

The Free Lunch

27. Parallel Programming I

Moore's Law and the Free Lunch, Hardware Architectures, Parallel Execution, Flynn's Taxonomy, Scalability: Amdahl and Gustafson, Data-parallelism, Task-parallelism, Scheduling

[Task-Scheduling: Cormen et al, Kap. 27] [Concurrency, Scheduling: Williams, Kap. 1.1 – 1.2]

The free lunch is over ⁴⁰

⁴⁰"The Free Lunch is Over", a fundamental turn toward concurrency in software, Herb Sutter, Dr. Dobb's Journal, 2005

Moore's Law



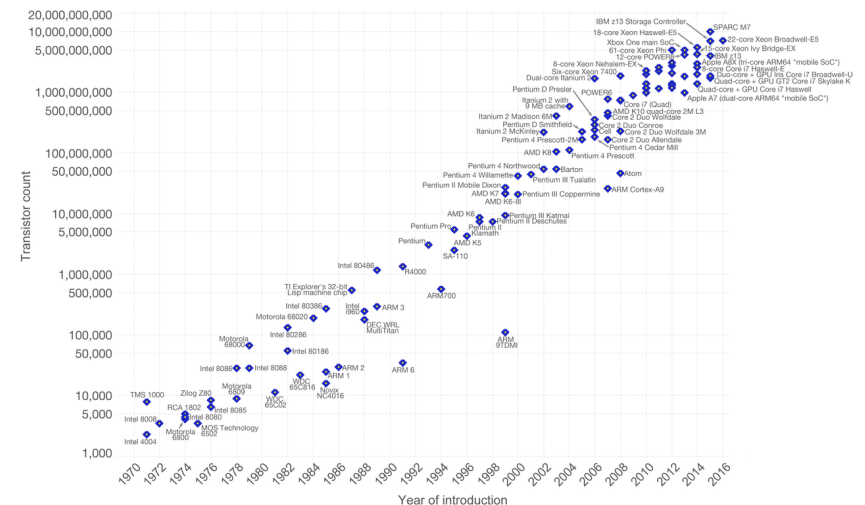
Gordon E. Moore (1929)

Observation by Gordon E. Moore:

The number of transistors on integrated circuits doubles approximately every two years.

Moore's Law – The number of transistors on integrated circuit chips (1971-2016)

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important as other aspects of technological progress – such as processing speed or the price of electronic products – are strongly linked to Moore's law.



Data source: Wikipedia (https://en.wikipedia.org/wiki/Transistor_count)
The data visualization is available at OurWorldInData.org. There you find more visualizations and research on this topic. Licensed under CC-BY-SA by the author Max Roser.

For a long time...

- the sequential execution became faster ("Instruction Level Parallelism", "Pipelining", Higher Frequencies)
- more and smaller transistors = more performance
- programmers simply waited for the next processor generation

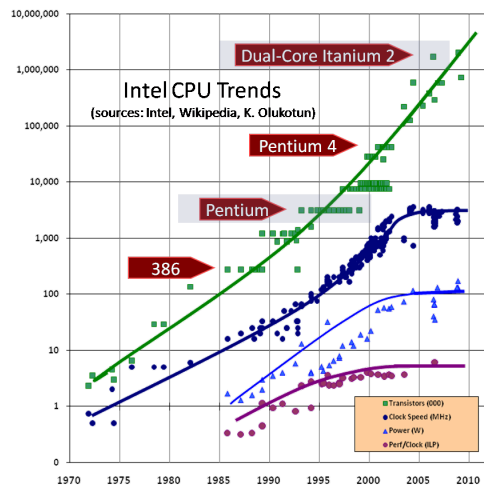
Today

- the frequency of processors does not increase significantly and more (heat dissipation problems)
- the instruction level parallelism does not increase significantly any more
- the execution speed is dominated by memory access times (but caches still become larger and faster)

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Trends



<http://www.gotw.ca/publications/concurrency-ddj.htm>

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Multicore

- Use transistors for more compute cores
- Parallelism in the software
- Programmers have to write parallel programs to benefit from new hardware

