

For a long time...

- the sequential execution became faster (Instruction Level Parallelism, Pipelining, Higher Frequencies)
- more and smaller transistors = more performance
- programmers simply waited for the next processor generation

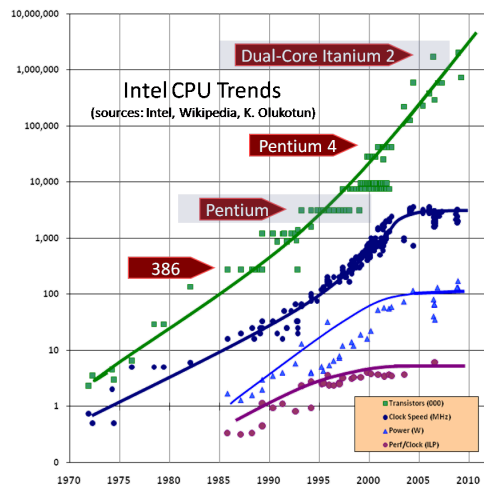
Today

- the frequency of processors does not increase significantly and more (heat dissipation problems)
- the instruction level parallelism does not increase significantly any more
- the execution speed is dominated by memory access times (but caches still become larger and faster)

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Trends



<http://www.govt.ca/publications/concurrency-ddj.htm>

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Multicore

- Use transistors for more compute cores
- Parallelism in the software
- Programmers have to write parallel programs to benefit from new hardware

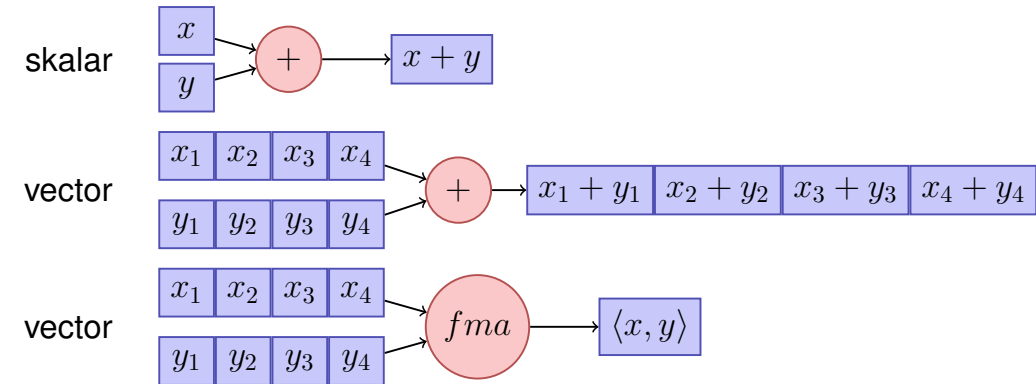
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Forms of Parallel Execution

- Vectorization
- Pipelining
- Instruction Level Parallelism
- Multicore / Multiprocessing
- Distributed Computing

Vectorization

Parallel Execution of the same operations on elements of a vector (register)



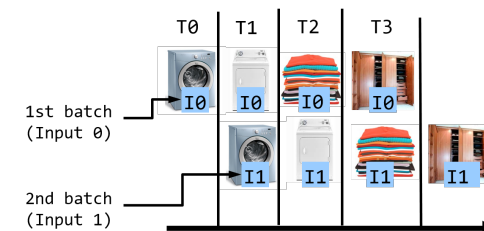
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Home Work



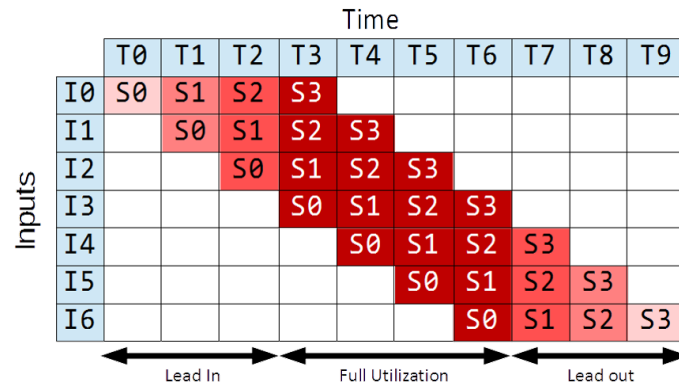
More efficient



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Pipeline



Throughput

- Throughput = Input or output data rate
- Number operations per time unit
- larger throughput is better
- Approximation

