ETH
Eidgenössische Technische Hochschule Zürich
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Informatik für Mathematiker und Physiker HS16

Exercise Sheet 10

Submission deadline: 15:15 - Tuesday 6th December, 2016
Course URL: http://lec.inf.ethz.ch/ifmp/2016/

Initial Remark

In parallel to this exercise sheet there is the Graded Homework. Even though we recommend solving this exercise sheet now, we will make sure that you will be able to continue in the later weeks even if you don't solve this exercise sheet now. i.e. you will have to make sure to solve this sheet for the exam, but it is up to you whether you want to do this now (and get feedback from your TA) or later on. Furthermore, in week 11 we will not hand out a regular exercise sheet so that you can focus entirely on the Graded Homework then.

Assignment 1 – Working with Recursive Functions (4 points)

a) State PRE- and POST-conditions for the following function: [based on: Exam Summer 2011, ex.5]

```
unsigned int f(const unsigned int n) {
  if (n <= 1) return n;
  else return f(n-1) + n;
}
```

b) What does the following function output for the given `main` function? [based on: Exam Summer 2015, ex. 3.b]

```
// PRE: ...
// POST: ...
unsigned int f (const unsigned int i, const unsigned int b) {
  if (i == 0) return 0;
  return 1 + f (i/b, b);
}

int main() {
  std::cout << f (1000, 2);
  return 0;
}
```

c) Binomial coefficients can be defined in multiple ways. For example:

$$\binom{n}{k} := \begin{cases} 0 & \text{if } n < k \\ 1 & \text{if } n \geq k, k = 0 \\ \frac{n}{k}\binom{n-1}{k-1} & \text{if } n \geq k, k > 0 \end{cases}$$

State expressions `expr1`, ..., `expr5` so that the resulting function computes the binomial coefficient and thus fulfills the given POST-condition! [based on Script Exercise 125.a]

```
// POST: returns the binomial coefficient of n and k.
unsigned int binomial (unsigned int n, unsigned int k) {
  if ([expr1]) return [expr2];
  if ([expr3]) return [expr4];
  return [expr5];
}
```

d) A natural number $n \geq 1$ is called simple if it is of the form $n = 2^k * 3^l$ for some natural numbers $k, l \geq 0$. This means that n is called simple if and only if n is a product of a power of 2 with a power of 3. For example, $18 = 2 \cdot 3^2$ and $24 = 2^3 \cdot 3$ are simple. But for example $10 = 2 \cdot 5$ is not simple.

The function below tests for a given `n` whether it is simple or not. State expressions `expr1` and `expr2` so that the resulting function fulfills the PRE- and POST-conditions! [Exam Summer 2013, ex. 3]

```
// PRE: n > 0
// POST: returns true if and only if n is simple, that means n is of
//       the form 2^k * 3^l for some natural numbers k,l >= 0
bool is_simple (unsigned int n) {
  if (n == 1) return [expr1];
  else        return [expr2];
}
```

This exercise can be handed in via Codeboard! However, if you prefer, you can also hand in your solutions on paper as before.

Submission: https://codeboard.ethz.ch/ifmp16E10T1

Assignment 2 – Subset Sum (4 points)

The **subset sum problem** is the following: you are given n integers $a_1, a_2, \ldots, a_n$ and an integer t. The question is whether there exists a subset $S \subseteq \{1, 2, \ldots, n\}$ such that

$$t = \sum_{i \in S} a_i \qquad \text{(or } t = 0 \text{ if } S = \emptyset\text{)}$$

This problem is well-known in theoretical computer science as one of the many known **NP-complete** problems. Roughly speaking, an NP-complete problem is a hard problem in the sense that so far, and despite substantial efforts, no one has found an efficient algorithm for solving it; moreover, it is unlikely that such an algorithm will ever be found, since this would imply the existence of an efficient algorithm for *all* NP-complete problems (and there are hundreds of them).

Now what does "efficient" really mean? There is a precise definition, but in many cases, one would already be quite happy with something a bit faster than the obvious. In case of the subset sum problem, the obvious is to go through all subsets $S \subseteq \{1, 2, \ldots, n\}$, and for each of them check whether the elements in S sum up to t. As the number of subsets is 2^n, this will be very slow already for moderate values of n. For example, suppose that $n = 100$, and that you could check 10^{15} subsets per second (this is a very optimistic estimate of what the currently fastest computer in the world can do). As there are $2^{100} \approx 10^{30}$ subsets, you would still need 10^{15} seconds, or 31 million years, to complete the task.