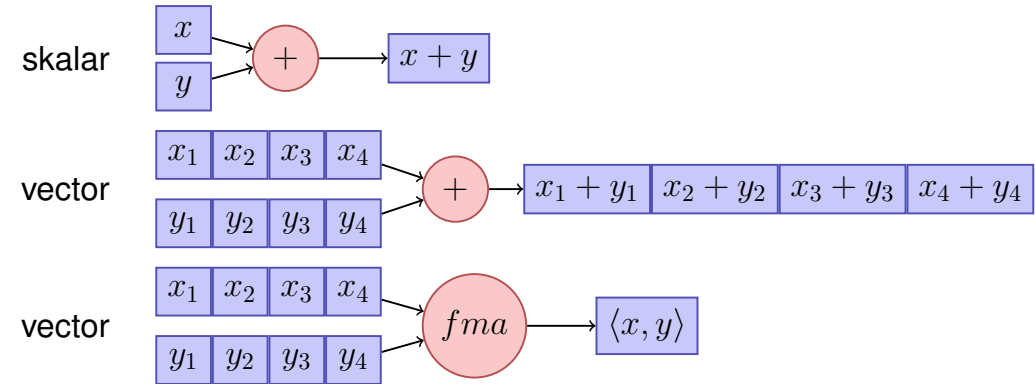
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Forms of Parallel Execution

- Vectorization
- Pipelining
- Instruction Level Parallelism
- Multicore / Multiprocessing
- Distributed Computing

Vectorization

Parallel Execution of the same operations on elements of a vector (register)



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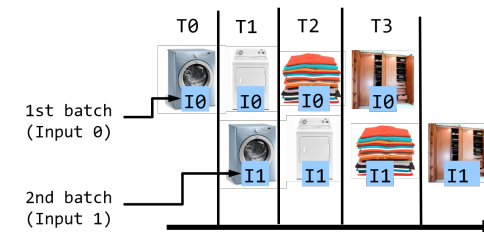
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Home Work



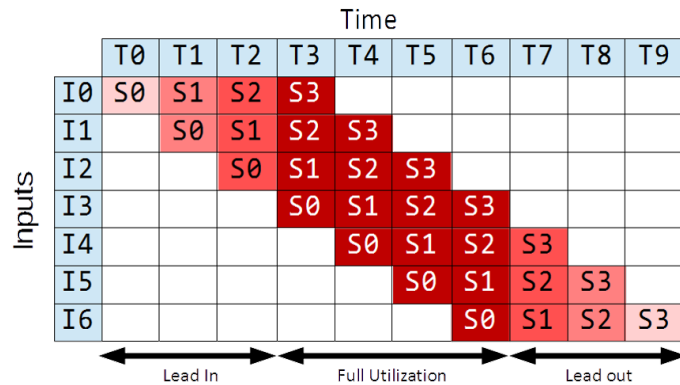
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More efficient



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Pipeline



Balanced / Unbalanced Pipeline

A pipeline is called balanced, if each step takes the same computation time.

Software-Pipelines are often unbalanced.

In the following we assume that each step of the pipeline takes as long as the longest step.

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Throughput

- Throughput = Input or output data rate
- Number operations per time unit
- larger throughput is better
-

$$\text{throughput} = \frac{1}{\max(\text{computationtime}(\text{stages}))}$$

ignores lead-in and lead-out times

Latency

- Time to perform a computation
- $\text{latency} = \# \text{stages} \cdot \max(\text{computationtime}(\text{stages}))$

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Homework Example

Washing $T_0 = 1h$, Drying $T_1 = 2h$, Ironing $T_2 = 1h$, Tidy up $T_3 = 0.5h$

- Latency $L = 8h$
- In the long run: 1 batch every $2h$ ($0.5/h$).

Throughput vs. Latency

- Increasing throughput can increase latency
- Stages of the pipeline need to communicate and synchronize: overhead

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Pipelines in CPUs

Fetch Decode Execute Data Fetch Writeback

Multiple Stages

- Every instruction takes 5 time units (cycles)
- In the best case: 1 instruction per cycle, not always possible (“stalls”)

Parallelism (several functional units) leads to *faster execution*.

ILP – Instruction Level Parallelism

Modern CPUs provide several hardware units and execute independent instructions in parallel.