$$\text{throughput} = \frac{1}{\max(\text{computationtime}(\text{stages}))}$$

ignores lead-in and lead-out times

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Latency

- Time to perform a computation
- Pipeline latency only constant when Pipeline is balanced: sum of all operations over all stages
- Unbalanced Pipeline
 - First batch as with the balanced pipeline
 - In a balanced version, latency = $\# \text{stages} \cdot \max(\text{computationtime}(\text{stages}))$

Homework Example

Washing $T_0 = 1h$, Drying $T_1 = 2h$, Ironing $T_2 = 1h$, Tidy up $T_3 = 0.5h$

- Latency first batch: $L = T_0 + T_1 + T_2 + T_3 = 4.5h$
- Latency second batch: $L = T_1 + T_1 + T_2 + T_3 = 5.5h$
- In the long run: 1 batch every $2h$ ($0.5/h$).

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Throughput vs. Latency

- Increasing throughput can increase latency
- Stages of the pipeline need to communicate and synchronize: overhead

Pipelines in CPUs

Fetch

Decode

Execute

Data Fetch

Writeback

Multiple Stages

- Every instruction takes 5 time units (cycles)
- In the best case: 1 instruction per cycle, not always possible (“stalls”)

Paralellism (several functional units) leads to *faster execution*.

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ILP – Instruction Level Parallelism

Modern CPUs provide several hardware units and execute independent instructions in parallel.

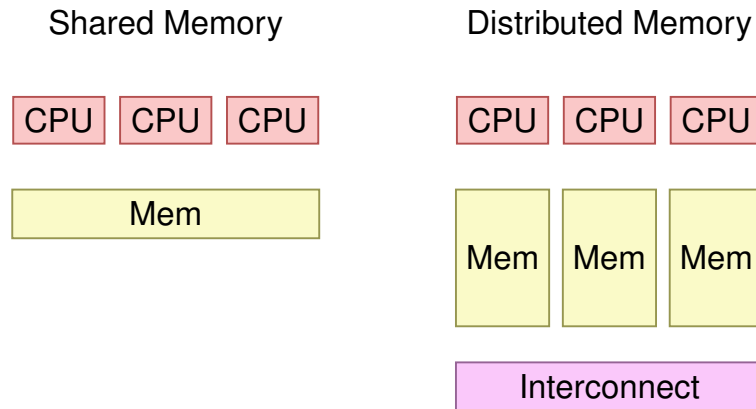
- Pipelining
- Superscalar CPUs (multiple instructions per cycle)
- Out-Of-Order Execution (Programmer observes the sequential execution)
- Speculative Execution

27.2 Hardware Architectures

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Shared vs. Distributed Memory



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Shared vs. Distributed Memory Programming

- Categories of programming interfaces
 - Communication via message passing
 - Communication via memory sharing
- It is possible:
 - to program shared memory systems as distributed systems (e.g. with message passing MPI)
 - program systems with distributed memory as shared memory systems (e.g. partitioned global address space PGAS)

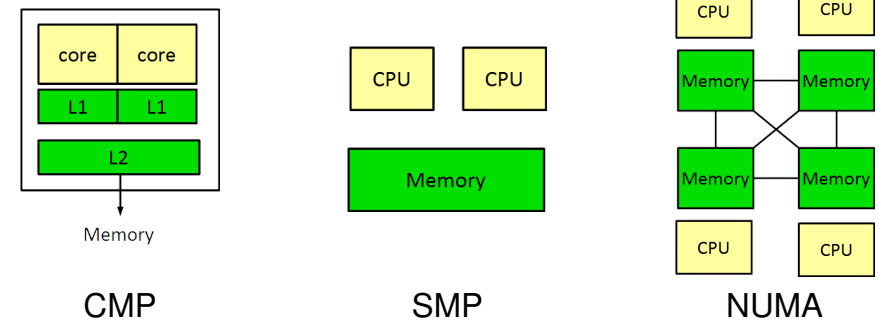
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Shared Memory Architectures

