Datenstrukturen und Algorithmen

Exercise 7

FS 2018

Program of today

1. Feedback of last exercise(s)
2. Repetition theory

Feedback

Open hashing:

- $h'(k) = \lceil \ln(k+1) \rceil \bmod q$

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- $s(j,k) = ((k \cdot j) \bmod q) + 1$

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- $s(j,k) = ((k \cdot j) \bmod q) + 1 \rightarrow$ not suitable: 1 if k is multiple of q, and range $p-q$ is not covered

Feedback

Coocoo hashing

- $h_1(k) = k \bmod 5$, $h_2(k) = \lfloor k/5 \rfloor \bmod 5$
- add 27, 2, 32

```
T_1: __, __, 27, __, __        T_2: __, __, __, __, __

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T_1: __, __, 27, __, __        T_2:  2, 32, __, __, __
```

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Coocoo hashing

- $h_1(k) = k \bmod 5$, $h_2(k) = \lfloor k/5 \rfloor \bmod 5$
- add 7: infinite loop

```
     T_1: __, __, 27, __, __    T_2:  2, 32, __, __, __
 7:  T_1: __, __,  7, __, __    T_2: 27, 32, __, __, __
 2:  T_1: __, __,  2, __, __    T_2: 27,  7, __, __, __
32:  T_1: __, __, 32, __, __    T_2:  2,  7, __, __, __
27:  T_1: __, __, 27, __, __    T_2:  2, 32, __, __, __
 7:  ...
```

Feedback

Finding a Sub-Array

```
// calculating hash_a, hash_b, c_to_k
It1 window_end = from;
for(It2 current = begin; current != end;
    ++current, ++window_end) {
  if(window_end == to) return to;
  hash_b = (C * hash_b % M + *current) % M;
  hash_a = (C * hash_a % M + *window_end) % M;
  c_to_k = c_to_k * C % M;
}
```

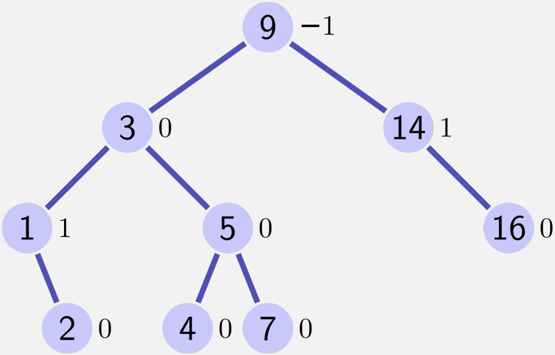
Feedback

Finding a Sub-Array

```
// looking for b and updating hash_a
for(It1 window_begin = from;
        ; ++window_begin, ++window_end) {
  if(hash_a == hash_b)
    if(std::equal(window_begin, window_end, begin, end))
      return window_begin;
  if(window_end == to) return to;
  hash_a = (C * hash_a % M + *window_end
             + (M - c_to_k) * *window_begin % M) % M;
}
```

AVL insertion

- Given an AVL tree, is there an order that produces the same tree and does not cause any rotations



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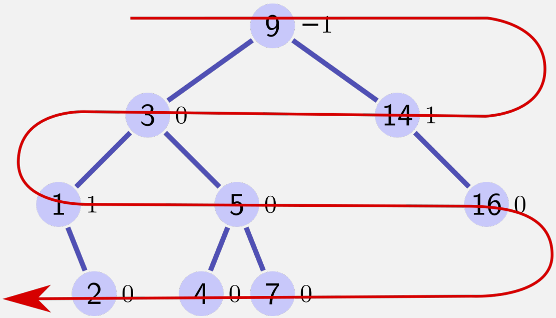

AVL insertion - sketch of proof

- Any sequence that keeps the height order intact is fine
- Proof?
- By induction over the height of the tree.

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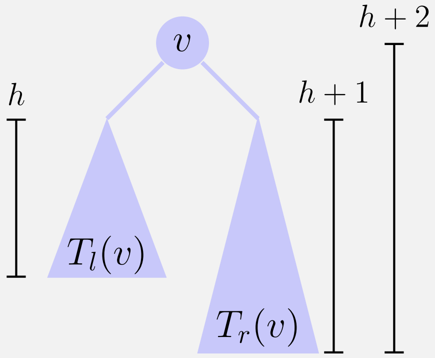
- By induction over the height of the tree.
- Hypothesis: Keys at height h and lower are placed in the same place and do not cause insertion.
- Step: Show that the traversal is the same as in the original tree, yields same position. Use AVL property of T to show that there will not be a height difference bigger than 1, and therefore no rotation.