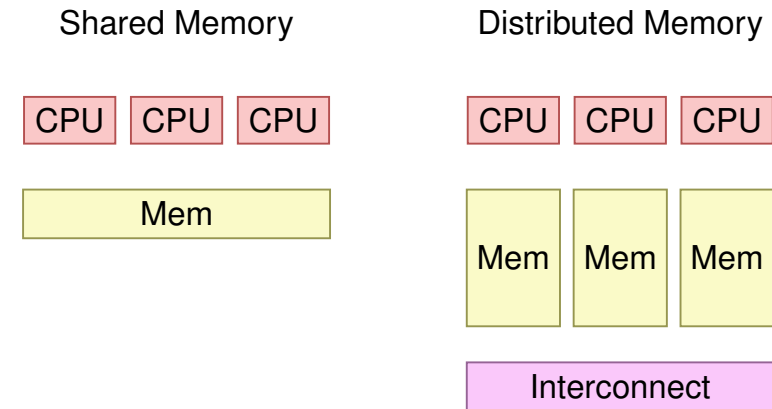
- Pipelining
- Superscalar CPUs (multiple instructions per cycle)
- Out-Of-Order Execution (Programmer observes the sequential execution)
- Speculative Execution ()

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27.2 Hardware Architectures

Shared vs. Distributed Memory



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Shared vs. Distributed Memory Programming

- Categories of programming interfaces
 - Communication via message passing
 - Communication via memory sharing
- It is possible:
 - to program shared memory systems as distributed systems (e.g. with message passing MPI)
 - program systems with distributed memory as shared memory systems (e.g. partitioned global address space PGAS)

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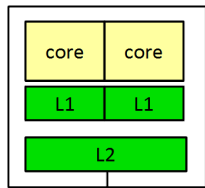
Shared Memory Architectures

- Multicore (Chip Multiprocessor - CMP)
- Symmetric Multiprocessor Systems (SMP)
- Simultaneous Multithreading (SMT = Hyperthreading)
 - one physical core, Several Instruction Streams/Threads: several virtual cores
 - Between ILP (several units for a stream) and multicore (several units for several streams). Limited parallel performance.
- Non-Uniform Memory Access (NUMA)

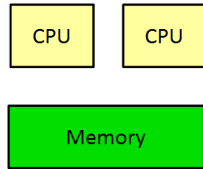
Same programming interface

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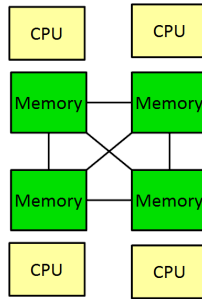
Overview



CMP



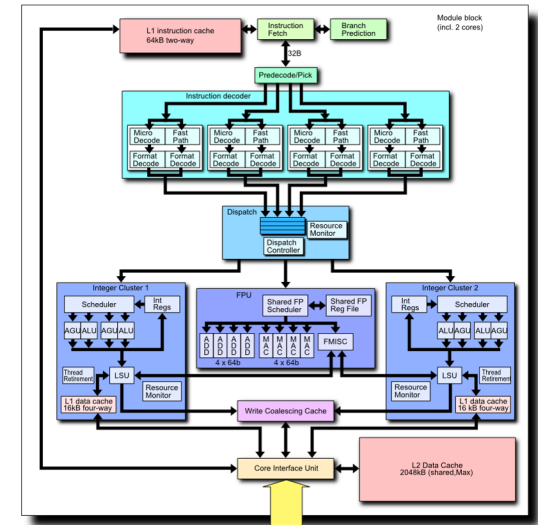
SMP



NUMA

An Example

- AMD Bulldozer: between CMP and SMT
- 2x integer core
 - 1x floating point core



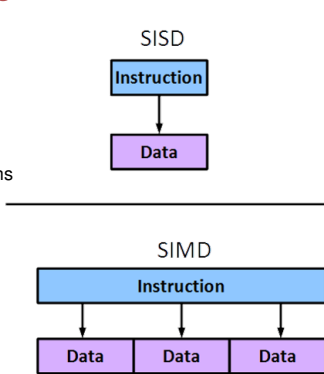
Flynn's Taxonomy

Single-Core

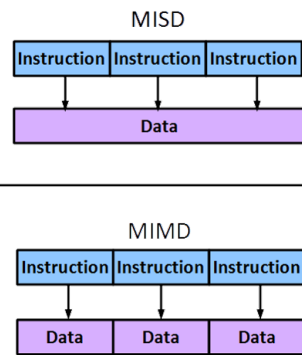
Fehlertoleranz

SI = Single Instruction
MI = Multiple Instructions

SD = Single Data
MD = Multiple Data



Vector Computing / GPU

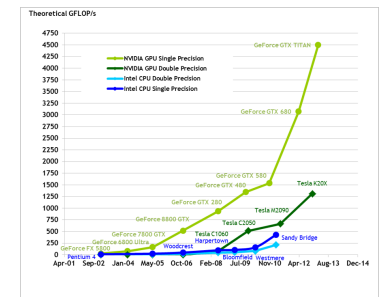


Multi-Core

Massively Parallel Hardware

[General Purpose] Graphical Processing Units ([GP]GPUs)