- Multicore (Chip Multiprocessor - CMP)
- Symmetric Multiprocessor Systems (SMP)
- Simultaneous Multithreading (SMT = Hyperthreading)
 - one physical core, Several Instruction Streams/Threads: several virtual cores
 - Between ILP (several units for a stream) and multicore (several units for several streams). Limited parallel performance.
- Non-Uniform Memory Access (NUMA)

Same programming interface

Overview



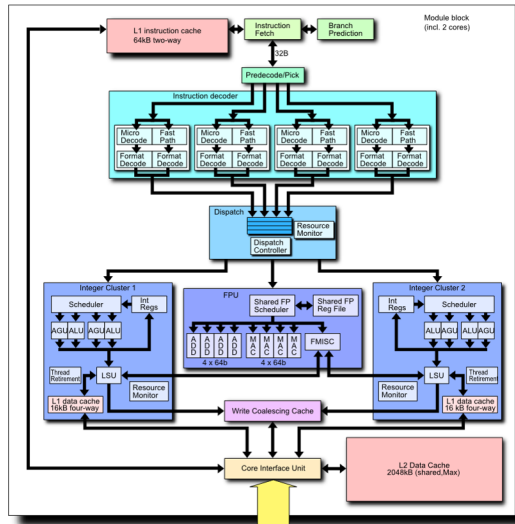
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An Example

AMD Bulldozer: between CMP and SMT

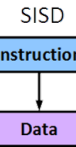
- 2x integer core
- 1x floating point core



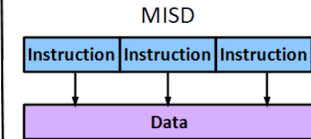
Wikipedia
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Flynn's Taxonomy

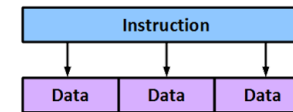
Single-Core



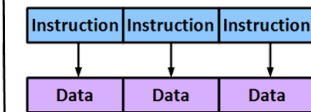
Fault-Tolerance



SIMD



MIMD



Vector Computing / GPU

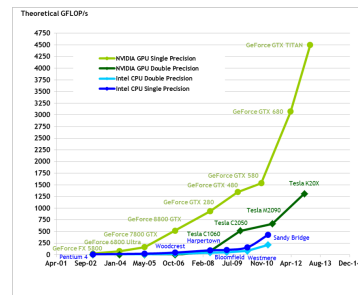
Multi-Core

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Massively Parallel Hardware

[General Purpose] Graphical Processing Units ([GP]GPUs)

- Revolution in High Performance Computing
 - Calculation 4.5 TFlops vs. 500 GFlops
 - Memory Bandwidth 170 GB/s vs. 40 GB/s
- SIMD
 - High data parallelism
 - Requires own programming model. Z.B. CUDA / OpenCL



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27.3 Multi-Threading, Parallelism and Concurrency

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Processes and Threads

- Process: instance of a program
 - each process has a separate context, even a separate address space
 - OS manages processes (resource control, scheduling, synchronisation)
- Threads: threads of execution of a program
 - Threads share the address space
 - fast context switch between threads

