8. Functions II

Stepwise Refinement, Scope, Libraries and Standard Functions

Stepwise Refinement

- A simple technique to solve complex problems

Solve the problem step by step. Start with a coarse solution on a high level of abstraction (only comments and abstract function calls).

- At each step, comments are replaced by program text, and functions are implemented (using the same principle again).

- The refinement also refers to the development of data representation (more about this later).

- If the refinement is realized as far as possible by functions, then partial solutions emerge that might be used for other problems.

- Stepwise refinement supports (but does not replace) the structural understanding of a problem.

Example Problem

Find out if two rectangles intersect!
Coarse Solution

(int include directives omitted)

```c
int main()
{
    // input rectangles
    // intersection?
    // output solution
    return 0;
}
```

Refinement 1: Input Rectangles

Width $w$ and height $h$ may be negative.

```c
int main()
{
    std::cout << "Enter two rectangles [x y w h each] \n";
    int x1, y1, w1, h1;
    std::cin >> x1 >> y1 >> w1 >> h1;
    int x2, y2, w2, h2;
    std::cin >> x2 >> y2 >> w2 >> h2;
    // intersection?
    // output solution
    return 0;
}
```
Refinement 2: Intersection? and Output

```c
int main()
{
    input rectangles ✓
    bool clash = rectangles_intersect (x1,y1,w1,h1, x2,y2,w2,h2);
    if (clash)
        std::cout << "intersection!\n";
    else
        std::cout << "no intersection!\n";
    return 0;
}
```

Refinement 3: Intersection Function...

```c
bool rectangles_intersect (int x1, int y1, int w1, int h1,
        int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```

Refinement 3: . . . with PRE and POST

```c
// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, 
// where w1, h1, w2, h2 may be negative. 
// POST: returns true if (x1, y1, w1, h1) and 
// (x2, y2, w2, h2) intersect 
bool rectangles_intersect (int x1, int y1, int w1, int h1,
        int x2, int y2, int w2, int h2)
{
    return false; // todo
}
```
Refinement 4: Interval Intersection

Two rectangles intersect if and only if their $x$ and $y$-intervals intersect.

\[
\begin{align*}
(x_1, y_1) & \quad w_1 \quad h_1 \\
(x_2, y_2) & \quad w_2 \quad h_2 \\
[x_1, x_1 + w_1] & \quad [y_1, y_1 + h_1] \\
[x_2, x_2 + w_2] & \quad [y_2, y_2 + h_2]
\end{align*}
\]

// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
// w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect

```cpp
bool rectangles_intersect (int x1, int y1, int w1, int h1,
                         int x2, int y2, int w2, int h2)
{
    return intervals_intersect (x1, x1 + w1, x2, x2 + w2)
        && intervals_intersect (y1, y1 + h1, y2, y2 + h2);
}
```

Refinement 4: Interval Intersections

// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
// with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect

```cpp
bool intervals_intersect (int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2);
}
```

Refinement 5: Min and Max

// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
// with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect

```cpp
bool intervals_intersect (int a1, int b1, int a2, int b2)
{
    return max(a1, b1) >= min(a2, b2)
        && min(a1, b1) <= max(a2, b2);
}
```
Refinement 5: Min and Max

// POST: the maximum of x and y is returned
int max (int x, int y) {  
  if (x>y) return x; else return y;
}

// POST: the minimum of x and y is returned
int min (int x, int y) {  
  if (x<y) return x; else return y;
}

Function intervals_intersect ✓
Function rectangles_intersect ✓
Function main ✓
already exists in the standard library

Result
Clean solution of the problem
Useful functions have been implemented
intervals_intersect
rectangles_intersect
Intersection

Look what we have achieved step by step!

```
#include<iostream>
#include<algorithm>

// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
// with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect (int a1, int b1, int a2, int b2) {  
  return std::max(a1, b1) >= std::min(a2, b2)  
    && std::min(a1, b1) <= std::max(a2, b2);
}

// PRE: (x1, y1, w1, h1), (x2, y2, w2, h2) are rectangles, where
// w1, h1, w2, h2 may be negative.
// POST: returns true if (x1, y1, w1, h1),(x2, y2, w2, h2) intersect
bool rectangles_intersect (int x1, int y1, int w1, int h1,  
  int x2, int y2, int w2, int h2) {  
  return intervals_intersect (x1, x1 + w1, x2, x2 + w2)  
    && intervals_intersect (y1, y1 + h1, y2, y2 + h2);
}
```

// Back to Intervals

// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
// with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect (int a1, int b1, int a2, int b2) {  
  return std::max(a1, b1) >= std::min(a2, b2)  
    && std::min(a1, b1) <= std::max(a2, b2); ✓
}

#include<iostream>
#include<algorithm>

// PRE: [a1, b1], [a2, b2] are (generalized) intervals,
// with [a,b] := [b,a] if a>b
// POST: returns true if [a1, b1],[a2, b2] intersect
bool intervals_intersect (int a1, int b1, int a2, int b2) {  
  return std::max(a1, b1) >= std::min(a2, b2)  
    && std::min(a1, b1) <= std::max(a2, b2);
}

int main () {  
  std::cout << "Enter two rectangles [x y w h each]\n";  
  int x1, y1, w1, h1;  
  std::cin >> x1 >> y1 >> w1 >> h1;  
  int x2, y2, w2, h2;  
  std::cin >> x2 >> y2 >> w2 >> h2;  
  bool clash = rectangles_intersect (x1,y1,w1,h1,x2,y2,w2,h2);  
  if (clash)  
    std::cout << "intersection!\n";  
  else  
    std::cout << "no intersection!\n";
  return 0;
}
Where can a Function be Used?

