12. Dictionaries

Dictionary, Self-ordering List, Implementation of Dictionaries with Array / List /Skip lists. [Ottman/Widmayer, Kap. 3.3,1.7, Cormen et al, Kap. Problem 17-5]

Dictionary

ADT to manage keys from a set $\mathcal{K}$ with operations

- `insert`(k, D): Insert $k \in \mathcal{K}$ to the dictionary D. Already exists $\Rightarrow$ error messsage.
- `delete`(k, D): Delete k from the dictionary D. Not existing $\Rightarrow$ error message.
- `search`(k, D): Returns `true` if $k \in D$, otherwise `false`

Idea

Implement dictionary as sorted array

Worst case number of fundamental operations

Search	$\mathcal{O}(\log n)$ ☺
Insert	$\mathcal{O}(n)$ ☹
Delete	$\mathcal{O}(n)$ ☹

Other idea

Implement dictionary as a linked list

Worst case number of fundamental operations

Search	$\mathcal{O}(n)$ ☹
Insert	$\mathcal{O}(1)$[13] ☺
Delete	$\mathcal{O}(n)$ ☹

[13] Provided that we do not have to check existence.

Self Ordered Lists

Problematic with the adoption of a linked list: linear search time

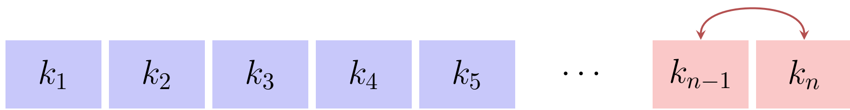
Idea: Try to order the list elements such that accesses over time are possible in a faster way

For example

- Transpose: For each access to a key, the key is moved one position closer to the front.
- Move-to-Front (MTF): For each access to a key, the key is moved to the front of the list.

Transpose

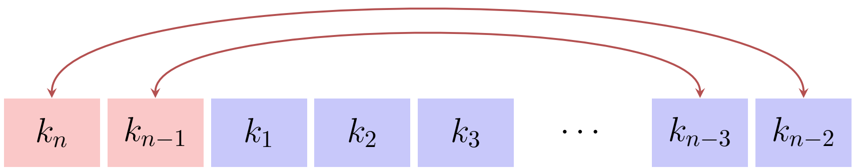
Transpose:



Worst case: Alternating sequence of n accesses to k_{n-1} and k_n. Runtime: $\Theta(n^2)$

Move-to-Front

Move-to-Front:



Alternating sequence of n accesses to k_{n-1} and k_n. Runtime: $\Theta(n)$

Also here we can provide a sequence of accesses with quadratic runtime, e.g. access to the last element. But there is no obvious strategy to counteract much better than MTF..

Analysis

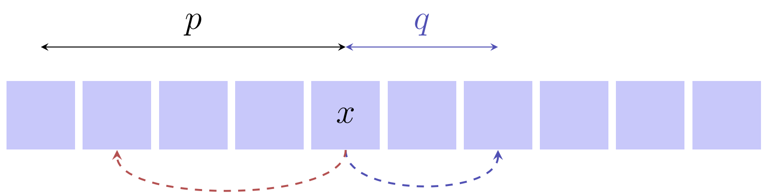
Compare MTF with the best-possible competitor (algorithm) A. How much better can A be?

Assumption: MTF and A may only move the accessed element. MTF and A start with the same list. Let M_k and A_k designate the lists after the kth step. $M_0 = A_0$.

Analysis

Costs:

- Access to x: position p of x in the list.
- No further costs, if x is moved before p
- Further costs q for each element that x is moved back starting from p.



Amortized Analysis

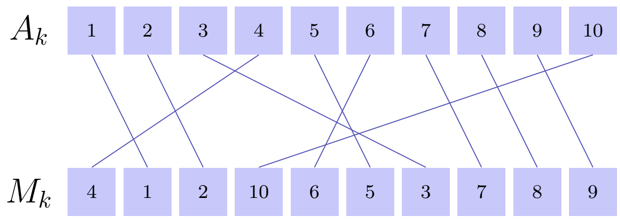
Let an arbitrary sequence of search requests be given and let $G_k^{(M)}$ and $G_k^{(A)}$ the costs in step k for Move-to-Front and A, respectively. Want estimation of $\sum_k G_k^{(M)}$ compared with $\sum_k G_k^{(A)}$.