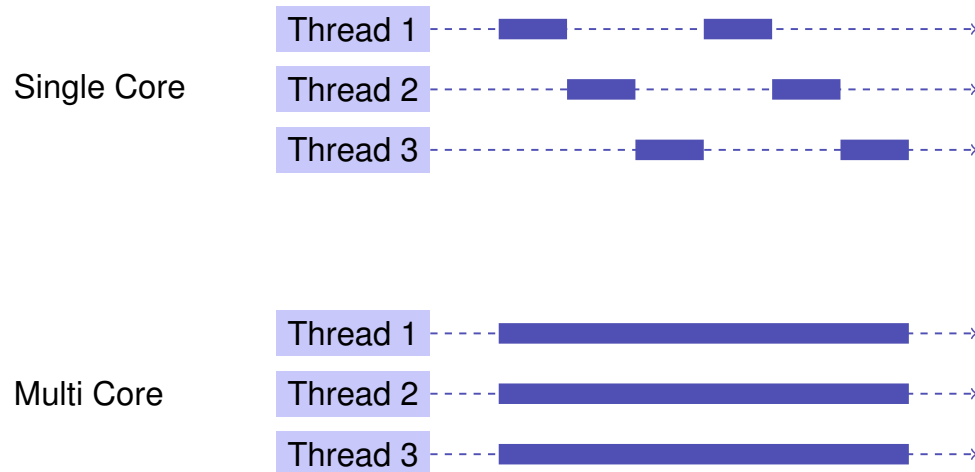
Why Multithreading?

- Avoid “polling” resources (files, network, keyboard)
- Interactivity (e.g. responsivity of GUI programs)
- Several applications / clients in parallel
- Parallelism (performance!)

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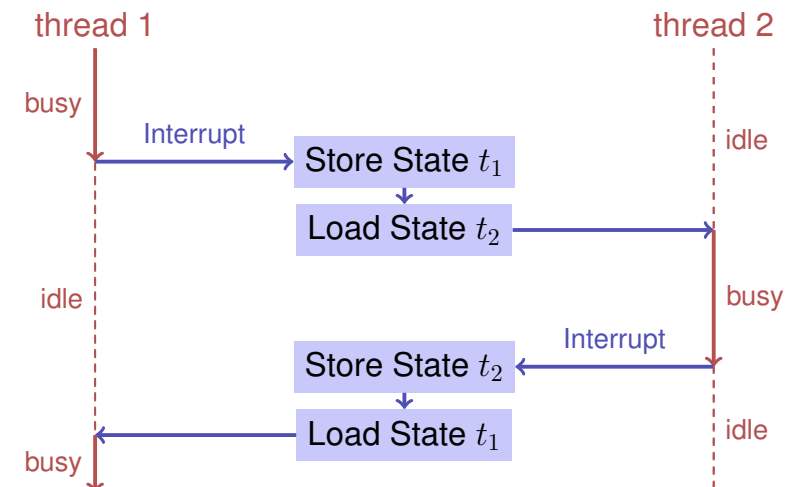
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Multithreading conceptually



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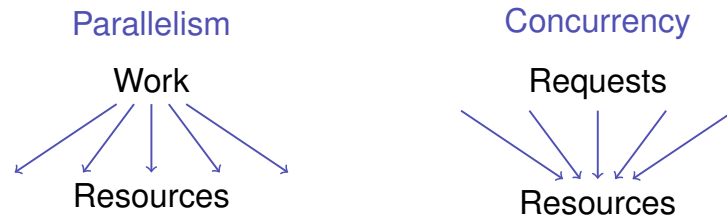
Thread switch on one core (Preemption)



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Parallelität vs. Concurrency

- **Parallelism:** Use extra resources to solve a problem faster
- **Concurrency:** Correctly and efficiently manage access to shared resources
- Begriffe überlappen offensichtlich. Bei parallelen Berechnungen besteht fast immer Synchronisierungsbedarf.



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Thread Safety

Thread Safety means that in a concurrent application of a program this always yields the desired results.

Many optimisations (Hardware, Compiler) target towards the correct execution of a *sequential* program.

Concurrent programs need an annotation that switches off certain optimisations selectively.

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Example: Caches

- Access to registers faster than to shared memory.
- Principle of locality.
- Use of Caches (transparent to the programmer)

If and how far a cache coherency is guaranteed depends on the used system.



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27.4 Scalability: Amdahl and Gustafson

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Scalability

In parallel Programming:

- Speedup when increasing number p of processors
- What happens if $p \rightarrow \infty$?
- Program scales linearly: Linear speedup.