- Revolution in High Performance Computing
 - Calculation 4.5 TFlops vs. 500 GFlops
 - Memory Bandwidth 170 GB/s vs. 40 GB/s
- SIMD
 - High data parallelism
 - Requires own programming model. Z.B. CUDA / OpenCL



27.3 Multi-Threading, Parallelism and Concurrency

Processes and Threads

- Process: instance of a program
 - each process has a separate context, even a separate address space
 - OS manages processes (resource control, scheduling, synchronisation)
- Threads: threads of execution of a program
 - Threads share the address space
 - fast context switch between threads

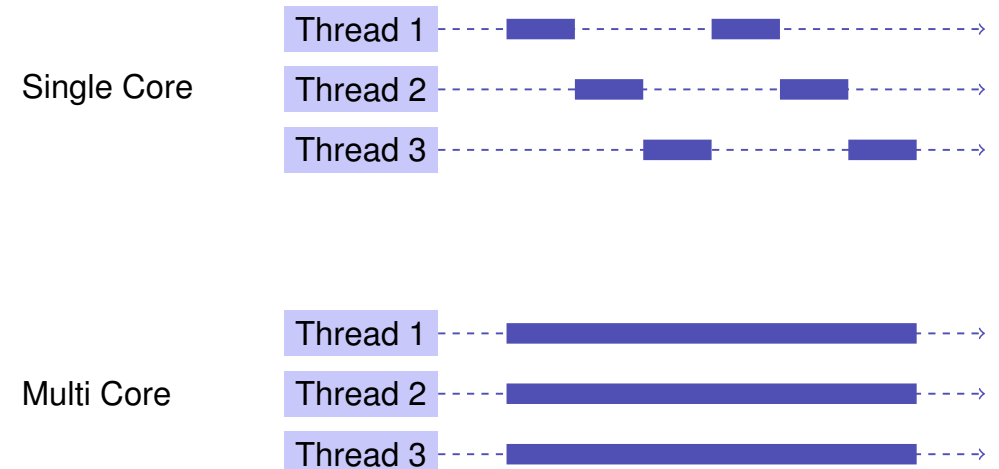
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Why Multithreading?

- Avoid “polling” resources (files, network, keyboard)
- Interactivity (e.g. responsivity of GUI programs)
- Several applications / clients in parallel
- Parallelism (performance!)

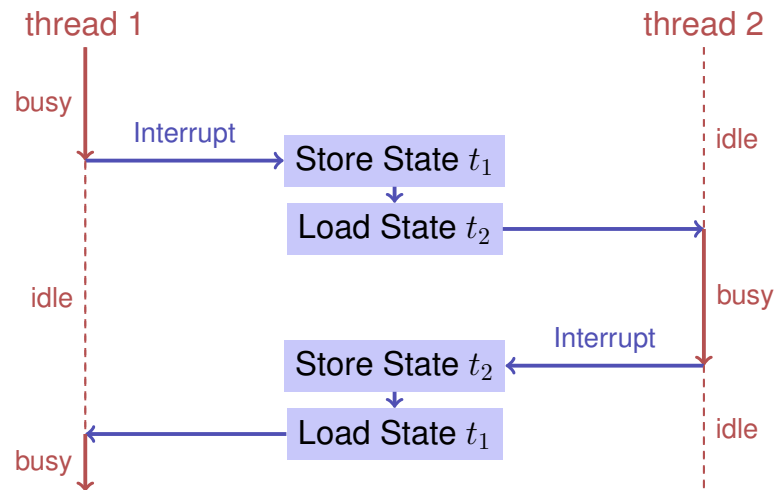
Multithreading conceptually



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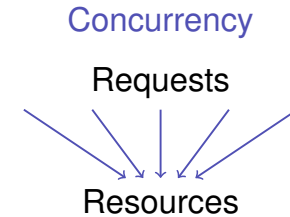
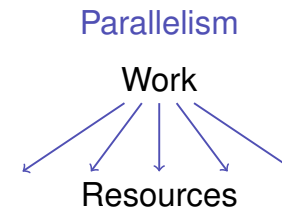
Thread switch on one core (Preemption)



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Parallelität vs. Concurrency

- **Parallelism:** Use extra resources to solve a problem faster
- **Concurrency:** Correctly and efficiently manage access to shared resources
- Begriffe überlappen offensichtlich. Bei parallelen Berechnungen besteht fast immer Synchronisierungsbedarf.