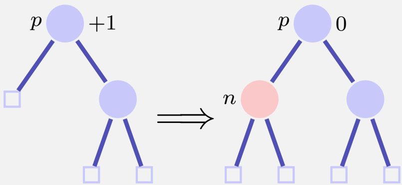
2. Repetition theory

AVL Condition

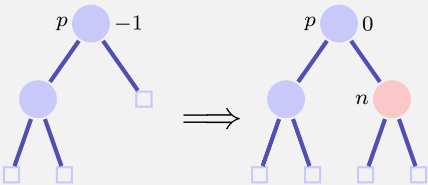
AVL Condition: for each node v of a tree $\mathrm{bal}(v) \in \{-1, 0, 1\}$



Balance at Insertion Point



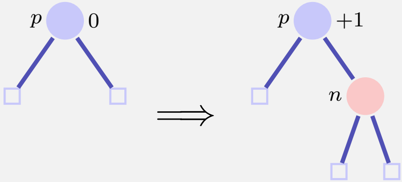
case 1: $\mathrm{bal}(p) = +1$



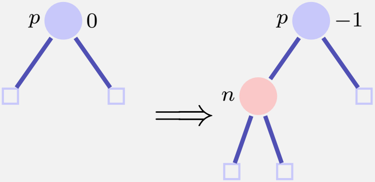
case 2: $\mathrm{bal}(p) = -1$

Finished in both cases because the subtree height did not change

Balance at Insertion Point



case 3.1: $\mathrm{bal}(p) = 0$ right



case 3.2: $\mathrm{bal}(p) = 0$, left

Not finished in both case. Call of `upin(p)`

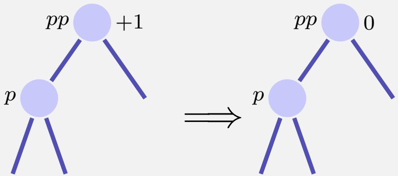
upin(p) - invariant

When **`upin(p)`** is called it holds that

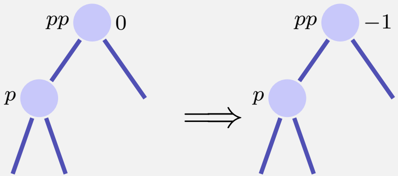
- the subtree from p has grown and
- $\mathrm{bal}(p) \in \{-1, +1\}$

upin(p)

Assumption: p is left son of pp[1]



case 1: $\mathrm{bal}(pp) = +1$, done.



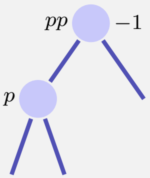
case 2: $\mathrm{bal}(pp) = 0$, `upin(pp)`

In both cases the AVL-Condition holds for the subtree from pp

[1] If p is a right son: symmetric cases with exchange of $+1$ and -1

upin(p)

Assumption: p is left son of pp



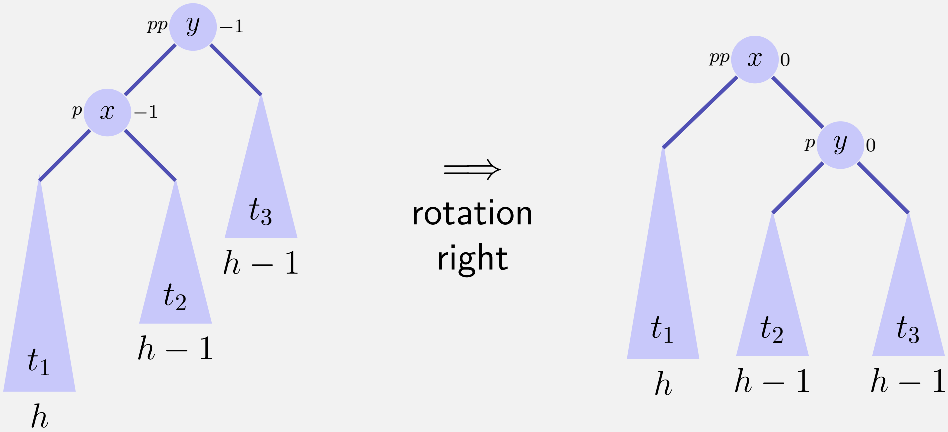
case 3: $\mathrm{bal}(pp) = -1,$

This case is problematic: adding n to the subtree from pp has violated the AVL-condition. Re-balance!