Parallel Performance

Given a fixed amount of computing work W (number computing steps)

Sequential execution time T_1

Parallel execution time on p CPUs

- Perfection: $T_p = T_1/p$
- Performance loss: $T_p > T_1/p$ (usual case)
- Sorcery: $T_p < T_1/p$

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Parallel Speedup

Parallel speedup S_p on p CPUs:

$$S_p = \frac{W/T_p}{W/T_1} = \frac{T_1}{T_p}.$$

- Perfection: linear speedup $S_p = p$
- Performance loss: sublinear speedup $T_p > T_1/p$ (the usual case)
- Sorcery: superlinear speedup $T_p < T_1/p$

Efficiency: $E_p = S_p/p$

Reachable Speedup?

Parallel Program

Parallel Part	Seq. Part
80%	20%

$$T_1 = 10$$

$$T_8 = ?$$

$$T_8 = \frac{10 \cdot 0.8}{8} + 10 \cdot 0.2 = 1 + 2 = 3$$

$$S_8 = \frac{T_1}{T_8} = \frac{10}{3} = 3.33$$

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Amdahl's Law: Ingredients

Computational work W falls into two categories

- Paralellisable part W_p
- Not parallelisable, sequential part W_s

Assumption: W can be processed sequentially by one processor in W time units ($T_1 = W$):

$$T_1 = W_s + W_p$$
$$T_p \geq W_s + W_p/p$$

Amdahl's Law

$$S_p = \frac{T_1}{T_p} \leq \frac{W_s + W_p}{W_s + \frac{W_p}{p}}$$

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Amdahl's Law

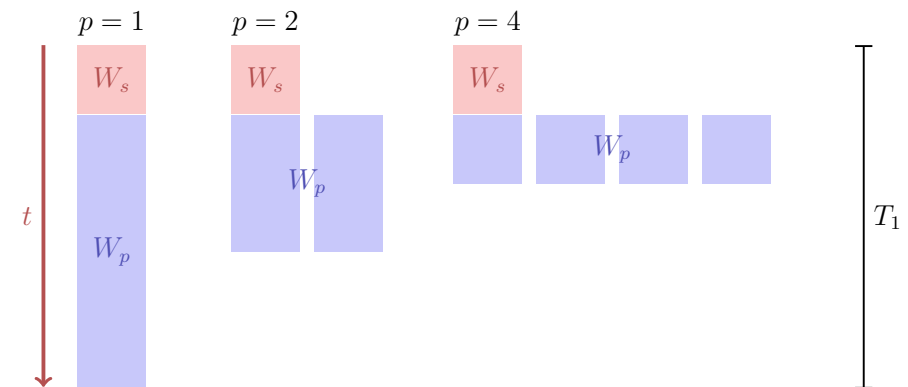
With sequential, not parallelizable fraction λ : $W_s = \lambda W$,
 $W_p = (1 - \lambda)W$:

$$S_p \leq \frac{1}{\lambda + \frac{1-\lambda}{p}}$$

Thus

$$S_\infty \leq \frac{1}{\lambda}$$

Illustration Amdahl's Law



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Amdahl's Law is bad news

All non-parallel parts of a program can cause problems

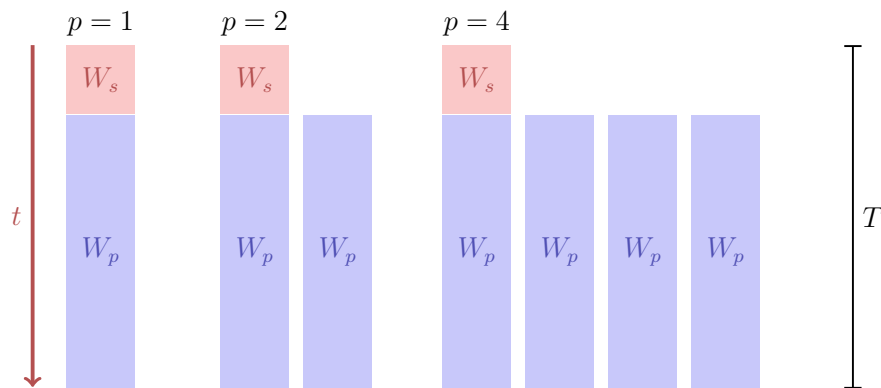
Gustafson's Law

- Fix the time of execution
- Vary the problem size.
- Assumption: the sequential part stays constant, the parallel part becomes larger