$\Rightarrow$ Amortized analysis with potential function Φ.

Potential Function

Potential function Φ = Number of inversions of A vs. MTF.

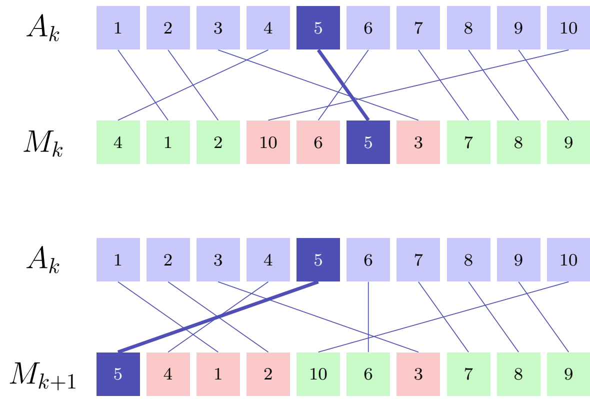
Inversion = Pair x, y such that for the positions of a and y
$p^{(A)}(x) < p^{(A)}(y) \wedge p^{(M)}(x) > p^{(M)}(x)$ or
$p^{(A)}(x) > p^{(A)}(y) \wedge p^{(M)}(x) < p^{(M)}(x)$



#inversion = #crossings

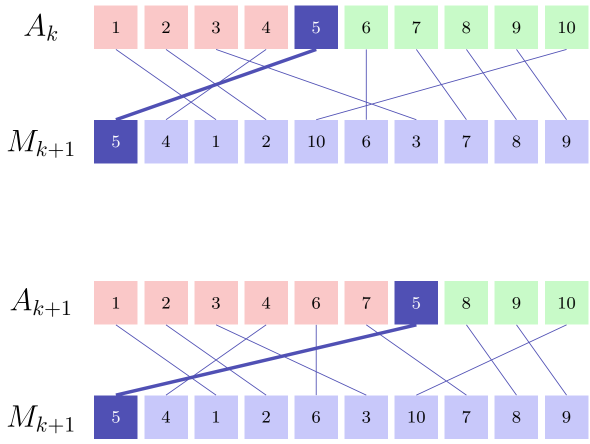
Estimating the Potential Function: MTF

- Element i at position $p_i := p^{(M)}(i)$.
- access costs $C_k^{(M)} = p_i$.
- x_i: Number elements that are in M before p_i and in A after i .
- MTF removes x_i inversions.
- $p_i - x_i - 1$: Number elements that are in M are before p_i and in A are before i.
- MTF generates $p_i - 1 - x_i$ inversions.



Estimating the Potential Function: A

- (Wlog) element i at position i.
- $X_k^{(A)}$: number movements to the back (otherwise 0).
- access costs for i: $C_k^{(A)} = i$
- A increases the number of inversions by $X_k^{(A)}$.



Estimation

$$\Phi_{k+1} - \Phi_k = \leq -x_i + (p_i - 1 - x_i) + X_k^{(A)}$$

Amortized costs of MTF in step k:

$$\begin{aligned} a_k^{(M)} &= C_k^{(M)} + \Phi_{k+1} - \Phi_k \\ &\leq p_i - x_i + (p_i - 1 - x_i) + X_k^{(A)} \\ &= (p_i - x_i) + (p_i - x_i) - 1 + X_k^{(A)} \\ &\leq C_k^{(A)} + C_k^{(A)} - 1 + X_k^{(A)}. \end{aligned}$$

