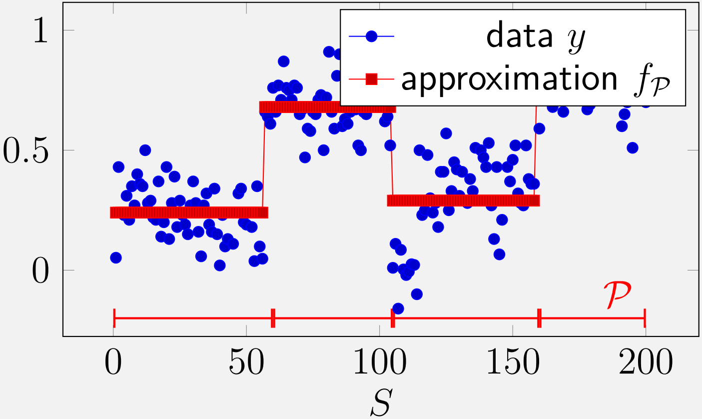
  - Consider all entries to find one with a longest sequence. From there, we can reconstruct the solution by following the corresponding predecessors.

Piecewise Constant Approximation



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- $\mathcal{P}$: (set of intervals I_i, such that $\cup_i I_i = S$).
- **Goal:** find the partition $\hat{\mathcal{P}}$ such that $H_{\gamma,y}(\hat{\mathcal{P}})$ is minimal
- Utilize: efficient computation of the mean using prefix sums (exercise 1): $\mu_I = \frac{1}{|I|} \sum_{i \in I} y_i$

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- **Goal:** find the partition $\hat{\mathcal{P}}$ such that $H_{\gamma,y}(\hat{\mathcal{P}})$ is minimal
- **Dynamic programming**: definition of the table, computation of an entry, calculation order, extracting solution
- Utilize*: $H_{\gamma,y}(\mathcal{P} \cup \{[l,r)\}) = H_{\gamma,y}(\mathcal{P}) + \gamma + e_{[l,r)}$

*Assumption: $\mathcal{P} \cup \{[l,r)\}$ is a partition

Questions?