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Thread Safety

Thread Safety means that in a concurrent application of a program this always yields the desired results.

Many optimisations (Hardware, Compiler) target towards the correct execution of a *sequential* program.

Concurrent programs need an annotation that switches off certain optimisations selectively.

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Example: Caches

- Access to registers faster than to shared memory.
- Principle of locality.
- Use of Caches (transparent to the programmer)

If and how far a cache coherency is guaranteed depends on the used system.



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Scalability

27.4 Scalability: Amdahl and Gustafson

In parallel Programming:

- Speedup when increasing number p of processors
- What happens if $p \rightarrow \infty$?
- Program scales linearly: Linear speedup.

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Parallel Performance

Given a fixed amount of computing work W (number computing steps)

Sequential execution time T_1

Parallel execution time on p CPUs

- Perfection: $T_p = T_1/p$
- Performance loss: $T_p > T_1/p$ (usual case)
- Sorcery: $T_p < T_1/p$

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Parallel Speedup

Parallel speedup S_p on p CPUs:

$$S_p = \frac{W/T_p}{W/T_1} = \frac{T_1}{T_p}$$

- Perfection: linear speedup $S_p = p$
- Performance loss: sublinear speedup $S_p < p$ (the usual case)
- Sorcery: superlinear speedup $S_p > p$

Efficiency: $E_p = S_p/p$

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Reachable Speedup?

Parallel Program

Parallel Part	Seq. Part
80%	20%

$$T_1 = 10$$

$$T_8 = ?$$

$$T_8 = \frac{10 \cdot 0.8}{8} + 10 \cdot 0.2 = 1 + 2 = 3$$

$$S_8 = \frac{T_1}{T_8} = \frac{10}{3} \approx 3.3 < 8 \quad (!)$$

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Amdahl's Law: Ingredients

Computational work W falls into two categories

- Parallellisable part W_p
- Not parallelisable, sequential part W_s

Assumption: W can be processed sequentially by *one* processor in W time units ($T_1 = W$):

$$T_1 = W_s + W_p$$

$$T_p \geq W_s + W_p/p$$

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Amdahl's Law

$$S_p = \frac{T_1}{T_p} \leq \frac{W_s + W_p}{W_s + \frac{W_p}{p}}$$

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Amdahl's Law

With sequential, not parallelizable fraction λ : $W_s = \lambda W$,
 $W_p = (1 - \lambda)W$:

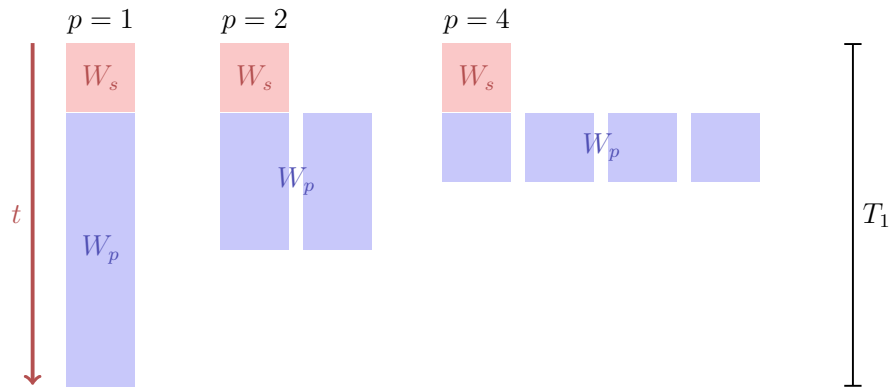
$$S_p \leq \frac{1}{\lambda + \frac{1-\lambda}{p}}$$