```cpp
#include<iostream>

int main()
{
    std::cout << f(1); // Error: f undeclared
    return 0;
}

int f (int i) // Scope of f starts here
{
    return i;
}
```

Scope of a Function

- is the part of the program where a function can be called
- is defined as the union of all scopes of its declarations (there can be more than one)

Declaration of a function: like the definition but without `{ ... }`.

```cpp
double pow (double b, int e);
```

This does not work...

```cpp
#include<iostream>

int main()
{
    std::cout << f(1); // Error: f undeclared
    return 0;
}

int f (int i) // Scope of f starts here
{
    return i;
}
```

...but this works!

```cpp
#include<iostream>

int f (int i); // Gueltigkeitsbereich von f ab hier

int main()
{
    std::cout << f(1);
    return 0;
}

int f (int i)
{
    return i;
}
```
Forward Declarations, why?

Functions that mutually call each other:

```c
int g(...); // forward declaration

int f (...) // f valid from here
{
    g(...) // ok
}
```

```c
int g (...)
{
    f(...) // ok
}
```

Reusability

- Functions such as `rectanges` and `pow` are useful in many programs.
- “Solution”: copy-and-paste the source code
- Main disadvantage: when the function definition needs to be adapted, we have to change all programs that make use of the function

Level 1: Outsource the Function

```c
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e)
{
    double result = 1.0;
    if (e < 0) { // b^e = (1/b)^(-e)
        b = 1.0/b;
        e = -e;
    }
    for (int i = 0; i < e; ++i)
        result *= b;
    return result;
}
```

Level 1: Include the Function

```c
// Prog: callpow2.cpp
// Call a function for computing powers.
#include <iostream>
#include "math.cpp"
int main()
{
    std::cout << pow( 2.0, -2) << "\n";
    std::cout << pow( 1.5, 2) << "\n";
    std::cout << pow( 5.0, 1) << "\n";
    std::cout << pow(-2.0, 9) << "\n";
    return 0;
}
```
Disadvantage of Including

- `#include` copies the file (math.cpp) into the main program (callpow2.cpp).
- The compiler has to (re)compile the function definition for each program.
- This can take long for many and large functions.

Level 2: Separate Compilation

of math.cpp independent of the main program:

```cpp
double pow(double b, int e) {
    ...}
```

Level 2: Separate Compilation

Declaration of all used symbols in so-called header file.

```cpp
// PRE: e >= 0 || b != 0.0
// POST: return value is b^e
double pow(double b, int e);
```

Level 2: Separate Compilation

of the main program, independent of math.cpp, if a declaration of math is included.

```cpp
#include <iostream>
#include "math.h"
int main() {
    std::cout << pow(2, -2) << "\n";
    return 0;
}
```
The linker unites... 

The linker unites... 

... what belongs together 

math.o + callpow3.o = Executable callpow

Availability of Source Code? 

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Source code is generally available.

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Observation 

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math.cpp (source code) is not required any more when the math.o (object code) is available.

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Many vendors of libraries do not provide source code. Header files then provide the only readable informations.

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### Libraries

- Logical grouping of similar functions
  - pow
  - exp
  - log
  - sin
  - math.h
  - math.cpp

### Name Spaces...

```cpp
#include <cmath>
#include "ifmpmath.h"

int main()
{
  double x = std::pow (2.0, -2); // <cmath>
  double y = ifmp::pow (2.0, -2); // ifmpmath.h
}
```

### ...Avoid Name Conflicts

- Avoid Name Conflicts

### Functions from the Standard Library

- Help to avoid re-inventing the wheel (such as with `ifmp::pow`);
- Lead to interesting and efficient programs in a simple way;
- Guarantee a quality standard that cannot easily be achieved with code written from scratch.
Prime Number Test with $\sqrt{n}$

$n \geq 2$ is a prime number if and only if there is no $d$ in $\{2, \ldots, n-1\}$ dividing $n$.

```c
unsigned int d;
for (d=2; n % d != 0; ++d);
```

Prime Number test with $\sqrt{n}$

$n \geq 2$ is a prime number if and only if there is no $d$ in $\{2, \ldots, \lfloor \sqrt{n} \rfloor \}$ dividing $n$.

```c
unsigned int bound = std::sqrt(n);
unsigned int d;
for (d = 2; d <= bound && n % d != 0; ++d);
```

- This works because `std::sqrt` rounds to the next representable double number (IEEE Standard 754).
- Other mathematical functions (`std::pow`,...) are almost as exact in practice.

Functions Should be More Capable! Swap ?

```c
void swap (int x, int y) {
    int t = x;
    x = y;
    y = t;
}
int main(){
    int a = 2;
    int b = 1;
    swap (a, b);
    assert (a==1 && b==2); // fail! 😞
}
```
Functions Should be More Capable! Swap?

// POST: values of x and y are exchanged
void swap (int& x, int& y) {
    int t = x;
    x = y;
    y = t;
}

int main(){
    int a = 2;
    int b = 1;
    swap (a, b);
    assert (a==1 && b==2); // ok! 😊
}

Sneak Preview: Reference Types

- We can enable functions to change the value of call arguments.
- Not a new concept for functions but rather a new class of types

Reference types (e.g. int&)

253

254