Well, for smaller values of n, up to $n = 10$, say, the obvious method is not so bad after all and finishes almost instantly on a normal computer. Write a program `subset_sum.cpp` that first asks the user for the `int` t, and then inputs 10 `int`s. Then it shall determine and output whether the number t occurs among the subset sums of the 10 ints (in the I/O example below this is the case because `3 == 1 + 2`). The assignment here is to implement the obvious method in form of the following function:

```
// PRE: [begin, end) is a valid range, representing a set X of integers
// POST: returns whether t = sum(S), for some subset S of X, where sum(S) is
//       the sum of all elements of S (or 0 if S is empty).
bool is_subset_sum (int t, const int* begin, const int* end);
```

You may use one or more helper functions.

I/O-Examples (Explanation: http://lec.inf.ethz.ch/ifmp/2016/codeboard.html)

```
3
1 0 0 2 0 0 0 0 0 0
1
```

Submission: https://codeboard.ethz.ch/ifmp16E10T2

Assignment 3 – Prefix Trees (4 points)

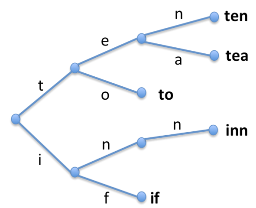
[based on Exam Summer 16, ex. 6]

The following EBNF defines a language for the description of prefix trees. These can be used to efficiently store a set of words by sharing common beginnings. An expression in parentheses stands for a prefix tree, characters designate edge labels. An illustrating example is shown below (bold: the words stored in the leaves of the prefix tree; every leaf of the tree corresponds to a stored word).

```
Tree = '(' Branch { Branch } ')'
Branch = Label [ Tree ]
Label = 'a' | 'b' | ... | 'z'
```

Example:

(t (e (n a) o) i (n (n) f)) $\Longleftrightarrow$



a) Which of the following strings are valid `Trees` according to the EBNF?

(i) a

(ii) (a (b) c)

(iii) (a ((b) c) d)

(iv) (a b (c d e) f g (h i j) k)

b) Fill in the gaps such that the following `main` function returns the depth of a valid prefix tree that is provided at the input stream according to this EBNF. The depth of a tree is the length of the longest path from the root to a leaf. In the example above, the depth is 3, and this corresponds to the length of the longest stored words `ten`, `tea`, and `inn`. In the following code the definition of `lookahead`, all `#include<...>`, and a separate declaration of `Branch` before `Tree` are not printed due to space constraints but are present in the program!

```
// POST: leading whitespace characters are extracted from is, and the
//       first non-whitespace character is returned (0 if there is none)
char lookahead (std::istream& is);

// PRE: Tree = '(' Branch { Branch } ')'
// POST: Extracts tree from is and returns its depth
int Tree (std::istream& is) {
  char c; is >> c; // extract '('
```

```
  int depth = [expr1];
  while (lookahead(is) != ')') {
    const int bdepth = [expr2];
    if ([expr3]) [expr4];
  }
  is >> c; // extract ')'
  return depth;
}

// PRE: Branch = Label [ Tree ]
// POST: Extracts single branch from is and returns its depth
int Branch (std::istream& is) {
  char c; is >> c; // extract label
  if (lookahead (is) == '(') return [expr5];
  return 1;
}
int main() {
  const int depth = Tree(std::cin);
  std::cout << "Longest stored word has length " << depth << "\n";
  return 0;
}
```

This exercise can be handed in via Codeboard! However, if you prefer, you can also hand in your solutions on paper as before.

Submission: https://codeboard.ethz.ch/ifmp16E10T3

Assignment 4 – Bridges (4 points) [based on: Exam Summer 2015, (new EBNF)]

The following EBNF describes simple viaducts (connections of bridges)

```
viaduct = bridge { bridge }
bridge = "<" landbridge ">" | "<" riverbridge ">"
landbridge = "-" { "-" }
riverbridge = "^" { "^" }
```

For example, `<---->< --><^^^><--->` describes a valid `viaduct` according to the above EBNF. Answer the following questions:

a) List the terminal symbols in the above EBNF!

b) List the nonterminal symbols in the above EBNF!

c) How do you have to extend the above EBNF to allow for riverbridges to consist of either one or two bridge pieces `"^"` but not more?

d) Write the above EBNF as a BNF.

This exercise can be handed in via Codeboard! However, if you prefer, you can also hand in your solutions on paper as before.

Submission: https://codeboard.ethz.ch/ifmp16E10T4