Thus

$$S_\infty \leq \frac{1}{\lambda}$$

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Illustration Amdahl's Law



Amdahl's Law is bad news

All non-parallel parts of a program can cause problems

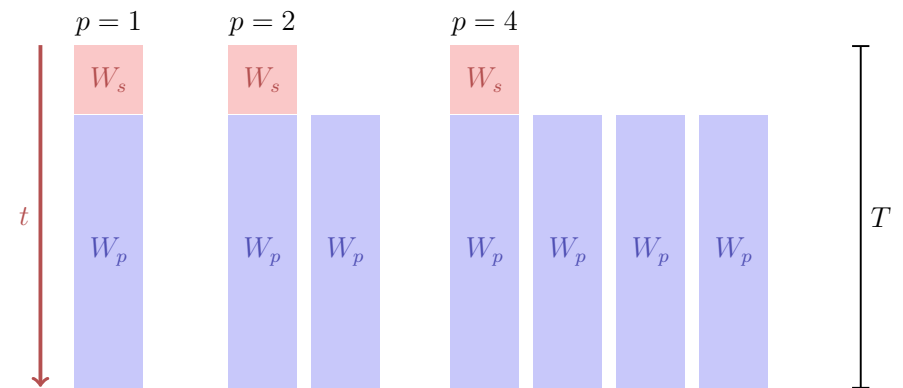
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Gustafson's Law

- Fix the time of execution
- Vary the problem size.
- Assumption: the sequential part stays constant, the parallel part becomes larger

Illustration Gustafson's Law



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Gustafson's Law

Work that can be executed by one processor in time T :

$$W_s + W_p = T$$

Work that can be executed by p processors in time T :

$$W_s + p \cdot W_p = \lambda \cdot T + p \cdot (1 - \lambda) \cdot T$$

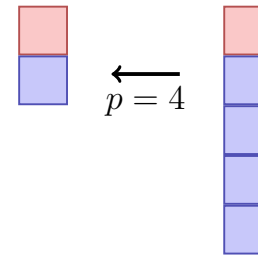
Speedup:

$$\begin{aligned} S_p &= \frac{W_s + p \cdot W_p}{W_s + W_p} = p \cdot (1 - \lambda) + \lambda \\ &= p - \lambda(p - 1) \end{aligned}$$

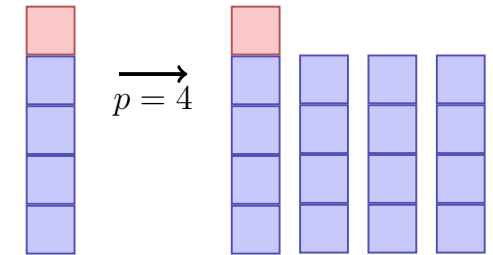
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Amdahl vs. Gustafson

Amdahl



Gustafson



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Amdahl vs. Gustafson

The laws of Amdahl and Gustafson are models of speedup for parallelization.

Amdahl assumes a fixed *relative* sequential portion, Gustafson assumes a fixed *absolute* sequential part (that is expressed as portion of the work W_1 and that does not increase with increasing work).

The two models do not contradict each other but describe the runtime speedup of different problems and algorithms.

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27.5 Task- and Data-Parallelism

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Parallel Programming Paradigms

- **Task Parallel:** Programmer explicitly defines parallel tasks.
- **Data Parallel:** Operations applied simultaneously to an aggregate of individual items.

Example Data Parallel (OMP)

```
double sum = 0, A[MAX];
#pragma omp parallel for reduction (+:ave)
for (int i = 0; i < MAX; ++i)
    sum += A[i];
return sum;
```

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Example Task Parallel (C++11 Threads/Futures)

```
double sum(Iterator from, Iterator to)
{
    auto len = from - to;
    if (len > threshold){
        auto future = std::async(sum, from, from + len / 2);
        return sumS(from + len / 2, to) + future.get();
    }
    else
        return sumS(from, to);
}
```

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Work Partitioning and Scheduling

- Partitioning of the work into parallel task (programmer or system)
 - One task provides a unit of work
 - Granularity?
- Scheduling (Runtime System)
 - Assignment of tasks to processors
 - Goal: full resource usage with little overhead

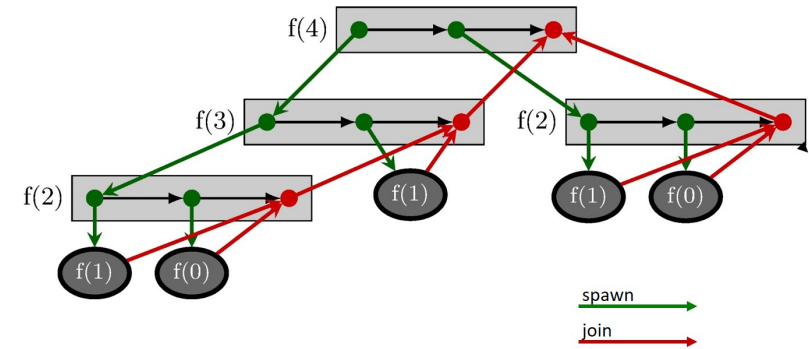
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Example: Fibonacci P-Fib

```

if  $n \leq 1$  then
  return  $n$ 
else
   $x \leftarrow$  spawn P-Fib( $n - 1$ )
   $y \leftarrow$  spawn P-Fib( $n - 2$ )
  sync
  return  $x + y$ ;
  