Estimation

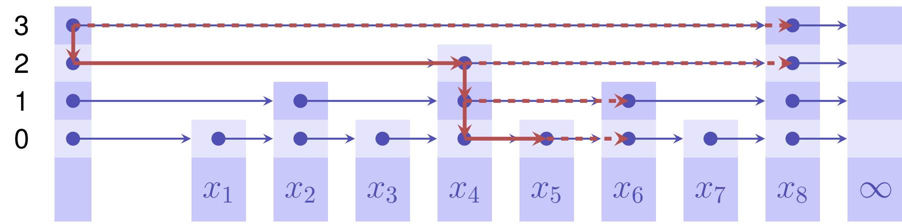
Summing up costs

$$\begin{aligned} \sum_k G_k^{(M)} = \sum_k C_k^{(M)} \leq \sum_k a_k^{(M)} &\leq \sum_k 2 \cdot C_k^{(A)} - 1 + X_k^{(A)} \\ &\leq \sum_k 2 \cdot C_k^{(A)} + X_k^{(A)} \leq 2 \cdot \sum_k C_k^{(A)} + X_k^{(A)} \\ &= 2 \cdot \sum_k G_k^{(A)} \end{aligned}$$

In the worst case MTF requires at most twice as many operations as the optimal strategy.

Cool idea: skip lists

Perfect skip list



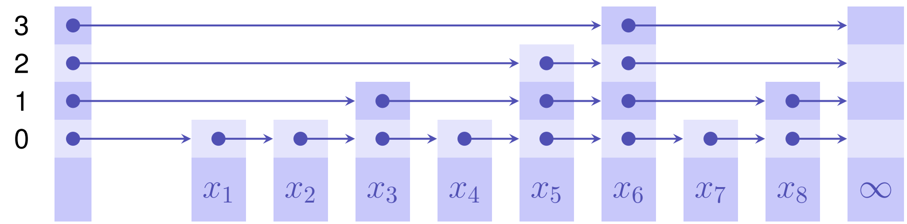
$x_1 \leq x_2 \leq x_3 \leq \cdots \leq x_9$.
Example: search for a key x with $x_5 < x < x_6$.

Analysis perfect skip list (worst cases)

Search in $\mathcal{O}(\log n)$. Insert in $\mathcal{O}(n)$.

Randomized Skip List

Idea: insert a key with random height H with $\mathbb{P}(H = i) = \frac{1}{2^{i+1}}$.



Analysis Randomized Skip List

Theorem

The expected number of fundamental operations for Search, Insert and Delete of an element in a randomized skip list is $\mathcal{O}(\log n)$.

The lengthy proof that will not be presented in this courseobserves the length of a path from a searched node back to the starting point in the highest level.

13. C++ advanced (III): Functors and Lambda

13.1 Appendix to previous C++ chapters

Appendix about Move-Semantics

```
// nonsense implementation of a "vector" for demonstration purposes
class vec{
public:
  vec () {
        std::cout << "default constructor\n";}
  vec (const vec&) {
        std::cout << "copy constructor\n";}
  vec& operator = (const vec&) {
        std::cout << "copy assignment\n"; return *this;}
  ~vec() {}
};
```


How many Copy Operations?

```
vec operator + (const vec& a, const vec& b){
    vec tmp = a;
    // add b to tmp
    return tmp;
}

int main (){
    vec f;
    f = f + f + f + f;
}
```

Output
default constructor
copy constructor
copy constructor
copy constructor
copy assignment

4 copies of the vector

Appendix about Move-Semantics

```
// nonsense implementation of a "vector" for demonstration purposes
class vec{
public:
  vec () { std::cout << "default constructor\n";}
  vec (const vec&) { std::cout << "copy constructor\n";}
  vec& operator = (const vec&) {
        std::cout << "copy assignment\n"; return *this;}
  ~vec() {}
  // new: move constructor and assignment
  vec (vec&&) {
        std::cout << "move constructor\n";}
  vec& operator = (vec&&) {
        std::cout << "move assignment\n"; return *this;}
};
```

How many Copy Operations?

```
vec operator + (const vec& a, const vec& b){
    vec tmp = a;
    // add b to tmp
    return tmp;
}

int main (){
    vec f;
    f = f + f + f + f;
}
```

Output
default constructor
copy constructor
copy constructor
copy constructor
move assignment

