

Datenstrukturen und Algorithmen

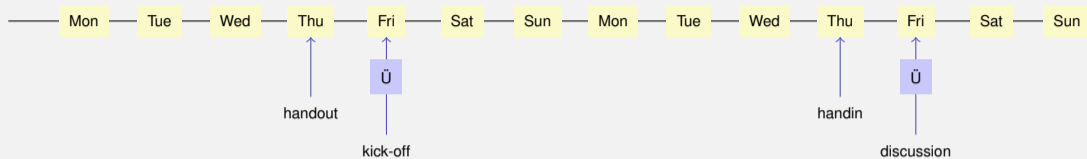
Exercise 1

FS 2019

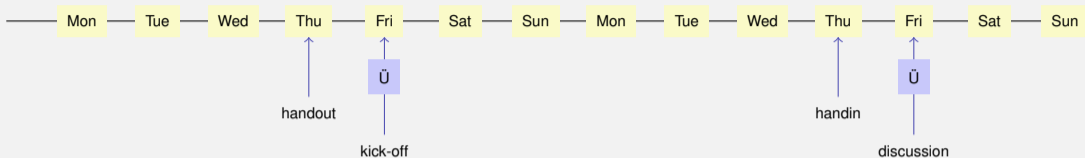
Schedule for today

- 1 Exercise Process
- 2 Repetition Theory
 - Aysmptotic Running Time
- 3 Programming Exercise

Process for the exercises



Process for the exercises



■ Thursday:

- Handout of new exercise sheet (online per Code Expert).
- Submission of old exercise sheet (online per Code Expert).

■ Friday during exercise class:

- Kick-off presentation of new exercise sheet.
- Discussion of old exercise sheet.
- Opportunity to ask questions about lecture and exercises.

2. Repetition Theory

Warm-up

- What is a problem?

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- What is an algorithm?

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 - well-defined computing procedure to compute output data from input data.

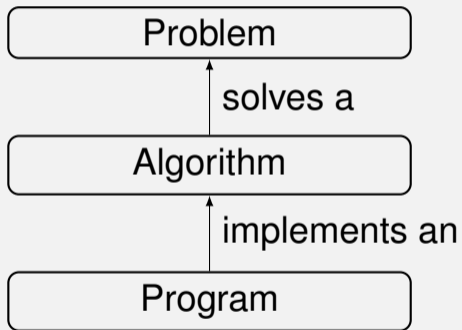
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- What is a problem?
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- What is a program?

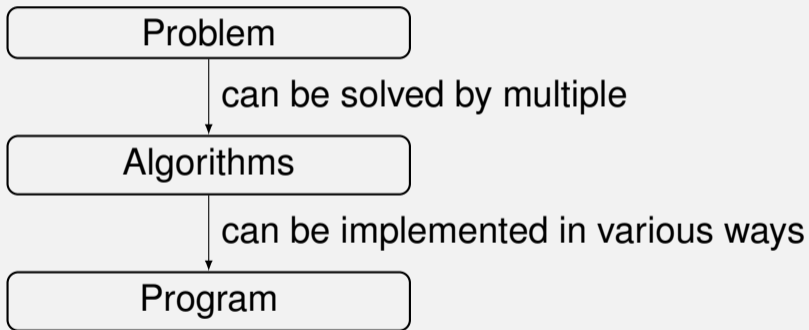
Warm-up

- What is a problem?
- What is an algorithm?
 - well-defined computing procedure to compute output data from input data.
- What is a program?
 - Concrete implementation of an algorithm

Warm-up



Warm-up



Efficiency

Problem	Complexity	Minimal (asymptotic) cost over all algorithms that solve the problem.
Algorithm	Cost	Number of elementary operations
Program	Computing time	Measurable value on an actual machine.

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- Estimation of cost or computing time depending on the input size, denoted by n .

Asymptotic behavior

- What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

Asymptotic behavior

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- Sets of functions!