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Illustration Gustafson's Law



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Gustafson's Law

Work that can be executed by one processor in time T :

$$W_s + W_p = T$$

Work that can be executed by p processors in time T :

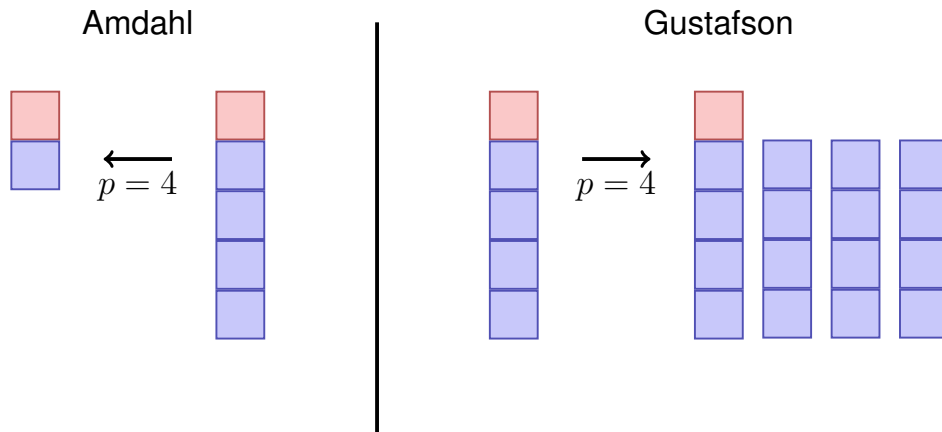
$$W_s + p \cdot W_p = \lambda \cdot T + p \cdot (1 - \lambda) \cdot T$$

Speedup:

$$\begin{aligned} S_p &= \frac{W_s + p \cdot W_p}{W_s + W_p} = p \cdot (1 - \lambda) + \lambda \\ &= p - \lambda(p - 1) \end{aligned}$$

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Amdahl vs. Gustafson



27.5 Task- and Data-Parallelism

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Parallel Programming Paradigms

- **Task Parallel:** Programmer explicitly defines parallel tasks.
- **Data Parallel:** Operations applied simultaneously to an aggregate of individual items.

Example Data Parallel (OMP)

```
double sum = 0, A[MAX];  
#pragma omp parallel for reduction (+:ave)  
for (int i = 0; i < MAX; ++i)  
    sum += A[i];  
return sum;
```

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Example Task Parallel (C++11 Threads/Futures)

```
double sum(Iterator from, Iterator to)
{
    auto len = from - to;
    if (len > threshold){
        auto future = std::async(sum, from, from + len / 2);
        return sumS(from + len / 2, to) + future.get();
    }
    else
        return sumS(from, to);
}
```

Work Partitioning and Scheduling

- Partitioning of the work into parallel task (programmer or system)
 - One task provides a unit of work
 - Granularity?
- Scheduling (Runtime System)
 - Assignment of tasks to processors
 - Goal: full resource usage with little overhead

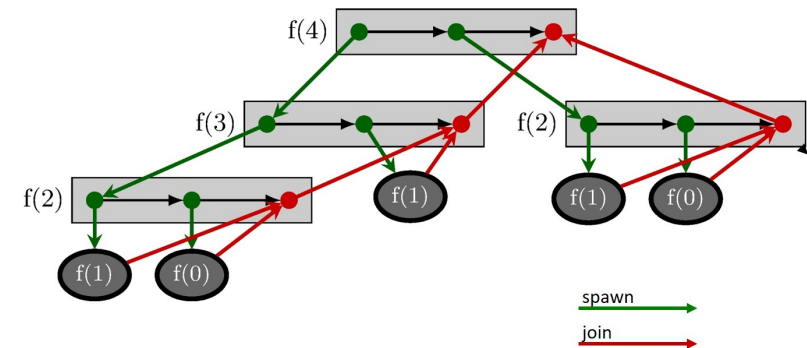
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Example: Fibonacci P-Fib

```
if  $n \leq 1$  then
    return  $n$ 
else
     $x \leftarrow \text{spawn P-Fib}(n-1)$ 
     $y \leftarrow \text{spawn P-Fib}(n-2)$ 
    sync
    return  $x + y$ ;
```