3 copies of the vector

How many Copy Operations?

```
vec operator + (vec a, const vec& b){
    // add b to a
    return a;
}

int main (){
    vec f;
    f = f + f + f + f;
}
```

Output
default constructor
copy constructor
move constructor
move constructor
move constructor
move assignment

1 copy of the vector

Explanation: move semantics are applied when an x-value (expired value) is assigned. R-value return values of a function are examples of x-values.

http://en.cppreference.com/w/cpp/language/value_category

How many Copy Operations?

```
void swap(vec& a, vec& b){
    vec tmp = a;
    a=b;
    b=tmp;
}

int main (){
    vec f;
    vec g;
    swap(f,g);
}
```

Output
default constructor
default constructor
copy constructor
copy assignment
copy assignment

3 copies of the vector

Forcing x-values

```
void swap(vec& a, vec& b){
    vec tmp = std::move(a);
    a=std::move(b);
    b=std::move(tmp);
}
int main (){
    vec f;
    vec g;
    swap(f,g);
}
```

Output
default constructor
default constructor
move constructor
move assignment
move assignment

0 copies of the vector

Explanation: With std::move an l-value expression can be transformed into an x-value. Then move-semantics are applied. http://en.cppreference.com/w/cpp/utility/move

13.2 Functors and Lambda-Expressions

Functors: Motivation

A simple output filter

```
template <typename T, typename function>
void filter(const T& collection, function f){
    for (const auto& x: collection)
        if (f(x)) std::cout << x << " ";
    std::cout << "\n";
}
```


Functors: Motivation

```
template <typename T, typename function>
void filter(const T& collection, function f);

template <typename T>
bool even(T x){
    return x % 2 == 0;
}

std::vector<int> a {1,2,3,4,5,6,7,9,11,16,19};
filter(a,even<int>); // output: 2,4,6,16
```


Functor: object with overloaded operator ()

```
class LargerThan{
  int value; // state
  public:
  LargerThan(int x):value{x}{};

  bool operator() (int par){
    return par > value;
  }
};
```

Functor is a callable object. Can be understood as a stateful function.

```
std::vector<int> a {1,2,3,4,5,6,7,9,11,16,19};
int value=8;
filter(a,LargerThan(value)); // 9,11,16,19
```

Functor: object with overloaded operator ()

```
template <typename T>
class LargerThan{
    T value;
public:
    LargerThan(T x):value{x}{};

    bool operator() (T par){
        return par > value;
    }
};
```

also works with a template, of course

```
std::vector<int> a {1,2,3,4,5,6,7,9,11,16,19};
int value=8;
filter(a,LargerThan<int>(value)); // 9,11,16,19
```


The same with a Lambda-Expression

```
std::vector<int> a {1,2,3,4,5,6,7,9,11,16,19};
int value=8;

filter(a, [value](int x) {return x>value;} );
```


Sum of Elements – Old School

```
std::vector<int> a {1,2,3,4,5,6,7,9,11,16,19};
int sum = 0;
for (auto x: a)
  sum += x;
std::cout << sum << "\n"; // 83
```


Sum of Elements – with Functor

```
template <typename T>
struct Sum{
    T & value = 0;
    Sum (T& v): value{v} {}

    void operator() (T par){
        value += par;
    }
};

std::vector<int> a {1,2,3,4,5,6,7,9,11,16,19};
int s=0;
Sum<int> sum(s);
sum = std::for_each(a.begin(), a.end(), sum);
std::cout << s << "\n"; // 83
```

Sum of Elements – with Λ

```
std::vector<int> a {1,2,3,4,5,6,7,9,11,16,19};
int s=0;

std::for_each(a.begin(), a.end(), [&s] (int x) {s += x;} );

std::cout << s << "\n";
```


Sorting, different

```
// pre: i >= 0
// post: returns sum of digits of i
int q(int i){
    int res =0;
    for(;i>0;i/=10)
        res += i % 10;
    return res;
}

std::vector<int> v {10,12,9,7,28,22,14};
std::sort (v.begin(), v.end(),
  [] (int i, int j) { return q(i) < q(j);}
);
```

Now $v = 10, 12, 22, 14, 7, 9, 28$ (sorted by sum of digits)

Lambda-Expressions in Detail

```
[value] (int x) ->bool {return x>value;}
```

- `[value]`: capture
- `(int x)`: parameters
- `->bool`: return type
- `{return x>value;}`: statement

Closure

```
[value] (int x) ->bool {return x>value;}
```

- Lambda expressions evaluate to a temporary object – a closure
- The closure retains the execution context of the function, the captured objects.
- Lambda expressions can be implemented as functors.