```

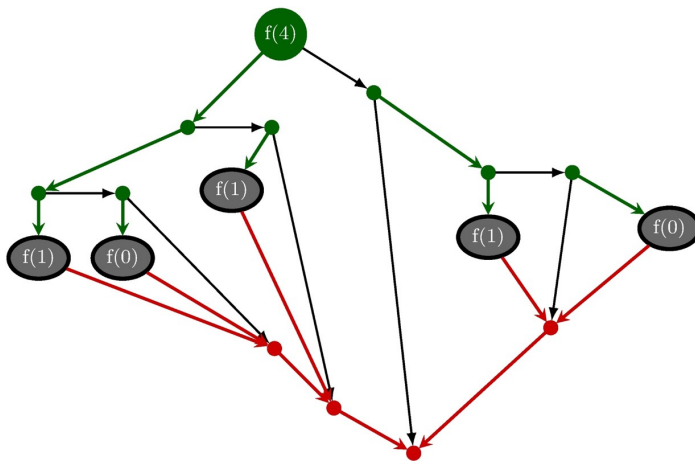
P-Fib Task Graph



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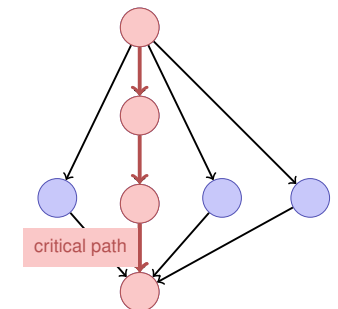
P-Fib Task Graph



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Question

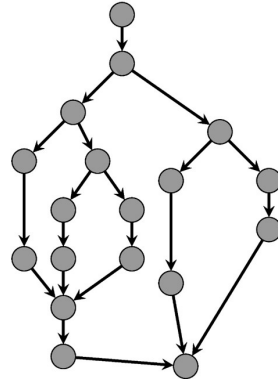
- Each Node (task) takes 1 time unit.
- Arrows depict dependencies.
- Minimal execution time when number of processors = ∞ ?



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Performance Model

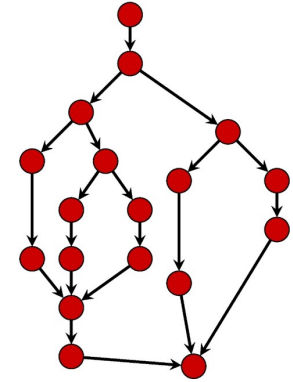
- p processors
- Dynamic scheduling
- T_p : Execution time on p processors



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Performance Model

- T_p : Execution time on p processors
- T_1 : *work*: time for executing total work on one processor
- T_1/T_p : Speedup



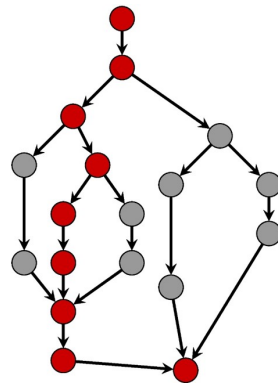
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Performance Model

- T_∞ : *span*: critical path, execution time on ∞ processors. Longest path from root to sink.
- T_1/T_∞ : *Parallelism*: wider is better
- Lower bounds:

$$T_p \geq T_1/p \quad \text{Work law}$$

$$T_p \geq T_\infty \quad \text{Span law}$$



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Greedy Scheduler

Greedy scheduler: at each time it schedules as many as available tasks.

Theorem

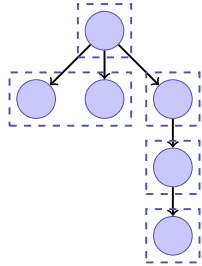
On an ideal parallel computer with p processors, a greedy scheduler executes a multi-threaded computation with work T_1 and span T_∞ in time

$$T_p \leq T_1/p + T_\infty$$

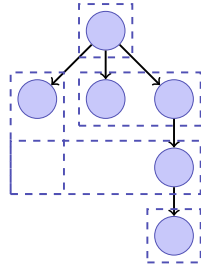
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Beispiel

Assume $p = 2$.



$$T_p = 5$$



$$T_p = 4$$

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Proof of the Theorem

Assume that all tasks provide the same amount of work.