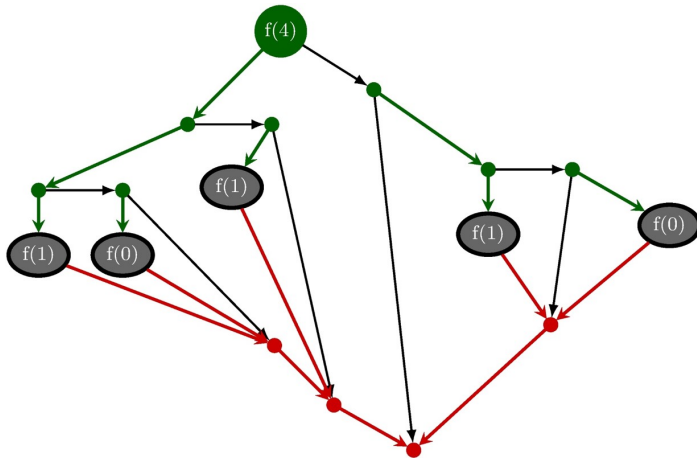
P-Fib Task Graph



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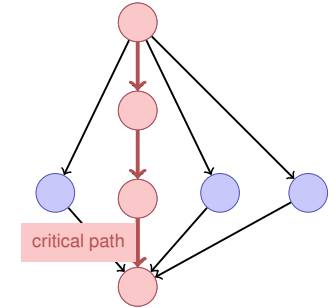
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P-Fib Task Graph



Question

- Each Node (task) takes 1 time unit.
- Arrows depict dependencies.
- Minimal execution time when number of processors = ∞ ?

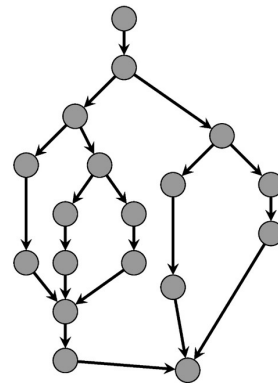


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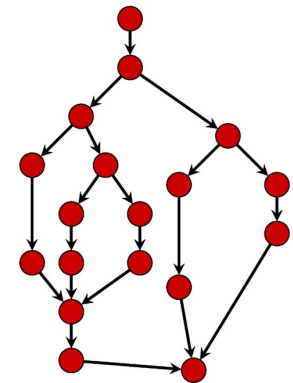
Performance Model

- p processors
- Dynamic scheduling
- T_p : Execution time on p processors



Performance Model

- T_p : Execution time on p processors
- T_1 : **work**: time for executing total work on one processor
- T_1/T_p : Speedup



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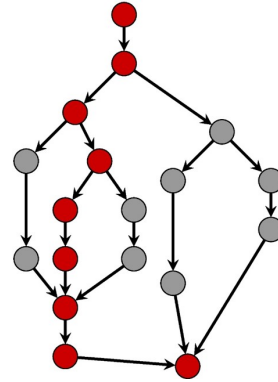
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Performance Model

- T_∞ : **span**: critical path, execution time on ∞ processors. Longest path from root to sink.
- T_1/T_∞ : **Parallelism**: wider is better
- Lower bounds:

$$T_p \geq T_1/p \quad \text{Work law}$$

$$T_p \geq T_\infty \quad \text{Span law}$$



Greedy Scheduler

Greedy scheduler: at each time it schedules as many as available tasks.

Theorem

On an ideal parallel computer with p processors, a greedy scheduler executes a multi-threaded computation with work T_1 and span T_∞ in time

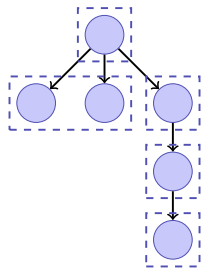
$$T_p \leq T_1/p + T_\infty$$

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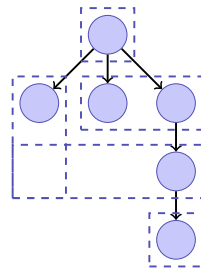
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Beispiel

Assume $p = 2$.



$$T_p = 5$$



$$T_p = 4$$

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Proof of the Theorem

Assume that all tasks provide the same amount of work.