Simple Lambda Expression

```
[]()->void {std::cout << "Hello World";}
```

call:

```
[]()->void {std::cout << "Hello World";}();
```


Minimal Lambda Expression

```
[]{}
```

- Return type can be inferred if ≤ 1 return statement.

  ```
  []() {std::cout << "Hello World";}
  ```

- If no parameters and no return type, then () can be omitted.

  ```
  []{std::cout << "Hello World";}
  ```

- `[...]` can never be omitted.

Examples

```
[](int x, int y) {std::cout << x * y;} (4,5);
```

Output: 20

Examples

```
int k = 8;
[](int& v) {v += v;} (k);
std::cout << k;
```

Output: 16

Examples

```
int k = 8;
[](int v) {v += v;} (k);
std::cout << k;
```

Output: 8

Capture – Lambdas

For Lambda-expressions the capture list determines the context accessible

Syntax:

- `[x]`: Access a copy of x (read-only)
- `[&x]`: Capture x by reference
- `[&x,y]`: Capture x by reference and y by value
- `[&]`: Default capture all objects by reference in the scope of the lambda expression
- `[=]`: Default capture all objects by value in the context of the Lambda-Expression

Capture – Lambdas

```
int elements=0;
int sum=0;
std::for_each(v.begin(), v.end(),
  [&] (int k) {sum += k; elements++;} // capture all by reference
)
```


Capture – Lambdas

```
template <typename T>
void sequence(vector<int> & v, T done){
  int i=0;
  while (!done()) v.push_back(i++);
}

vector<int> s;
sequence(s, [&] {return s.size() >= 5;} )
```

now v = 0 1 2 3 4

The capture list refers to the context of the lambda expression.

Capture – Lambdas

When is the value captured?

```
int v = 42;
auto func = [=] {std::cout << v << "\n"};
v = 7;
func();
```

Output: 42

Values are assigned when the lambda-expression is created.

Capture – Lambdas

(Why) does this work?

```
class Limited{
  int limit = 10;
 public:
  // count entries smaller than limit
  int count(const std::vector<int>& a){
    int c = 0;
    std::for_each(a.begin(), a.end(),
        [=,&c] (int x) {if (x < limit) c++;}
    );
    return c;
  }
};
```

The `this` pointer is implicitly copied by value

Capture – Lambdas

```
struct mutant{
  int i = 0;
  void do(){ [=] {i=42;}();}
};

mutant m;
m.do();
std::cout << m.i;
```

Output: 42

The `this` *pointer* is implicitly copied by value

Lambda Expressions are Functors

```
[x, &y] () {y = x;}
```

can be implemented as

```
unnamed {x,y};
```

with

```
class unnamed {
  int x; int& y;
  unnamed (int x_, int& y_) : x (x_), y (y_) {}
  void operator () () {y = x;}};
};
```

Lambda Expressions are Functors

```
[=] () {return x + y;}
```

can be implemented as

```
unnamed {x,y};
```

with

```
class unnamed {
  int x; int y;
  unnamed (int x_, int y_) : x (x_), y (y_) {}
  int operator () () {return x + y;}
};
```


Polymorphic Function Wrapper std::function

```
#include <functional>

int k= 8;
std::function<int(int)> f;
f = [k](int i){ return i+k; };
std::cout << f(8); // 16
```

Kann verwendet werden, um Lambda-Expressions zu speichern.

Other Examples

`std::function<int(int,int)>`;

`std::function<void(double)>` ...

http://en.cppreference.com/w/cpp/utility/functional/function