- Complete step: p tasks are available.
- incomplete step: less than p steps available.

Assume that number of complete steps larger than $\lfloor T_1/p \rfloor$. Executed work $\geq \lfloor T_1/p \rfloor \cdot p + p = T_1 - T_1 \bmod p + p > T_1$. Contradiction. Therefore maximally $\lfloor T_1/p \rfloor$ complete steps.

We now consider the graph of tasks to be done. Any maximal (critical) path starts with a node t with $\deg^-(t) = 0$. An incomplete step executes all available tasks t with $\deg^-(t) = 0$ and thus decreases the length of the span. Number incomplete steps thus limited by T_∞ .

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Consequence

if $p \ll T_1/T_\infty$, i.e. $T_\infty \ll T_1/p$, then $T_p \approx T_1/p$.

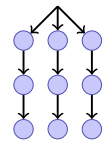
Example Fibonacci

$T_1(n)/T_\infty(n) = \Theta(\phi^n/n)$. For moderate sizes of n we can use a lot of processors yielding linear speedup.

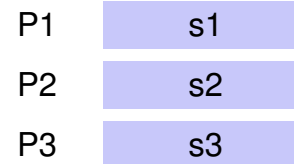
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Granularity: how many tasks?

- #Tasks = #Cores?
- Problem if a core cannot be fully used
- Example: 9 units of work. 3 core.
Scheduling of 3 sequential tasks.

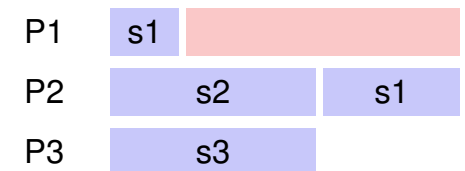


Exclusive utilization:



Execution Time: 3 Units

Foreign thread disturbing:

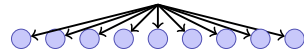


Execution Time: 5 Units

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Granularity: how many tasks?

- #Tasks = Maximum?
- Example: 9 units of work. 3 cores. Scheduling of 9 sequential tasks.



Exclusive utilization:

P1	s1	s4	s7
P2	s2	s5	s8
P3	s3	s6	s9

Execution Time: $3 + \varepsilon$ Units

Foreign thread disturbing:

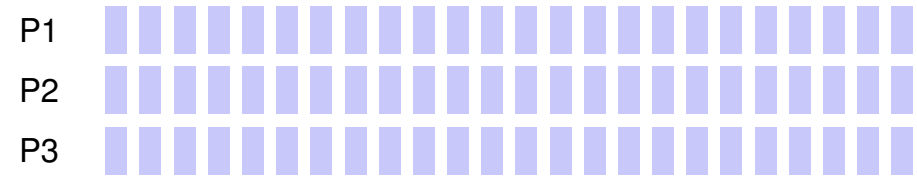
P1	s1			
P2	s2	s4	s5	s8
P3	s3	s6	s7	s9

Execution Time: 4 Units. Full utilization.

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Granularity: how many tasks?

- #Tasks = Maximum?
- Example: 10^6 tiny units of work.



Execution time: dominiert vom Overhead.

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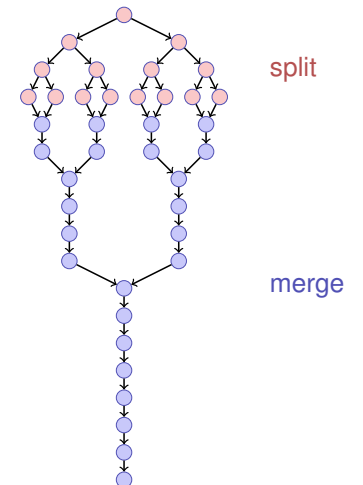
Granularity: how many tasks?

Answer: as many tasks as possible with a sequential cutoff such that the overhead can be neglected.

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Example: Parallelism of Mergesort

- Work (sequential runtime) of Mergesort $T_1(n) = \Theta(n \log n)$.
- Span $T_\infty(n) = \Theta(n)$
- Parallelism $T_1(n)/T_\infty(n) = \Theta(\log n)$ (Maximally achievable speedup with $p = \infty$ processors)



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