- Complete step: p tasks are available.
- incomplete step: less than p steps available.

Assume that number of complete steps larger than $\lfloor T_1/p \rfloor$.
 Executed work $\geq P \cdot (\lfloor T_1/p \rfloor \cdot p) = T_1 - T_1 \bmod p + p \geq T_1$.
 Contradiction. Therefore maximally $\lfloor T_1/p \rfloor$ complete steps.

Each incomplete step executed at any time all available tasks t with $\deg^-(t) = 0$ and decreases the length of the span. Otherwise the chosen span would not have been maximal. Number of incomplete steps thus maximally T_∞ .

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Consequence

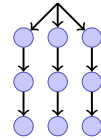
if $p \ll T_1/T_\infty$, i.e. $T_\infty \ll T_1/p$, then $T_p \approx T_1/p$.

Example Fibonacci

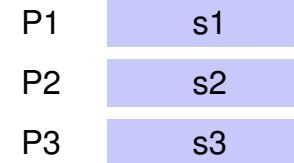
$T_1(n)/T_\infty(n) = \Theta(\phi^n/n)$. For moderate sizes of n we can use a lot of processors yielding linear speedup.

Granularity: how many tasks?

- #Tasks = #Cores?
- Problem if a core cannot be fully used
- Example: 9 units of work. 3 core.
Scheduling of 3 sequential tasks.

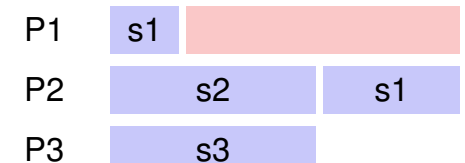


Exclusive utilization:



Execution Time: 3 Units

Foreign thread disturbing:



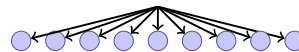
Execution Time: 5 Units

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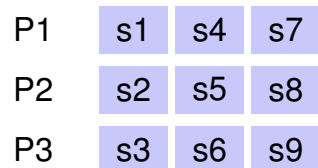
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Granularity: how many tasks?

- #Tasks = Maximum?
- Example: 9 units of work. 3 cores.
Scheduling of 9 sequential tasks.

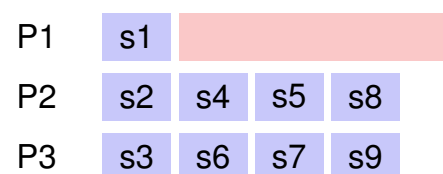


Exclusive utilization:



Execution Time: $3 + \varepsilon$ Units

Foreign thread disturbing:

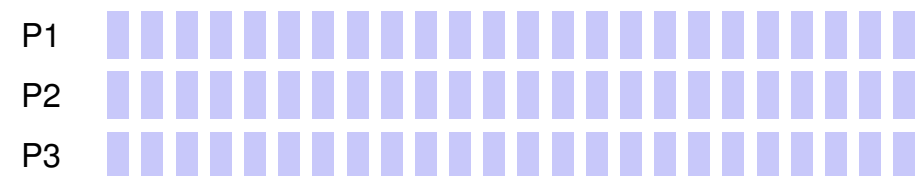


Execution Time: 4 Units. Full utilization.

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Granularity: how many tasks?

- #Tasks = Maximum?
- Example: 10^6 tiny units of work.



Execution time: dominiert vom Overhead.

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Granularity: how many tasks?

Answer: as many tasks as possible with a sequential cutoff such that the overhead can be neglected.

Example: Parallelism of Mergesort

- Work (sequential runtime) of Mergesort $T_1(n) = \Theta(n \log n)$.
- Span $T_\infty(n) = \Theta(n)$
- Parallelism $T_1(n)/T_\infty(n) = \Theta(\log n)$
(Maximally achievable speedup with $p = \infty$ processors)