Asymptotic behavior

■ What are $\Omega(g(n))$, $\Theta(g(n))$, $\mathcal{O}(g(n))$?

→ Sets of functions!

Repetition, sets A, B :

subset $A \subseteq B$

proper subset $A \subsetneq B$

intersection $A \cap B$

Asymptotic behavior

Given: function $f : \mathbb{N} \rightarrow \mathbb{R}$.

Definition:

$$\mathcal{O}(g) = \{f : \mathbb{N} \rightarrow \mathbb{R} \mid \exists c > 0, n_0 \in \mathbb{N} : 0 \leq f(n) \leq c \cdot g(n) \forall n \geq n_0\}$$

$$\Omega(g) = \{f : \mathbb{N} \rightarrow \mathbb{R} \mid \exists c > 0, n_0 \in \mathbb{N} : 0 \leq c \cdot g(n) \leq f(n) \forall n \geq n_0\}$$

$$\Theta(g) = \mathcal{O}(g) \cap \Omega(g)$$

Useful information for the exercise

Theorem

- 1 $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f \in \mathcal{O}(g), \mathcal{O}(f) \subsetneq \mathcal{O}(g).$
- 2 $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = C > 0$ (C constant) $\Rightarrow f \in \Theta(g).$
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Example

- 1 $\lim_{n \rightarrow \infty} \frac{n}{n^2} = 0 \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2).$
- 2 $\lim_{n \rightarrow \infty} \frac{2n}{n} = 2 > 0 \Rightarrow 2n \in \Theta(n).$
- 3 $\frac{n^2}{n} \xrightarrow[n \rightarrow \infty]{} \infty \Rightarrow n \in \mathcal{O}(n^2), \mathcal{O}(n) \subsetneq \mathcal{O}(n^2).$

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Quiz

$1 \in \mathcal{O}(15)$?

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$1 \in \mathcal{O}(15)$? ✓ better $1 \in \mathcal{O}(1)$

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Quiz: A good strategy?

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Komplexität	(speed $\times 10$)	(speed $\times 100$)
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------------	--	--

n		
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n^2	$n \rightarrow 3.16 \cdot n$	$n \rightarrow 10 \cdot n$
2^n	$n \rightarrow n + 3.32$	$n \rightarrow n + 6.64$

¹To see this, you set $f(n') = c \cdot f(n)$ ($c = 10$ or $c = 100$) and solve for n'

Asymptotic Running Times with Θ

```
void run(int n){  
    for (int i = 1; i<n; ++i)  
        for (int j = 1; j<n; ++j)  
            op();  
}
```

How often is `op()` called?

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```
void run(int n){  
    for(int i = 1; i <= n; ++i)  
        for(int j = 1; j*j <= n; ++j)  
            for(int k = n; k >= 2; --k)  
                op();  
}
```

How often is `op()` called?

3. Programming Exercise

Sum in sub-interval (naive algorithm)

Input: A sequence of n numbers $(a_0, a_1, \dots, a_{n-1})$ and a sub-interval

$$I = [x_0, x_1]$$

Output: $\sum_{i=x_0}^{x_1} a_i.$

$\mathcal{S} \leftarrow 0$

for $i \in \{x_0, \dots, x_1\}$ **do**

$\mathcal{S} \leftarrow \mathcal{S} + a_i$

return \mathcal{S}

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Idea

- Use the prefix sum to compute the sum of arbitrary sub-intervals with constant complexity
- Generalization to two dimensions.

Multidimensional vectors

Definition

```
std::vector< std::vector<int> > my_vec( n_rows,  
std::vector<int>(n_cols,init_value) );
```

Indexing

```
my_vec[row][col]
```

Classes

```
class Insurance { // Definition
public: // public section
    Insurance(double rate) {rate_ = rate;} // Konstruktor
    double get_rate() {return rate_;} // member function
private: // private section
    double rate_; // data member
};

int main() {
    Insurance insurance(2.);
    std::cout << insurance.get_rate();
    return 0;
}
```

Questions or Suggestions?