Two cases $\mathrm{bal}(p) = -1$, $\mathrm{bal}(p) = +1$

Rotationen

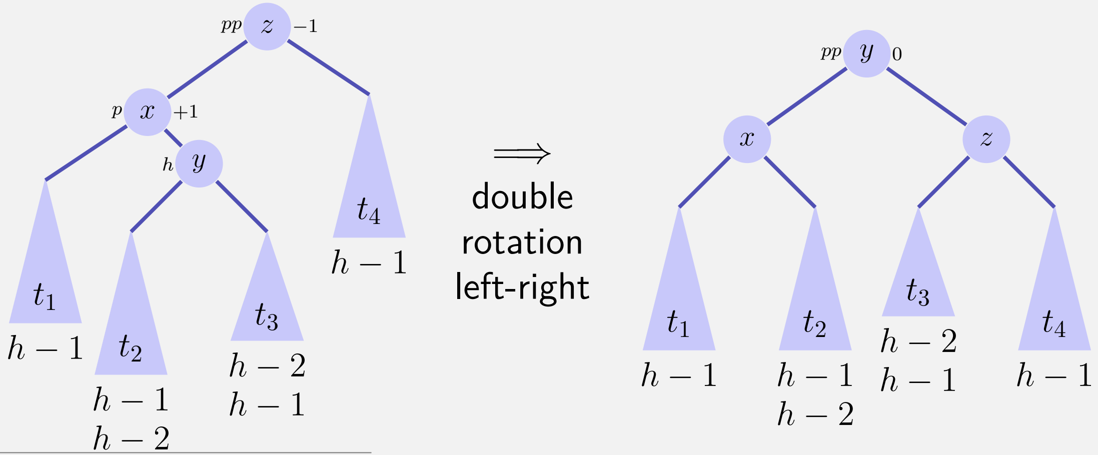
case 1.1 $\mathrm{bal}(p) = -1$. [2]



[2] p right son: $\mathrm{bal}(pp) = \mathrm{bal}(p) = +1$, left rotation

Rotationen

case 1.2 $\mathrm{bal}(p) = +1.$ [3]



[3] p right son: $\mathrm{bal}(pp) = +1$, $\mathrm{bal}(p) = -1$, double rotation right left

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Longest ascending Sequence in matrix

Given $n \times m$ matrix A:

9	27	42	41	48
35	39	8	3	5
12	49	2	38	4
15	47	29	28	6
19	1	25	33	10

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Wanted longest ascending sequence:

$$4, 6, 28, 29, 47, 49$$

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- What is the meaning of each entry?
  - In $T[x][y]$ is the length of the longest ascending sequence that ends in $A[x][y]$
  - In $S[x][y]$ are the coordinates of the predecessor in ascending sequence (if exists)

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- Arbitrary order, if entry is already computed skip it otherwise compute for smaller neighbor recursively.

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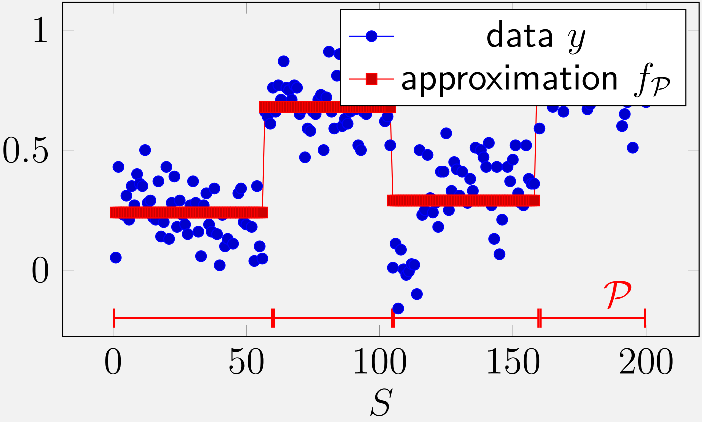
    - Consider all entries to find one with a longest sequence. From there, we can reconstruct the solution by following the corresponding predecessors.

Piecewise Constant Approximation



Piecewise Constant Approximation

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- $\mathcal{P}$: (set of intervals I_i, such that $\cup_i I_i = S$).
- **Goal:** find the partition $\hat{\mathcal{P}}$ such that $H_{\gamma,y}(\hat{\mathcal{P}})$ is minimal
- Utilize: efficient computation of the mean using prefix sums (exercise 1): $\mu_I = \frac{1}{|I|} \sum_{i \in I} y_i$

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- **Goal:** find the partition $\hat{\mathcal{P}}$ such that $H_{\gamma,y}(\hat{\mathcal{P}})$ is minimal
- **Dynamic programming**: definition of the table, computation of an entry, calculation order, extracting solution
- Utilize[§]: $H_{\gamma,y}(\mathcal{P} \cup \{[l,r)\}) = H_{\gamma,y}(\mathcal{P}) + \gamma + e_{[l,r)}$

[§]Assumption: $\mathcal{P} \cup \{[l,r)\}$ is a partition

Questions?